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HIGGS PRODUCTION AT e^+e^- COLLIDERS (II). Ward identities for $\gamma^*H\gamma$ and $Z^*H\gamma$ Green functions

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In the literature there are confusing and sometimes wrong statements about the $\gamma^*H\gamma$ and $Z^*H\gamma$ irreducible Green functions. To clarify this issue we derive Ward-Slavnov-Taylor identities involving these functions. These relations confirm our previous calculation of the $e^+e^- \to H\gamma$ cross section.

1. Introduction

In a recent work [1], which in the following will be denoted by I, we have calculated the cross section for Higgs (H) and gamma (γ) production at electron-positron colliders. In I we have shown that the $e^+e^- \to H\gamma$ amplitude receives contributions from the $Z^*H\gamma$ and $\gamma^*H\gamma$ Green functions and from W and Z box diagrams. To be precise $T_Z^{\rho\mu}$ and $T_G^{\rho\mu}$ denote the one-particle irreducible Green function with the outgoing particles (H and γ) on-shell and the incoming one (Z^* or γ^*) off-shell. We denote by q the incoming momentum and by k the outgoing photon momentum; ρ and μ are the Lorentz indices of the incoming and outgoing vector particles. Working in the 't Hooft-Feynman gauge and using an on-shell renormalization scheme [2] we have shown in I that at one-loop level, $T_A^{\rho\mu}$ (A = Z or G) are not gauge invariant with respect to the photon, i.e.

$$k_\mu T_A^{\rho\mu} = X^\rho \neq 0 . \tag{1}$$

Let us stress that there is no reason to expect gauge invariance for $T_A^{\rho\mu}$. Only amplitudes which correspond to physical processes must be gauge invariant and this requirement is satisfied by eq. (1). In fact, if the incoming particle is on shell we have shown in I that $X^\rho = 0$. Clearly, these on shell Green's functions enter the amplitude for the processes $Z \to H\gamma$ and $H \to \gamma\gamma$.

After completing our paper [1] we became aware of a recent and similar calculation by Bergström and Hulth [3] which contradicts our previous statement.

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These authors [3] calculate $T_G^{\rho\mu}$ in the non-linear gauge of Fujikawa [4] and obtained an apparently gauge invariant result, namely

$$k_\mu T_G^{\rho\mu} = 0 . \tag{2}$$

They claim [3], wrongly as we shall see, that the same result is obtained in the linear 't Hooft-Feynman (HF) gauge. Then, based on this result, Bergström and Hulth calculated $T_Z^{\rho\mu}$ in the HF gauge.

Our aim in this paper, which may be regarded as an extension of I, is to try to clarify all these issues related to the gauge properties of $T_A^{\rho\mu}$. To do that we derive Ward-Slavnov-Taylor identities [5] for the $Z^*H\gamma$ and $\gamma^*H\gamma$ Green functions. On the basis of those relations and working at one-loop level we will recover our previous results. However, as we shall see, in the non-linear gauge [4] eq. (2) is valid. This illustrates our previous statement since it shows that $k_\mu T_A^{\rho\mu}$ is gauge dependent.

2. Slavnov-Taylor identities

The simplest way (see e.g. ref. [6]) of deriving Slavnov-Taylor identities is to use the fact that Green functions are invariant under the Becchi-Rouet-Stora (BRS) [7] transformation. Let us denote by c_A, c_Z and $c^{\pm}$ the Faddeev-Popov (FP) ghosts associated with the fields A_μ, Z_μ and $W_\mu^{\pm}$ respectively. For our purposes, it is sufficient to consider that under a BRS transformation we have

$$\begin{aligned}
\delta A_\mu &= \partial_\mu c_A \theta + iec^- W_\mu^+ \theta - iec^+ W_\mu^- \theta ,\\
\delta Z_\mu &= \partial_\mu c_Z \theta - ig\cos\theta_W c^- W_\mu^+ \theta + ig\cos\theta_W c^+ W_\mu^- \theta ,\\
\delta \bar{c}_A &= \partial^\mu A_\mu \theta ,\\
\delta \bar{c}_Z &= \partial^\mu Z_\mu \theta - M_Z \phi_Z \theta ,
\end{aligned} \tag{3}$$

where θ is a space-time independent anticommuting Grassmann variable and ϕ_Z is the would be Goldstone boson associated with the Z.

In the HF gauge the gauge fixing term $\mathscr{L}_{gf}$ is

$$\mathscr{L}_{gf} = -\tfrac{1}{2}\left(\partial^\mu A_\mu\right)^2 - |F_+|^2 - \tfrac{1}{2}|F_Z|^2 , \tag{4}$$

with

$$F_+ = \partial^\mu W_\mu^+ - iM_W \phi^+ , \tag{5a}$$

$$F_Z = \partial^\mu Z_\mu - M_Z \phi_Z . \tag{5b}$$

From $\mathscr{L}_{gf}$ one can derive the lagrangian for the FP ghosts $\mathscr{L}_{FP}$ and, once this is done, the BRS transformation follows from the requirement of invariance of the effective action.

Consider the Green function $\langle 0|TA_\mu \bar{c}_A|0\rangle$. Since

$$\delta\langle 0|TA_\mu \bar{c}_A|0\rangle = 0,$$

eqs. (3) lead to

$$\langle 0|TA_\mu \partial^\nu A_\nu|0\rangle\theta = \langle 0|T\partial_\mu c_A \bar{c}_A|0\rangle\theta + ie\langle 0|Tc^- W_\mu^+ \bar{c}_A|0\rangle\theta - ie\langle 0|Tc^+ W_\mu^- \bar{c}_A|0\rangle\theta, \tag{6}$$

which means

$$k^\nu G_{\mu\nu}(k) = -k_\mu \Delta_{c_A}(k) + F_\mu \Delta_{c_A}(k). \tag{7}$$

In the equation above $G_{\mu\nu}$ and Δ_{c_A} are the photon and c_A propagators, respectively, and the quantity F_μ, arising from the last two terms of eq. (6), does not play any role in the following. Notice that it is at least of order e^2.

Starting with $\delta\langle 0|TZ_\mu \bar{c}_Z|0\rangle = 0$ it is straightforward to obtain

$$k^\nu G_{\mu\nu}^Z(k) = -k_\mu \Delta_{c_Z}(k) + E_\mu \Delta_{c_Z}(k), \tag{8}$$

where E_μ is at least of order g^2. It is now easy to derive a Ward identity for $\gamma^* H\gamma$. In fact, from

$$\delta\langle A_\rho \bar{c}_A H\rangle = 0,$$

where H is the Higgs field and we have omitted the time order and the vacuum states, one obtains

$$\langle A_\rho\, \partial^\mu A_\mu\, H\rangle\theta = \langle \partial_\rho c_A\, \bar{c}_A H\rangle\theta + ie\langle c^- W_\rho^+ \bar{c}_A H\rangle\theta - ie\langle c^+ W_\rho^- \bar{c}_A H\rangle\theta - \langle A_\rho \bar{c}_A \delta H\rangle. \tag{9}$$

After multiplying by the inverse Higgs propagator the last term of eq. (9) vanishes, because the Higgs is on-shell. Hence, we have

$$-ik^\mu G_{\rho\rho'}(q) G_{\mu\mu'}(k) T_G^{\rho'\mu'} = iq_\rho \Delta_{c_A}(q)\Delta_{c_A}(k) T_G + \Delta_{c_A}(k) T_\rho, \tag{10}$$

where T_G and T^ρ are defined in figs. 1 and 2 respectively. After multiplication by $G^{-1\rho\rho'}$ and the use of eqs. (7) and (8) we finally obtain

$$k_\mu T_G^{\rho\mu} = -q^\rho T_G - iG^{-1\rho\rho'}(q) T_{\rho'} + F_\mu T_G^{\rho\mu} + F^\rho T_G,$$

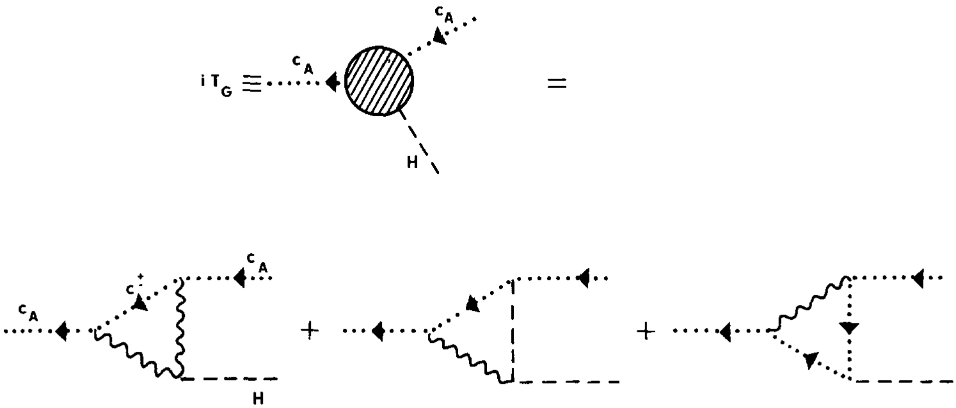

Fig. 1. One-loop contributions to T_{G} (cf. eq. (11)).

which in the lowest order is

$$k_\mu T_{\mathrm{G}}^{\rho\mu} = -q^\rho T_{\mathrm{G}} + q^2 T_{\mathrm{G}}^{\rho}, \tag{11}$$

with T_{G} and T_{G}^{ρ} given by the loop diagrams of figs. 1 and 2. Evaluating them we have

$$k_\mu T_{\mathrm{G}}^{\rho\mu} = \frac{3e^2 g}{16\pi^2 M_{\mathrm{W}}} \frac{\Delta S}{\beta - \beta_{\mathrm{H}}} k_\mu \left[q^\mu q^\rho - q^2 g^{\rho\mu} \right], \tag{12}$$

which is precisely the result* obtained in I (cf. eq. (2.24), there)

The derivation of the Ward-Slavnov-Taylor identity for $Z^*H\gamma$ is very similar. From

$$\delta \langle Z_\rho \bar{c}_A H \rangle = 0,$$

one obtains

$$\begin{aligned} \langle Z_\rho\, \partial^\nu A_\nu\, H \rangle \theta = {} & \langle \partial_\rho\, c_Z \bar{c}_A H \rangle \theta - ig \cos\theta_{\mathrm{W}} \langle c^- W_\mu^+ \bar{c}_A H \rangle \theta \\ & + ig \cos\theta_{\mathrm{W}} \langle c^+ W_\rho^- \bar{c}_A H \rangle \theta - \langle Z_\rho \bar{c}_A \delta H \rangle, \end{aligned} \tag{13}$$

which in the lowest order gives

$$k_\mu T_{\mathrm{Z}}^{\rho\mu} = -q^\rho T_{\mathrm{Z}} + \left(q^2 - M_{\mathrm{Z}}^2 \right) T_{\mathrm{Z}}^{\rho}. \tag{14}$$

* The functions S and L are defined in I, $\Delta S = S(\beta) - S(\beta_{\mathrm{H}})$ with $\beta = q^2/M_{\mathrm{W}}^2$ and $\beta_{\mathrm{H}} = M_{\mathrm{H}}^2/M_{\mathrm{W}}^2$, and similarly for ΔL.

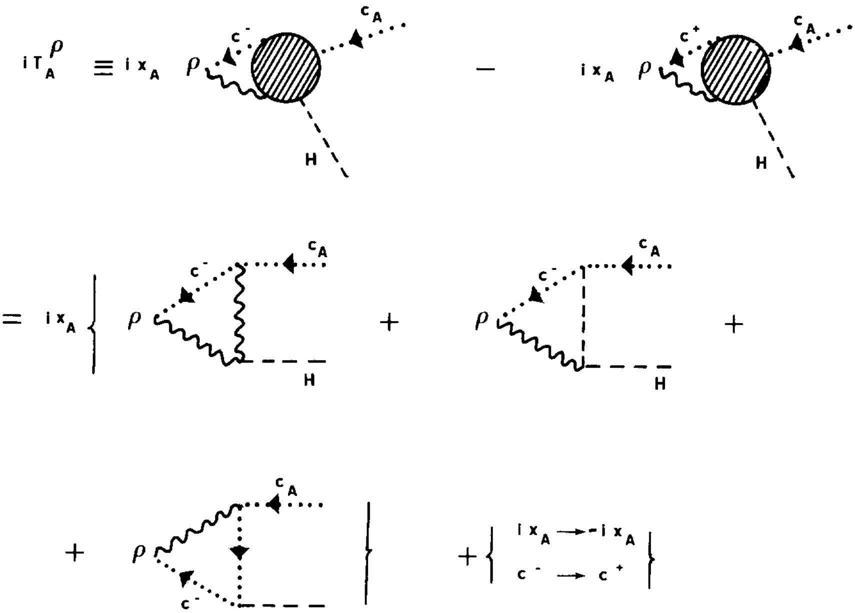

Fig. 2. One-loop contributions to T_A (cf. eqs. (11) and (14)). $x_G = e$ and $x_Z = -g\cos\theta_W$.

The one loop result for T_Z^ρ is

$$T_Z^\rho = \frac{eg^2\cos\theta_W}{16\pi^2 M_W}\frac{\Delta S}{\beta-\beta_H}(3k-q)^\rho, \tag{15}$$

which can be trivially obtained from T_G^ρ substituting one power of e by $-g\cos\theta_W$. However for T_Z, represented in fig. 3, the situation is slightly different. Because the coupling $\bar{c}_Z c^+\phi^-$ exists whereas $\bar{c}_A c^+\phi^-$ does not exist, we have six triangle diagrams rather than the three we had before. Furthermore, the sum of these diagrams is divergent and the result becomes finite with the introduction of a counterterm, T_Z (count.), which is:

$$T_Z(\text{count.}) = -\frac{eg^2 M_W}{2\cos\theta_W}\frac{1}{16\pi^2}\Gamma\left(\tfrac{1}{2}\varepsilon\right)\left(M_W^2\right)^{-\varepsilon/2}. \tag{16}$$

This counterterm amplitude can be easily deduced from the counterterms, $\mathscr{L}^c$, developed by $\mathscr{L}_{FP}$. Adding eq. (16) to the triangle amplitudes we get

$$T_Z = \frac{eg^2}{16\pi^2 M_W}(\beta-\beta_H)^{-1}\left[\Delta S\left(3q\cdot k - q^2 + M_Z^2\right)\cos\theta_W\right.$$
$$\left. + \left(\tfrac{1}{2}\Delta L - \Delta S\right)k\cdot q/\cos\theta_W\right]. \tag{17}$$

Finally, inserting eqs. (15) and (17) into eq. (14) we again reproduce our previous result [1] (cf. eq. (2.23) there).

So far we have been using the linear HF gauge. Let us repeat the previous analysis in the non-linear gauge [4]. In this gauge $\mathscr{L}_{\mathrm{gf}}$ differs from the one given in eq. (4) because in eq. (5a) the derivative is replaced by a covariant derivative, i.e.

$$F_+ \to F'_+ = F_+ + ieA^\mu W^+_\mu \,. \tag{18}$$

Clearly, the extra term in the gauge function F'_+ will lead to additional couplings for the ghosts. In particular, for our problem, the important point is to realize that the $\bar{c}^{\pm} c_A W_\mu^{\mp}$ triple vertice is modified in such a way that it becomes symmetric in the momenta of the two outgoing ghosts, i.e.

$$\pm iep^\mu \to \pm ie(p+p')^\mu \,.$$

Bearing in mind this difference, the first and the third loop diagrams of fig. 2 are changed, and we obtain

$$T^\rho_{\mathrm{G}} = \frac{e^2 g}{16\pi^2 M_{\mathrm{W}}} q^\rho \frac{\Delta S}{\beta - \beta_{\mathrm{H}}} \,, \tag{19}$$

$$T^\rho_{\mathrm{Z}} = -\frac{eg^2 \cos\theta_{\mathrm{W}}}{16\pi^2 M_{\mathrm{W}}} q^\rho \frac{\Delta S}{\beta - \beta_{\mathrm{H}}} \,. \tag{20}$$

Similarly, evaluating the triangles of figs. 1 and 3 we derive

$$T_{\mathrm{G}} = \frac{e^2 g}{16\pi^2 M_{\mathrm{W}}} q^2 \frac{\Delta S}{\beta - \beta_{\mathrm{H}}} \,, \tag{21}$$

$$T_{\mathrm{Z}} = \frac{eg^2 \cos\theta_{\mathrm{W}}}{16\pi^2 M_{\mathrm{W}}} \left(-q^2 + M_{\mathrm{Z}}^2\right) \frac{\Delta S}{\beta - \beta_{\mathrm{H}}} \,, \tag{22}$$

respectively. Hence, in the non-linear gauge we obtain

$$k_\mu T^{\rho\mu}_{\mathrm{G}} = k_\mu T^{\rho\mu}_{\mathrm{Z}} = 0 \,, \tag{23}$$

which could be interpreted as a signal of electromagnetic gauge invariance. However this is particular to this gauge and the general statement must be that $k_\mu T^{\rho\mu}_A$ are gauge dependent quantities. Notice that the change in the gauge fixing function, $F_+ \to F'_+ = F_+ + \Delta F_+$, does not alter eqs. (3). So, the Slavnov-Taylor identities given by eqs. (11) e (14) remain valid.

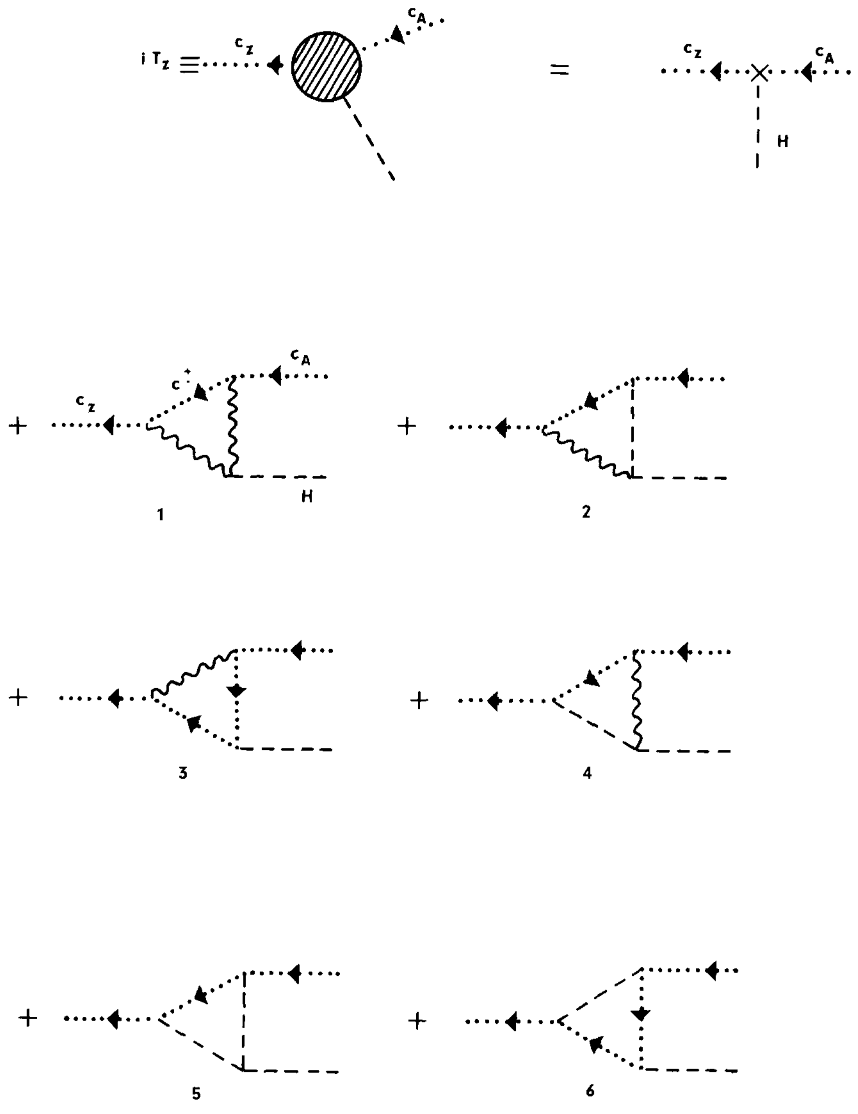

Fig. 3. One-loop contributions to T_Z (cf. eq. (14)).

3. Conclusions

Using the HF gauge and the non-linear gauge we derived Slavnov-Taylor identities for the one particle irreducible three-point functions $T_A^{\rho\mu}$. We summarize our conclusions as follows:

(i) In the HF gauge and in the lowest order we obtained $k_\mu T_A^{\rho\mu} = X^\rho \neq 0$ in agreement with the results of I.

(ii) In the non-linear gauge [4] the introduction of the electromagnetic covariant derivative in F'_+ induces electromagnetic gauge invariance in $T_A^{\rho\mu}$.

(iii) From (i) and (ii) it follows that one must take care when using results valid with a particular $\mathscr{L}_{\mathrm{gf}}$ to simplify calculations done with other gauge fixing condition.

Two last comments are in order. Firstly in the calculation of $\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{H}\gamma)$ the box diagrams play another role besides solving the technical problem of the gauge invariance [1]. It should be clear from I that away from the Z resonance their contribution to σ is as important as the one stemming from $T_A^{\rho\mu}$. Secondly, let us stress that the W's contribution to $T_Z^{\rho\mu}$ is not finite. As we have explained in detail in I there is a counterterm. This is again seen in the calculation of T_Z which becomes finite because there is a $\bar{c}_Z c_A H$ counterterm.

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