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ELECTROWEAK RADIATIVE CORRECTIONS AT LEP ENERGIES

A. Barroso[1)], W. Beenakker[2)], F.A. Berends[2)], W. Blum[3)], F. Boudjema[4)], G.J.H. Burgers[5)], H. Burkhardt[6)], M. Caffo[7)], J.P. Cole[8)], N. Dombey[4)], F. Dydak[5)], J. Fleischer[9)], E. Gonzales[10)], W. Hollik[11)], S. Jadach[3)], F. Jegerlehner[9)], R. Kleiss[5)], L. Lanceri[12)], B.W. Lynn[13)], C. Mana[14)], M. Martinez[14)], E. Neugebauer[6)], D. Perret-Gallix[15)], R. Ragazzon[12)], P. Ratoff[16)], J.C. Romão[1], J. Salicio[10)], A. Shapira[17)], E. Simopoulou[18)], R.G. Stuart[3)], W.L. van Neerven[2)], C. Verzegnassi[12)], H. Wachsmuth[5)] and Z. Wąs[19)]

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1) CFMC/CFN, Lisbon University, Portugal.
2) Instituut Lorentz, Leiden University, The Netherlands.
3) Max Planck Institut für Physik und Astrophysik, Munich, Fed. Rep. Germany.
4) Physics Division, University of Sussex, Brighton, UK.
5) CERN, Geneva, Switzerland.
6) Fachbereich Physik, Siegen University, Fed. Rep. Germany.
7) Istituto di Fisica, Bologna University, Italy.
8) Lancing College, Lancing, UK.
9) Fakultät für Physik, Bielefeld University, Fed. Rep. Germany.
10) JEN Madrid, Spain.
11) II. Institut für Theoretische Physik, Hamburg University, Fed. Rep. Germany.
12) Istituto di Fisica, Trieste University, Italy.
13) SLAC, Stanford, Calif., USA.
14) DESY, Hamburg, Fed. Rep. Germany.
15) LAPP, Annecy-Le-Vieux, France.
16) Nuclear Physics Laboratory, Oxford University, UK.
17) Weizmann Institute, Rehovot, Israel.
18) Nuclear Research Centre Demokritos, Athens, Greece.
19) Institute of Nuclear Physics, Kraków, Poland.

PREFACE

In the frame of the LEP 200 Workshop, a study group was set up to look into the problem of electroweak radiative corrections. Since these corrections were dealt with at the earlier LEP Workshop [LEP 86] only as a side issue, and since their relevance extends to the entire energy range of LEP, the study group decided to have a global view of the matter, not confined to the high-energy regime alone.

The study group set itself three tasks: first, to review and discuss the physics reward of electroweak radiative corrections; second, to attempt to unify the language and concepts in the calculation of electroweak radiative corrections; third, to make progress in the practical applications of electroweak radiative corrections—in their availability, in their technical implementation in the analysis of e^+e^- data, and in the way that data should be presented.

This report summarizes the work of the study group. It also aims at familiarizing the non-expert reader with the field. Several contributions to the subject, which are of a more technical nature, are appended as annexes.

1. INTRODUCTION

1.1 Overview of electroweak radiative corrections

One of the reasons for the current belief that the electroweak interactions are described by a spontaneously broken gauge theory is the fact that such theories are renormalizable: for any observable quantity its theoretical prediction can be calculated to — in principle — an arbitrary order of perturbation theory, in terms of a finite set of input parameters. The predictions to 'first non-trivial order of perturbation theory', the 'Born approximation' predictions, can usually be calculated fairly simply. In contrast, the effects of the higher orders of the perturbation expansion, called 'radiative corrections' are usually complicated to calculate, and smaller in size. On the other hand, it is the existence and consistency of the radiative corrections which give the Standard Electroweak $SU(2)_L \otimes U(1)$ Model the character of a quantum field theory.

The physics motivation for the study of electroweak radiative corrections (hereafter called EWRC) is twofold:

i) High-precision measurements of electroweak observables can test the validity of EWRC as implied by the Standard Electroweak Model, and hence test the model at the quantum level. An experimental verification of EWRC would constitute an important milestone in the tests of $SU(2)_L \otimes U(1)$. In principle, although not comparable in precision, this programme is analogous to the measurement of, for example, the Lamb shift in the hydrogen atom, as a test of QED. Alternatively, while leaving the Standard Electroweak Model intact, a discrepancy between measured and calculated EWRC may signify the existence of heavy unknown particles (see Section 7).

ii) Possible 'new physics' in the sense of departures from the Standard Electroweak Model as a spontaneously broken gauge theory of the electroweak interaction of fundamental fermions (e.g. compositeness, technicolour), or in the sense of the appearance of new particles and/or symmetries, will probably manifest itself as small deviations from the predictions of the model. Hence the latter have to be known accurately, including EWRC.

1.2 Why worry about electroweak radiative corrections at LEP?

The reason why we should worry about EWRC at LEP energies is simple: EWRC can be large, of O(1). *Any* quantitative measurement at LEP energies, even at moderate precision, faces the problem of EWRC.

In e^+e^- physics, a sample of events of the type $e^+e^- \to ab$ invariably entails another sample of *bremsstrahlung* events of the type $e^+e^- \to ab\gamma$, $ab\gamma\gamma$, ... These radiative events constitute part of the EWRC. The trouble is their frequent occurrence, and that their relative amount strongly depends on the cuts applied to the data. The importance of EWRC is shown in Figs. 1a,b, which compare the ratio $R_{\mu\mu} = \sigma(e^+e^- \to \mu^+\mu^-)/\sigma_{\mathrm{point}}$, and the forward–backward asymmetry $A_{FB}(e^+e^- \to \mu^+\mu^-)$, as a function of $\sqrt{s}$, in Born approximation and with $O(\alpha)$ EWRC included. No experimental cuts are applied. The importance of taking EWRC into account for any measurement at LEP seems apparent, although this argument chiefly applies to photon *bremsstrahlung*.

Another demonstration of the need of EWRC can be found in the Z energy region: the large peak cross-section enables measurements with unparalleled precision: m_Z to ± 20 MeV [Altarelli 86], the polarized left–right asymmetry A_{LR} to ± 0.003 [Blockus 86]. It is a challenge to the computation of EWRC to match such splendid experimental precision.

1.3 A guided tour through the terminology

This subsection recalls the main vocabulary used in EWRC, with the aim of arriving at a more uniform language. This seems the more appropriate when looking at the somewhat confusing terminology which is used in the literature.

Physical observables are calculated in perturbation expansion. The lowest non-trivial order of perturbation expansion is called 'Born approximation', also 'tree level' or 'zero-loop level'. We prefer the term 'Born approximation'. The lowest non-trivial order of QED cross-sections is typically $O(\alpha^2)$ but not necessarily so.

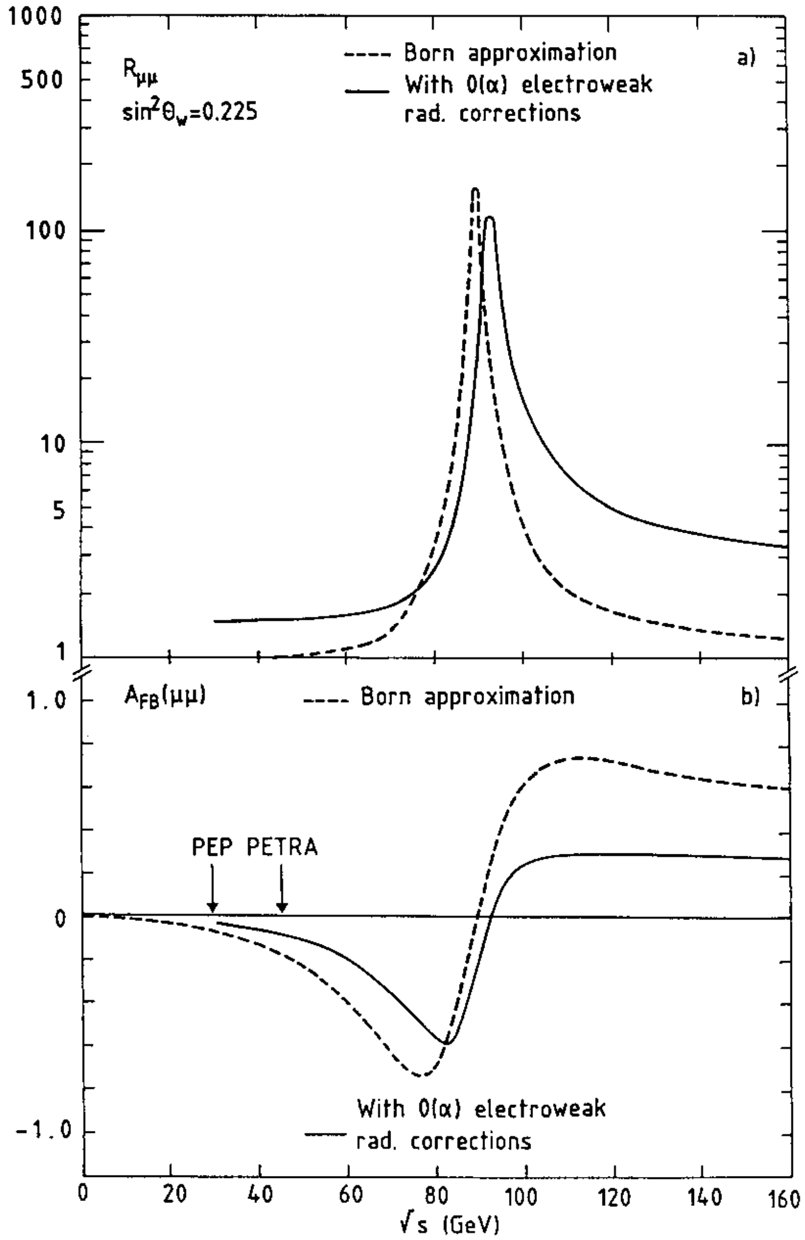

Fig. 1 The ratio $R_{\mu\mu} = \sigma(e^+e^- \to \mu^+\mu^-)/\sigma_{point}$, and the forward-backward asymmetry $A_{FB}(e^+e^- \to \mu^+\mu^-)$, as a function of $\sqrt{s}$, in Born approximation and with O(α) EWRC included. The input parameters are α, G_μ, $\sin^2\theta_w$ = 0.225, m_H = 100 GeV, and m_t = 35 GeV. No experimental cuts are applied.

If calculations include higher orders of the perturbation expansion they 'include EWRC'. Rather than the absolute order (because of the dependence of the meaning on the process under consideration) we prefer to give the *relative* order with respect to the lowest non-trivial order:

'O(α) corrected' $\equiv$ Born approximation + O(α) corrections;
'O(α^2) corrected' $\equiv$ Born approximation + O(α) corrections + O(α^2) corrections;
etc.

The orders are always counted in powers of α (in contrast to older papers on QED radiative corrections where the counting is done in powers of e). Virtual $O(\alpha)$ corrections are also called one-loop corrections; virtual $O(\alpha^2)$ corrections are also called two-loop corrections, and so on.

We now concentrate on $O(\alpha)$ EWRC. During the last few years it has become customary to divide them into two classes: the first group contains those diagrams which involve an extra photon which has been added to the Born diagrams, either in the form of a real bremsstrahlung photon, or as a virtual photon loop. They are called 'QED corrections' or else 'photonic corrections'. The other group contains all other diagrams. We call them 'non-QED corrections' or else 'weak corrections'. The subset of those diagrams which involve corrections to the self-energy of the gauge bosons, is frequently referred to as 'oblique corrections'. This division is also shown in Table 1, which lists in more detail the categories of radiative corrections within each class.

Table 1

Preferred classification of $O(\alpha)$ electroweak radiative corrections

<table>
<tr><td colspan="4">EWRC</td></tr>
<tr><td colspan="2">QED RC (Photonic RC)
[diagrams involving an extra real or virtual γ]</td><td colspan="2">Non-QED RC (Weak RC)
[all other diagrams]</td></tr>
<tr><td>Real γ
(Hard bremsstrahlung,
Soft bremsstrahlung)</td><td>Virtual γ
(Fermion self-energy,
Vertex corrections,
Box diagrams)</td><td>γ self-energy,
Z, W self-energy,
γ–Z mixing
[Oblique corrections]</td><td>Virtual Z, W,
etc.</td></tr>
</table>

There is a theoretical and an experimental reason for the above classification. Firstly, the QED radiative corrections form a gauge-invariant subset within the totality of EWRC. Secondly, the QED radiative corrections are dependent on the cuts applied to the data, whereas the non-QED radiative corrections are not.

While the classification shown in Table 1 describes the consensus achieved within our study group we are quite aware that not everybody will like it. The main point of disagreement concerns the photon self-energy which we classify as a 'non-QED correction', whereas it is traditionally classified as a 'QED correction'. For those who prefer to stick to the historical classification, and consider the photon self-energy as part of the QED correction, we suggest the term 'purely weak corrections' (comprising Z and W self-energy, γ–Z mixing, virtual Z and W, etc.) for all corrections other than QED.

The QED corrections (in our preferred sense) are considered well known and not very interesting since they contain no 'new physics'. On the other hand, they give the largest contribution to the total EWRC. Once a suitable renormalization scheme has been adopted (see Section 2) the effects of the non-QED corrections are typically much smaller than those of the QED corrections. This experience motivates a simpler and restricted treatment of EWRC, in which only QED corrections are applied (see Section 6).

For certain high-precision measurements, $O(\alpha)$ EWRC are considered inadequate, and $O(\alpha^2)$ EWRC have to be applied. The calculation of $O(\alpha^2)$ EWRC is a rather tedious enterprise because of the rapidly growing number of diagrams to be considered. For practical purposes, no complete calculation of all $O(\alpha^2)$ diagrams is made but an approximation as shown in Table 2 is done which is considered good enough for all practical purposes. The approach adopted in practice is that to $O(\alpha^2)$, only the QED corrections are explicitly calculated.

Table 2

Classification of $O(\alpha^2)$ electroweak radiative corrections

$O(\alpha^2)$ EWRC $\equiv$ $O(\alpha)$ EWRC $\otimes$ $O(\alpha)$ EWRC

$O(\alpha^2)$ QED RC *[applied]*	$O(\alpha^2)$ non-QED RC *[neglected]*

$O(\alpha)$ QED RC $\otimes$ $O(\alpha)$ non-QED RC
[in part implicitly included by a suitable choice of renormalization scheme]

2. THE 'BEST' RENORMALIZATION SCHEME

The calculation of EWRC involves the choice of a set of independent input parameters, and the choice of a renormalization scheme in order to deal, in a well-defined way, with various divergences (ultraviolet and infrared divergences, mass singularities). Although all renormalization schemes are in principle equivalent, the results in a given order of perturbation in different schemes will deviate from each other, because of higher-order contributions. From a practical point of view it is preferable to choose a scheme where the $O(\alpha)$ corrections are small, though this is no guarantee for small higher-order effects.

In QED, the favoured scheme is the 'on-shell (OS) scheme': the physical fermion (e, μ, ...) mass is defined as the pole position of the fermion propagator, and the only coupling constant $\alpha = e^2/4\pi$ is defined in the Thomson limit ($Q^2 = 0$) where the photon and the fermions are on mass shell.

In the $SU(2)_L \otimes U(1)$ Standard Model the situation is more complex. The Lagrangian has in its manifest $SU(2)_L \otimes U(1)$ symmetric form the input parameters g_2,g_1 [the SU(2) and U(1) gauge coupling constants], μ^2,λ (the second- and fourth-order coefficients in the Higgs potential), and g_f (the fermion–Higgs Yukawa coupling constants). Other than in QED, none of these parameters can be directly measured in present-day experiments. An alternative set of independent parameters,

$$\alpha,\ m_W,\ m_Z,\ m_H,\ m_f \tag{1}$$

has the advantage that each quantity can—in principle—be measured. The so-far unknown Higgs mass m_H is treated as a free parameter. The fermion mass parameter m_f is a shorthand notation for the fermion masses and the weak quark mixing angles.

The renormalization scheme which makes use of set (1) as parameters, and establishes their physical meaning by appropriate renormalization conditions, is called the 'on-shell (OS) scheme', in analogy to QED. It is the most direct and natural extension of the QED renormalization scheme. All parameters are defined as on-shell quantities: α in the Thomson limit, and the masses as pole positions of the corresponding propagators.

What is then the role of the popular electroweak mixing parameter $\sin^2\theta_w$? The simplest and most natural definition of this parameter is in terms of the physical W and Z masses:

$$\sin^2\theta_w = 1 - m_W^2/m_Z^2\,. \tag{2}$$

By definition, relation (2) is valid to all orders of perturbation theory. The parameter $\sin^2\theta_w$ is not an independent quantity and can, in principle, be avoided completely. However, for historical and practical reasons, $\sin^2\theta_w$ is retained as a bookkeeping device in the neutral-current phenomenology.

The advantages of the OS scheme are:

i) The input parameters have a clear physical meaning, and can be measured directly. Except m_H and the top quark mass m_t, all parameters are known.

ii) The Thomson cross-section formula from which α is obtained is exact to all orders of perturbation theory.

iii) The separation between the QED corrections and the non-QED corrections (which is important for the Monte Carlo implementation of EWRC, see Section 4) is automatic.

The OS scheme also has drawbacks:

i) The experimental precision of m_W is not as good (even assuming an error of ± 100 MeV as can be achieved at LEP 200) as to make the uncertainty in EWRC negligible.

ii) The fine-structure constant α is defined as a low-energy parameter, at $Q^2 = 0$, whereas m_W and m_Z are high-energy parameters. The renormalization of the Thomson α therefore induces large logarithmic corrections from the photon self-energy, typically $(\alpha/\pi)\log(m_W/m_f)$, in the EWRC at the W mass scale.

The first drawback can easily be overcome: employing the OS scheme one can calculate the muon decay width Γ_μ in terms of the parameter set (1), including $O(\alpha)$ EWRC. On the other hand, Γ_μ is related to the Fermi coupling constant G_μ by the following relation which includes — for historical reasons — $O(\alpha)$ QED corrections:

$$\Gamma_\mu = (G_\mu^2 m_\mu^5/192\pi^3)(1 - 8m_e^2/m_\mu^2)[1 + (25/4 - \pi^2)\,\alpha/2\pi]\,. \tag{3}$$

Utilizing the thus-determined coupling constant,

$$G_\mu = (1.16637 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2}\,,$$

one obtains from $\Gamma_\mu = \Gamma_\mu(\alpha, m_W, m_Z, m_H, m_f)$ the relation

$$G_\mu = (\pi\alpha/\sqrt{2})[m_W^2(1 - m_W^2/m_Z^2)(1 - \Delta r)]^{-1}\,, \tag{4}$$

where $\Delta r = \Delta r(\alpha, m_W, m_Z, m_H, m_f)$ is the $O(\alpha)$ non-QED correction of muon decay in the OS scheme. Relation (4) allows replacement of m_W by the precisely measured G_μ.

We denote the Fermi coupling constant by G_μ rather than G_F, in order to underline the origin of its numerical value from the muon-decay width. There are minor ambiguities in the literature about the extraction of G_μ from the muon lifetime, owing to neglect of higher-order terms in the QED correction. The differences are, however, numerically unimportant at the present level of precision.

The parameter set α, G_μ, m_Z, m_H, m_f then comprises the best measured quantities, and only m_H and the quark masses give rise to a theoretical uncertainty in EWRC.

The second drawback of the OS scheme seems to be more controversial. On the one hand, the large logarithmic corrections which are due to the definition of α at $Q^2 = 0$, can be 'absorbed' by replacing the Thomson α by a running QED coupling constant, evaluated at the W mass scale:

$$\alpha(m_W^2) = \alpha(0)/[1 - \Pi^\gamma_{QED}(m_W^2)]\,,$$

where $\Pi^\gamma_{QED}(Q^2)$ denotes the fermion-loop corrections to the photon self-energy. On the other hand, this introduces an uncertainty in the parameter $\alpha(m_W^2)$ from the quark loops in the photon self-energy, mostly from the unknown m_t. The contribution from light quarks is also uncertain because of the unknown quark masses and QCD radiative corrections. Using a dispersion relation, it is customarily related to available data on $e^+e^- \to$ hadrons at low energies. The experimental error of these data may well be the ultimate limit of the precision with which EWRC can be calculated.

Another problem occurs in QED corrections where large corrections arise from the soft real and virtual photon emission. Here the Thomson α is more adequate than $\alpha(m_W^2)$. Also in Bhabha scattering at small angles the Thomson α is more appropriate.

We favour the following procedure as a viable way out of this dilemma:

i) Calculate within the OS scheme the $O(\alpha)$ corrections to the gauge boson propagators,

$$\begin{aligned} \gamma &: 1/q^2 \to 1/[q^2 + \Sigma^\gamma(q^2)] \\ W &: 1/[q^2 - m_W^2 + i m_W \Gamma_W] \to 1/[q^2 - m_W^2 + \Sigma^W(q^2)] \\ Z &: 1/[q^2 - m_Z^2 + i m_Z \Gamma_Z] \to 1/[q^2 - m_Z^2 + \Sigma^Z(q^2)] , \end{aligned} \tag{5}$$

with the renormalized self-energies $\Sigma(q^2)$ in terms of α, m_W, m_Z, m_H, and m_f.

ii) Replace in the Born approximation expressions each boson propagator by the expressions (5).

iii) Use the relation (4) to replace m_W by α, G_μ, and m_Z.

iv) Add the remaining non-QED corrections.

v) Add the QED corrections.

This procedure leaves the non-QED corrections small while retaining the Thomson α and G_μ as input parameters.

The effect of employing the 'wrong' renormalization scheme can be seen in Fig. 1a, where α, G_μ, $\sin^2\theta_w$, m_H, and m_f have been used as input parameters: notice the large displacement of the Z peak due to $O(\alpha)$ non-QED corrections.

3. QED RADIATIVE CORRECTIONS

As already mentioned above, the QED radiative corrections form a gauge-invariant subset of all EWRC. They are considered not very interesting but are at LEP energies large in magnitude, and hence must receive a lot of attention.

In calculating QED radiative corrections one usually restricts oneself to the $O(\alpha)$ correction. When the result of this calculation is not too large (say, 10% of the Born value), the $O(\alpha^2)$ corrections are in general considered negligible. It is known though that stringent cuts applied to the data, which leave little room for the emission of bremsstrahlung photons, imply large and negative corrections. In such a situation one has to calculate the $O(\alpha^2)$ corrections. In the particular case of bremsstrahlung emission, there is another way of incorporating higher-order corrections, namely by exponentiating a certain part of the $O(\alpha)$ QED correction, a technique pioneered by Yennie, Frautschi and Suura [Yennie 61].

The ultraviolet divergences in EWRC, which arise from the high-momentum domain in loop integrals, are removed by the renormalization procedure. The virtual QED and the non-QED corrections then modify the Born cross-section (which we take as an example of a physical observable) as follows: $d\sigma/d\Omega = (d\sigma^{Born}/d\Omega)(1 + \delta_{QED}^{virt} + \delta_{non\text{-}QED}^{virt})$. Both corrections are ultraviolet-finite. However, δ_{QED}^{virt} still contains an infrared divergence which has its origin in the low-energy region. The Bloch–Nordsieck theorem [Bloch 37], generalized by Kinoshita [Kinoshita 62] and Lee and Nauenberg [Lee 64], ensures that the infrared divergences from the emission of real and virtual photons cancel each other.

In practice, the bremsstrahlung emission of photons is divided into two classes: 'soft' bremsstrahlung ($E_\gamma < k_1$) and 'hard' bremsstrahlung ($E_\gamma > k_1$), where k_1 is an arbitrary cut-off energy chosen smaller than E_γ^{min}, the detection threshold of the energy of photons in the experimental apparatus employed. The soft bremsstrahlung correction, together with the virtual photon correction, leads to a finite correction of the cross-section. The hard bremsstrahlung leads to events with additional photons in the final state. In the following, we concentrate on the bremsstrahlung part of the QED radiative correction. For the discussion, we refer to the specific case of muon-pair creation: $e^+e^- \rightarrow \mu^+\mu^-$.

In the Standard Electroweak Model the bremsstrahlung matrix element is described by eight Feynman graphs, obtained by attaching photons to each $e^{\pm}$ or $\mu^{\pm}$ line in the γ and Z exchange diagrams. The differential cross-section is a complicated expression. For two specific kinematical situations, however, the expressions become much simpler: in the 'soft photon limit' ($E_\gamma \ll E_e$), where the single bremsstrahlung cross-section becomes proportional to the Born cross-section, and for hard photon emission in the 'ultrarelativistic limit' ($E_e \gg m_e, m_\mu$). Calculational techniques have been given in the literature [Berends 82] for these cases, and compact formulae exist for all standard reactions.

Other than in the case of soft photon emission where the photon is emitted nearly isotropically, the analytical integration of the differential cross-section for hard photon emission is only sometimes possible. When all kinds of cuts are applied to the data the integration boundaries become too complicated for an analytical calculation of the total cross-section.

4. THE IMPLEMENTATION OF ELECTROWEAK RADIATIVE CORRECTIONS

There are two approaches to obtaining the hard bremsstrahlung correction. The first is to perform a numerical integration over the phase space allowed for by the experimental cuts applied to the data, by means of a standard multidimensional integration routine [Berends 73]. Care must be taken that strong peaks in the differential cross-section are adequately dealt with. The other approach is the use of a Monte Carlo event generator [Berends 81, 81a, 82a, 83]: a program generates a set of four-momenta of the final-state particles, including the real photon(s), in such a way as to reproduce the differential cross-section for hard bremsstrahlung. The events are either generated in the full phase space, or in a large part of the phase space which is restricted by one or two 'natural' variables only, such as a minimum scattering angle, or a maximum photon energy. Besides generating a large number of events, the program should also yield the bremsstrahlung cross-section for the phase space covered by the events. Since the events are not weighted the application of cuts on the event sample is simple: the events not satisfying the selection criteria are rejected.

In practice, one not only considers an event generator for photons with $E_\gamma > k_1$ but one also generates a set of four-momenta of the final-state particles without a bremsstrahlung photon when $E_\gamma < k_1$. The generation is done according to the analytical expressions which result after the $O(\alpha)$ non-QED, virtual QED, and soft bremsstrahlung ($E_\gamma < k_1$) corrections have been applied. The event generator is normalized such that the ratio of soft- to hard-photon events is given by the theoretically-known respective cross-sections.

Although an event generator allows a great flexibility in the implementation of EWRC, it has one disadvantage with respect to a numerical integration program. In the latter there is more freedom to vary the cut-off energy k_1. The larger k_1 becomes, the less good is the soft-photon approximation of the bremsstrahlung cross-section. On the other hand, the smaller k_1 becomes, the larger becomes the sum of the $O(\alpha)$ virtual QED and soft-photon corrections. The latter sum is a (potentially) large negative correction, which is compensated by the positive correction due to hard bremsstrahlung, so as to yield an overall QED correction of reasonable size. It may even happen that if the cut-off energy k_1 becomes too small, the $O(\alpha)$ QED corrected cross-section becomes negative in some part of the phase space. In order to generate events, however, a positive cross-section is mandatory. If this problem occurs and no suitable k_1 can be found, one has to go beyond the $O(\alpha)$ approximation in the Monte Carlo generator.

Another problem related to the introduction of k_1 can arise when certain distributions are generated where soft bremsstrahlung events play a dominant role. For example, consider the study of $d\sigma/d\zeta$ in $e^+e^- \rightarrow \mu^+\mu^-$, where ζ is the acollinearity angle of the muon pair. For very small ζ (i.e. nearly back-to-back muons) soft bremsstrahlung events become predominant, and the $O(\alpha)$ approximation may not apply any longer. Since the sum of the $O(\alpha)$

virtual- and soft-photon corrections is negative, the number of events in the interval $[0, \zeta]$ is underestimated. This means that in the angular interval $[\zeta, \pi]$ the number of events is overestimated, since the total number of events in $[0, \pi]$ is adequate. In the case of a particular interest in the bin $[0, \zeta]$, higher-order corrections should be taken into account in the event generator, for example by extending the whole procedure to $O(\alpha^2)$, including the generation of double bremsstrahlung events.

As an alternative to calculating the QED corrections to $O(\alpha^2)$, techniques exist to re-sum the leading logarithmic QED corrections to all orders of perturbation. It was noticed by Greco and Rossi [Greco 67] that the so-called 'coherent state formalism' not only reproduces the exponentiated form of the soft bremsstrahlung photon correction [Yennie 61], but also yields a correction for hard collinear photons emitted from any external leg of a diagram, in exponentiated form [Caffo 85]. The latter feature is important because the emission of hard collinear photons is steeply peaked under zero angle, which is difficult to deal with in a Monte Carlo generator with adequate precision.

5. HOW TO PRESENT THE DATA?

Once the theoretical calculation of EWRC is available, incorporating the special experimental conditions, theoretical expectations can be compared with experimental measurements. If several experiments perform the same measurement, there will be a strong interest not only in comparing the physics conclusions but also the data from which they have been obtained. This will be the case, in particular, when experimental results do not agree: is the source of the disagreement in the data, or in the radiative corrections, or in the analysis of the data? So one wants to see the data themselves published as has always been the case. The only question is: before or after radiative corrections?

Before we turn to the situation at LEP energies, we recall the situation at PETRA energies. The experimenters chose to publish data after the subtraction of $O(\alpha)$ QED corrections. That is the event sample was the one which one would find if the photon radiation had been switched off. The theory could be compared with the thus obtained 'data' at the level of the Born approximation. Non-QED radiative corrections were too small to be considered if the OS scheme was employed. The argument in favour of this procedure was that at the level of the Born approximation the dependence of the data on the selection criteria was eliminated, and hence data from different detectors could be directly compared. Moreover, the 'interesting' physics could be studied without disturbance from the 'known' physics of the QED radiative corrections.

While this is a valid point of view it is not easy to see how this procedure could be continued at LEP energies because there the QED corrections can be of $O(1)$! Are 'data' after 100% corrections still 'data'? Secondly, the radiative corrections applied by one experiment may for some reason be different from those applied by another experiment although 'they used the same program'. Needless to say, the problem of the correctness of the radiative corrections is much aggravated by their magnitude.

We list in Table 3 possible ways of presenting data together with their pros and cons. The first column suggests the presentation of 'raw' data, i.e. of data corrected for apparatus acceptance, bad channels, and insensitive regions, but before any radiative correction. Although the data are model independent, they are strongly dependent on the experimental cuts via the QED radiative effects, and hence a comparison cannot be made between one experiment and another. We discard the option to present raw data.

The last column suggests the presentation of data after the totality of the EWRC has been applied, i.e. in the form to be compared with the Born approximation prediction. Whilst this procedure is transparent and hence attractive, the fact remains that the applied radiative corrections are large and, chiefly via the non-QED corrections, model dependent. We feel that data should not be published with EWRC applied which are calculated in today's accepted framework of the Standard Electroweak Model, with everyone's preferred values for m_H and m_t. We also discard this option.

The third column, QED removed, is in essence the extrapolation of the PETRA procedure to LEP energies. A slight model dependence would arise from QED corrections to Z exchange diagrams which rely on the Standard

Table 3

Options of presenting data at LEP energies

	'Raw' data	Canonical cuts applied[a)]	QED removed	Born approximation
Pros	Model independent	Model independent Analytical formulae for EWRC possible?	Insensitive to non-QED corrections, hence largely model independent	Clear, simple, independent of exp. cuts
Cons	Dependent on exp. cuts	Difficult since consensus required	QED corrections are large, slight model dependence	Large, model-dependent corrections

a) Recommended procedure

Electroweak Model. Yet it would be a viable solution, in principle, if the QED corrections were not so large. Still, if we adopted this procedure for LEP, could we accept even larger corrections at future e^+e^- colliders in the TeV energy region? Since we feel that published 'data' should be close to what has actually been measured, we also discard this option.

The consensus within the study group was a preference for the use of 'canonical cuts'. Data should be presented (see, however, the clarification below!) before EWRC but after an agreed set of canonical cuts have been applied. Canonical cuts are thought to be cuts which satisfy the following criteria:

i) the phase space within the cuts is well within the acceptance of all relevant detectors;
ii) the cuts have a clear and unambiguous physical meaning;
iii) the cuts should be chosen such as not to cause $O(\alpha)$ QED corrections to become too large, so that the $O(\alpha^2)$ corrections may be assumed to be small and hence negligible.

The comparison between various experiments will be quite simple. Also the comparison between theory and experiment may be facilitated: if a set of canonical cuts exists it becomes worthwhile to construct a numerical integration program specialized to these cuts.

Our recommendation of the use of canonical cuts does not, of course, restrict anybody's freedom to publish whatever is deemed appropriate, with any cuts applied or with any method of analysis preferred. We only urge the experimental teams to publish *in addition* their data with canonical cuts applied. This implies no restriction whatsoever to employing all the strong features of each detector in the main analysis.

Returning once again to the specific case of muon-pair production it seems that good candidates for a set of canonical cuts are the acollinearity angle ζ, the muon momentum p_μ, and the scattering angle θ with respect to the beam line. Thus all events where the muons have a scattering angle $\theta > \theta^{cut}$, momentum $p_\mu > p_\mu^{cut}$, and acollinearity angle $\zeta < \zeta^{cut}$, fall within the canonical region.

6. APPROXIMATIONS AND 'RULES OF THUMB'

Within the complete set of EWRC, the numerically most important contributions arise from photon bremsstrahlung from the initial electron and positron legs (assuming that a convenient renormalization scheme has been chosen such as to minimize the non-QED corrections). Hence the obvious approximation to calculating the entire set of corrections is to restrict oneself to the initial-state bremsstrahlung correction only. It depends on the experimental precision of the particular experiment whether or not more elaborate radiative corrections are needed.

The photon radiation from the initial legs gives rise to a large logarithmic correction of the size $(\alpha/\pi)t = 0.056$, with $t = \ln m_Z^2/m_e^2 = 24.2$. Other terms in the perturbative series are of the size $(\alpha/\pi)^n t^m$. Since $t \gg 1$, the effective expansion parameter $(\alpha/\pi)t$ is also large, and the influence of orders beyond $O(\alpha)$ are to be considered in experiments which measure quantities which vary strongly with $\sqrt{s}$. A typical experiment of this type is the measurement of the line-shape of the Z resonance.

The 'leading log approximation' just retains the leading logarithmic terms, often summed over all orders of perturbation. The summation techniques are either based on the exponentiation of the $O(\alpha)$ QED correction (excluding the photon self-energy part, according to our definition given above), or on QCD inspired approaches employing the Altarelli–Parisi evolution equations for the initial lepton states [Greco 67; Tsai 83; Kuraev 85; Altarelli 86; Nicrosini 86].

In the following, we give a few 'rules of thumb' on the approximate use of EWRC. Their justification is *a posteriori*, i.e. from comparison with the result of exact calculations. Their numerical accuracy is at the level of 20%.

Rule of Thumb No. 1

Non-QED radiative corrections will be largely absorbed by the use of $\alpha(m_W^2)$, G_μ, m_Z (or, equivalently: G_μ, m_W, m_Z) in the Born approximation expressions. Remaining non-QED corrections are of the order of 1%.

Rule of Thumb No. 2

For QED radiative corrections, use the Thomson α in the QED formulae. If the $O(\alpha)$ QED correction changes a cross-section by a fraction x ($x \lesssim 0.3$) then the $O(\alpha^2)$ QED correction will cause a further change by a fraction $\simeq x^2/2$.

Rule of Thumb No. 3

The peak cross-section at the Z pole gets reduced by QED corrections by a factor $f = (\Gamma_Z/m_Z)^\beta \simeq 0.7$, where $\beta = (2\alpha/\pi)(\ln m_Z^2/m_e^2 - 1) \simeq 0.1$. The position of the maximum of the cross-section is shifted to $\sqrt{s} = m_Z + \Delta E$, where $\Delta E = \pi\beta\Gamma_Z/8 \simeq 120$ MeV.

Rule of Thumb No. 4

An observable $\mathcal{A}$ changing linearly with $\sqrt{s}$ around the Z pole will change by initial-state QED corrections from $\mathcal{A}(m_Z)$ to $\mathcal{A}(m_Z - 2\Delta E)$, where $\Delta E \simeq 120$ MeV (see rule of thumb No. 3).

7. THE PHYSICS REWARD OF ELECTROWEAK RADIATIVE CORRECTIONS

The EWRC constitute a considerable obstacle between data taking and physics analysis at LEP energies. They also merit attention in their own right because of their physics content. This latter feature has been highlighted already at the LEP I Physics Workshop, in particular in the contribution of Lynn, Peskin and Stuart [Lynn 86]. Because of the importance of the subject, but also because of its relation to a precise measurement of the W mass, accessible only at LEP 200, we recall here the main arguments.

The W and Z bosons couple to all particles which take part in the weak interaction. There is a fair chance that expected but hitherto unobserved particles (top quark, Higgs boson) will be produced directly at LEP. However, if these or other novel particles are too heavy to be directly produced, they might still be 'observable' through their contributions to the EWRC. Actually the heavier the respective particles, or the larger the mass splitting within isospin multiplets, the larger their effects. Hence the experimental measurement of EWRC provides a window on 'new physics' in the mass range above 100 GeV, which is otherwise not accessible at LEP and is complementary to the searches for the direct production of new particles.

In order to profit from the physics reward of EWRC there is a price to pay: not the large QED radiative corrections but the small non-QED corrections are the interesting part, with effects of relative size $\alpha/\pi \simeq 0.002$.

This means first that precision experiments are called for, and second that the logarithmically enhanced QED radiative corrections of size $(\alpha/\pi)\ln m_Z^2/m_e^2 \simeq 0.056$ have to be handled with care. In the following it will be argued that the measurement of the left-right asymmetry A_{LR}, at the Z pole, and the measurement of m_W have a considerable potential for insight into 'new physics'.

In a renormalization scheme with α, G_μ, m_Z, m_H, and m_f as the input parameters, other precisely measurable quantities such as A_{LR} and m_W can be calculated. As is shown elsewhere in these Proceedings [Roudeau 87], m_W can be measured at LEP 200 to ± 100 MeV. The quantity $A_{LR} = (\sigma_L - \sigma_R)/(\sigma_L + \sigma_R)$ is the asymmetry of the annihilation cross-sections of left- and right-handed electrons colliding with unpolarized positrons, and hence is accessible with longitudinally polarized beams only. The feature which singles out A_{LR}, as compared to other measurable asymmetries, is that it is independent of the fermion type in the final state, at the Z pole:

$$A_{LR}^{Born}(m_Z) = 2v_e a_e/(v_e^2 + a_e^2) .$$

Hence A_{LR} can be measured in an inclusive mode, rendering the statistical precision superior to any other asymmetry measurement. Experimentally, an overall precision of $\Delta A_{LR} \simeq \pm 0.003$ seems within reach [Blockus 86]. QED radiative corrections — which are almost negligible for A_{LR} [Böhm 82] because of its very mild dependence on $\sqrt{s}$ — are ignored in the following.

In terms of the precisely measured input parameters α, G_F, and m_Z — the Z mass is assumed to be ultimately measured to ± 20 MeV [Altarelli 86] — the predictions for A_{LR} and m_W depend on m_t and m_H when $O(\alpha)$ non-QED radiative corrections are included, in the framework of the Electroweak Standard Model. The dependence of the predictions on m_t and m_H within the presently favoured domain are shown in Fig. 2, together with the expected

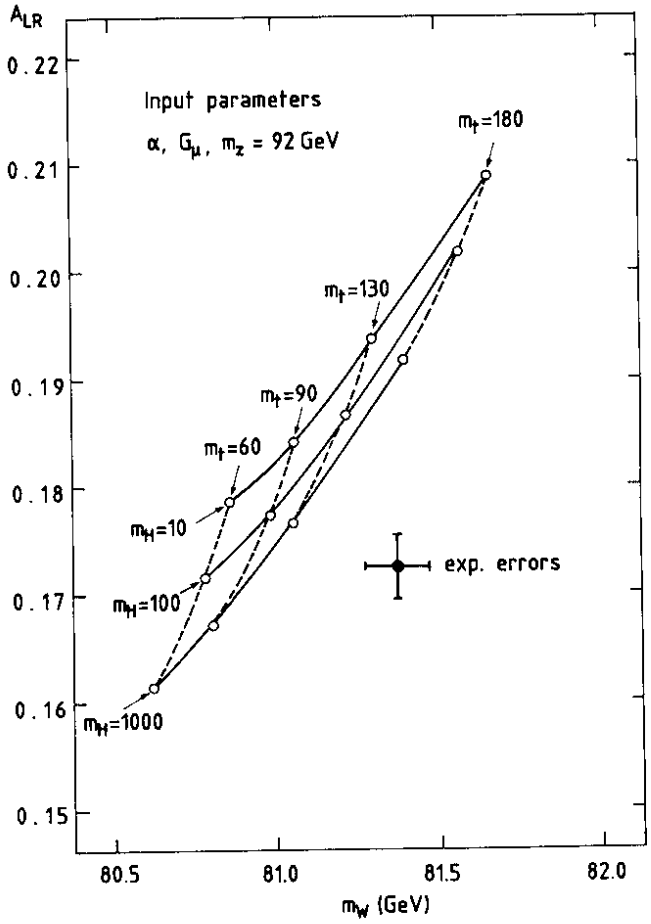

Fig. 2 Variation of the predictions for the left-right asymmetry A_{LR}, and for m_W, as a function of m_t and m_H, in comparison with the expected experimental precision of A_{LR} and m_W.

experimental error. One concludes that each A_{LR} and m_W restricts the allowed range of m_t and m_H considerably, and even more so if both quantities are measured. Optimistically, m_W 'measures' m_t to $\pm$ 20 GeV, and A_{LR} narrows the allowed range of m_H to ± 200 GeV. All this is made possible by experimental errors which are expected to be small as compared to the O(α) non-QED radiative corrections caused by the top quark and the Higgs boson.

Similarly, the effects of new heavy quarks and leptons, or of supersymmetric particles, would show up in the radiative corrections of A_{LR} and m_W [Lynn 86]. This discovery potential, however, can be fully exploited only if polarized beams at $\sqrt{s} = m_Z$ become available at LEP.

8. INVENTORY OF EXISTING CALCULATIONS

The study group saw as one of its main tasks a review of the likely measurements at LEP together with the expected precision, and the preparation of Monte Carlo event generators for the various processes including EWRC with adequate accuracy. Table 4 gives an overview of the availability of EWRC, as of October 1986.

For the process $e^+e^- \to \bar{f}f$, where f is a light fermion other than an electron, many authors (including Berends, Böhm, Greco, Hollik, Jadach, Kleiss, Lynn, Pancheri, Srivastava, and Stuart) have contributed to the complete EWRC at O(α). The most versatile Monte Carlo generator which is available at present is BREMMUS. Beyond O(α), analytic expressions exist for the distortion of the Z renonance shape due to initial-state QED radiation (in leading log approximation at O(α^2) by Kuraev and Fadin [Kuraev 85], and Altarelli and Martinelli [Altarelli 86]; in leading log approximation to all orders by exponentiating the infrared divergent O(α) correction, by Greco [Greco 86]; complete O(α^2) by Berends, Burgers and van Neerven, see Annex A, and by Nicrosini and Trentadue [Nicrosini 86]).

For Bhabha scattering, $e^+e^- \to e^+e^-$, analytic formulae exist which are complete to O(α) [Böhm 86], and have large logarithmic contributions resummed to all orders [Greco 86a, and references quoted therein]. The formulae, however, do not include hard bremsstrahlung, so their use is limited to essentially collinear e^-e^+ pairs. A Monte Carlo generator, comprising the complete EWRC at O(α), is available (Berends, Hollik and Kleiss). At present, the option of initial-state longitudinal polarization is being implemented in this Monte Carlo generator.

The process $e^+e^- \to \tau^+\tau^-$ is of interest because it offers an easy way to measure the final-state polarization asymmetry A_{pol}. A Monte Carlo generator is prepared by Jadach, Stuart and Wąs, which will include the complete O(α) EWRC, and all major τ decay modes [Jadach 84; see also Annex B]. The attainable experimental precision is such that ultimately O(α^2) QED corrections may have to be included.

Electroweak radiative corrections to heavy fermion production, $e^+e^- \to \bar{F}F$, are being calculated by Beenakker and Hollik (see Annex C). Analytical formulae for the one-loop corrections and soft bremsstrahlung exist. A Monte Carlo generator, TIPTOP, which simulates O(α) initial-state bremsstrahlung effects in heavy fermion production, has recently been made available by Jadach and Kühn [Jadach 86].

The O(α) EWRC for W production, $e^-e^+ \to W^+W^-$, is being worked upon by Gaemers and Kunszt, making use of earlier work by Lemoine and Veltman [Lemoine 80], and Phillipe [Phillipe 82]. At present, only O(α) hard bremsstrahlung corrections are available in the form of a Monte Carlo generator.

Higgs particle creation via $e^-e^+ \to H^0\mu^+\mu^-$ has been studied by Fleischer and Jegerlehner. A Monte Carlo generator including complete O(α) EWRC is in preparation. Figure 3 shows the cross-section for Higgs-boson production (m_H = 50 GeV) before and after O(α) EWRC, where a cut-off $E_\gamma < 0.25\ E_e$ for bremsstrahlung photons has been applied. A Monte Carlo generator including O(α) initial-state QED radiation has been used by Berends and Kleiss [Berends 85].

Higgs particle creation via $e^-e^+ \to H^0\gamma$ has been studied by Barroso, Pulido and Romão, including one-loop non-QED corrections [Barroso 86].

Radiative Z production, $e^+e^- \to Z\gamma$, has attracted a lot of interest. This process was originally suggested as a means of 'counting' the number of neutrino families [Ma 78, Barbiellini 81], via the cross-section for single-photon production in $e^+e^- \to Z\gamma$, with $Z \to \nu\bar{\nu}$. The question of O(α) QED corrections of the cross-section has been

Table 4

Status of EWRC of various processes[a)]

Process	$O(\alpha)$ EWRC	$O(\alpha^2)$ EWRC	Comments
$e^+e^- \to \bar{f}f$ (f = light fermion other than e; initial-state e can be longitudinally polarized)	Done[b)]	At work	$O(\alpha)$: non-QED part needs to be checked. $O(\alpha^2)$: QED part only; analytic formulae for the Z resonance shape; a MC generator including $O(\alpha^2)$ QED should exist by the end of 1987.
$e^+e^- \to e^+e^-$	Done	Needed (?)	Also existing are analytic formulae in leading log approximation.
$e^+e^- \to e^+e^-$ (Initial-state e can be longitudinally polarized)	At work	Needed (?)	
$e^+e^- \to \tau^+\tau^-$ (Initial-state e can be longitudinally polarized)	At work	Needed	$O(\alpha)$: QED part complete; non-QED part at work; complete $O(\alpha)$ should exist by the end of 1986. One-prong τ decays included. $O(\alpha^2)$ QED corrections from $e^+e^- \to \bar{f}f$ applicable.
$e^+e^- \to \bar{F}F$ (F = heavy fermion other than τ)	At work	Not needed	One-loop corrections and soft bremsstrahlung available in analytical form; hard bremsstrahlung missing. A MC generator with initial-state bremsstrahlung is available.
$e^+e^- \to W^+W^-$	At work	Not needed	Hard bremsstrahlung done; one-loop corrections and soft bremsstrahlung in analytical form; radiative corrections for decays for on-shell W's only.
$e^+e^- \to H^0\mu^+\mu^-$	At work	Not needed	Hard bremsstrahlung done for the initial state; one-loop corrections and soft bremsstrahlung done; a MC generator should exist by the end of 1986
$e^+e^- \to H^0\gamma$	At work	Not needed	One-loop non-QED corrections exist in analytical form.
$e^+e^- \to Z\gamma$	At work	Needed (?)	One-loop electroweak corrections as well as soft bremsstrahlung exist in analytical form; an estimate of hard bremsstrahlung is available in the form of a MC generator.
$e^+e^- \to e^+e^-\gamma$	At work	Not needed (?)	

a) The initial-state electrons and positrons are unpolarized unless stated otherwise.

b) 'Done' means 'available in the form of a Monte Carlo generator'.

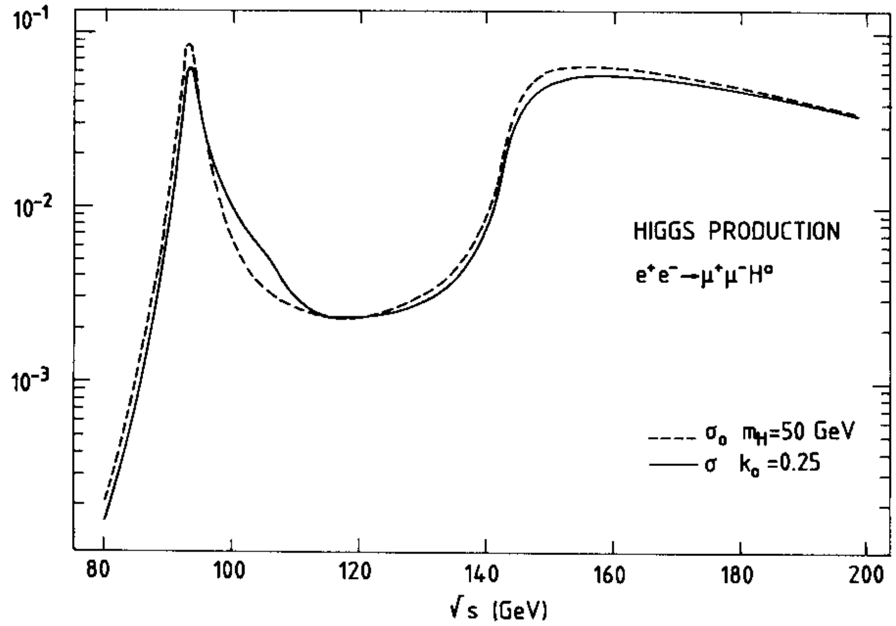

Fig. 3 Cross-section of $\sigma(e^+e^- \to H^0\mu^+\mu^-)$ as a function of $\sqrt{s}$. The dotted line shows the Born approximation; the full line includes O(α) EWRC, with a cut-off $E_\gamma < 0.25\ E_e$ for bremsstrahlung photons. The Higgs boson mass is $m_H = 50$ GeV.

addressed by Berends, Burgers and van Neerven [Berends 86], Boudjema, Dombey and Cole [Boudjema 86], Igarashi and Nakazawa [Igarashi 86], Mana, Martinez and Cornet [Mana 86], and Bento, Romão and Barroso [Bento 86, Romão 87]. An estimate of O(α) hard bremsstrahlung corrections to the (dominant) Z exchange graphs of $e^-e^+ \to \nu\bar{\nu}\gamma$ is shown in Fig. 4: the cross-section for single-photon production is reduced by a significant amount,

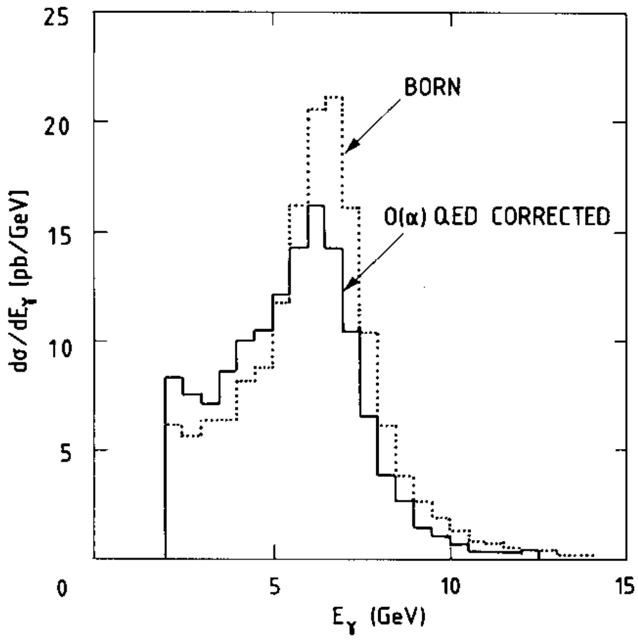

Fig. 4 Photon energy spectrum from $e^+e^- \to \nu\bar{\nu}\gamma$, in Born approximation (dotted line) and after O(α) hard bremsstrahlung corrections.

on the scale of the 30% variation from three to four neutrino generations. The one-loop QED and soft bremsstrahlung corrections exist in analytical form only. Also, the O(α) non-QED corrections as published by Böhm and Sack [Böhm 86a], and discussed by Boudjema, Cole and Dombey (see Annex D), exist in analytical form only. The inclusion of O(α) EWRC to $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ in a Monte Carlo generator is an urgent task. Also, in view of the large O(α) QED correction, O(α^2) QED corrections may have to be considered.

The dominant background of the neutrino counting process is radiative Bhabha scattering, where both final-state electrons remain undetected: $e^+e^- \rightarrow (e^-e^+)\gamma$. A Monte Carlo generator for the process $e^+e^- \rightarrow e^-e^+\gamma$ including O(α) EWRC is under preparation by Kleiss. Calculations of the cross-section of radiative Bhabha scattering at the Born approximation level have recently been performed by Caffo, Gatto and Remiddi (see Annex E), Mana and Martinez [Mana 86a], and Karlen [Karlen 86]. Mana and Martinez, and Karlen, have developed a Monte Carlo generator which is particularly designed to handle radiative Bhabha scattering at very small electron scattering angle. At the Born approximation level, there is good numerical agreement between the three calculations. However, Karlen points out that the O(α) QED corrections to radiative Bhabha scattering are sizeable, and have to be taken into account in a quantitative analysis of the neutrino counting experiment. This claim is not supported by Mana and Martinez, and obviously deserves clarification. Notice that radiative Bhabha scattering might also be of interest for a luminosity measurement.

Finally, there is good news from a sector which has particular relevance to the energy range of LEP 200. Lynn, Kennedy and Verzegnassi (see Annex F) have studied the shift in the W mass due to a very heavy top quark, at O(α^2). This question was triggered by the large influence of m_t on the W mass at O(α), see Section 7. For m_t = 200 GeV, the calculation yields an O(α^2) correction of 18 MeV only, so that O(α^2) corrections from m_t can be safely neglected, compared to the expected experimental precision of $\pm$ 100 MeV.

9. CONCLUSION AND RECOMMENDATIONS

Electroweak radiative corrections, in particular their QED part, are large at LEP energies. Their application by means of well-tested Monte Carlo generators is inescapable for any quantitative analysis.

Electroweak radiative corrections not only allow significant tests of the Standard Electroweak $SU(2)_L \otimes U(1)$ Model beyond the Born Approximation level, but also offer an opportunity to spot particles which are too heavy to be observed directly, through their virtual effect in loop diagrams.

Although a great deal of work has been done already to calculate O(α) [and O(α^2) where needed] electroweak radiative corrections, and to make them available in the form of Monte Carlo generators, a lot remains to be done. This includes cross-checking of the results of different authors.

In order to make the comparison of the results of calculations easier, the use of the on-shell renormalization scheme is recommended, with α, G_μ, m_Z, m_H, and m_f as input parameters.

Experimental teams should feel committed to also present their data with canonical cuts applied. We urge a Workshop of experts to be held, in order to define such canonical cuts.

Finally, we urge the creation of a library of 'standard' Monte Carlo generators for events which have electroweak radiative corrections included.

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ANNEX A

ON SECOND ORDER QED CORRECTIONS TO THE Z^0 RESONANCE SHAPE

F.A. Berends, G.J.H. Burgers and W.L. van Neerven

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Since the $O(\alpha)$ correction to the Z resonance shape is sizeable, higher order corrections have to be considered. This had been done in the Physics at LEP report by Altarelli and Martinelli[1] and by Greco[2]. The former authors present results for the first order corrections plus second order leading logarithmic correction whereas the latter takes higher order effects into account by exponentiating the infrared singular part of the first order correction.

Here we present an exact $O(\alpha^2)$ calculation of the initial state radiative corrections. Results are given for the total cross section of the reaction

$$e^+e^- \to \gamma,\ Z \to \mu^+\mu^- \ . \tag{1}$$

Once an event generator is available experimental cuts can be considered in the calculation.

Since we want to be able to make eventually an event generator for $O(\alpha^2)$ corrections we have to improve on the ingredients of the $O(\alpha)$ event generators. More specifically we need the following information:

1. The total cross section for (1) is needed with $O(\alpha^2)$ virtual corrections and including single or double soft photon emission:

$$\sigma = \sigma^0 \left[1+\delta_1(\varepsilon) + \delta_2(\varepsilon)\right] \tag{2}$$

where

$$\sigma^0(s) = \frac{4\pi\alpha^2}{3s}\left[1+\frac{2(s-M^2)sc_V^{\ 2}}{|z(s)|^2}+\frac{s^2(c_V^{\ 2}+c_A^{\ 2})^2}{|z(s)|^2}\right] \tag{3}$$

$$z(s) = s-M^2+iM\Gamma \tag{4}$$

$$\delta_1(\varepsilon) = \beta\ \ell n\ \varepsilon + \delta_1^{\ V} \tag{5}$$

$$\beta = \frac{2\alpha}{\pi}\left(\ell n \frac{s}{m^2} -1\right) \tag{6}$$

$$\delta_1^{\ V} = \frac{\alpha}{\pi}\left(\frac{3}{2}\ \ell n \frac{s}{m^2} + \frac{\pi^2}{3} -2\right) \tag{7}$$

The invariant mass of the μ pair defines ε:

$$s' = m_{\mu\mu}^{\ \ 2} = s(1-\varepsilon) \tag{8}$$

For single soft photon emission $\varepsilon = k_1/E$, where k_1 is the maximum soft photon energy. For double soft photon emission the maximum total photon energy is εE, neglecting ε^2 terms.

The $O(\alpha^2)$ correction is

$$\delta_2(\varepsilon) = \frac{1}{2}\beta^2 \ln^2\varepsilon + \beta\delta_1^{\,v} \ln \varepsilon + \delta_2^{\,v} \tag{9}$$

where (the results originate from ref. 3)

$$\begin{aligned}\delta_2^{\,v} = \frac{\alpha}{\pi}^2 &\{-\frac{1}{18}\ln^3 \frac{s}{m^2} + [\frac{119}{72} - 2\zeta(2)]\ln^2 \frac{s}{m^2} \\ &+ [-\frac{2275}{432} + \frac{37}{6}\zeta(2) + 3\zeta(3)]\ln \frac{s}{m^2} - \frac{6}{5}[\zeta(2)]^2 \\ &- \frac{9}{2}\zeta(3) - 6\zeta(2)\ln 2 - \frac{79}{24}\zeta(2) + \frac{1279}{108}\}\end{aligned} \tag{10}$$

with $\zeta(2) = \pi^2/_6$

$\zeta(3) = 1.20205$

For small ε it has been shown that the corrections can be summed into the form:

$$\sigma = \sigma^0(1+\delta_1^{\,v} + \delta_2^{\,v} + \dots)\varepsilon^\beta \tag{11}$$

Here we know $\delta_1^{\,v}$ and $\delta_2^{\,v}$, but not the higher order terms. We assumed ε to be so small that the energy loss does not effect $\sigma^0(s)$. When the energy loss matters other formulae can be used. Since we will also consider hard bremsstrahlung, we can take ε as small as we want.

2. The second ingredient is $\frac{d\sigma}{ds'}$ in some form.
   In lowest order it is just another form of the single bremsstrahlung spectrum

$$\frac{d\sigma}{dk} = \beta\,\frac{1+(1-k)^2}{2k}\,\sigma_0(s^1) = \beta\,\frac{1+z^2}{2v}\,\sigma_0(s') \tag{12}$$

where kE is the photon energy.

Considering virtual corrections to single bremsstrahlung and adding the effect of the emission of a second photon, we get another distribution $\frac{d\sigma}{dv}$, where $\frac{s'}{s} = 1-v = z$, which is an order in α higher than eq. (12):

$$\frac{d\sigma}{dv} = \{\frac{1+z^2}{v}\left[\frac{1}{2}\beta^2 \ln v + \frac{1}{2}\beta\delta_1{}^v + \left(\frac{\alpha}{\pi}\right)^2 A\right]$$

$$+ (1+z)\left(\frac{\alpha}{\pi}\right)^2 B + z\left(\frac{\alpha}{\pi}\right)^2 C\}\sigma_0(s') \tag{13}$$

The full distribution $d\sigma/dv$ (lowest order plus first order correction) is the sum of eqs. (12) and (13). In eq. 13 we introduced the quantities:

$$A = -\ln^2\frac{s}{m^2}\ln z + \ln\frac{s}{m^2}\left[Li_2(v) - \frac{1}{2}\ln^2 z\right.$$

$$\left. + \ln z \ln v + 7/_2 \ln z\right] + \frac{1}{2}\ln^2 z \ln v$$

$$+ \frac{1}{2}Li_2(v)\ln z - \frac{1}{6}\ln^3 z + \zeta(2)\ln z - \frac{3}{2}Li_2(v)$$

$$- \frac{3}{2}\ln z \ln v - \frac{17}{6}\ln z + \frac{1}{6}\ln^2 z - \frac{1}{v}\ln^2 z$$

$$- \frac{1}{3} - \frac{2}{3v}\ln z - \frac{1}{3v^2}\ln^2 z \tag{14}$$

$$B = \ln^2\frac{s}{m^2}\left(\frac{1}{2}\ln z - 1\right) + \ln\frac{s}{m^2}\left[\frac{1}{4}\ln^2 z - \ln z + \frac{7}{2}\right]$$

$$+ \frac{3}{2}Li_3(v) - 2\, S_{12}(v) - \ln v\, Li_2(v) - \zeta(2)$$

$$+ \frac{1}{6}\ln^2 z + \frac{1}{2}\ln z \ln v - \frac{1}{4}\ln^2 v + \frac{5}{2}\ln z$$

$$+ \frac{3}{2}\ln v - \frac{1}{6}] \tag{15}$$

$$C = 2\ln^2\frac{s}{m^2} + \ln\frac{s}{m^2}\left(\ln z - \frac{13}{2}\right) + \frac{16}{3}\zeta(2)$$

$$- \frac{25}{6}Li_2(v) - \frac{13}{12}\ln^2 z - 4\ln z \ln v$$

$$+ \frac{3}{2}\ln^2 v - \frac{5}{6}\ln z - \frac{15}{6}\ln v - \frac{2}{3} \tag{16}$$

These formulae are based amongst others on results of ref. 4. When v is small, or $z \approx 1$ the $\frac{1}{v}$ term dominates and in the coefficient of 1/v A can be negleted. This result is the same as differentiating eq. (11) with respect to ε and expanding the result up to $O(\alpha^2)$. The $O(\alpha)$ term is the same as eq. (12), the $O(\alpha^2)$ as eq. (13) in the $z \to 1$ limit.

We see however from eq. (13) that for larger v values the distribution deviates from the one implied by soft photon emission alone c.f. eq. (11).

By exponentiating the infrared parts we may write for $d\sigma/dv$

$$
\begin{aligned}
\frac{d\sigma}{dv} = \beta\, v^{\beta}\, \frac{1+z^2}{2v}\, [1+\delta_1{}^v + \delta_2{}^v + \ldots]\sigma_0(s') \\
+ \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{1+z^2}{v} A + (1+z)B+zC\right]\sigma_0(s') \\
+ \left(\frac{\alpha}{\pi}\right)^3 [\ldots] + \ldots \qquad (17)
\end{aligned}
$$

By expanding eq. (17) in α the $O(\alpha)$ term equals eq. (12) and the $O(\alpha^2)$ equals eq. (13).

We now have the ingredients to calculate the radiative corrections to $\sigma^0(s)$ for some energy $s = 4E^2$.

Up to $O(\alpha^2)$ the corrected cross section is given by

$$
\sigma = \sigma^0 \left[1+\delta_1(\varepsilon) + \delta_2(\varepsilon)\right] + \int_{\varepsilon}^{1-\frac{4\mu^2}{s}} \frac{d\sigma}{dv}\, dv \ , \qquad (18)
$$

where $\frac{d\sigma}{dv}$ is given by the sum of eqs. (12) and (13). The result of (18) is the cross section including an exact evaluation of $O(\alpha)$ and $O(\alpha^2)$ initial state corrections, due to virtual and real photons. It should be noted at this point that in principle there is another $O(\alpha^2)$ radiative correction namely due to the emission of an e^+e^- pair instead of a $\gamma\gamma$ pair. We have omitted this contribution for the moment since it is small with respect to the double bremsstrahlung correction.

We evaluate (18) numerically by subtracting in the integrand $d\tilde{\sigma}/dv$, a distribution which behaves like $d\sigma/dv$ for small v and which can be easily integrated analytically.

In fact we evaluate (18) for a number of cases:

a. Only $O(\alpha)$ i.e. $\delta_2(\varepsilon) = 0$ and $\frac{d\sigma}{dv}$ given by (12).

b. $O(\alpha)$ and $O(\alpha^2)$ i.e. δ_1 and δ_2 , $\frac{d\sigma}{dv}$ given by the sum of eqs. (12) and (13).

c. An exponentiated form of $O(\alpha)$ i.e. the first term of eq. (18) is replaced by eq. (11) with $\delta_2{}^v = 0$ and $\frac{d\sigma}{dv}$ is given by (17) with $\delta_2{}^v = 0$ and the explicit $\left(\frac{\alpha}{\pi}\right)^2$ term omitted. In this case one could directly take $\varepsilon = 0$ because (17) is integrable.

d. An exponentiated form of $O(\alpha^2)$ i.e. the first term of (18) is replaced by (11) and $d\sigma/dv$ is replaced by (17).

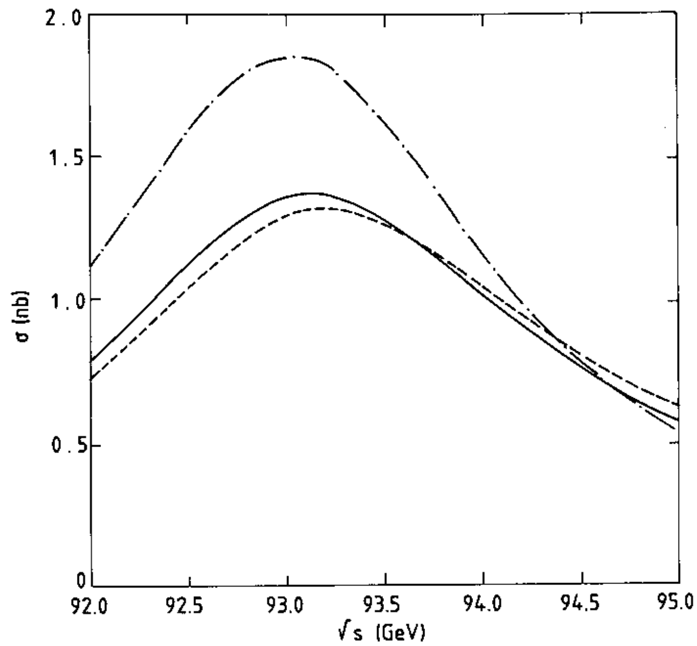

Fig. 1: The total cross section for $e^+e^- \to \mu^+\mu^-$ as a function of the c.m.s. energy. The dash-dotted, dashed, and solid lines represent the Born approximation, the $O(\alpha)$ corrected, and up to $O(\alpha^2)$ corrected calculations.

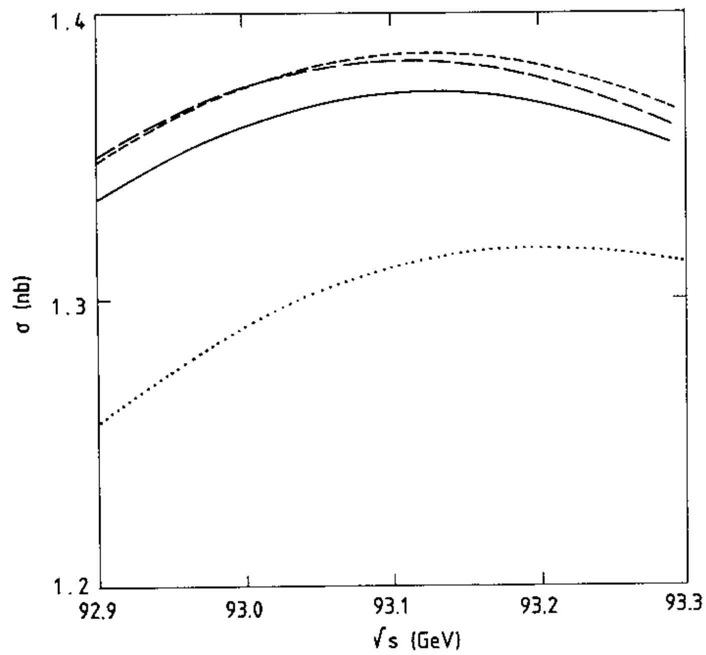

Fig. 2: The total cross section for $e^+e^- \to \mu^+\mu^-$ around the top. The dotted line represents the $O(\alpha)$ corrected cross-section. The curve with large dashes represents the up to $O(\alpha^2)$ corrected cross section. The other dashed line is the $O(\alpha)$ exponentiated form (case c) whereas the solid line represents case d, the $O(\alpha^2)$ exponentiated expression.

The numerical results are shown for the resonance region in Figs. 1 and 2. Also results are given for an energy above resonance. Here $d\sigma/dv$ in lowest order is compared with a first order corrected $d\sigma/dv$ (Fig. 3).

In the region of the resonance the small v region is most important in the integral, since $\sigma(s')$ falls off sharply with decreasing energy. So in this case the most important terms in (13) are the β^2 and β terms. Since the first order exponentiated expression, listed under case c, takes these terms into account it is not surprising that the results do not differ very much quantitatively.

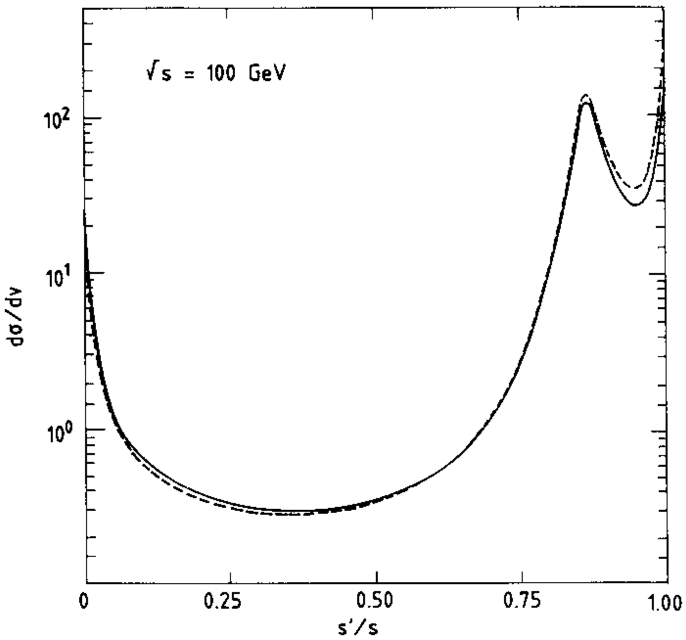

Fig. 3: The energy loss spectrum $d\sigma/dv$ in Born approximation (dashed line) and with $O(\alpha)$ corrections (solid line), at c.m.s. energy 100 GeV.

In fact a comparison between cases b, c and d gives an indication of the theoretical accuracy of the radiative corrections.

We find that for a peak value in nb. of 1.38 the uncertainty due to radiative corrections is 0.01 nb. For the position of the peak we find similarly an uncertainty of 15MeV. The peak values for various assumptions are (in nb):

lowest order : 1.86
$+ O(\alpha)$: 1.32
$+ O(\alpha^2)$: 1.38

The position is in these three cases respectively (in GeV): 93.020, 93.204, 93.116. All calculations were performed with
$M_z = 93$ GeV, $\Gamma_z = 2.5$ GeV and $\sin^2\theta = 0.223$.

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ANNEX B
SYSTEMATIC UNCERTAINTIES IN THE MEASUREMENT OF $A_{pol}(m_Z)$ WITH τ PAIRS

S. Jadach and Z. Wąs

The strategy of precision tests of the Standard Electroweak Model is to predict theoretically the value of some precisely measurable quantities — including electroweak radiative corrections — using for example α, G_μ, and m_Z as input parameters, and to test their consistency with the measured values. Besides the left-right asymmetry A_{LR}, which offers the best sensitivity but requires polarized beams, and the forward-backward asymmetry A_{FB}, the τ spin polarization asymmetry A_{pol} in the reaction $e^+e^- \rightarrow \tau^+\tau^-$ is of particular interest [1]. It offers the same sensitivity to 'new physics' as A_{LR}, and requires no beam polarization. Hence the measurement of A_{pol} is of great importance although the precision of its measurement cannot match that of A_{LR}. All the quantities discussed are measured on top of the Z resonance. This note discusses briefly the systematic uncertainties of the measurement of $A_{pol}(m_Z)$, in particular $O(\alpha)$ QED radiative corrections. The systematic uncertainties may be divided into three categories: i) QED bremsstrahlung, ii) imprecise knowledge of the physics of τ decays, and iii) contamination due to background from unwanted τ decay modes or other processes.

It was shown [2] that the value of $\langle P_\tau \rangle$, in the case of unpolarized beams, is not directly affected by QED bremsstrahlung. In fact $\langle P_\tau \rangle$ is changed by less than 0.005, very little in comparison with its value $\langle P_\tau \rangle \simeq 0.16$ and very little in terms of $\sin^2\theta_w$. One could expect a larger effect due to initial-state bremsstrahlung and the related reduction of $\sqrt{s}$ if $\langle P_\tau \rangle$ was strongly dependent on $\sqrt{s}$. This is, however, not the case here. To some extent, however, this happens for polarized beams, see Ref. [2]. Let us note that this mechanism, owing to the strong dependence of A_{FB} on $\sqrt{s}$, shifts A_{FB} by -3.5%*), a very large amount as compared to $A_{FB} \sim 2\%$, and in terms of $\sin^2\theta_w$. The experimentally measured value of $\langle P_\tau \rangle$ is influenced by QED bremsstrahlung indirectly, and in a different way from A_{FB}. The $\langle P_\tau \rangle$ is measured ($\langle P_\tau \rangle_{meas}$) using the slope of the pion energy distribution in the $\tau \rightarrow \pi\nu$ decay channel; owing to the loss in the c.m.s. energy $\sqrt{s}$ (both the initial- and the final-state bremsstrahlung contribute), the π momentum distribution gets distorted (more steep) leading to $\langle P_\tau \rangle_{meas}$ different from $\langle P_\tau \rangle$ by about -0.03. This effect, when compared with $\langle P_\tau \rangle = 0.16$ ($\sin^2\theta_w = 0.23$), and in terms of $\sin^2\theta_w$, is much less dramatic than the shift of A_{FB}.

In conclusion, we would like to stress that in order to obtain $\sin^2\theta_w$ with comparable precision from $\langle P_\tau \rangle$ and A_{FB}, the QED bremsstrahlung effect must be known in the case of A_{FB} with much higher precision than for $\langle P_\tau \rangle$. So far, no study has been done on the dependence of the QED bremsstrahlung effects on $\langle P_\tau \rangle$ on kinematical cut-offs, but it seems that they should diminish with stronger cut-offs, as opposed to the situation in the case of A_{FB}. As for radiative QED corrections to the $\tau \rightarrow \pi\nu$ decay, to our knowledge there is nothing in the literature, but generally these effects are expected to be of the order of $\alpha \ln(m_\tau/m_\pi)$ and therefore very small. The strongest QED bremsstrahlung effect may show up in $\tau \rightarrow e\nu\bar{\nu}$ decay. Here many analytical results from $\mu \rightarrow e\nu\bar{\nu}$ may be used, but nothing in the form of a Monte Carlo event generator exists as yet.

Figure 1 shows a comparison of the π momentum spectrum in $\tau \rightarrow \pi\nu$ decay, in Born approximation and after $O(\alpha)$ EWRC are applied. The cut-offs used in the latter case are rather strong, yielding closely similar values for P_τ if analysed in Born approximation, or after $O(\alpha)$ EWRC.

The main uncertainty due to the unknown aspects of the τ decay mechanism is related to the fact that the $V-A$ nature of τ decay is not confirmed by experiment precisely enough. Let us note that if τ decay is used as a spin polarimeter then one obtains as a result not really $\langle P_\tau \rangle$ but rather $\varrho\langle P_\tau \rangle$ where $\varrho = -2g_Vg_A/(g_V^2 + g_A^2)$, $g_{V,A}$ being charged-current couplings in τ decay. If $\langle P_\tau \rangle$ is to be measured model independently then the value of ϱ must be

*) This is countered by another contribution from the interference of initial- and final-state bremsstrahlung, which changes A_{FB} typically by $+1.5\%$ to $+3\%$, depending on cut-offs.

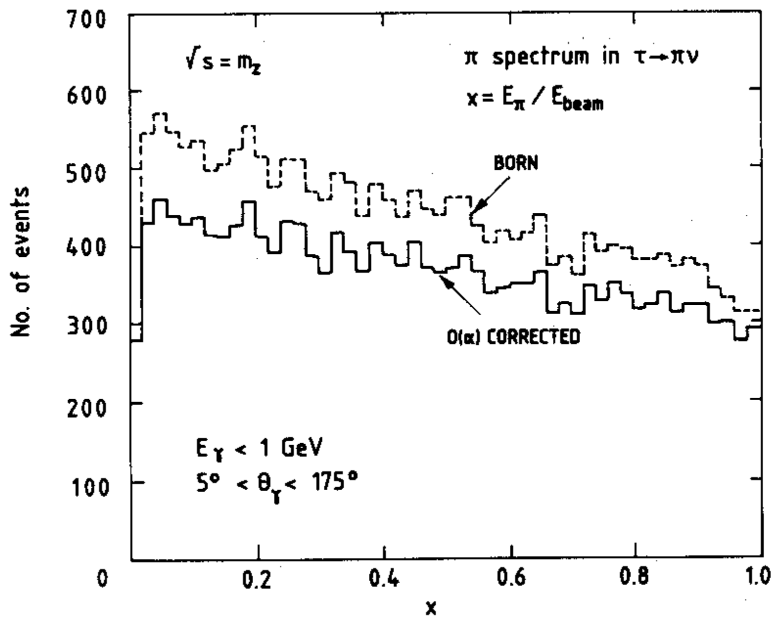

Fig. 1 Normalised π momentum spectrum in $\tau \to \pi\nu$ decay, in Born approximation, and after O(α) EWRC are applied.

known from independent sources. Most probably the experimental error on ϱ will still be large by the time of the LEP experiments, and we will have to assume $\varrho = 1$.

The background problems depend on the quality of the detector and we are not entering into these questions. Here we limit ourselves to some rather general and detector-independent aspects of the background problems. First, it should be noted that, in comparison with PETRA/PEP experiments, backgrounds from other processes, namely from Bhabha and $\gamma\gamma$ physics, are fortunately much less important. On the other hand, the problem of the contamination of one τ-decay channel by another will look similar. The common assumption that it is not worth including the decays $\tau \to e\nu\bar{\nu}$, $\mu\nu\bar{\nu}$, $\varrho\nu$ as spin analysers, because the reduction of the statistical error on $\langle P_\tau \rangle$ is less than a factor of 2, might have to be reconsidered. The inclusion of these other τ decay modes in the $\langle P_\tau \rangle$ measurement may significantly reduce not only the statistical error but also the overall systematic error [3].

For the analysis of τ pair events, a Monte Carlo generator is required which includes all the features discussed above. A good step in this direction has already been made [4], and some results presented here were based on the results from this program. However, it is still not complete and requires inclusion of some missing O(α) non-QED corrections, of some multipion decay modes of τ, and a more refined QED bremsstrahlung. A study of the impact of changes of the Higgs mass, etc., on $\langle P_\tau \rangle$ is in progress.

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ANNEX C
ELECTROWEAK ONE-LOOP CORRECTIONS TO HEAVY-FERMION PAIR PRODUCTION

W. Beenakker and W. Hollik

The one-loop corrections to $e^+e^- \rightarrow F\bar{F}$ for charged heavy fermions F can be classified in the following way:

i) QED corrections, consisting of initial- and final-state bremsstrahlung, together with the virtual photon corrections: vertex corrections and box diagrams with internal photon lines. It is sometimes convenient to include the fermionic part of the photon vacuum polarization in this subclass. These corrections have been calculated in Refs. [2–4] and, partly, in Ref. [1].

ii) Non-QED corrections (sometimes also referred to as 'purely weak corrections'), consisting of the Z boson self energy, the γ–Z mixing energy, vertex corrections not from virtual photon exchange, and massive ZZ and WW box diagrams [5–10].

For a completion of the fermionic sector in the Electroweak Standard Model the top quark has still to be discovered experimentally. In the case of a fourth generation also a new sequential heavy lepton may be around in the energy range covered by LEP II. For the experimental investigation of cross-sections, forward–backward asymmetries, polarization asymmetries, etc., and for comparison with the model predictions, it will become necessary to extend the radiative-correction calculations to the case of a heavy fermion pair in $e^+e^- \rightarrow F\bar{F}$.

Finite mass effects give drastic reductions of cross-sections and asymmetries if one is near to threshold. But also the calculation of radiative corrections becomes more cumbersome for the following reasons:

i) The evaluation of the vertex and box diagrams with heavy fermions yields lengthy expressions which are less transparent than in the light fermion case, where one has only vector and axial-vector form factors in a compact and handy form (see, for example, Ref. [11]).

ii) The coupling of Higgs bosons to heavy fermions is no longer negligible. Therefore one has to respect the Higgs contributions in the vertex corrections and fermion self energies. For a renormalizable gauge also with unphysical Higgs bosons $\phi^\pm$, χ, the additional vertex and fermion self-energy diagrams are depicted in Figs. 1

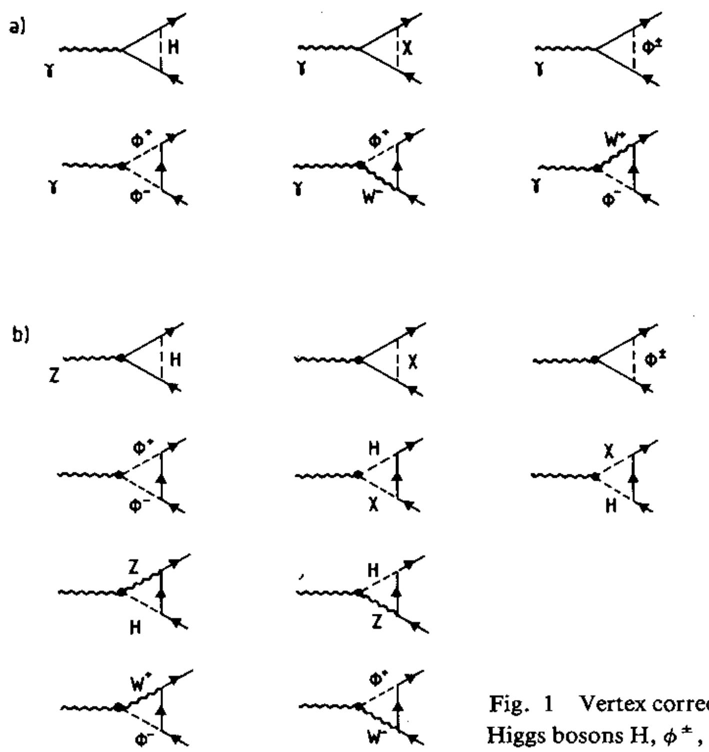

Fig. 1 Vertex corrections from the Higgs bosons H, $\phi^\pm$, and χ.

Fig. 2 Fermion self-energy diagrams from the Higgs bosons H, $\phi^{\pm}$, and χ.

and 2. In box diagrams the exchange of virtual Higgs bosons can again be neglected since they have to couple also to the electron line leading to a suppression factor m_e/m_W. The renormalization is performed in the on-shell scheme, as described in detail in Ref. [11] for the light fermion case, now extended to the heavy fermion one [12].

The renormalization conditions are the on-shell subtractions of the self energies, the vanishing of the γ-Z mixing for on-shell photons, and the definition of the electric charge in the Thomson limit. The input quantities are therefore the physical masses

$$m_W,\ m_Z,\ m_H,\ m_f$$

and the fine structure constant α in the Thomson limit.

In Figs. 3-5 we display the case of $t\bar{t}$ productions with either $m_t = 40$ GeV or $m_t = 50$ GeV in terms of the integrated cross-section σ and the forward-backward asymmetry

$$A_{FB} = (1/\sigma)\{\int_{\cos\theta>0} d\sigma - \int_{\cos\theta<0} d\sigma\}, \qquad \theta = \sphericalangle(e^-,F).$$

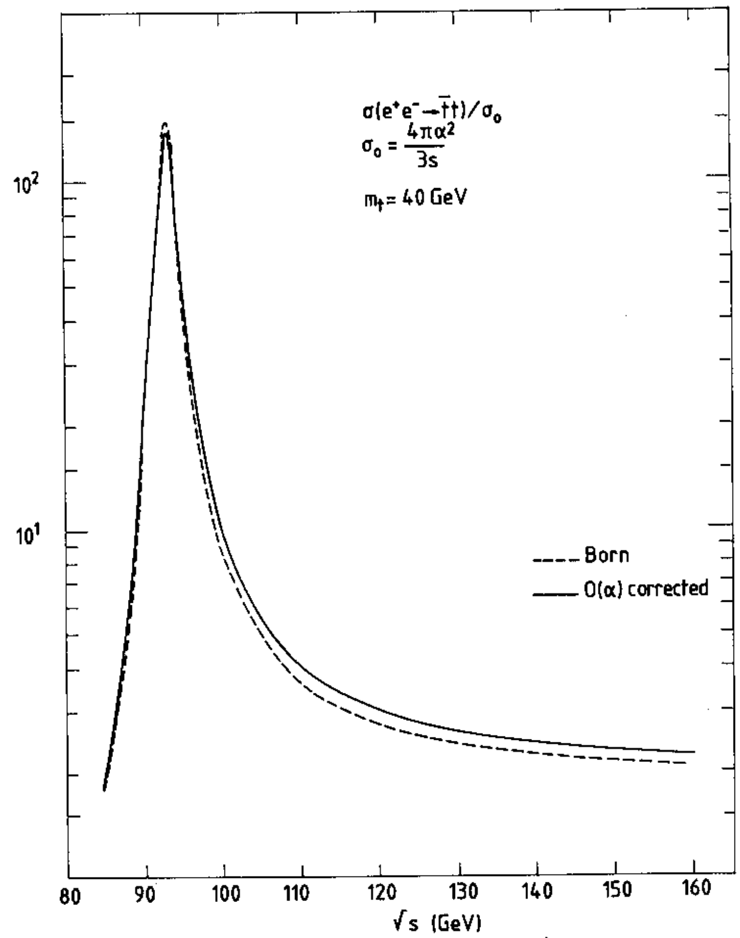

Fig. 3 Cross-section ratio $\sigma(e^+e^- \to \bar{t}t)\sigma_{point}$, as a function of $\sqrt{s}$, in Born approximation and after $O(\alpha)$ corrections are applied [$m_t = 40$ GeV).

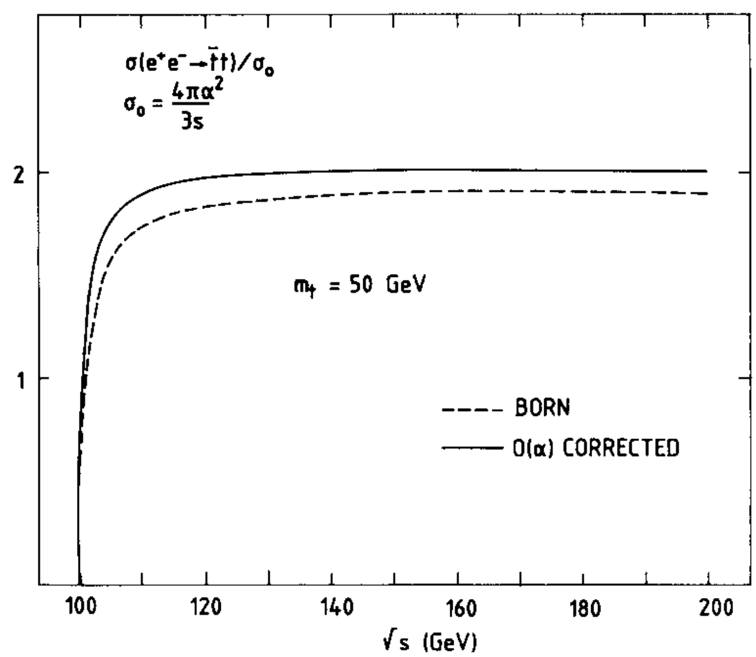

Fig. 4 Cross-section ratio $\sigma(e^+e^- \to \bar{t}t)/\sigma_{point}$, as a function of $\sqrt{s}$, in Born approximation and after O(α) corrections are applied [m_t = 50 GeV).

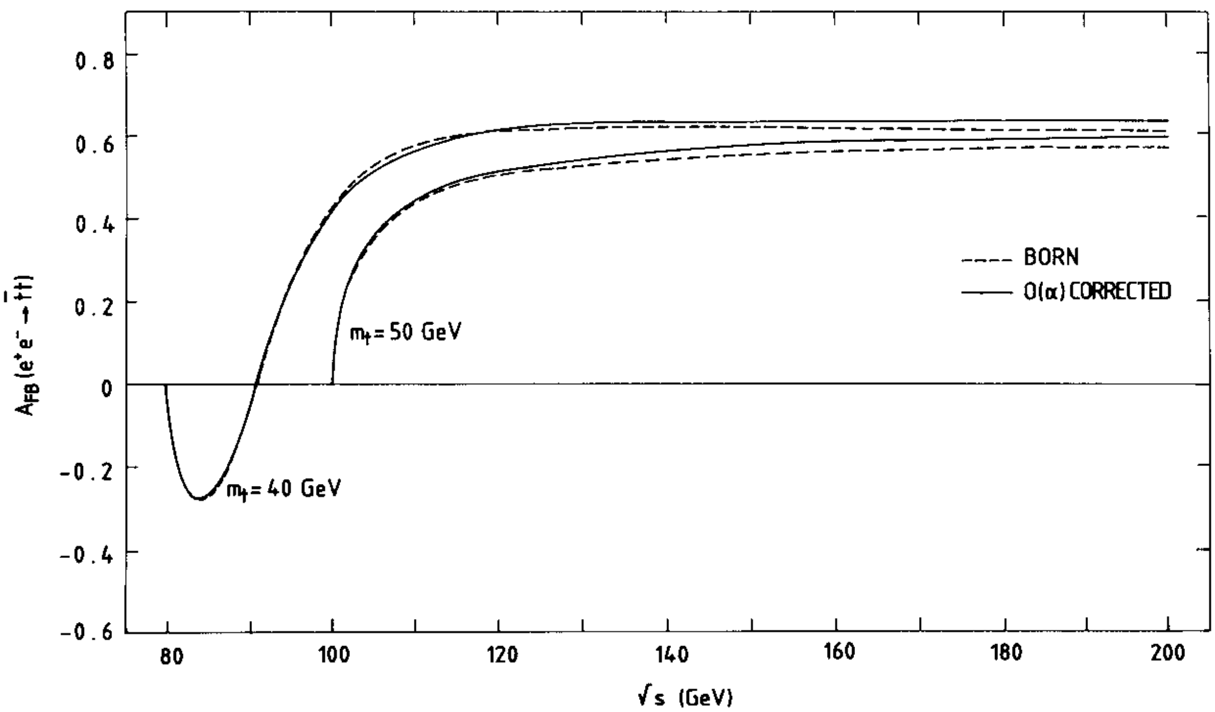

Fig. 5 Forward–backward asymmetry of $e^+e^- \to \bar{t}t$, as a function of $\sqrt{s}$, in Born approximation and after O(α) are corrections applied, for m_t = 40 GeV and m_t = 50 GeV.

As in the case of a light fermion pair the weak corrections are dominated by the Z boson self energy, where the largest part could be absorbed by defining a running $\alpha(s)$

$$\alpha(s) = \alpha(0)/[1 - \Pi^{\gamma}_{QED}(s)]$$

with the QED photon vacuum polarization Π^{γ}_{QED}. The residual corrections are of the order of 1–2% depending slightly on the Higgs mass. They can best be exhibited on top of the Z peak (if $m_f < m_Z/2$), where the large Z boson self energy vanishes owing to the on-shell renormalization condition.

Large corrections arise from the QED subclass, in particular from initial-state bremsstrahlung together with the virtual photonic vertex correction. The dominant initial-state radiation is the same as in the case of light fermion production. Since the QED corrections depend on the experimental cuts they have not been included in Figs. 3–6.

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ANNEX D

RADIATIVE Z PRODUCTION AT LEP I

F. Boudjema, J. Cole and N. Dombey

1. BORN APPROXIMATION

At tree level the $e^+e^- \to Z\gamma$ cross section is given by

$$\frac{d\sigma_B}{dx} = \frac{\pi\alpha^2 F(s_\theta)}{s^2(s-M_z^2)}\left[2\frac{s^2+M_z^4}{1-v^2x^2} - (s-M_z^2)^2\right]$$

where $x = \cos\theta$, $s_\theta = \sin^2\theta_W$, $F(s_\theta) = \dfrac{2s_\theta^2 - s_\theta + \frac{1}{4}}{s_\theta(1-s_\theta)}$

and $v = \surd(1 - 4m^2/s)$, neglecting the electron mass in the denominator.

2. RENORMALISATION SCHEME

We outline the renormalisation scheme (RS) we have used in the calculation of the radiative corrections (RC) to the process $e^+e^- \to Z\gamma$.

In this paper we use the RS based on Ref.[1]. The merits of this scheme are

(i) it is an on-shell, gauge invariant scheme,

(ii) all Green's functions calculated in this scheme are finite, enabling their values to be taken from previous calculations,

(iii) the counterterms are readily divided into photonic and weak parts enabling the corresponding Green's functions to be similarly divided.

The outline of the scheme is as follows. First the parameters of the electroweak Lagrangian are rescaled, generating counterterms in the usual way. Then renormalisation conditions are chosen such that the renormalised masses are equal to the physical masses, the value of the Weinberg angle is defined by $\cos\theta_W = M_W / M_Z$.

The other coupling is the renormalised electric charge is equal to the physical value determined at low energy by Thomson scattering. In other words the values of M_W, M_Z and α are used as input parameters. In addition the scheme gives us rules for renormalising external lines to any diagram. The details may be found in Ref.[1].

The RC for the processes we are considering can be divided into several groups. First we can separate the photonic and weak corrections defined in the following way: photonic RC are those diagrams involving either virtual photons or Bremsstrahlung diagrams whereas weak RC are any other diagram. Our scheme enables each set to be calculated as both infrared (IR) and ultraviolet (UV) finite.

Next the virtual RC of each of these groups is divided into subgroups consisting of

(i) electron self energy corrections (Fig.1).
(ii) vertex corrections (Fig.2)
(iii) box diagrams (Fig.3)

and two groups which apply only to the weak corrections

(iv) boson self energy corrections and the $Z\gamma$ mixing amplitude (Fig.4)
(v) fermion triangle diagrams (Fig.5)

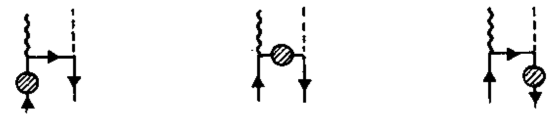


Fig. 1: Electron self energy corrections

Fig. 2: Vertex corrections

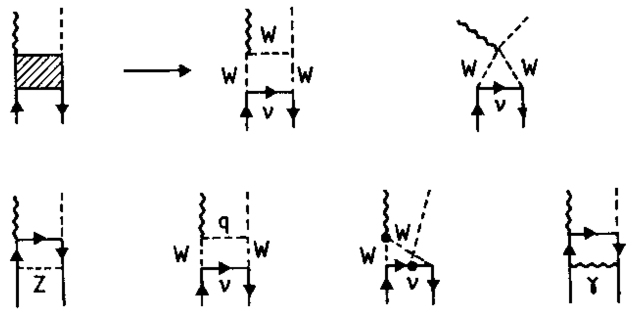


Fig. 3: Box diagrams

Fig. 4: Boson self energy corrections, and the $Z\gamma$ mixing amplitude

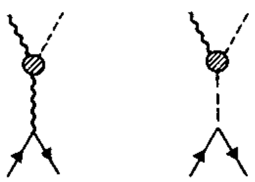
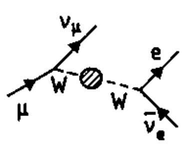


Fig. 5: Fermion triangle diagrams



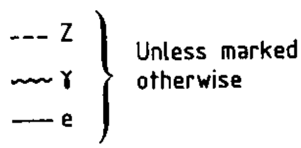


Fig. 6: W self energy diagram

This last group of diagrams requires additional consideration since they are diagrams which generate the well-known ABJ anomalies, although they are finite and so are not scheme dependent in the usual way. The treatment of such diagrams is given in Ref.[2] using symmetry arguments and in Ref.[3] using dimensional regularisation.

3.WEAK CORRECTIONS

A large RC comes, perhaps surprisingly, from the weak sector.This is the contribution of the Z^0 self-energy (see Ref.[1]) which is of order 7%. This diagram has a large value because of the fact that its counterterm contains the value of the unrenormalised photon self energy evaluated at $q^2 = 0$. This is essentially because the Z^0 and the photon are renormalised at different scales and yet are components of the same boson in the SU(2)xU(1) Lagrangian. Since M_z is taken as an input parameter the Z^0 is renormalised at the energy scale M_z whereas α is renormalised at zero energy.

Should we have a scheme in which α was renormalised at a scale M_z, the calculated value of the Z^0 self energy would not be large. Thus, as emphasised by Altarelli [4], this large weak correction is equivalent (at least to leading logarithm) to the running of the electromagnetic coupling constant α. Calculation of the RC to muon decay gives the same term in the W self energy diagram of Fig. 6. This in turn affects the calculation of the Fermi constant G_μ (cf the Δr of Sirlin [5]) .So, if the formula for the differential cross section is written down in terms of G_μ, for example in $e^+e^- \to Z\gamma$ we have

$$\frac{d\sigma_c}{dx} = \frac{G_\mu M_z^2 \alpha F'(s_\theta)}{s^2(s-M_z^2)}\left[2\frac{s^2+M_z^4}{1-v^2x^2} - (s-M_z^2)^2\right]$$

where $F'(s_\theta) = \sqrt{2}\,F(s_\theta)s_\theta(1-s_\theta)$

We have also calculated the weak corrections to $e^+e^- \to Z\gamma$. Apart from the scheme dependent large correction mentioned above (which becomes small if G_μ is used for the differential cross section formula) the weak corrections are small (of order 1%) in the standard model.

4.ELECTROMAGNETIC CORRECTIONS

We show in Ref.[6] that there is a low energy theorem for this process.This allows for a simple derivation of the IR contribution near threshold (where LEPI will be operating)as well as giving the dependence of the threshold amplitude in $\ln(M_z/m)$ where m is the electron mass. Adding the soft bremsstrahlung contribution in the usual way leads to the full one loop radiative correction

$$d\sigma/dx = d\sigma_c/dx(1-(\alpha/\pi)(\ln(s/m^2)\ln(s/4\omega_0)-3\ln(M_z/m)))$$

where ω_0 signifies the photon energy resolution.

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ANNEX E

EXPECTED COUNTING RATES FOR $e^+e^- \rightarrow (e^+e^-)\gamma$

M. Caffo

The availability of the neutral vector boson Z^0 has open the possibility of counting the number of light neutrinos. A definite measurement of this number could be done in LEP stage 1 experiments through the investigation of the process

(1) $$e^+e^- \rightarrow \bar{\nu}\nu\gamma$$

whose cross section is dominated by the Z^0 diagrams [1]. Signal (1) has to compete with a large background due to the process

(2) $$e^+e^- \rightarrow (e^+e^-)\gamma$$

for the kinematical configuration in which the final positron and electron go undetected in forward and backward cones of half opening angle θ_{cone} around the beam line, and the emitted photon is detected in an angular range $\theta_\gamma^{min} < \theta_\gamma < (180^\circ - \theta_\gamma^{min})$, where $\theta_\gamma^{min} > \theta_{cone}$.

The knowledge of this background could be relevant also at higher energies where processes of the kind

(3) $$e^+e^- \rightarrow X\gamma$$

where X is missing energy-momentum, could take place.

An accurate description of the kinematics and of the cross section for the process (2) is therefore given and recent results are rewieved.

In the following we shall use the notation:

(4) $$e^-(p_1)e^+(p_2) \rightarrow e^-(q_1)e^+(q_2)\gamma(k)$$

electron mass: m_e;
electromagnetic coupling constant: α;
beam energy: E, $\quad s = -(p_1+p_2)^2 = 4E^2, \quad p^2 = E^2 - m_e^2$;
photon energy: ω, $\quad x = \omega/E, \quad s_1 = -(q_1+q_2)^2 = s(1-x)$;
photon angle with the direction of the incident electron: θ_γ, $\quad y = cos\theta_\gamma$;
half opening angle of desappareance cone: θ_{cone};
minimum angle of photon detection:θ_γ^{min};
final electron energy: E_e, $\quad p_e^2 = E_e^2 - m_e^2$;
final electron angle with the direction of the incident electron: θ_e;
azimuthal angle of final electron: ϕ_e;
angle between photon and final electron directions: $\theta_{e,\gamma}$;
other invariant variables:
$t = -(p_2-q_2)^2, \quad t_1 = -(p_1-q_1)^2, \quad u = -(p_2-q_1)^2, \quad u_1 = -(p_1-q_2)^2,$
$k_- = -p_1.k, \quad k_+ = -p_2.k, \quad h_- = -q_1.k, \quad h_+ = -q_2.k$.

In the kinematical configuration relevant for the discussion of the background the invariant quantities s, s_1 and the absolute values of u, u_1 are always large and the fermion propagators $k_\pm, h_\pm$ are never too small in absolute value because of the restriction on the photon angle. On the contrary the other two invariant variables t and t_1 can reach very small values: in a typical configuration they can run over 15 orders of magnitude. So an accurate study of their behaviour is crucial for a correct integration. The variables t and t_1 are prevented from vanishing by the non zero values of the photon energy ω and angle θ_γ and can be appropriately written as function of the variables $x, \theta_\gamma, \theta_e$ and ϕ_e. The expression for t_1 is

$$t_1 \simeq -m_e^2 \frac{x^2 E(1+cos\theta_{e,\gamma})^2}{4E_e\left[1-\frac{x}{2}(1-cos\theta_{e,\gamma})\right]^2} - 4EE_e sin^2\left(\frac{\theta_e}{2}\right) \tag{5}$$

where

$$E_e \simeq \frac{E(1-x)}{\left[1-\frac{x}{2}(1-cos\theta_{e,\gamma})\right]} + \frac{m_e^2 x cos\theta_{e,\gamma}}{4E(1-x)} \tag{6}$$

$$cos\theta_{e,\gamma} = cos\theta_e cos\theta_\gamma + sin\theta_e sin\theta_\gamma cos\phi_e$$

The very steep dependence on θ_e in the second term of t_1 is very clear, but the first term, although very small is never vanishing for the indicated conditions. A corresponding expression holds for t, when the electron variables are consistently replaced by the positron ones. Therefore, when the photon is radiated in the positron hemisphere ($cos\theta_{e,\gamma} < 0$) the minimum of the absolute value of t_1 is much lower than the minimum of the absolute value of t, although for values of the electron angles

$$\theta_e \simeq 2arctg \frac{x sin\theta_\gamma}{2\left[1-\frac{x}{2}(1+cos\theta_\gamma)\right]}, \qquad \phi_e = \pi \tag{7}$$

the positron lies on its initial direction. Obviously, when the photon is radiated in the electron hemisphere ($cos\theta_{e,\gamma} > 0$) the roles of t and t_1 are exchanged.

As a last observation the value of θ_e in Eq. (7) shows that, for not too small values of x and θ_γ, the absolute values of t and t_1 are not simultaneously pathologically small.

In this kinematical configuration, therefore, the dominant contribution is given by the t-channel with virtual photon exchanged, so that the calculation for the background actually reduces to the evaluation of the QED part only. Indeed the contributions coming from Z^0 exchange have been computed and found to be negligible [7,9]. For hard ($\omega > m_e$), non collinear ($\theta_\gamma >> m_e/E_e$) photons the QED bremsstrahlung formula simplifies into [2,3]

$$\frac{d^2\sigma_0^{QED}}{dxdy} = \frac{\alpha^3}{\pi s}\int_0^{\theta_{cone}} sin\theta_e d\theta_e \int_0^{2\pi} d\phi_e \frac{EE_e x}{2-x(1-cos\theta_{e,\gamma})} X_0, \tag{8}$$

$$X_0 = \frac{1}{4} \quad A \quad W_{IR}$$

$$A = \frac{\left[ss_1(s^2+s_1^2)+tt_1(t^2+t_1^2)+uu_1(u^2+u_1^2)\right]}{ss_1tt_1}$$

$$W_{IR} = \left[\frac{s}{k_+k_-} + \frac{s_1}{h_+h_-} - \frac{t}{k_+h_+} - \frac{t_1}{k_-h_-} + \frac{u}{k_+h_-} + \frac{u_1}{k_-h_+}\right]$$

Note the lucky absence of double poles in t and t_1, which should be expected for a square matrix element. To obtain the background the integration of Eq. (8) has to be done also for the above specified range of $y = cos\theta_\gamma$ and for some chosen range of $x = \omega/E$. The whole operation requires a numerical treatement and much caution, even if the angle of the emitted photon is large enough for avoiding collinear singularity problems.

First attempts used an available Bhabha scattering Monte Carlo program [3], which requires a minimum scattering angle of both the final states e^+ and e^-, whereas the photon angle is unrestricted. The program is claimed to work correctly down to minimum scattering angles of 0.001 degrees. However, since the cross section is peaked very much near zero angle, the Monte Carlo generation technique becomes extremely inefficient and is limited by the available computer time. While for large photon energies results [4] in agreement with the more recent ones were obtained, for small photon energies such results [5] turned out to be too small. In Ref. [5] a sequence of minimum scattering angles were chosen and plotted against the number of accepted single photon events in order to obtain an extrapolation to zero minimum scattering angle. While this procedure is in principle correct, it suffers very much from lack of statistics and hence does not allow a precise extrapolation.

A different attempt with a newly written Monte Carlo program and using a square matrix element keeping also the electron mass gave too large values, but a revised version seems to give results more in shape [6].

An approach that overcomes all such difficulties has been developed [7] using the Monte Carlo program RIWIAD [8]. It consists on integrating only on a reduced region of the phase space, where the photon energy ω, the photon angle θ_γ and the electron angles θ_e and ϕ_e are good integration variables, and then to exploit the symmetry of the integrand to recover the full value of the integral. In fact, as the differential cross section for photon emission is symmetric for $\theta_\gamma \rightarrow (180° - \theta_\gamma)$, the integral over $\theta_\gamma^{min} < \theta_\gamma < 90°$ is equal to the integral over $90° < \theta_\gamma < (180° - \theta_\gamma^{min})$. In this second range of θ_γ the integration variables accurately map the peak due to $1/t_1$; less well they map the secondary peak due to $1/t$, but this contribute much less to the integral. One integrates only in the region $90° < \theta_\gamma < (180° - \theta_\gamma^{min})$ and then one multiplies the result by a factor of two. With this prescriptions the indicated integration variables are suitable for properly accounting for the dominant contribution of the region of very small values of θ_e.

Furthermore Eq. (8) is so compact that numerical contributions can be kept under control in every situation. In the expression for X_0 the square braket in A is composed of terms which are always positive and so can not originate loss in precision. On the contrary the expression for W_{IR} in the very peaked region of small t and t_1 exhibits quite large cancellations, checked to be of 8 digits at most. Since VAX double-precision arithmetic with 15 decimal digits has been used, this can not spoil the requested precision of one per cent.

All m_e^2 terms have been dropped from the numerator in Eq. (8), but they were kept in the careful evaluation of the kinematical variables. It was also observed that no term with $1/t_1^2$, which has the leading behaviour in the chosen configuration, is present in X_0. An explicit calculation, done by subtracting from the square matrix element exact in m_e the value of X_0, shows that, in the considered kinematical region (where the electron is almost aligned with the beam line and the photon in the positron emisphere) the leading m_e^2/t_1^2 term is given by

$$X_m = -\frac{m_e^2}{2t_1^2}\left[\frac{s_1^2+u_1^2}{k_+^2} + \frac{s^2+u^2}{h_+^2} + \frac{t^2 - 2ss_1 - 2uu_1}{k_+h_+}\right], \tag{9}$$

an analogous formula holding for the leading m_e^2/t^2 term in the symmetric situation with the gamma in the electron emisphere.

X_m is very strongly peaked at $\theta_e = 0$, where it almost compensate the X_0 peak, and its contribution to the cross section has been approximately evaluated analytically and is always negative, usually very small (a few percent at most) against that of X_0, to which it has to be summed, and its contribution to the cross section is

$$\sigma_m^{QED}(x_{max}) - \sigma_m^{QED}(x_{min}) = \frac{\alpha^3}{s}2\left\{\left[2x(x-4) + 8\ln(x)\right]\frac{y_{max}}{(y_{max}-1)}\right. \tag{10}$$
$$\left. -x(x-2)\ln(1-y_{max}) + 2Li_2\left(\frac{1+y_{max}}{1-y_{max}}(x-1)\right) - 2Li_2(x-1)\right\}_{x_{min}}^{x_{max}}$$

with $y_{max} = cos\theta_\gamma^{min}$ and Li_2 the Euler dilogarithm, the limit of applicability being

$$\omega < E\frac{sin\theta_{cone}}{sin\theta_\gamma^{min}}. \tag{11}$$

For values of ω larger than in Eq. (11), the value of Eq. (10) is however an upper limit. Numerically the X_m contribution amounts to a few percent of the X_0 contribution at most.

One has to note that this method uses very simple formulae, some known for a long time and all very easy to check. In the numerical treatement the region of integration is accurately described and the very limited number of terms allows a good control of precision loss.

For annihilation-channel dominated processes like $e^+e^- \rightarrow \bar{\nu}\nu\gamma$ the choice of α renormalized at the square Z^0 mass (α=1/128.5) is the obvious one. On the contrary for the amplitude of the process $e^+e^- \rightarrow (e^+e^-)\gamma$, strongly peaked corresponding to extremely small values of the momentum transfers t, t_1 (exceptional momenta in the terminology of renormalization group), the value $\alpha = 1/137$ seems to be more natural. In the first of Ref.s [7] the value $\alpha = 1/128.5$ was used for simplicity for both the neutrino counting reaction and the electromagnetic background. Guessing that the use of $\alpha = 1/137$ is more appropriate for the electromagnetic background, due to the dominance of the lowest transferred momenta, one has only to uniformly decrease the cross-section by the constant factor 0.825.

Recently some other calculations of the process $e^+e^- \rightarrow (e^+e^-)\gamma$ have been completed. One [9] is done with the helicity amplitude technique and with a mapping of the phase space variables in order to absorb the peak of the square matrix element. According to Ref. [9] a discrepancy of 10-20 % for photon energy over 1 GeV and larger for smaller photon energy exists with the values reported in the first of Ref.s [7]. A part from the different choice for the value of α (α=1/137 in Ref. [9]), the discrepancy is no longer claimed after the Aachen Conference. Finally an event generator program [10] and a Monte Carlo program [11] have provided results in agreement with Ref's [7].

In conclusion the results of Ref.s [7], confirmed by Ref.s [9,10,11] fix the values for the background process $e^+e^- \rightarrow (e^+e^-)\gamma$ in Born approximation.

However in Ref. [10] is given also an estimation for the order α correction which comes out to be larger than the lower order. As this estimation is obtained by keeping only the contributions supposed to be dominant, a complete calculation is required and a cross-check would not be superfluous. A confirmation of this result implies a reevaluation of the proposed experiments on neutrino counting.

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ANNEX F

$O(\alpha^2)$W MASS SHIFT FROM A VERY HEAVY TOP QUARK

B.W. Lynn, D. Kennedy and C. Verzegnassi

One of the most sensitive tests of the Standard Model[1] (and of electroweak theories in general) to one loop level will be the precision measurement of the W mass to better than 1% accuracy. As is known, the latter is related to the Fermi constant, the Z_0 mass and the electric charge by Sirlin's one-loop formula:[2]

$$M_W^2\left[1-\frac{M_W^2}{M_Z^2}\right] = \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{1-\Delta_r} \quad , \tag{1}$$

where Δ_r is the radiative correction, evaluated to one loop. Δ_r contains the still unknown parameters M_{Higgs} and M_{top}, so that its numerical value can only be given for fixed values of these quantities. Normally, one assumes $M_t \simeq 30$ GeV, $M_H \simeq 100$ GeV and finds[3]

$$\Delta_r\ (M_t = 30\ \text{GeV}\ , M_H = 100\ \text{GeV}\ ) \simeq 0.07 \quad . \tag{2}$$

In practice, this important correction stems mostly from oblique corrections, particularly fermionic vacuum polarization diagrams. More precisely, the value of Eq. (2) is mainly determined by renormalization of the running electric charge where in Euclidean metric with $q^2 = \vec{q}^2 - q_0^2 = -M_Z^2$

$$\alpha_{em}(-M_Z^2) = \frac{\alpha(0)}{1-\Delta_\alpha(-M_Z^2)} \quad , \tag{3}$$

with $\alpha^{-1}(0) \simeq 137.036$ and $\Delta_\alpha(-M_Z^2) \simeq .06$ Actually, one can write

$$\Delta_r = \Delta\alpha(-M_Z^2) - \frac{c_\theta^2}{s_\theta^2}\Delta_\rho(0) + \text{small contributions} \quad , \tag{4}$$

where $c_\theta = M_W/M_Z$, $s_\theta^2 = 1 - c_\theta^2$. The parameter $\Delta_\rho(0)$ gives the correction to the ρ parameter

$$\rho = 1 + \Delta_\rho(0) \quad , \tag{5}$$

and, if the top mass is equal to 30 GeV, $\Delta_\rho(0)$ is sensibly smaller than $\Delta_\alpha(-M_z^2)$.

In fact, $\Delta_\alpha(-M_Z^2)$ gives the leading logarithmic contribution $\sim \ell n(M_Z^2/m_f^2)$ to Δ_r. This is not the case of $\Delta_\rho(0)$, which is quadratic in the fermionic mass and proportional to m_f^2/M_Z^2. Thus for $m_f^2/M_Z^2 \ll 1$ one can discard $\Delta_\rho(0)$ and approximate Δ_r by its leading logarithmic term Δ_α. In this case, renormalization group arguments first introduced by Marciano and Sirlin[4] allow us to compute next order effects in Eq. (1) by simply expanding the Δ_α content of Δ_r through the related geometrical series. Thus, one easily computes the contribution to leading log to Eq. (1) from $O(\Delta_\alpha^2)$ and finds that it is small; *i.e.*, much smaller than the $O(\Delta_\alpha)$ term. This is a welcome indication that, as far as Eq. (1) is concerned, assuming $m_t \simeq 30$ GeV, higher order effects can probably be neglected.

The situation might be rather different if the top quark turned out to be substantially heavier; *e.g.*, of the order of $\simeq 200$ GeV. This is still not ruled out by the existing experimental evidence. A straightforward computation shows that in that case the numerical contribution of $\Delta_\rho(0)$ to Eq. (4) becomes almost of the same size (and opposite) to that of $\Delta_\alpha(-M_Z^2)$:

$$-\frac{c_\theta^2}{s_\theta^2}\,\Delta_\rho^{(top)}(0)\Bigg|_{m_t=200\ GeV} \simeq -\frac{c_\theta^2}{s_\theta^2}\left[\frac{3\alpha}{16\pi s_\theta^2 c_\theta^2}\,\frac{m_t^2}{M_Z^2}\right] \simeq -0.05 \quad . \tag{6}$$

If this were the case, one would have strong motivation to fear that next order contributions to Δ_r, *e.g.*, of the kind $\sim\ \Delta_\rho^2(0)$ and $\Delta_\alpha(-M_Z^2)\Delta_\rho(0)$ might be relevant. Since these contributions are *not* of the leading logarithmic kind, their coefficient will differ from that of Δ_α^2. In this case it is *not* correct to expand Eq. (1) including terms $\sim (\Delta_r)^2$ with Δ_r given in Eq. (4). The relevant terms must be evaluated by application of perturbation theory to the proper oblique corrections contributions involving the various vacuum polarizations in a renormalization scheme independent way. We have done this starting from a general approach wihch evaluates higher order corrections which will be illustrated elsewhere.[5] Here we only deal with the specific case of the $O(\alpha^2)$ heavy top corrections to the precise $W^\pm$ mass which will be of special interest for the W mass measurement to be carried through at LEP II.

Here we work in the renormalization scheme which uses $\alpha(0)$, the muon lifetime coefficient, $G_\mu(0)$ and the physical Z° mass M_Z as physical input parameters and start from the coupled Dyson's equations for the various gauge bosons propagators:

$$\begin{aligned} G_{WW} &= \frac{1}{M_W^2 + q^2 - \widetilde{\pi}_{WW}(q^2)} \quad , \\ G_{ZZ} &= \frac{1}{M_Z^2 + q^2 - \widetilde{\pi}_{ZZ}(q^2) - \dfrac{\widetilde{\pi}_{ZA}^2(q^2)}{q^2 - \widetilde{\pi}_{AA}(q^2)}} \quad , \end{aligned} \tag{7}$$

$$G_{AA} = \frac{1}{q^2 - \widetilde{\pi}_{AA}(q^2) - \dfrac{\widetilde{\pi}^2_{ZA}(q^2)}{M_Z^2 + q^2 - \widetilde{\pi}_{ZZ}(q^2)}} \quad ,$$

$$G_{ZA} = \frac{\widetilde{\pi}_{ZA}(q^2)}{[q^2 - \widetilde{\pi}_{AA}(q^2)]\,[M_Z^2 + q^2 - \widetilde{\pi}_{ZZ}(q^2)] - \widetilde{\pi}^2_{ZA}(q^2)} \quad ,$$

where the $\widetilde{\pi}_{ij}$'s are the $1PI$ vacuum polarizations for vector bosons $i,j = W^{\pm}$, Z, A (photon) which we write as

$$\widetilde{\pi}_{ij} \equiv \pi_{ij} + \text{counterterms} \, , \tag{8}$$

with π_{ij} calculated with the *bare* coupling constants. The specific choices of physical parameters are then used to fix the numerical value of diferent quantities which enter the oblique radiative corrections. In particular, we find:

$$\begin{aligned} \text{Re}\ \frac{\widetilde{\pi}_{ZA}(-M_Z^2)}{M_Z^2} &= \frac{\Delta_p(-M_Z^2)}{s_\theta\, c_\theta} \left[1 + \frac{\Delta_p(-M_Z^2)}{1-2s_\theta^2} + O(\alpha^2)\right] ; \\ \Delta_p(-M_Z^2) &\simeq \frac{s_\theta^2\, c_\theta^2}{1-2s_\theta^2}\ [\Delta_\alpha(-M_Z^2) - \Delta_\rho(0)] \quad . \end{aligned} \tag{9}$$

Defining the W mass as the pole of the W propagator and using consistently Eq. (7) leads us then to the following result:

$$\begin{aligned} M_W^2 &\simeq \tilde{c}_\theta^2 M_Z^2 \left\{1 - \frac{\tilde{s}_\theta^2}{1-2\tilde{s}_\theta^2}\ \left[\Delta_\alpha(-M_Z^2) - \frac{\tilde{c}_\theta^2}{\tilde{s}_\theta^2}\ \Delta_\rho(0)\right]\right. \\ &\quad - \frac{\tilde{s}_\theta^2(1-3\tilde{s}_\theta^2+3\tilde{s}_\theta^4)}{(1-2\tilde{s}_\theta^2)^3}\ \Delta_\alpha^2(-M_Z^2) + \frac{\tilde{c}_\theta^4(1-3\tilde{s}_\theta^2)}{(1-2\tilde{s}_\theta^2)^3}\ \Delta_\rho^2(0) \\ &\quad \left. + \frac{2\tilde{c}_\theta^2\, \tilde{s}_\theta^4\, \Delta_\alpha(-M_Z^2)\, \Delta_\rho(0)}{(1-2\tilde{s}_\theta^2)^3}\right\} ; \end{aligned} \tag{10}$$

$$\tilde{c}_\theta^2 = 1 - \tilde{s}_\theta^2 = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4\alpha\pi}{\sqrt{2}\, G_\mu M_z^2}}\right) \, . \tag{11}$$

which allows us to compute those effects coming from a heavy top quark to $O(\alpha^2)$ (one loop $\times$ one loop) terms.[♯1]

Note that the coefficient of Δ_α^2 in Eq. (10) is, as we expected from Marciano and Sirlin's arguments, that which corresponds to the geometrical series

♯1 One particle irreducible two-loop effects within $\Delta_\rho(0)$ have been computed[6] and found to be negligibly small.

expansion of the Δ_α content of $1/(1-\Delta_r)$. But the coefficients of Δ_ρ^2 and of $\Delta_\alpha\Delta_\rho$ are, as one might expect, quite different. For a top quark mass of 200 GeV, we find from Eq. (11)

$$\Delta M_W^{[top;O(\alpha^2)]} \simeq +18 \text{ MeV} , \tag{12}$$

and of these $\sim$ 18 MeV, $\sim$ 10 come from the interference $\sim \Delta_\alpha\Delta_\rho$, while $\sim$ 8 come from Δ_ρ^2. This $O(\alpha^2)$ contribution should be compared to that coming, for the same value of $m_t = 200$ GeV, from the $O(\alpha)$ term, which is of approximately +1 GeV.[2,3,7] Thus we conclude that such $O(\alpha^2)$ effect is completely negligible even at the required level of accuracy, which we assume to be of the order of $\sim$ 50 MeV. This result is rather important since, *a priori*, a larger effect might have been found[♮2] and thus it may be feared that a large uncertainty in the Standard Model prediction for the $W^\pm$ mass could come from higher order effects.

♮2 An *incorrect* calculation done by expanding Eq. (1) including terms $\sim (\Delta_r)^2$ with Δ_r given in Eq. (4) would have yielded the *incorrect* result $\Delta M_W^{[top;O(\alpha^2)]} \simeq -40$ MeV.

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