Electroweak Physics @ Lisbon (circa 1986)
(Contribution to Augusto Barroso Fest)

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24 October 2008

A. Barroso & J. C. Romão papers

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1) GAUGE SYMMETRY BREAKING IN COMPACT MULTIPLY CONNECTED MANIFOLDS.
By F. Freire, J.C. Romao, A. Barroso.
Phys.Lett.B206:491,1988.

2) NEUTRINO COUNTING AND A COMPOSITE Z BOSON.
By A. Barroso, Paulo Nogueira, J.C. Romao.
Phys.Lett.B196:547,1987.

3) ELECTROMAGNETIC PROPERTIES OF THE Z BOSON. 2. WARD IDENTITIES FOR Z GAMMA GAMMA AND Z Z GAMMA GREEN'S FUNCTIONS.
By A. Barroso, Paulo Nogueira, J.C. Romao.
Z.Phys.C33:243-246,1986.

4) ELECTROWEAK RADIATIVE CORRECTIONS AT LEP ENERGIES.
By A. Barroso, et al.,
Aachen ECFA Workshop 1986:15.

5) e+ e- ---> neutrino anti-neutrino gamma: THE IMPORTANCE OF AN EXACT CALCULATION.
By J.C. Romao, L. Bento, A. Barroso.
Phys.Lett.B194:440,1987.

6) RENORMALIZATION OF THE ELECTROWEAK THEORY IN THE NONLINEAR GAUGE.
By J.C. Romao & A. Barroso.
Phys.Rev.D35:2836,1987.

7) HIGGS PRODUCTION WITH POLARIZED e+ e- BEAMS.
By J.C. Romao & A. Barroso.
Phys.Lett.B185:195,1987.

8) HIGGS PRODUCTION AT e+ e- COLLIDERS. 2. WARD IDENTITIES FOR gamma* H gamma AND Z* H gamma GREEN'S FUNCTIONS.
By J.C. Romao & A. Barroso.
Nucl.Phys.B272:693,1986.

9) HIGGS PRODUCTION AT e+ e- COLLIDERS.
By A. Barroso, J. Pulido, J.C. Romao.
Nucl.Phys.B267:509-530,1986.

10) e+ e- ---> gamma + MISSING NEUTRALS: NEUTRINO VERSUS PHOTINO PRODUCTION.
By L. Bento, J.C. Romao, A. Barroso.
Phys.Rev.D33:1488,1986.

11) THE VACUUM OF SUPERSYMMETRIC SU(3) x SU(2) x U(1).
By A. Barroso & J.C. Romao.
Phys.Lett.B158:51,1985.

12) FLAVOR VIOLATION IN SUPERSYMMETRIC THEORIES.
By J.C. Romao, A. Barroso, M.C. Bento, G.C. Branco.
Nucl.Phys.B250:295,1985.

Summary

- Diagrams for $e^-e^+ \to H\gamma$
- The $ZH\gamma$ and $\gamma H\gamma$ three point functions
  - Diagrams with fermions
  - Diagrams with gauge bosons
  - Renormalization
  - Gauge Invariance
- Box diagrams
- Cross section for $e^-e^+ \to H\gamma$
- Ward Identities
- $H \to \gamma\gamma$ as a sub-product
- Conclusions

Diagrams for $e^- e^+ \to H\gamma$

A. Barroso et al. / Higgs production at e^+e^- colliders

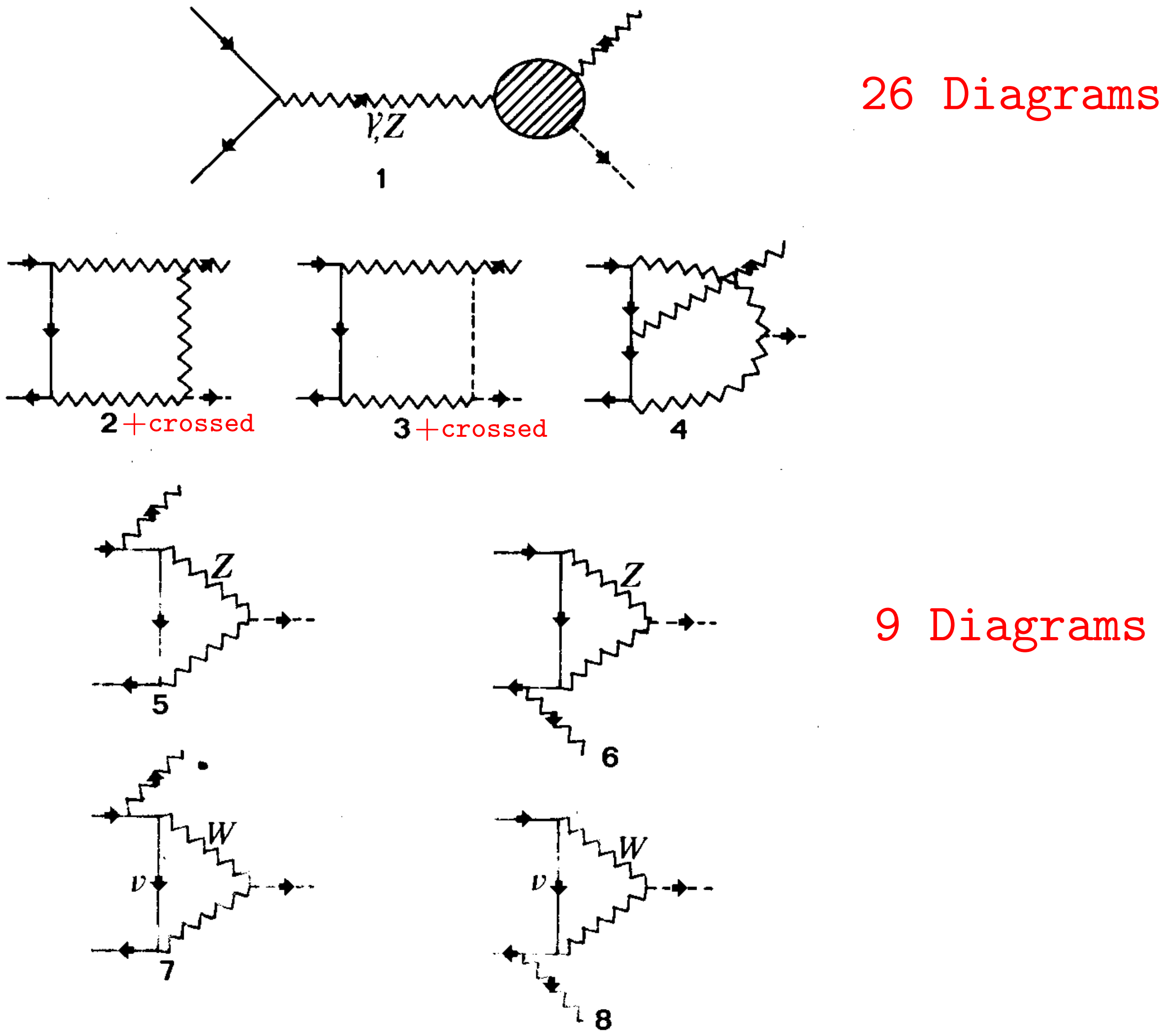


Fig. 4. Diagrams corresponding to the $e^+e^- \to H\gamma$ amplitude.

Main Issues

- Gauge (non)-invariance of the 3-point functions.

Contrary to what was stated in the literature at the time, we showed that the off-shell 3-point functions $\gamma^* H\gamma$ and $Z^* H\gamma$ were not gauge invariant.

- Renormalization of the 3-point functions.

While the 3-point function $\gamma^* H\gamma$ is finite (the divergences cancel out), the 3-point function $Z^* H\gamma$ needs to be renormalized. We explicitly showed how this leads to a finite result.

- Gauge invariance of the final result for $e^-e^+ \to H\gamma$.

We explicitly showed that the gauge non-invariant part of the diagrams with the 3-point functions cancels out with the gauge non-invariant part of the boxes, making the final result gauge invariant.

Input file for QGRAF

For this problem it is very useful the program `QGRAF` by Paulo Nogueira, http://cfif.ist.utl.pt/~paulo/. The input file for $ZH\gamma$ is:

```
output= 'ZHG.lista' ;

style= 'Styles/sum.sty' ;

model= 'Models/gws-tHooftFeynmanGauge';

in= Z;

out=H,A;

loops= 1;

loop_momentum= ;

options= onepi ;
```

The SM file for QGRAF

The Standard Model file in the Feynman-'t Hooft gauge is:

```
* -----------------------------------------------------*
* Last Update: 13.05.2008                              *
* -----------------------------------------------------*
*                  Higgs

  [H,H,+]

*   electron, muon, tau

  [e1,E1,-]
  [e2,E2,-]
  [e3,E3,-]
```

··· (Complete code at http://porthos.ist.utl.pt/CTQFT)

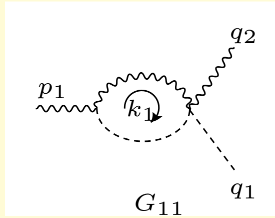
```
* Gauge Goldstone

  [GWP,WM,A]
  [WP,GWM,A]
  [GWP,GWM,A]
  [GWP,WM,Z]
  [WP,GWM,Z]
  [GWP,GWM,Z]
```

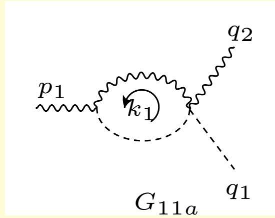
QGRAF: Output


```
# file generated by qgraf-3.1.1

 +(1)*
 prop(WM(1,-k1+p1),WP(2,k1-p1))*
 prop(GWM(3,-k1),GWP(4,k1))*
 vrtx(WP(2,k1-p1),GWM(3,-k1),Z(-1,p1))*
 vrtx(WM(1,-k1+p1),GWP(4,k1),H(-2,-q1),A(-4,-q2))
```



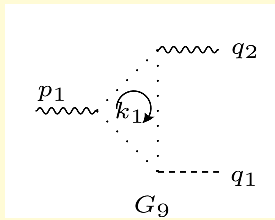

```
 +(1)*
 prop(WM(1,k1-p1),WP(2,-k1+p1))*
 prop(GWM(3,k1),GWP(4,-k1))*
 vrtx(GWP(4,-k1),WM(1,k1-p1),Z(-1,p1))*
 vrtx(WP(2,-k1+p1),GWM(3,k1),H(-2,-q1),A(-4,-q2))
```



··· (Complete output at http://porthos.ist.utl.pt/CTQFT)

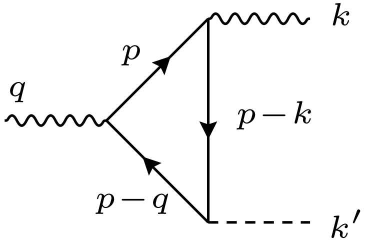
```
 -(1)*
 prop(cWM(1,k1),CWM(2,-k1))*
 prop(cWM(3,k1-p1),CWM(4,-k1+p1))*
 prop(cWM(5,k1-q1),CWM(6,-k1+q1))*
 vrtx(CWM(2,-k1),cWM(3,k1-p1),Z(-1,p1))*
 vrtx(CWM(6,-k1+q1),cWM(1,k1),H(-2,-q1))*
 vrtx(CWM(4,-k1+p1),cWM(5,k1-q1),A(-4,-q2))

# end
```

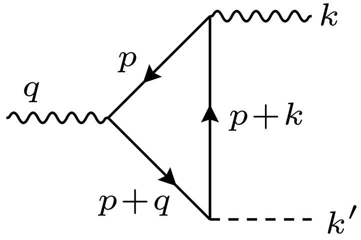


The Diagrams

With Fermions

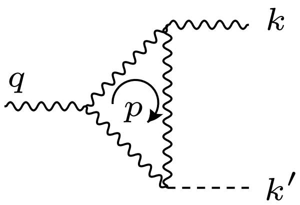


F_1

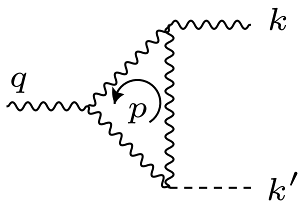


F_{1a}

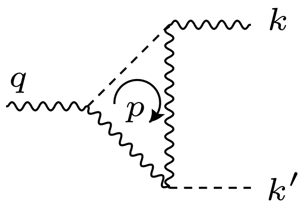
With Gauge Bosons



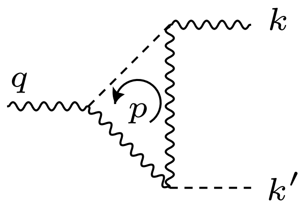
G_1



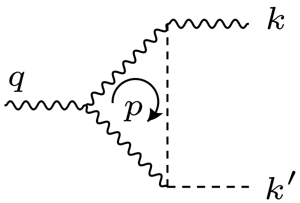
G_{1a}



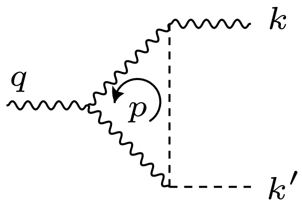
G_2



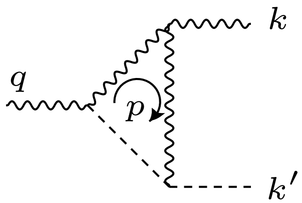
G_{2a}



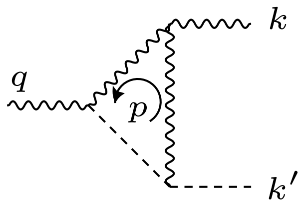
G_3



G_{3a}

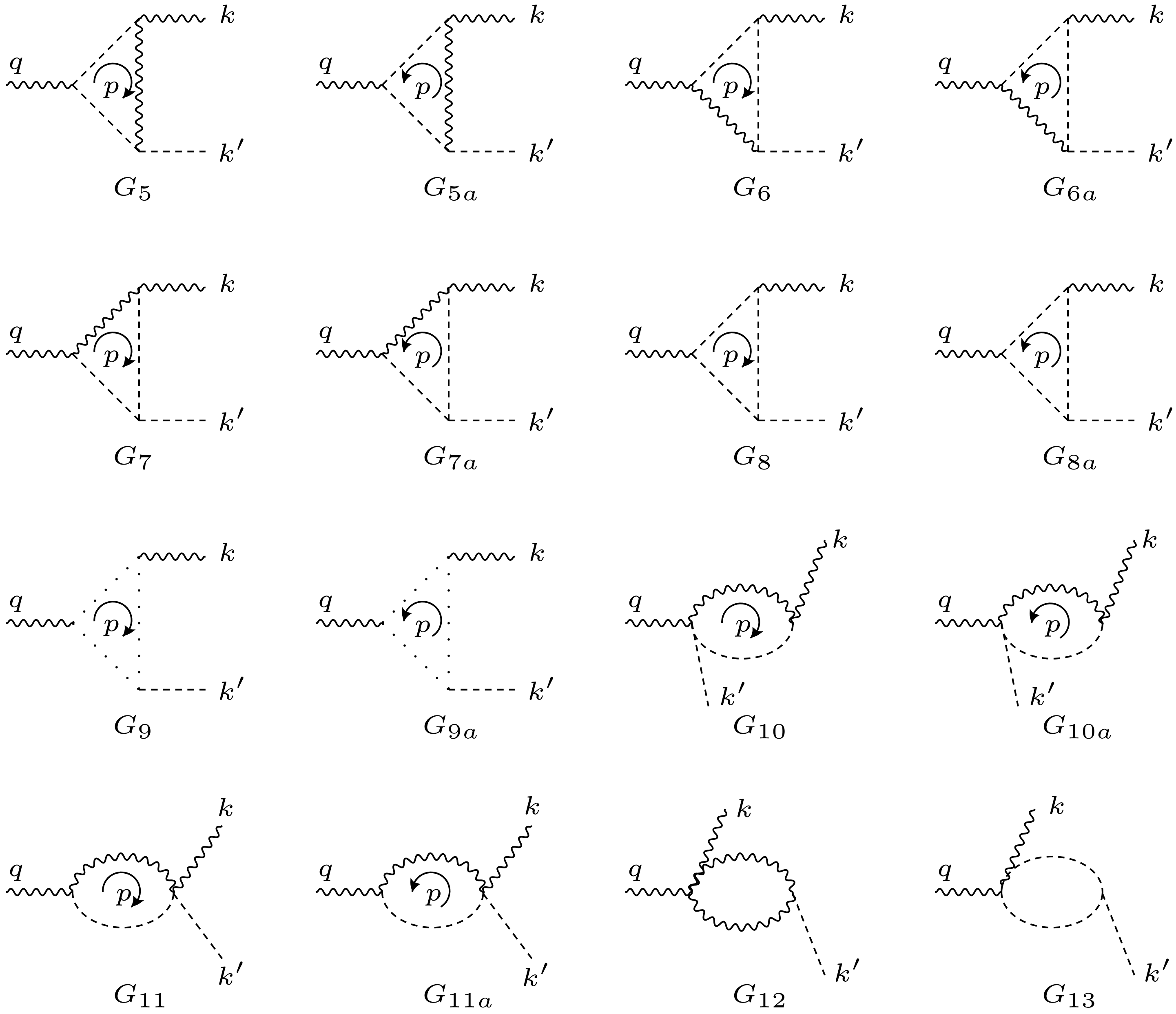


G_4

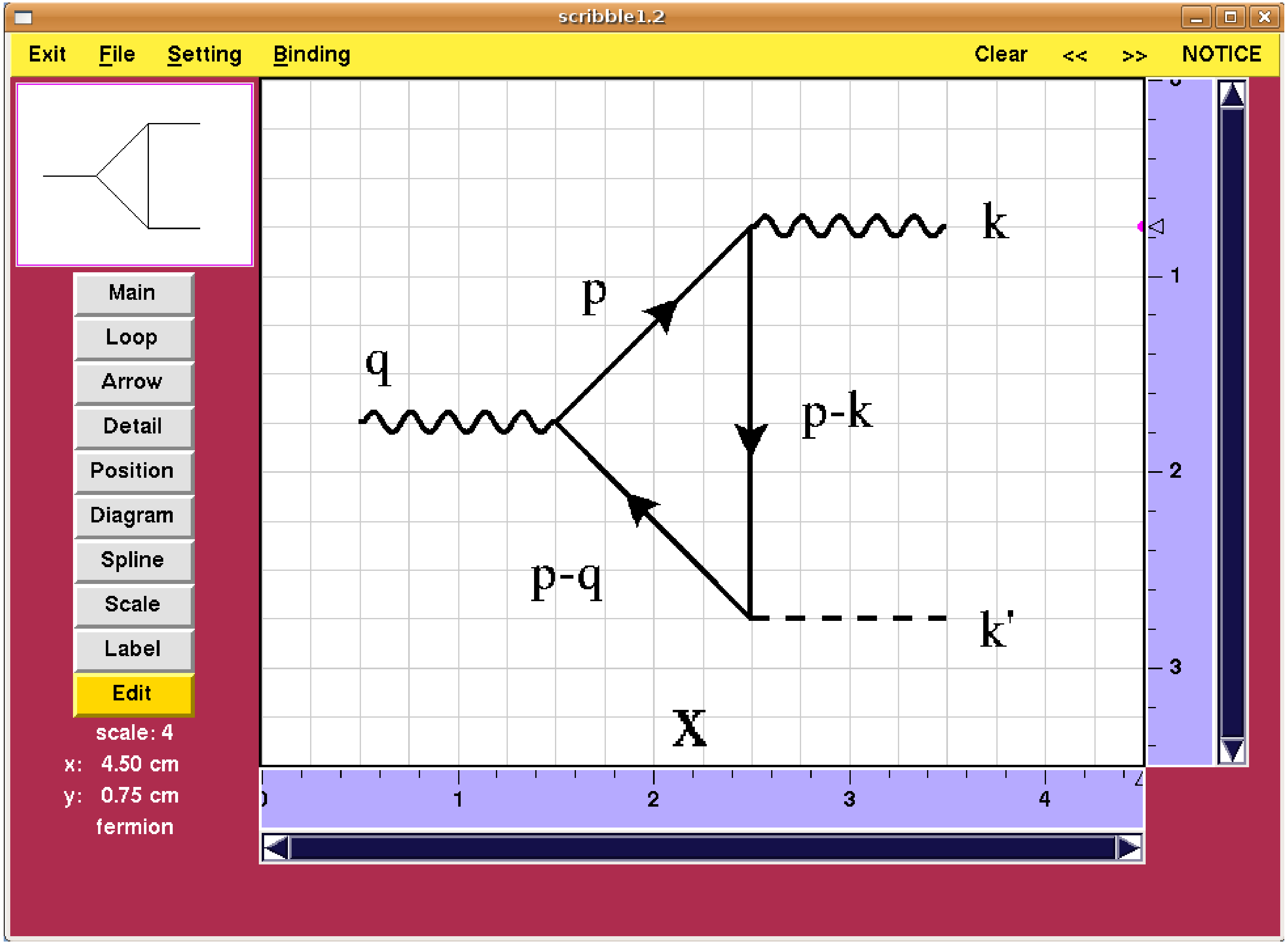


G_{4a}

The Diagrams · · ·



Scribble (by Paulo Nogueira & Marc Baillargeon)

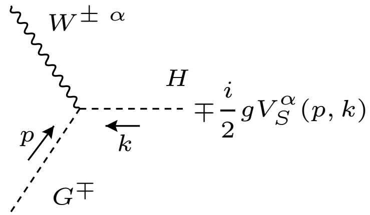


The Amplitudes for $\gamma^* H\gamma$ and $Z^* H\gamma$

Conventions:

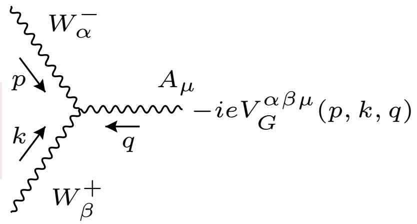
- To simplify we ommit the denominators of the propagators.

- $V_S^\alpha(p,k) = (p-k)^\alpha$ with p incoming charged scalar, k incoming Higgs, α Lorentz index of W^α.



- $W^{-\alpha}(p), W^{+\beta}(k), A^\mu(q)$ with incoming momenta:

$$V_G^{\alpha\beta\mu}(p,k,q) = [g_{\alpha\beta}(p-k)_\mu + g_{\beta\mu}(k-q)_\alpha + g_{\mu\alpha}(q-p)_\beta]$$



We get for $\gamma^* H\gamma$:

$$F_1 = (-ieQ_f)^2(-i\frac{g}{2}\frac{m_f}{m_W})i^3(-1)\mathrm{Tr}[(\not p + m_f)\gamma^\nu(\not p - \not q + m_f)(\not p - \not k_1 + m_f)\gamma^\mu]$$

$$F_{1a} = (-ieQ_f)^2(-i\frac{g}{2}\frac{m_f}{m_W})i^3(-1)\mathrm{Tr}[(\not p + m_f)\gamma^\mu(\not p + \not k_1 + m_f)(\not p + \not q + m_f)\gamma^\nu]$$

$$G_1 = igm_W(-ie)^2(-i)^3 V_{G\alpha\beta}{}^{\nu}(-p, p-q, q) V_G^{\beta\alpha\mu}(-p+k, p, -k)$$

$$G_{1a} = igm_W(-ie)^2(-i)^3 V_{G\beta\alpha}{}^{\nu}(p-q, -p, q) V_G^{\alpha\beta\mu}(p, -p+k, -k)$$

$$G_2 = G_{2a} = igm_W(-ie)(-ie)(-i)^2 i m_W^2 g^{\mu\nu}$$

The Amplitudes ···

$$G_3 = \frac{ig}{2} V_{S\beta}(p-k,-k')(-ie)^2 m_W (-i)^2 i\ V_G^{\mu\beta\nu}(-p,p-q,q)$$

$$G_{3a} = -\frac{ig}{2} V_{S\beta}(p-k,-k')(-ie)^2 m_W (-i)^2 i\ V_G^{\beta\mu\nu}(p-q,-p,q)$$

$$G_4 = -\frac{ig}{2} V_{S\beta}(-p+q,-k')(-ie)^2 m_W (-i)^2 i\ V_G^{\beta\nu\mu}(-p+k,p,-k)$$

$$G_{4a} = \frac{ig}{2} V_{S\beta}(-p+q,-k')(-ie)^2 m_W (-i)^2 i\ V_G^{\nu\beta\mu}(p,-p+k,-k)$$

$$G_5 = -\frac{ig}{2}(-ie)^2 m_W (-i) i^2 V_S^{\nu}(-p-q,-p) V_S^{\mu}(-p+q,-k')$$

$$G_{5a} = \frac{ig}{2}(-ie)^2 m_W (-i) i^2 V_S^{\nu}(-p,p-q) V_S^{\mu}(-p+q,-k')$$

$$G_6 = \frac{ig}{2}(-ie)^2 m_W (-i) i^2 V_S^{\mu}(p,-p+k) V_S^{\nu}(p-k,-k')$$

$$G_{6a} = -\frac{ig}{2}(-ie)^2 m_W (-i) i^2 V_S^{\mu}(-p+k,p) V_S^{\nu}(p-k,-k')$$

$$G_7 = G_{7a} = -\frac{ig}{2}\frac{m_H^2}{m_W}(-ie)^2 m_W^2 (i)^2 (-i) g^{\mu\nu}$$

The Amplitudes $\cdots$

$$G_8 = -\frac{ig}{2}\frac{m_H^2}{m_W}(-ie)^2 i^3 V_S^\mu(p,k-p)V_S^\nu(p-q,-p)$$

$$G_{8a} = -\frac{ig}{2}\frac{m_H^2}{m_W}(-ie)^2 i^3 V_S^\mu(-p+k,p)V_S^\nu(-p,p-k)$$

$$G_9 = -\frac{ig}{2}m_W(-ie)^2 i^3(-1)(p)^\nu(p-k)^\mu$$

$$G_{9a} = -\frac{ig}{2}m_W(-ie)^2 i^3(-1)(-p)^\mu(-p+q)^\nu$$

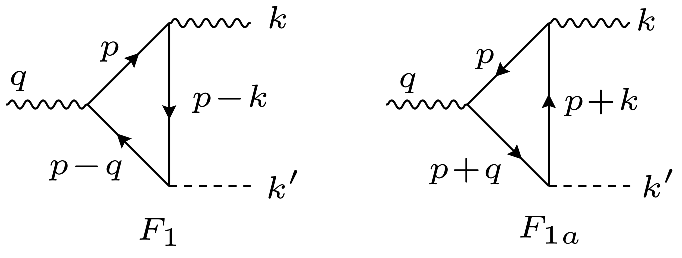
$$G_{10} = G_{10a} = G_{11} = G_{11a} = -\frac{ieg}{2}(-ie)m_W(-i)ig^{\mu\nu}$$

$$G_{12} = igm_W(-ie^2)(-i)^2 g_{\alpha\beta}\left(2g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}\right)$$

$$G_{13} = -\frac{ig}{2}\frac{m_H^2}{m_W}\; i^2\; 2ie^2\; g^{\mu\nu}$$

One–Loop in 1986: The Fermion Loop

The fermion loops can be evaluated



using standard techniques (see the Appendix of my text `OneLoop`)
We get for the amplitudes

$$iM^{\nu\mu}_{F_1} = (-ieQ_f)^2(-i\frac{g}{2}\frac{m_f}{m_W})i^3(-1)\int \frac{d^4p}{(2\pi)^4}\frac{\text{Tr}[(\not{p}+m_f)\gamma^\nu(\not{p}-\not{q}+m_f)(\not{p}-\not{k}+m_f)\gamma^\mu]}{[p^2-m_f^2][(p-q)^2-m_f^2][(p-k)^2-m_f^2]}$$

and

$$iM^{\nu\mu}_{F_2} = (-ieQ_f)^2(-i\frac{g}{2}\frac{m_f}{m_W})i^3(-1)\int \frac{d^4p}{(2\pi)^4}\frac{\text{Tr}[(\not{p}+m_f)\gamma^\mu(\not{p}+\not{k}+m_f)(\not{p}+\not{q}+m_f)\gamma^\nu]}{[p^2-m_f^2][(p+q)^2-m_f^2][(p+k)^2-m_f^2]}$$

With the change of variable $p \to -p$ in the second integral we can reduce both integrals to the same denominator.

Doing the trace of the Dirac matrices we get for the sum of both diagrams

$$iM_F^{\nu\mu} = \frac{e^2 g}{m_W} \int \frac{d^4p}{(2\pi)^4} \frac{N^{\nu\mu}}{[p^2 - m_f^2][(p-q)^2 - m_f^2][(p-k)^2 - m_f^2]}$$

with

$$N_{\nu\mu} = -4m_f^2 \left[g_{\mu\nu}(m_f^2 + k\cdot q) - k_\nu q_\mu + (-4g_{\mu\alpha}k_\nu + 2k_\alpha g_{\mu\nu})p^\alpha + 4p_\mu p_\nu - p^2 g_{\mu\nu} \right]$$

Using Feynman's trick to reduce to the same denominator and performing the Wick rotation we finally get (many pages of work !)

$$M_F^{\nu\mu} = \frac{e^2 g}{m_W} \frac{1}{16\pi^2} \left[(k\cdot q) g^{\mu\nu} - k^\nu q^\mu \right] Q_f^2 X_F(\beta', \beta'_H)$$

$$X_F(\beta', \beta'_H) = 4J_1(\beta', \beta'_H) - 16J_2(\beta', \beta'_H), \quad \beta' = \frac{q^2}{m_f^2}, \quad \beta'_H = \frac{m_H^2}{m_f^2}$$

and

$$J_{[1,2]}(\beta', \beta'_H) = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \, \frac{[1, x_1 x_2]}{1 - \beta' x_1(1-x_1) + (\beta' - \beta'_H) x_1 x_2}$$

Diagrams with Fermions in FeynCalc


```
(* Definitions *)
dm[mu_]:=DiracMatrix[mu]
dm[5]:=DiracMatrix[5]
ds[p_]:=DiracSlash[p]
mt[mu_,nu_]:=MetricTensor[mu,nu]
fv[p_,mu_]:=FourVector[p,mu]
epsilon[a_,b_,c_,d_]:=LeviCivita[a,b,c,d]
id[n_]:=IdentityMatrix[n]
sp[p_,q_]:=ScalarProduct[p,q]
li[mu_]:=LorentzIndex[mu]
prop[p_,m_]:=m + ds[p]
PV[k_,mu_]:=PolarizationVector[k,mu]

(* Diagrams F1 e F1a *)
numF1:=Tr[prop[p,mf] . dm[nu] . prop[p-q,mf] . prop[p-k,mf] .dm[mu]]
numF1a:=Tr[prop[-p,mf] . dm[mu] . prop[-p+k,mf] .
prop[-p+q,mf] . dm[nu]]


ampF:= mf (-1/2 ) Contract[(numF1+numF1a) PV[k,mu] ] \
FeynAmpDenominator[PropagatorDenominator[p,mf], \
PropagatorDenominator[p-q,mf],PropagatorDenominator[p-k,mf]]
```

Diagrams with Fermions with FeynCalc


```
(* Simplifications *)
onshell={sp[k,k]->0, sp[q,q]->q2}
kin={sp[k,q]->q2-mH^2/2}

resF:= (-I/Pi^2) OneLoop[p,ampF] /. onshell
ansF=PaVeReduce[resF]

aux =(Pair[LorentzIndex[nu], Momentum[Polarization[k, I]]]*
Pair[Momentum[k], Momentum[q]] -
Pair[LorentzIndex[nu], Momentum[k]]*
Pair[Momentum[q], Momentum[Polarization[k, I]]])

XF=Coefficient[ansF,aux]
```

Result:

$$\mathtt{aux} = \epsilon^\nu\ k\cdot q - k^\nu\ q\cdot\epsilon$$

```
               2                2    2    2                      2    2
XF = (4 mf   (2 q2 B0[mH  ,mf  ,mf  ] - 2 q2 B0[q2,mf  ,mf  ]

        2                        2      2                    2    2    2    2
 - (mH   - q2) (-2 + (-4 mf   + mH   - q2) C0[0, q2,mH  ,mf  ,mf  ,mf  ]))

            2        2
    / (mH   - q2)
```

Diagrams with Fermions: Numerics with LoopTools

Mathematica

```
stmp=OpenWrite["XF.f",FormatType -> FortranForm]
Write[stmp,"XF=",XF]
Close[stmp]
```

Fortran

```
      program ghgtest
...
*** Initialize LoopTools
#include "looptools.h"
      call ffini
      lbd2=1.d0
      call setlambda(lbd2)
```

··· (Complete code at http://porthos.ist.utl.pt/CTQFT)

```
        xf=(4*mf2*(2*q2*B0(mH2,mf2,mf2) - 2*q2*B0(q2,mf2,mf2) -
     &          (mH2 - q2)*(-2 + (-4*mf2 + mH2 - q2)*
     &          C0(0,q2,mH2,mf2,mf2,mf2))))/(mH2 - q2)**2
        xfpaper=4*(J1(betap,betapH)-4d0*J2(betap,betapH))
```

Result for $m_f = m_c$, $q^2 = (100 \text{ Gev})^2$

```
  mH                 XF                          XF Paper
 50.00  (-3.39604E-03, 1.45127E-03)  (-3.39604E-03, 1.45127E-03)
100.00  (-2.37761E-03, 9.07261E-04)  (-2.37761E-03, 9.07261E-04)
150.00  (-1.80151E-03, 6.45544E-04)  (-1.80151E-03, 6.45544E-04)
```

Diagrams with Gauge Bosons: $G_8 + G_{8a} + G_{13}$

Diagrams $G_8 + G_{8a} + G_{13}$ are proportional do m_H^2 and gauge invariant *per se*.

```
(* Diagrams G8 e G8a *)
numG8GHG:= (1/2 mH^2) VScalar[p,k-p,mu] VScalar[p-q,-p,nu]
numG8aGHG:=(1/2 mH^2) VScalar[-p+k,p,mu] VScalar[-p,p-q,nu]

ampG8GHG:=Contract[(numG8GHG+numG8aGHG) PV[k,mu]  ] \
FeynAmpDenominator[PropagatorDenominator[p,mW], \
PropagatorDenominator[p-k,mW],PropagatorDenominator[p-q,mW]]

resG8GHG:= (-I/Pi^2) OneLoop[p,ampG8GHG] /. onshell

(* Diagrams G13 *)
numG13GHG:= - mH^2 mt[mu,nu]

ampG13GHG:=Contract[numG13GHG PV[k,mu] ] \
FeynAmpDenominator[PropagatorDenominator[p,mW], \
PropagatorDenominator[p-q+k,mW]]

resG13GHG:= (-I/Pi^2) OneLoop[p,ampG13GHG] /. onshell
```

(Complete code at http://porthos.ist.utl.pt/CTQFT)

Diagrams with Gauge Bosons: $G_8 + G_{8a} + G_{13} \cdots$


```
(* Diagrams G8+G8a+G13 *)

ansG8GHG=PaVeReduce[resG8GHG]
ansG13GHG=PaVeReduce[resG13GHG]

ansG8G13GHG=Simplify[ansG8GHG+ansG13GHG \
 /. q2-> mH^2 + 2 ScalarProduct[q,k]]

auxG8G13GHG = Coefficient[ansG8G13GHG,aux]

XG8G13GHG:= Simplify[auxG8G13GHG /. ScalarProduct[q,k]->(q2-mH^2)/2]
```

we get the result

```
                       2           2    2    2                2    2
XG8G13GHG= -(2 mH  (q2 B0[mH , mW , mW ] - q2 B0[q2, mW , mW ] +

               2                 2              2    2    2    2
      (mH  - q2) (1 + 2 mW  C0[0, q2, mH , mW , mW , mW ])))\

           2       2
   / (mH  - q2)
```

(Comparison with Eq.(2.8) at http://porthos.ist.utl.pt/CTQFT)

Diagrams $G_1 + \cdots + G_7 + G_9 + \cdots + G_{12}$


```
(* Diagram G1+G1a *)
numG1:= (mW^2) WMWPA[-k+q1,k,-q1,b,a,mu] \
WMWPA[-k+q1+q2,k-q1,-q2,a,b,nu]
numG1a:= (mW^2) WMWPA[k,-k+q1,-q1,a,b,mu] \
WMWPA[k-q1,-k+q1+q2,-q2,b,a,nu]

ampG1:=Contract[(numG1+numG1a) PV[q1,mu] PV[q2,nu] ] \
FeynAmpDenominator[PropagatorDenominator[k,mW], \
PropagatorDenominator[k-q1,mW],PropagatorDenominator[k-q1-q2,mW]]

resG1:= (-I/Pi^2) OneLoop[k,ampG1] /. onshell
ansG1=PaVeReduce[resG1]
```

$\cdots$ (Complet program at http://porthos.ist.utl.pt/CTQFT)

```
(* Diagrams G12 and G12a *)
numG12:= (-mW^2) mt[a,b] (2 mt[a,b] mt[mu,nu]- \
         mt[a,mu] mt[b,nu] -mt[a,nu] mt[b,mu])

ampG12:=Contract[numG12 PV[q1,mu] PV[q2,nu] ] \
FeynAmpDenominator[PropagatorDenominator[k,mW], \
PropagatorDenominator[k+q1+q2,mW]]

resG12:= (-I/Pi^2) OneLoop[k,ampG12] /. onshell
ansG12=PaVeReduce[resG12]
```

Renormalization: Finiteness of Result for $\gamma^* H\gamma$

There is no counterterm for $\gamma^* H\gamma$, so the result has to be finite. Most of the diagrams are divergent and `FeynCalc` is great in helping us checking the result. Only B_0 is divergent with

$$\mathrm{Div}(B_0) = \Delta_\epsilon \equiv \frac{2}{\epsilon} - \gamma + \ln 4\pi$$

```
(* Test that the divergences in GHG cancel out *)
div={B0[m1_,m2_,m3_]->Div}

ansGGHG=ansG1GHG+ansG2GHG+ansG3GHG+ansG4GHG+ansG5GHG+ansG6GHG+\
    ansG7GHG+ansG9GHG+ansG10GHG+ansG11GHG+ansG12GHG+ansG8G13GHG

ansGkinGHG= ansGGHG /. kin

TestDivGHG:= Simplify[Coefficient[ansGkinGHG /.div ,Div]]

TestDivDiagGHG=Function[exp,test=exp /. kin; \
               test=Simplify[Coefficient[test /.div ,Div]];\
               test=Coefficient[test,aux5 mW^2]];
```

with the result:

```
  TestDivGHG= 0
```

Table of Divergencies: Coefficient of $-\frac{1}{4}m_W^2\Delta_\epsilon g^{\mu\nu}$

$$T_Z^{\mu\nu} = \frac{eg^2\cos\theta_W}{16\pi^2 m_W} m_W^2 (1+\tan^2\theta_W) 2B_0(0, m_W^2, m_W^2)$$

$Z^* H\gamma$ counter term

Diagram	$\gamma^* H\gamma$	$Z^* H\gamma$
F1+F1a	0	0
G1+G1a	36	36
G2+G2a	0	0
G3+G3a	-3	-3
G4+G4a	-3	$3\tan^2\theta_W$
G5+G5a	2	$1-\tan^2\theta_W$
G6+G6a	2	$-2\tan^2\theta_W$
G7+G7a	0	0
G9+G9a	-2	-2
G10+G10a	-4	$4\tan^2\theta_W$
G11+G11a	-4	$4\tan^2\theta_W$
G12	-24	-24
G8+G8a+G13	0	0
Counter Term	0	$-8(1+\tan^2\theta_W)$
Sum	0	0

Final Result for $\gamma^* H\gamma$ and $Z^* H\gamma$

The final result for the 3-point functions can be written as

$$T^{\nu\mu}_{\gamma^* H\gamma}\epsilon_\mu(k) = \frac{e^2 g}{16\pi^2 m_W}\left[c_5^{\Delta\gamma}\, g^{\nu\mu} + c_2^{\Delta\gamma}\, k^\nu q^\mu + c_3^{\Delta\gamma}\, q^\nu q^\mu\right]\epsilon_\mu(k)$$

$$T^{\nu\mu}_{Z^* H\gamma}\epsilon_\mu(k) = \frac{e g^2 \cos\theta_W}{16\pi^2 m_W}\left[d_5^{\Delta Z}\, g^{\nu\mu} + d_2^{\Delta Z}\, k^\nu q^\mu + d_3^{\Delta Z}\, q^\nu q^\mu\right]\epsilon_\mu(k)$$

Using FeynCalc we get

```
aux5=PolarizationVector[k,nu]
aux2=FourVector[k,nu] Pair[Momentum[q], Momentum[Polarization[k,I]]]
aux3=FourVector[q,nu] Pair[Momentum[q], Momentum[Polarization[k,I]]]

c5GHG = Simplify[Coefficient[ansGHG, aux5] ];
...
d3ZHG = Simplify[Coefficient[ansZHG, aux3] ];
```

with the following type of result:

```
In[5]:= c3GHG
              2                                 2     2     2
Out[5]= 3 mW   C0[0, q2, q2 - 2 k.q, mW  , mW  , mW  ]
```

Final Result for $\gamma^* H\gamma$ and $Z^* H\gamma$ $\cdots$

Now these results can be written as

$$T^{\nu\mu}_{\gamma^* H\gamma} = \frac{e^2 g}{16\pi^2 m_W}\left[c_1^{\Delta\gamma}\left(k\cdot q g^{\nu\mu} - k^\nu q^\mu\right) + c_3^{\Delta\gamma}\left(q^\nu q^\mu - q^2 g^{\nu\mu}\right)\right]$$

$$T^{\nu\mu}_{Z^* H\gamma} = \frac{e g^2 \cos\theta_W}{16\pi^2 m_W}\left[d_1^{\Delta Z}\left(k\cdot q g^{\nu\mu} - k^\nu q^\mu\right) + d_3^{\Delta Z} q^\nu q^\mu - d_4^{\Delta Z}\left(q^2 - m_Z^2\right) g^{\nu\mu}\right]$$

with

$$c_1^{\Delta\gamma} = -c_2^{\Delta\gamma}, \quad c_5^{\Delta\gamma} = c_1^{\Delta\gamma} k\cdot q - q^2 c_3^{\Delta\gamma}$$

$$d_1^{\Delta Z} = -d_2^{\Delta Z}, \quad d_5^{\Delta Z} = d_1^{\Delta Z} k\cdot q - d_4^{\Delta Z}\left(q^2 - m_Z^2\right)$$

```
d1ZHG = -d2ZHG
d4auxZHG = Simplify[ d1ZHG sp[k,q] -d5ZHG /. kin ]
d4ZHG= Coefficient[d4auxZHG,q2]

Testd4:= Simplify[d4auxZHG - d4ZHG*(q2 -mW^2 (1+tw2))]
```

```
In[6]:= Testd4

Out[6]= 0
```

Results for the Boxes

The calculation procedes as usual with `FeynCalc`. The important point is that now the result will contain a spinor line. In fact there are six independent `Standard Matrix Elements`, $ME[1], \ldots, ME[6]$ defined by

$$
\begin{aligned}
ME[1] &= \overline{v}(p_+)\gamma_\nu P_R\, u(p_-)\ \epsilon^\nu(k) \\
ME[2] &= \overline{v}(p_+)\gamma_\nu P_L\, u(p_-)\ \epsilon^\nu(k) \\
ME[3] &= \overline{v}(p_+)\gamma_\nu P_R\, u(p_-)\ k^\nu\ p_- \cdot \epsilon(k) \\
ME[4] &= \overline{v}(p_+)\gamma_\nu P_L\, u(p_-)\ k^\nu\ p_- \cdot \epsilon(k) \\
ME[5] &= \overline{v}(p_+)\gamma_\nu P_R\, u(p_-)\ k^\nu\ p_+ \cdot \epsilon(k) \\
ME[6] &= \overline{v}(p_+)\gamma_\nu P_L\, u(p_-)\ k^\nu\ p_+ \cdot \epsilon(k)
\end{aligned}
$$

These can be extracted with the `FeynCalc` code

```
var = Select[Variables[ansBox],\
(Head[#] === StandardMatrixElement) &]

Set @@ {var, {ME[1],ME[2],ME[3],ME[4],ME[5],ME[6]}}
```

An Example: Result for the Box of Diagram 7


```
numB7:= Spinor[pp,0] . dm[a] . dm[7] . prop[p-pp,0] . dm[a] . \
dm[7] . prop[pm-k,0] . dm[mu] . Spinor[pm,0]

ampB7:=  Contract[ (1/2) numB7 PV[k,mu] ] \
FeynAmpDenominator[PropagatorDenominator[p,mW], \
PropagatorDenominator[p+k-pp-pm,mW], \
PropagatorDenominator[p-pp,me], \
PropagatorDenominator[pm-k,0]]

resB7:= (-I/Pi^2) OneLoop[p,ampB7] /. onshell
ansB7=PaVeReduce[resB7]
```

```
                           2     2
        B0[-2 k.pm, me  , mW ] ME[2]
ansB7 = --------------------------- -
             2 (k.pp - pm.pp)
                                            2     2
  B0[-2 k.pm - 2 k.pp + 2 pm.pp, mW  , mW ] ME[2]
  ---------------------------------------------- -
                 2 (k.pp - pm.pp)
      2      2
  ((me   - mW ) C0[0, -2 k.pm, -2 k.pm - 2 k.pp + 2 pm.pp,
        2     2     2
      mW  , me  , mW ] ME[2]) / (2 (k.pp - pm.pp))
```

Final Results for the Boxes: Gauge (Non)-Invariance

The final results can be written in the following form:

$$
\begin{aligned}
T^\mu_{\text{BoxW}} &= \frac{eg^3 m_W}{16\pi^2}\overline{v}(p_+)\gamma_\nu P_L u(p_-)\Big[(k\cdot p_+ g^{\nu\mu} - k^\nu p_+^\mu)\, c_6^{\Box\text{W}} \\
&\quad + (k\cdot p_- g^{\nu\mu} - k^\nu p_-^\mu)\, c_4^{\Box\text{W}} + \left(c_2^{\Box\text{W}} - c_4^{\Box\text{W}} p_-\cdot k - c_6^{\Box\text{W}} p_+\cdot k\right) g^{\nu\mu}\Big] \\
T^\mu_{\text{BoxZ}} &= \frac{eg^3 m_Z}{16\pi^2\cos^3\theta_W}\overline{v}(p_+)\gamma_\nu(a_L P_L + a_R P_R)u(p_-)\Big[(k\cdot p_+ g^{\nu\mu} - k^\nu p_+^\mu)\, c_5^{\Box\text{Z}} \\
&\quad + (k\cdot p_- g^{\nu\mu} - k^\nu p_-^\mu)\, c_3^{\Box\text{Z}} + \left(c_1^{\Box\text{Z}} - c_3^{\Box\text{Z}} p_-\cdot k - c_5^{\Box\text{Z}} p_+\cdot k\right) g^{\nu\mu}\Big]
\end{aligned}
$$

The Z box has to be gauge invariant by itself, because it depends on m_Z.

```
GIBoxZ:=Simplify[c1Z - c3Z sp[k,pm] - c5Z sp[k,pp] /. me->0]
GIBoxW:=Simplify[c2W - c4W sp[k,pm] - c6W sp[k,pp] /. me->0]
```

```
GIBoxZ=0
                                                         2    2    2
       3 C0[0, 2 pm.pp, -2 (k.pm + k.pp - pm.pp),mW  ,mW  ,mW  ]
GIBoxW= ---------------------------------------------------------
                                   2
```

The Gauge Non Invariant part of the 3-point Functions

Now the gauge non invariant part of the diagrams with the 3-point functions is

$$\begin{aligned} T^\mu_\Delta &= \frac{eg^3}{16\pi^2 m_W}\overline{v}(p_+)\gamma_\nu \Big[P_L(-c_3^{\Delta\gamma}(g_V-g_A) - d_4^{\Delta Z}(g_V+g_A)) \\ &\qquad +P_R(-c_3^{\Delta\gamma}(g_V-g_A) - d_4^{\Delta Z}(g_V-g_A))\Big]\, u(p_-) \\ &= \frac{eg^3}{16\pi^2 m_W}\overline{v}(p_+)\gamma_\nu P_L u(p_-)\ 2c_3^{\Delta\gamma}\, g_A \end{aligned}$$

where we have used $c_3^{\Delta\gamma} = -d_4^{\Delta Z}$

```
In[14]:= c3GHG+d4ZHG /. kin

Out[14]= 0
```

Therefore the final check of gauge invariance of $e^-e^+ \to H\gamma$ is

```
GNIBoxW:=Simplify[c2W - c4W sp[k,pm] - c6W sp[k,pp]]
kinxs={sp[pm,pp]->s/2,sp[k,pp]->(s-2 kpm-mH2)/2,sp[k,pm]->kpm,q2->s}

TestGI:= Simplify[Simplify[GNIBoxW*mW + Simplify[c3GHG (-1/2/mW)\
 /. kin] /. kinxs] /. {mH^2->mH2,me->0}]
TestGI=0
```

Cross Section

After all these checks we can write for the cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{64\pi s} \frac{s - m_H^2}{2s} \sum_{\text{spins}} \left| \sum_i T_i^\mu \epsilon_\mu \right|^2$$

with

$$T_i^\mu = \frac{eg^3}{16\pi^2 m_W^3} \overline{v}(p_+)\gamma_\nu \left(a_{iL} P_L + a_{iR} P_R\right) u(p_-) \left[\left(k \cdot p_+ \, g^{\nu\mu} - k^\nu p_+^\mu\right) G_i^+ + \left(k \cdot p_- \, g^{\nu\mu} - k^\nu p_-^\mu\right) G_i^-\right]$$

and

Diagram	a_{iL}	a_{iR}	G_i^+	G_i^-
Δ_γ	$\sin^2\theta_W$	$\sin^2\theta_W$	$m_W^2 \frac{c_1^\Delta}{s}$	$m_W^2 \frac{c_1^\Delta}{s}$
Δ_Z	$g_V^e + g_A^e$	$g_V^e - g_A^e$	$m_W^2 \frac{d_1^\Delta}{s - m_Z^2 + i m_Z \Gamma_Z}$	$m_W^2 \frac{d_1^\Delta}{s - m_Z^2 + i m_Z \Gamma_Z}$
Box_W	1	0	$m_W^4 c_6^W$	$m_W^4 c_4^W$
Box_Z	$(g_V^e + g_A^e)^2$	$(g_V^e - g_A^e)^2$	$m_Z^4 c_5^Z$	$m_Z^4 c_3^Z$

Using FeynCalc to do Traces


```
AL[mu_, rho_] :=  Sum[al[i] ((sp[k, pp] mt[mu, rho] - fv[k, rho] \
fv[pp, mu]) Gp[i] + (sp[k, pm] mt[mu,rho] - fv[k,rho] fv[pm, mu]) \
Gm[i]), {i, 1, 4}]

...

ARc[mu_, rho_] :=  Sum[ar[i] ((sp[k, pp] mt[mu, rho] - fv[k, rho] \
fv[pp, mu]) Gpc[i] + (sp[k, pm] mt[mu,rho] - fv[k,rho] fv[pm, mu]) \
Gmc[i]), {i, 1, 4}]

res:=Contract[ Tr[ds[pp] . dm[rho] . (AL[mu,rho] dm[7] + AR[mu,rho]\
     dm[6]) . ds[pm] . dm[a] . (ALc[nu,a] dm[7] + ARc[nu,a] dm[6])]
      (- mt[mu,nu])]
...

simp2={Gp[i_]->ReGp[i] + I ImGp[i],Gpc[i_]->ReGp[i] - I ImGp[i], \
Gm[i_]->ReGm[i] + I ImGm[i],Gmc[i_]->ReGm[i] - I ImGm[i]}

...

res= res /. onshell;
res= res /. kinxs
res= Simplify[res /. simp]
res= Simplify[res/mW2^3 /. simp2]
res= Simplify[res /. simp3]
```

Output to Fortran


```
(* Separate the contributions from various Diagrams *)

TG=Simplify[res /. {al[2]->0,ar[2]->0,al[3]->0,al[4]->0,ar[4]->0}]
TZ=Simplify[res /. {al[1]->0,al[3]->0,al[4]->0,ar[4]->0}]
TGTZ=Simplify[res -TG -TZ /. {al[3]->0,al[4]->0,ar[4]->0}]
BW=Simplify[res /. {al[1]->0,al[2]->0,ar[2]->0,al[4]->0,ar[4]->0}]
BZ=Simplify[res /. {al[1]->0,al[2]->0,ar[2]->0,al[3]->0}]
BWBZ=Simplify[res - BW -BZ /. {al[1]->0,al[2]->0,ar[2]->0}]
TGZBWZ=Simplify[res- TG-TZ-TGTZ-BW-BZ-BWBZ]
TestSeparation:=Simplify[res-TG-TZ-TGTZ-BW-BZ-BWBZ-TGZBWZ]

stmp=OpenWrite["functions.f",FormatType -> FortranForm]
Write[stmp,"TG=",TG]
Write[stmp,"TZ=",TZ]
Write[stmp,"TGTZ=",TGTZ]
Write[stmp,"BW=",BW]
Write[stmp,"BZ=",BZ]
Write[stmp,"BWBZ=",BWBZ]
Write[stmp,"TGZBWZ=",TGZBWZ]
Close[stmp]
```

Complete code at http://porthos.ist.utl.pt/CTQFT

```
TestSeparation=0
```

Fortran **Code**


```
*----------------------------------------------------------------------*
* This program calculates  e^- e^+ -> Higgs gamma. The notation *
* and other explanations can be  found at the page:              *
*                                                                *
*      http://porthos.ist.utl.pt/CTQFT                           *
*     (See also Nuclear Physics B267(1986)509)                   *
*                                                                *
*    Needs: LoopTools, CUBA integration package, gauss.f for     *
*    gaussian integration.                                       *
*                                                                *
*    Author: Jorge Crispim Romao                                 *
*    email:jorge.romao@ist.utl.pt                                *
*----------------------------------------------------------------------*
      program eEHG
      implicit none
...

*** Initialize LoopTools

#include "looptools.h"


...

      MSquared=TG+TZ+TGTZ+BW+BZ+BWBZ+TGZBWZ
...
```

Final Plot for the 1986 data

$$m_Z = 92.9 \text{ GeV}, m_W = 80.4 \text{ GeV}, m_t = 40 \text{ GeV}, \sin^2\theta_W = 0.21$$

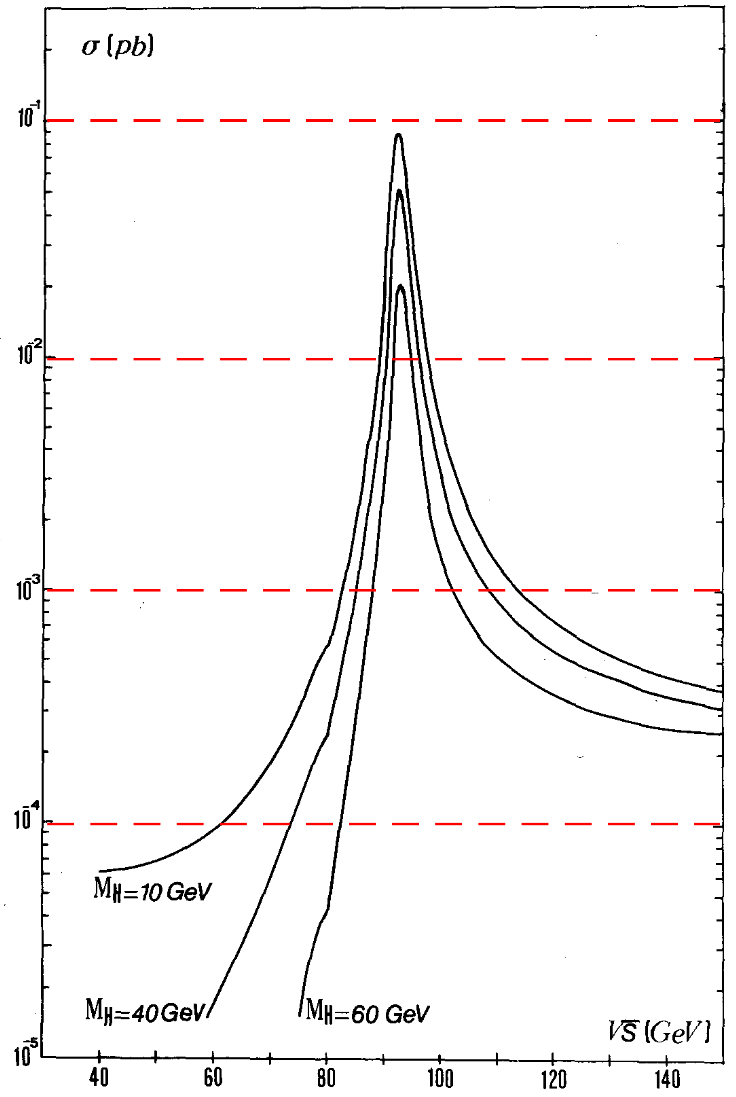
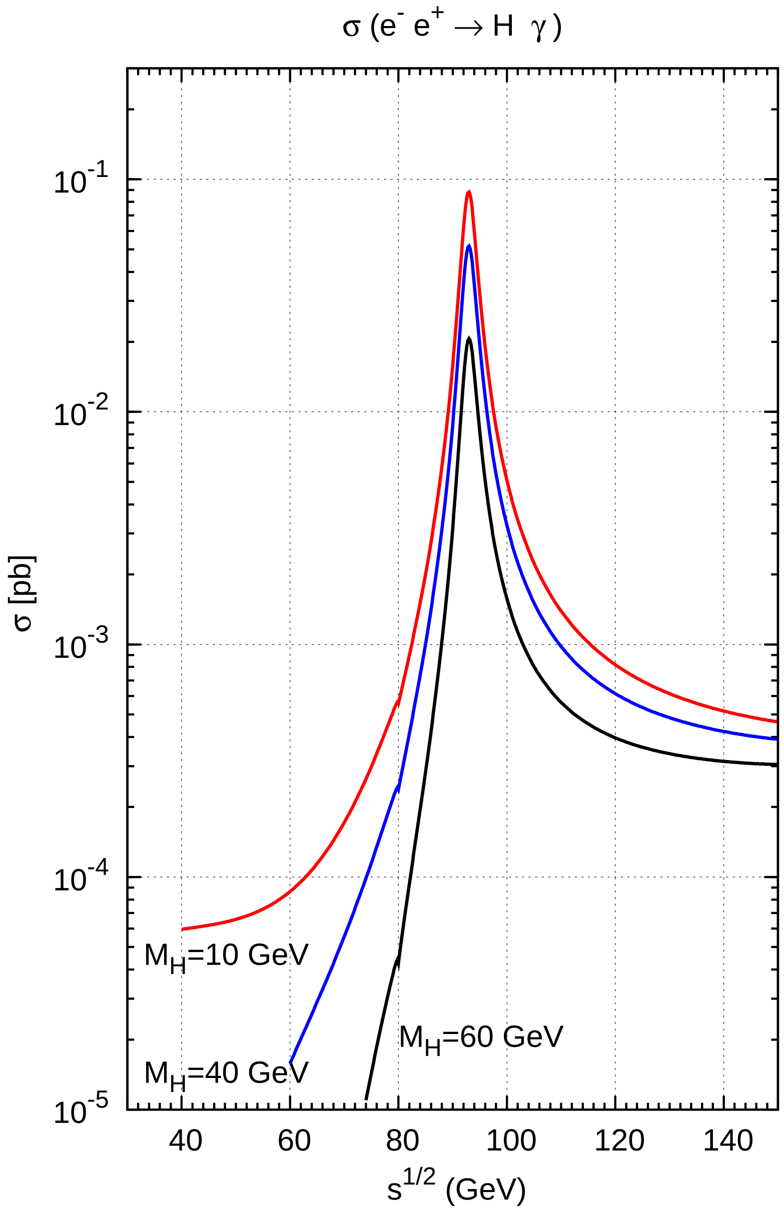
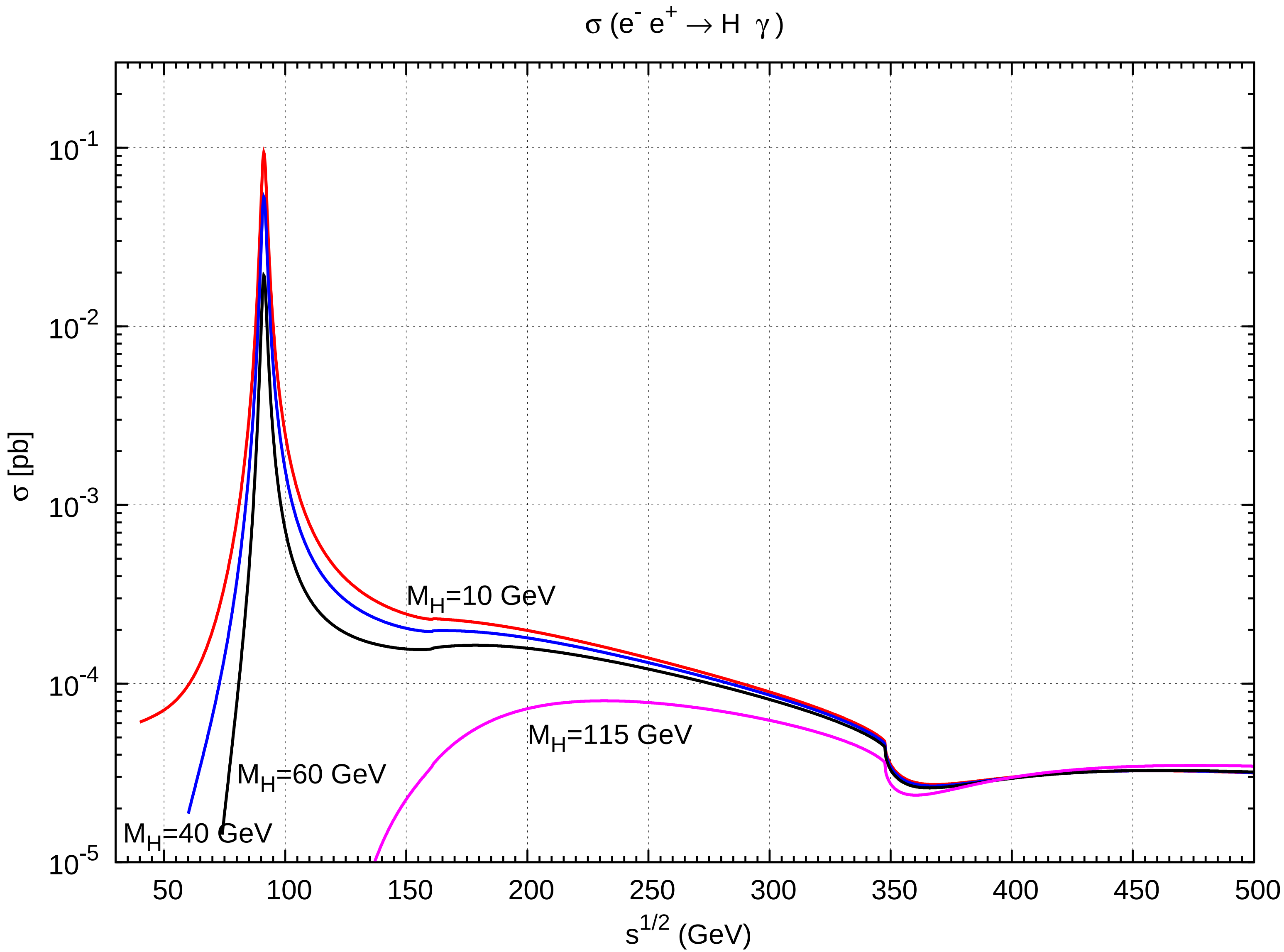


Fig. 8. Total $e^+e^- \to H\gamma$ cross section.



Final Plot for 2008 data

$$m_Z = 91.187 \text{ GeV}, m_W = 80.4 \text{ GeV}, m_t = 172 \text{ GeV}, \sin^2\theta_W = 0.2319$$



Ward Identities for $\gamma^* H\gamma$ (and $Z^* H\gamma$) NPB272(1986)693

The Ward identities are:

$$k_\mu T_{\rm G}^{\rho\mu} = -q^\rho T_{\rm G} - i\, G^{-1\ \rho\rho'}(q) T_{{\rm G}\rho'} + F_\mu T_{\rm G}^{\rho\mu} + F^\rho T_{\rm G}$$

and in lowest order

$$k_\mu T_{\rm G}^{\rho\mu} = -q^\rho T_{\rm G} - q^2 T_{\rm G}^\rho$$

with

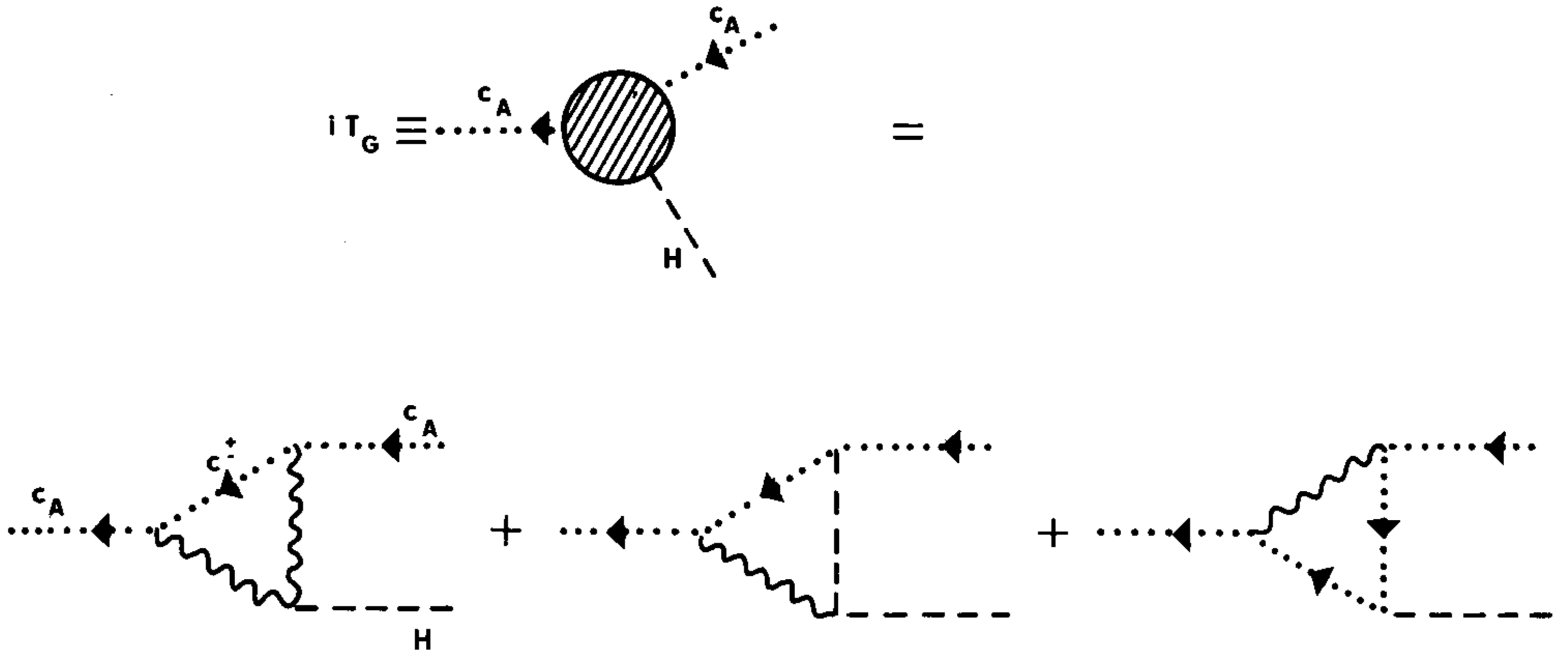


Fig. 1. One-loop contributions to T_G (cf. eq. (11)).

Ward Identities · · ·

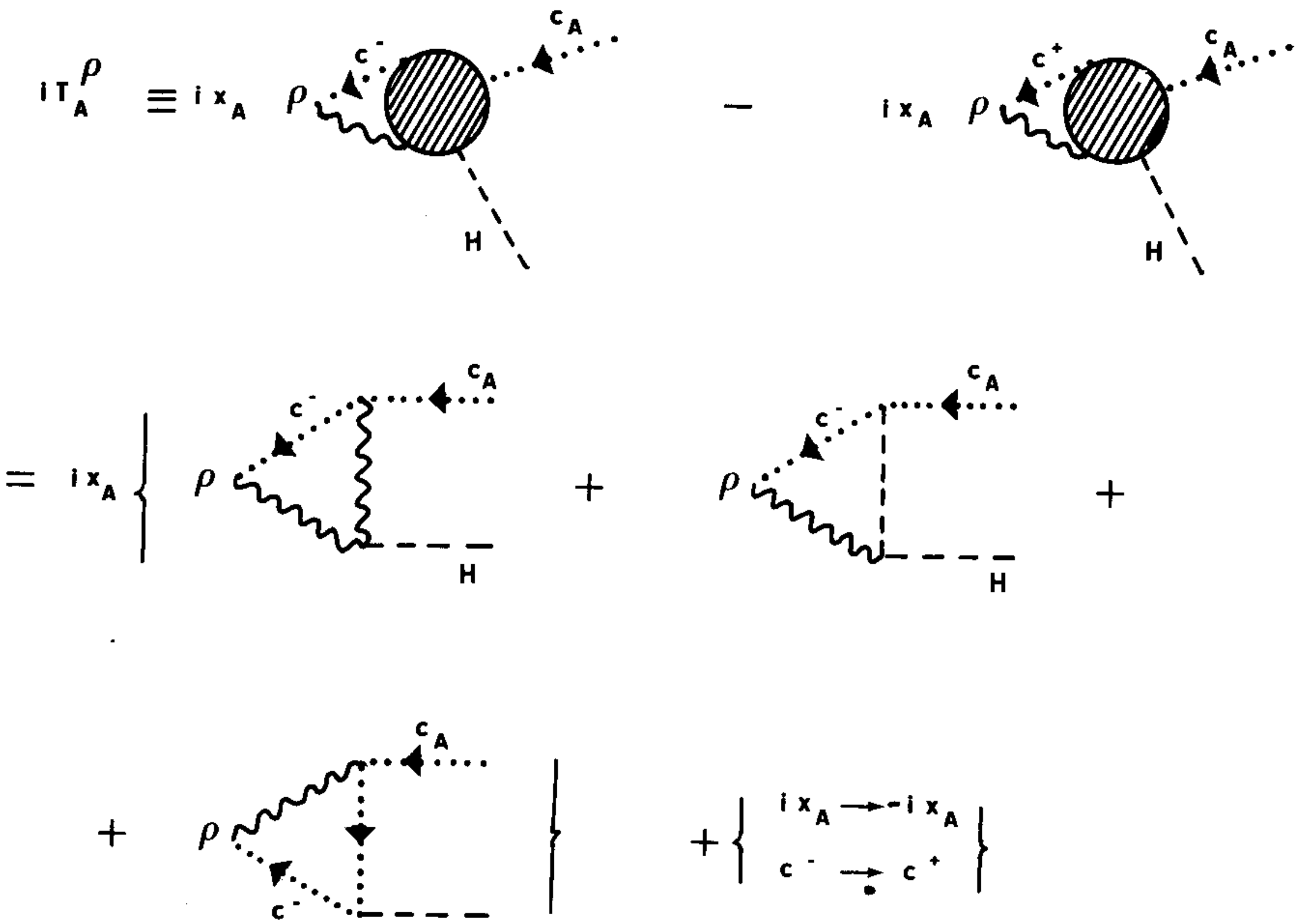


Fig. 2. One-loop contributions to T_A (cf. eqs. (11) and (14)). $x_G = e$ and $x_Z = -g\cos\theta_W$.

Ward Identities · · ·

Doing the one-loop diagrams we get the final result

$$k_\mu T_{\rm G}^{\rho\mu} = \frac{e^2 g}{16\pi^2 m_W} c_3^{\Delta\gamma}\, k_\mu \left(q^\rho q^\mu - q^2 g^{\rho\mu} \right)$$

which is precisely what we got in the first paper.

Final Comments:

- $Z^* H\gamma$ proceeds in the same way.
- These one-loop diagrams can most easily be done with `FeynCalc`
- In the non-linear gauge of Fujikawa

  $$k_\mu T_{\rm G}^{\rho\mu} = 0$$

  as Bergstrom and Hulth showed. However, this does not mean that the same is true in the Feynman-'t Hooft gauge as they claimed.
- Anyway, away from $\sqrt{s} = m_Z$ the boxes are important and should be evaluated.

The Decay Width for $H \to \gamma\gamma$

The decay width is

$$\Gamma = \frac{1}{16\pi}\frac{1}{m_H}\overline{|M|^2}\ \frac{1}{2}$$

where the factor $1/2$ is due to the presence of two identical particles in the final state. Due to gauge invariance all the diagrams should be written in the form

$$M_i = \frac{e^2 g}{m_W}\frac{1}{16\pi^2}\left[\epsilon_1(q_1)\cdot\epsilon_2(q_2)\ q_1\cdot q_2 - \epsilon_1(q_1)\cdot q_2\ \epsilon_1(q_2)\cdot q_1\right] Q_i^2 X_i$$

We get therefore

$$\Gamma = \frac{\alpha^2 g^2 m_H^3}{1024\pi^3 m_W^2}\sum_i \left|Q_i^2 X_i\right|^2 = \frac{\alpha^2 G_F m_H^3}{128\sqrt{2}\pi^3}\sum_i \left|Q_i^2 X_i\right|^2$$

where

$$\sum_{\lambda_1,\lambda_2}\left|\epsilon_1(q_1)\cdot\epsilon_2(q_2)\ q_1\cdot q_2\ - \epsilon_1(q_1)\cdot q_2\ \epsilon_1(q_2)\cdot q_1\right|^2 = 2\ (q_1\cdot q_2)^2 = \frac{1}{2}m_H^4$$

Final Result for the Amplitude

Using FeynCalc we get

```
In[3]:=  XF

                           2                                    2    2    2    2
Out[3]= -2 tauF - 4 mf   (-1 + tauF) C0[0, 0, mH , mf , mf , mf ]

In[4]:=  XG

                              2                                    2    2    2    2
Out[4]= 2 + 3 tauW + 6 mW   (-2 + tauW) C0[0, 0, mH , mW , mW , mW ]
```

with

$$\tau_f = \frac{4m_f^2}{m_H^2}, \quad \tau_W = \frac{4m_W^2}{m_H^2}$$

Tests: Comparison with the Literature

- **Barroso, Pulido, Romão**, Nucl. Phys B267 (1986), 509

$$\begin{aligned} X_F &= -4\left[J_1(0,4/\tau_f) - 4J_2(0,4/\tau_f)\right] \\ X_G &= 4\left[4J_1(0,4/\tau_W) - (6+4/\tau_W)J_2(0,4/\tau_W)\right] \end{aligned}$$

- **Gunion, Haber, Kane, Dawson**, Higgs Hunter's Guide

$$\begin{aligned} X_F &= -2\tau_f\left[1 + (1-\tau_f)f(\tau_f)\right] \\ X_G &= 2 + 3\tau_W + 3\tau_W(2-\tau_W)f(\tau_W) \end{aligned}$$

with

$$f(\tau) = \begin{cases} \left[\sin^{-1}(\sqrt{1/\tau})\right]^2, & \text{se} \quad \tau \geq 1 \\ -\frac{1}{4}\left[\ln(\eta_+/\eta_-) - i\pi\right]^2, & \text{se} \quad \tau < 1 \end{cases}$$

$$\eta_\pm = 1 \pm \sqrt{1-\tau}, \quad \tau_f = \frac{4m_f^2}{m_H^2}, \quad \tau_W = \frac{4m_W^2}{m_H^2}$$

Tests: Comparison with the Literature · · ·

We can use FeynCalc to check the results with the code:

```
(* Comparison with NPB267(1986)509 *)

subJ[m_] := {J1 -> -m^2 C0[0, 0, mH^2, m^2, m^2, m^2],
  J2 -> tau/4 (-1/2 - m^2 C0[0, 0, mH^2, m^2, m^2, m^2])}

XFBPR=Simplify[-4 ( J1 -4 J2) /. subJ[mf]]
XGBPR= Simplify[ 4 (4 J1 -( 6 + 4/tau) J2 )  /. subJ[mW]]

TestXFBPR:=Simplify[XF-XFBPR]
TestXGBPR:=Simplify[XG-XGBPR]
```

```
(* Comparison with Higgs Hunter's Guide *)

subftau[m_]:= {ftau[m] -> -2 m^2 C0[0, 0, mH^2, m^2, m^2, m^2] /tau}

XFHHG=Simplify[-2 tau (1 + (1-tau) ftau[mf]) /. subftau[mf]]
XGHHG= Simplify[ 2 + 3 tau + 3 tau (2-tau) ftau[mW]
/. subftau[mW]]

TestXFHHG:=Simplify[XF-XFHHG]
TestXGHHG:=Simplify[XG-XGHHG]
```

Final result for $H \to \gamma\gamma$

The final result is

$$\Gamma = \frac{\alpha^2 G_F m_H^3}{128\sqrt{2}\pi^3} \sum_i \left| Q_i^2 X_i \right|^2$$

with

$$X_F = -2\tau_f - 4m_f^2(-1+\tau_f)C_0(0,0,m_H^2,m_f^2,m_f^2,m_f^2)$$

$$X_G = 2 + 3\tau_W + 6m_W(-2+\tau_W)C_0(0,0,m_H^2,m_W^2,m_W^2,m_W^2)$$

Branching Ratios

We have for the various partial widths:

- $H \to f\overline{f}$

$$\Gamma(H \to f\overline{f}) = \frac{G_F\, m_H m_f^2}{4\pi\sqrt{2}} N_c \beta^3, \quad \beta = \sqrt{1 - 4m_f^2/m_H^2}$$

- $H \to WW^* \; (m_W < m_H < 2m_W)$

$$\Gamma(H \to WW^*) = \frac{3G_F^2\, m_W^4 m_H}{16\pi^3} F(x,\delta), \quad x = \frac{m_W}{m_H}, \quad \delta = \frac{\Gamma_W}{m_H}$$

$$F(x,\delta) = \int_{2x}^{1+x^2} dy\, \frac{y^2 - 4x^2}{(1-y)^2 + x^2\delta^2} \left(y^2 - 12x^2 y + 8x^2 + 12x^4\right)$$

- $H \to ZZ^* \; (m_Z < m_H < 2m_Z)$

$$\Gamma(H \to ZZ^*) = \frac{G_F^2\, m_Z^4 m_H}{64\pi^3}\left(7 - \frac{40}{3}\sin^2\theta_W + \frac{160}{9}\sin^4\theta_W\right) F(x',\delta'),\; x' = \frac{m_Z}{m_H},\; \delta' = \frac{\Gamma_Z}{m_H}$$

Branching Ratios ...

- $H \to \gamma\gamma$

$$\Gamma(H \to \gamma\gamma) = \frac{G_F\, m_H^3}{8\pi\sqrt{2}} \frac{\alpha^2}{16\pi^2} \sum_i |Q_i X_i|^2$$

- $H \to Z\gamma$

$$\Gamma(H \to Z\gamma) = \frac{G_F\, m_H^3}{4\pi\sqrt{2}} \frac{\alpha^2}{16\pi^2} \left(1 - \frac{m_Z^2}{m_H^2}\right)^3 |Y_F + Y_W|^2$$

$$Y_F = \sum_f N_{cf} \frac{Q_f g_V^f}{\sin\theta_W \cos\theta_w} I_F, \quad Y_G = \frac{1}{\tan\theta_W} I_W$$

where

$$\begin{aligned} I_F &= \frac{8 m_f^2 m_Z^2}{(m_H^2 - m_Z^2)^2} \left[B_0(m_H^2, m_f^2, m_f^2) - B_0(m_Z^2, m_f^2, m_f^2)\right] \\ &\quad - \frac{4 m_F^2}{m_H^2 - m_Z^2} \left[-2 + \left(-4m_f^2 + m_H^2 - m_Z^2\right) C_0(m_Z^2, 0, m_H^2, m_f^2, m_f^2, m_f^2)\right] \end{aligned}$$

and

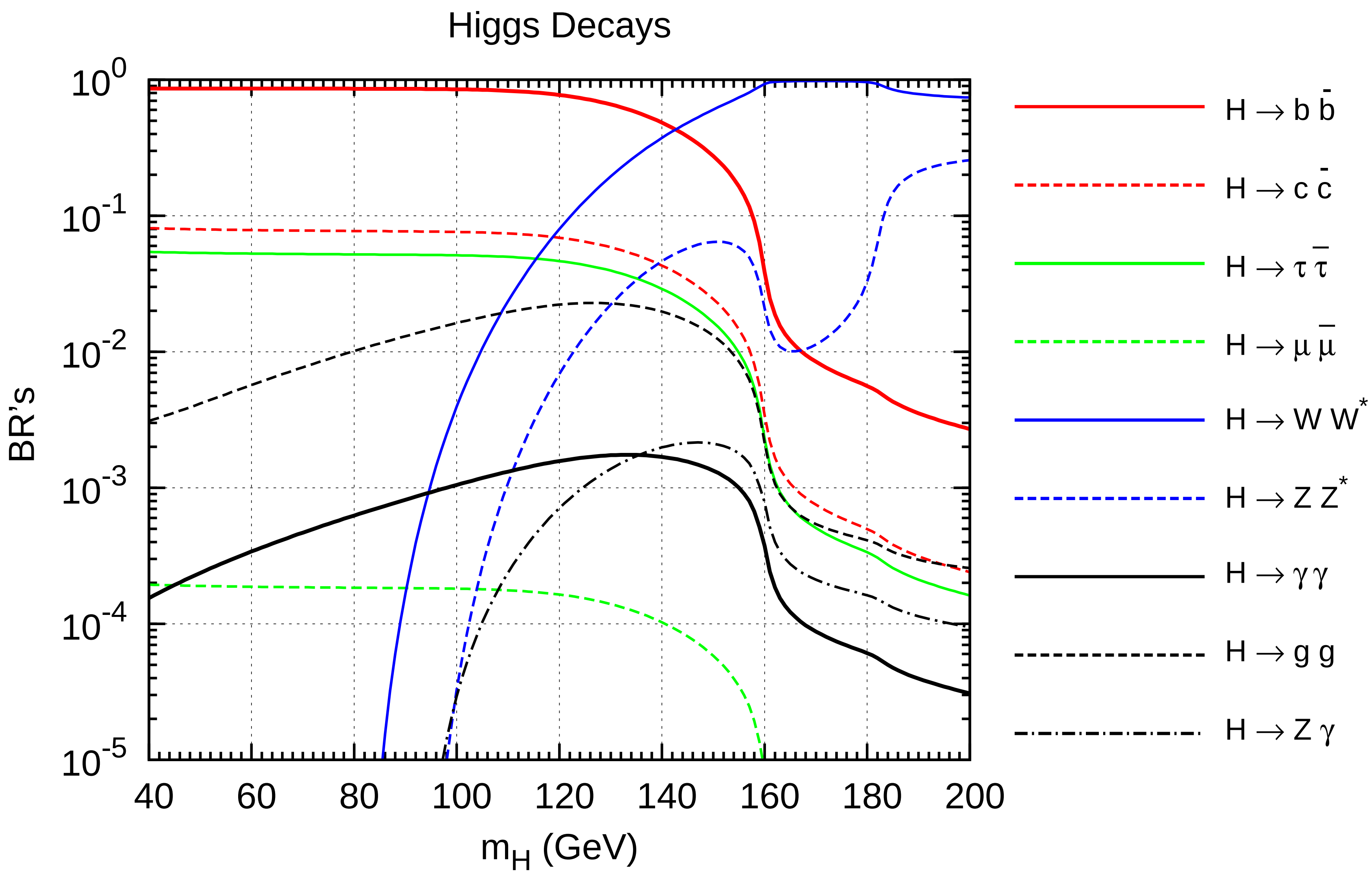
$$I_W = -\frac{1}{(m_H^2 - m_Z^2)^2}\left[m_H^2(1-\tan^2\theta_W) - 2m_W^2(-5+\tan^2\theta_W)\right] m_Z^2 \Delta B_0$$
$$-\frac{1}{m_H^2 - m_Z^2}\left[m_H^2(1-\tan^2\theta_W) - 2m_W^2(-5+\tan^2\theta_W)\right.$$
$$\left.+2m_W^2\Big((-5+\tan^2\theta_W)(m_H^2 - 2M_W^2) - 2m_Z^2(-3+\tan^2\theta_W)\Big) C_0\right]$$

$$\Delta B_0 = B_0(m_H^2, m_W^2, m_W^2) - B_0(m_Z^2, m_W^2, m_W^2), \quad C_0 = C_0(m_Z^2, 0, m_H^2, m_W^2, m_W^2, m_W^2)$$

- $H \to gg$

$$\Gamma(H \to gg) = \frac{\alpha_s^2 G_F m_H^3}{64\pi^3\sqrt{2}} \sum_{i=u,d,s,c,t,b} |X_{F_i})|^2$$

Final Graph


Higgs Decays
BR's
m_H (GeV)
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
40
60
80
100
120
140
160
180
200
H → b b̄
H → c c̄
H → τ τ̄
H → μ μ̄
H → W W*
H → Z Z*
H → γ γ
H → g g
H → Z γ

Bibliography

- **Barroso, Pulido, Romão**, `Nucl. Phys. B267 (1986), 509`
- **Barroso, Romão**, `Nucl. Phys. B272 (1986), 693`
- **Gunion, Haber, Kane, Dawson**, `Higgs Hunter's Guide`
- **Web Page for Computational Methods in QFT:** http://porthos.ist.utl.pt/CTQFT
- **J. C. Romão**, **Modern Techniques for One-Loop Calculations** http://porthos.ist.utl.pt/OneLoop/one-loop.pdf
- **QGRAF:** http://cfif.ist.utl.pt/~paulo/
- **scribble:** `Ask Paulo Nogueira`