THE VACUUM OF SUPERSYMMETRIC SU(3) × SU(2) × U(1)

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We show that in supersymmetric SU(3)×SU(2)×U(1) it is possible to break SU(2) with an Higgs singlet and at the same time to avoid vacuum expectation values for s-fermions. With rather simple constraints we obtain important restrictions on the values of the model parameters.

Spontaneously broken supersymmetric gauge theories coupled to $N = 1$ supergravity (see ref. [1] for a recent review) give rise at low energies to effective global supersymmetric models. In these models supersymmetry (SUSY) is explicitly broken by a constrained set of soft operators [1]. Given this framework, one is faced with a SUSY version of the standard $SU_c(3) \times SU_L(2) \times U_Y(1)$, which at the weak scale has to be spontaneously broken into $SU_c(3) \times U_{em}(1)$. If, the $SU_c(3) \times SU_L(2) \times U_Y(1)$ model is embedded in a grand unified gauge model (GUT) the most promising scheme to achieve the weak breaking is via radiative corrections [2]. An alternative way, using a singlet Higgs superfield [3], might destroy [4,5] the hope of understanding the hierarchy problem. However, considering the present (lack of) evidence for GUT's it is perfectly reasonable to explore the low energy consequences of SUSY $SU_c(3) \times SU_L(2) \times U_Y(1)$ without taking into account grand unification. Then, a singlet Higgs superfield is introduced to trigger the $SU_L(2)$ breaking.

The purpose of this letter is to examine models of this type and in particular to study their Higgs sector. Our motivation to address this problem was provided by a recent work of Frère et al. [6]. There [6] it is shown that the vacuum of these SUSY models violates charge symmetry. Hence, on the basis of this disease these models seemed to be excluded. However, we will prove that this pathological behaviour, rather than being general, it is specific of their [6] choice of the f function.

The Higgs sector of SUSY $SU_c(3) \times SU_L(1) \times U_Y(1)$ has three chiral superfields, namely $H(1,2,-\frac{1}{2})$, $H^c(1,2,\frac{1}{2})$ and $Y(1,1,0)$. In brackets we indicate the field transformation properties under the gauge groups [‡1]. The potential part of the lagrangian, L_{Pot}, is

$$L_{Pot} = -\sum_A \left| \frac{\partial f}{\partial z_A} \right|^2 - \sum_{j=1,3} \tfrac{1}{2}(D \cdot D)_j\,, \tag{1}$$

where $z_A(\chi_{LA})$ denotes the scalar (spinor) component of a general chiral superfield and the first sum is over all chiral superfields. In the D terms the index j refers to the gauge group and for each group we have

$$D_j = g_j z^{*A} T_A^B z_B\,, \tag{2}$$

where T are the group generators and g_j the coupling constants. Without introducing mass terms for the gauginos, which are unimportant for our study, the supersymmetry breaking part of the lagrangian is

$$L_{SB} = -m_{3/2}^2 \sum_A z_A^* z_A$$
$$- m_{3/2} \left[(A-3) f(z) + \sum_A \frac{\partial f}{\partial z_A} z_A + \text{h.c.} \right]. \tag{3}$$

If, h, h^c and y are the scalar components of the Higgs superfields and $\tilde{L}$, $\tilde{\ell}^c$, etc. denote the scalar partners

‡1 For further notation and details of the model see for instance ref. [7].

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of the matter superfields the f function can be written as

$$f=\beta(y\epsilon^{\alpha\beta}h_\alpha h^c_\beta-\epsilon y)+g_i\epsilon^{\alpha\beta}h_\alpha\widetilde{\mathrm{L}}_{\beta i}\widetilde{\ell}^c_i$$
$$+g_{ui}\epsilon^{\alpha\beta}h^c_\beta\widetilde{\mathrm{Q}}_{\alpha i}\widetilde{\mathrm{u}}^c_i+g_{di}\epsilon^{\alpha\beta}h_\alpha\widetilde{\mathrm{Q}}_{\beta i}\widetilde{\mathrm{d}}^c_i\ . \tag{4}$$

The index i is a generation label and the matter superfields are $\mathrm{L}_i(1,2,-\frac{1}{2})$, $\ell^c_i(1,1,1)$, $\mathrm{Q}_i(3,2,\frac{1}{6})$, $\mathrm{d}^c_i(\bar{3},1,\frac{1}{3})$ and $\mathrm{u}^c_i(\bar{3},1,-\frac{2}{3})$. Inserting eq. (4) into eqs. (1) and (3) it is easy to obtain the general expression for the potential V.

As in the standard electroweak model, the spontaneous breaking of $\mathrm{SU_L}(2)$ is obtained if the Higgs bosons develop a vacuum expectation value, VEV, i.e.

$$\langle h\rangle=\begin{pmatrix}v_1\\0\end{pmatrix}\ ,\quad \langle h^c\rangle=\begin{pmatrix}0\\v_2\end{pmatrix}\ ,\quad \langle y\rangle=v'\ . \tag{5}$$

However, V depends on other scalar bosons, the partners of leptons and quarks. So, it is necessary to make sure that the minimum of V does not occur when some of these bosons have a VEV. Clearly, $\langle\tilde{\mathrm{e}}\rangle\neq 0$ would be physically unacceptable since it would signal a breaking of charge symmetry. Examining the behaviour of V when the s-electron, $\tilde{\mathrm{e}}$, and the s-neutrino, $\tilde{\nu}$, acquire a non-zero VEV we conclude that for $|A|>3$ the minimum of V has $\langle\tilde{\mathrm{e}}\rangle\neq 0$. To be specific, one can show that, for $|A|>3$,

$$V_{\min}=\epsilon m^2_{3/2}[1-\epsilon^{-1}(m_{3/2}/g_\mathrm{e})^2u^2(u^2-1)]<0, \tag{6}$$

where

$$v_1=\langle\tilde{\mathrm{e}}\rangle=\langle\tilde{\mathrm{e}}^c\rangle=-(\beta/g_\mathrm{e})u\ ,$$

and u satisfies the equation

$$u^2-\tfrac{1}{2}Au+\tfrac{1}{2}=0\ . \tag{7}$$

Notice that $\epsilon m^2_{3/2}>0$ is the value of V corresponding to zero VEVs for all fields. We can also see that this problem is more important for the s-electrons because of the smallness of g_e. This confirms the result of Frère et al. [6] despite the fact that our function f is not the same as they have used. It is precisely this difference in the f function – with a term linear in y replacing a y^3 term – that enables us to find minima of V that break $\mathrm{SU_L}(2)$ with $A<3$. On the contrary, to break $\mathrm{SU_L}(2)$ with the potential used in ref. [6] one needs $A>3$ [4]. It is interesting to point out that there is another reason to exclude values of A greater than 3. In fact, even with zero VEVs for the s-leptons, the absolute minimum of V corresponds to $v_1=v_2=0$ and $v'\neq 0$ and so does not break $\mathrm{SU_L}(2)$. It is quite easy to show that a similar problem occurs if $A<1$. Hence, models of the type that we are considering must have $1<A<3$.

Using the complete potential for the first generation of s-leptons we checked numerically that: (i) For $|A|<\sqrt{8}$ there are no minima that break electric charge; (ii) For $\sqrt{8}<|A|<3$ there is a minimum that breaks electric charge but it is not the absolute minimum. Hence from now on we shall consider $1<A<3$ and so, the VEVs of all scalar fields except h, h^c and y is zero. Then, the general expression for V is [7]

$$V=\beta^2|\epsilon^{\alpha\beta}h_\alpha h^c_\beta-\epsilon|^2+\beta^2|y\epsilon^{\alpha\beta}h^c_\beta|^2$$
$$+\beta^2|y\epsilon^{\alpha\beta}h_\alpha|^2+m^2_{3/2}(h^{*\alpha}h_\alpha+h^{c*\alpha}h^c_\alpha+y^*y)$$
$$+\beta m_{3/2}[Ay\epsilon^{\alpha\beta}h_\alpha h^c_\beta-(A-2)\epsilon y+\mathrm{c.c.}]$$
$$+\tfrac{1}{2}(D^aD^a+D'D')\ , \tag{8}$$

with

$$D^a=g_2(h^+\tfrac{1}{2}\sigma^a h+h^{+c}\tfrac{1}{2}\sigma^a h^c)\ , \tag{9a}$$

and

$$D'=g_1(-\tfrac{1}{2}h^+h+\tfrac{1}{2}h^{c+}h^c)\ . \tag{9b}$$

Writing

$$v_1=\sqrt{2}v\cos\theta\ ,\quad v_2=\sqrt{2}v\sin\theta\ ,$$

it is easy to see that the minimum of V corresponds to $\theta=\pi/4$ and to v and v' which are solutions of the equations

$$\hat{v}[(\hat{v}^2-x)+\hat{v}'^2+A\hat{v}'+1]=0\ , \tag{10a}$$

$$(2\hat{v}'+A)\hat{v}^2+\hat{v}'-(A-2)x=0\ , \tag{10b}$$

with

$$\hat{v}=(\beta/m_{3/2})v\ ,\quad \hat{v}'=(\beta/m_{3/2})v',\quad x=\epsilon\beta^2/m^2_{3/2}\ . \tag{11}$$

Eqs. (10) have two solutions. The first one, corresponding to $\hat{v}=0$ and $\hat{v}'=(A-2)x$ does not break $\mathrm{SU_L}(2)$ and leads to

$$V^{\mathrm{NB}}_{\min}=(m^4_{3/2}/\beta^2)x^2[1-(A-2)^2]\ . \tag{12}$$

Finally, assuming $\hat{v}\neq 0$, eq. (10a) gives

$$\hat{v}=(x-1-\hat{v}'^2-Av')^{1/2}\equiv[f(x,A)]^{1/2}\ , \tag{13}$$

with $\hat{v}'$ solution of the cubic equation

$$2\hat{v}'^3 + 3A\hat{v}'^2 + (1 - 2x + A^2)\hat{v}' + (A - 2x) = 0\,. \quad (14)$$

The value of V at this $SU_L(2)$ breaking minimum is

$$V^B_{min} = (m^4_{3/2}/\beta^2)[-\tfrac{1}{2}A\hat{v}'^3 - \tfrac{1}{2}\hat{v}'^2(A^2 + 1 - 2x)$$
$$- \tfrac{3}{2}\hat{v}'(A - 2x) + 2x - 1]\,. \quad (15)$$

As we have said before, the requirement $V^B_{min} < V^{NB}_{min}$ restricts A to vary between the limits one and three. It is also possible to see that for the allowed range of A this condition excludes certain values of x roughly below $x = 1$.

To tighten up the constraints on the model parameters it is interesting to consider the mass matrix for the neutral Higgs scalars. After diagonalization, the masses of the five neutral Higgs bosons are

$$m^2_{1\pm} = \tfrac{1}{2}m^2_{3/2}\{a + b \pm [(a - b)^2 + 4c^2]^{1/2}\}\,, \quad (16a)$$

$$m^2_{2\pm} = \tfrac{1}{2}m^2_{3/2}[2a - 1 \pm (1 + 4d^2)^{1/2}]\,, \quad (16b)$$

$$m^2_3 = m^2_Z + 2m^2_{3/2}(1 + \hat{v}'^2)\,, \quad (16c)$$

with

$$a = 2x - 1 - 2\hat{v}'^2 - 2A\hat{v}'\,, \quad b = 2x - 2A\hat{v}'\,,$$

$$c = -\sqrt{2}A\hat{v}\,, \quad d = 2\sqrt{2}\hat{v}\hat{v}' - c\,.$$

On the other hand, let us recall that the masses of the s-fermions, $\tilde{f}$, are

$$m^2_{\tilde{f}} = m^2_{3/2} + m^2_f \pm A m_{3/2} m_f\,. \quad (17)$$

Looking at eqs. (16), it is interesting to remark that it is possible to have a light neutral Higgs. However, the condition $v_1 = v_2$ does not allow us to implement the Glashow mechanism [8] to explain the decay $\Upsilon \to \gamma\eta$ [9].

We now have all the ingredients to restrict the model parameters. To do that we shall use the following constraints:

(i) From the measurement [10] of the W mass it follows that $v = 250$ GeV.

(ii) Experimental searches [11] for s-fermions imply $m_{3/2} > 20$ GeV.

(iii) Taking the top mass as $m_t = 40$ GeV [12] we use eq. (17) to restrict $m^2_{\tilde{t}} \geqslant 0$.

(iv) Similarly, we use eqs. (16) to impose m^2_{1-} and $m^2_{2-} \geqslant 0$;

(v) We decide that perturbation theory is valid, i.e., $\beta < 1$.

Then eqs. (11) and (13) imply

$$\beta = (m_{3/2}/v)\,[f(x, A)]^{1/2} < 1\,.$$

(vi) A varies in the range $1 < A < 3$.

In fig. 1 the triangular shaped area corresponds to the values of $m_{3/2}$ and x that, for $A = 2$, satisfy all criteria. The vertical line comes from condition (ii) while the horizontal line is a consequence of the restriction (iv). If ones tries to tighten up this constraint requiring m^2_{1-} and m^2_{2-} greater than 10 GeV/c^2, only a very small region of the $m_{3/2}$, x plane is further ruled out. In fact, now the horizontal boundary is the dash–dotted line. Finally, the other limiting curve represents the criterium (v) with $\beta^2 = 1$. If, instead of $\beta^2 \leqslant 1$ we require $\beta^2 \leqslant \frac{1}{4}$ the full curve is replaced by the dashed curve. This implies a considerable reduction of the allowed region of the parametric space. For other values of A one obtains figures with a simi-

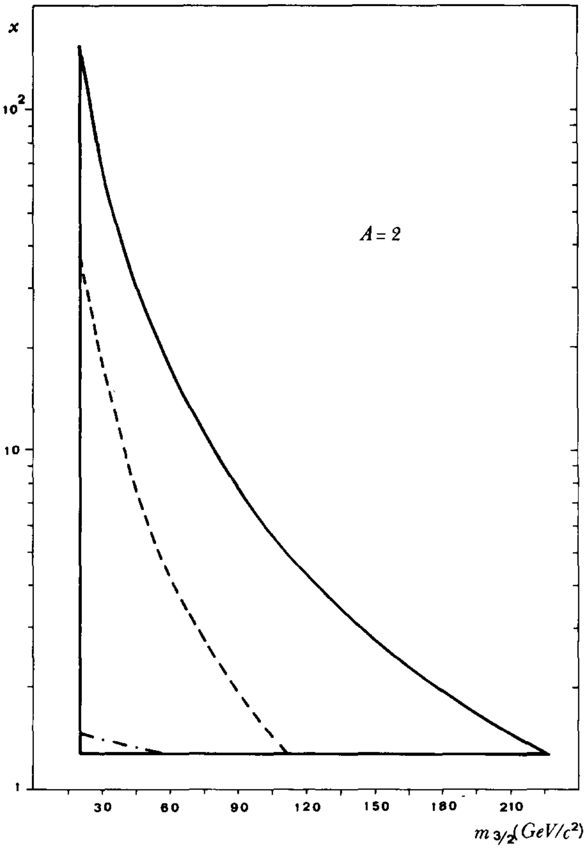

Fig. 1. The values of x and $m_{3/2}$ outside the triangular area are excluded.

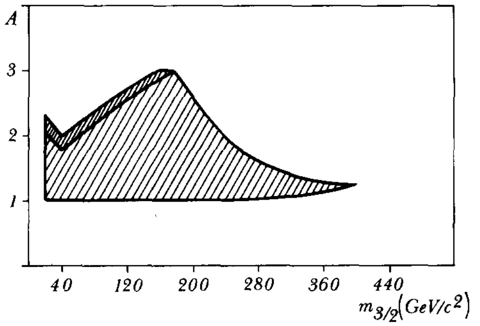

Fig. 2. Gravitino mass $m_{3/2}$ as a function of A. Values outside the hatched area are not allowed.

lar shape but with different areas. This can be seen in fig. 2 where the crossed area represents the possible values of A and gravitino mass. The vertical and horizontal lines and the right-hand-side boundary are due to the criteria (ii), (iv) and (v), respectively. The top boundary was derived using the condition (iii), i.e., $m_{\tilde{t}} > 0$. As before, the use of a stronger bound, $m_{\tilde{t}} \geqslant 20\,\mathrm{GeV}/c^2$, has a very small effect. This is also shown in fig. 2.

We summarize our conclusions as follows:

(i) Using an Higgs singlet it is possible to break $SU_L(2)$ and at the same time to avoid VEVs for the s-fermions. To implement this scheme the potential must not contain cubic terms [‡2] in y and $1 < A < 3$. Let us remark that this problem with the vacuum of SUSY is not specific of the present model. On the contrary, it was shown before [13] that models with $SU_L(2)$ breaking induced by radiative corrections are also affected by this illness.

(ii) With rather simple constraints one obtains important restrictions on the values of the model parameters. In other words, even without calling upon SUSY to explain any fact, the parameters are already restricted. In particular, the gravitino mass, should be less than a few hundred GeV ($m_{3/2} < 400$ GeV/c^2)! Since $m_{3/2}$ sets the mass scale for the partners of the fermions one can say that SUSY is on the brink of being either discovered or ruled out. It is gratifying to see that the same conclusion was recently obtained [14] in the framework of grand unified SUSY.

[‡2] This requires the introduction of another mass scale, ϵ.

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