Renormalization of the electroweak theory in the nonlinear gauge

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We study several problems associated with the perturbative expansion of the electroweak theory in the nonlinear gauge. In this gauge a consistent quantization can be obtained if we generalize the usual Faddeev-Popov method. The new ghost vertices are displayed and we show that a more complicated ghost sector derogates some of the merits attributed to this gauge-fixing term.

I. INTRODUCTION

After the discovery at CERN (Ref. 1) of the W and Z bosons and with their forthcoming operation of LEP the Glashow-Weinberg-Salam (GWS) theory is entering an era of detailed experimental checking. This, in turn, implies the evaluation of several measurable quantities beyond the tree level. The amount of work involved in the calculations is not trivial but it is a worthwhile effort since it will, hopefully, prove that a non-Abelian gauge principle is at work in nature.

Following the leading work[2] of almost 15 years ago it became normal practice to quantize the GWS theory using a gauge-fixing term $\mathscr{L}_{\rm gf}$ which cancels the mixing between the gauge fields $(W_\mu^\pm, Z_\mu)$ and the scalar fields $(\phi^\pm, \phi_Z)$. The study of these R_ξ gauges is by now well known (e.g., Ref. 3) and for practical calculations the use of the 't Hooft—Feynman gauge $\xi=1$, has clear advantages. From $\mathscr{L}_{\rm gf}$ the Faddeev-Popov[4] (FP) procedure gives the ghost Lagrangian and once this is obtained we have the correct effective Lagrangian to start perturbation theory. Feynman rules can be simply written down and a renormalization program can be implemented. It is interesting to point out two important properties of $\mathscr{L}_{\rm gf}$ that greatly simplify the renormalization scheme. The linear gauge-fixing term is not renormalized and it has a global $\mathrm{SU}(2)\times\mathrm{U}(1)$ invariance.

Recently, several authors[5–8] have advocated the use of a nonlinear R_ξ gauge $\mathscr{L}_{\rm gf}^{\rm nl}$, first introduced by Fujikawa.[9] The reason to recommend this gauge-fixing condition is essentially the simplification that it introduces in some calculations, especially those that involve the photon A^μ couplings. Using $\mathscr{L}_{\rm gf}^{\rm nl}$ the Feynman rules are slightly simpler and in particular there is no $A^\mu W_\mu^\pm \phi^\pm$ vertex. Furthermore, in this approach the electromagnetic U(1) symmetry is preserved and this might be of some advantage since the Ward identities for the electromagnetic vertices are the ones known from quantum electrodynamics. Finally, let us remark that the study of a gauge-fixing term of a spontaneously broken theory that preserves the unbroken gauge symmetry could be of a more general interest. In fact, when (and if) any spontaneously broken grand unified theory is taken seriously its gauge-fixing term must preserve the standard $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ invariance.

All these favorable attributes of the nonlinear gauge are well known. However, the study of the GWS theory using $\mathscr{L}_{\rm gf}^{\rm nl}$ is also full of difficulties. To expose them and to show how the problems can be overcome is the aim of this work. The first difficulty, which is not generally appreciated and has to be balanced against the simplification of having fewer Feynman diagrams, concerns a more complicated structure of the counterterms. In Sec. II we give an example that illustrates this problem and at the same time indicate how the full counterterm Lagrangian can be derived. The remainder of the paper deals with a more serious difficulty that one has to face if the GWS theory is quantized with $\mathscr{L}_{\rm gf}^{\rm nl}$. Using an example we show that the FP method does not give a renormalizable theory. We then continue to prove that a sensible theory is obtained if one adopts the point of view that the quantum Lagrangian is a local functional of the classical and ghost fields invariant under the Becchi-Rouet-Stora (BRS) transformations.[10] This unorthodox way of quantizing perturbative gauge theories has been summarized in a recent review article.[11] Our work here may be regarded as an application of this method.

II. PROBLEMS WITH THE NONLINEAR GAUGE

In order to do a perturbative expansion of a gauge theory it is necessary to add a gauge-fixing term $\mathscr{L}_{\rm gf}$ such that one can define the gauge-field propagators. For the GWS theory and working in the 't Hooft—Feynman gauge the nonlinear gauge fixing is

$$\mathscr{L}_{\rm gf}^{\rm nl}=-\tfrac{1}{2}\mathscr{F}_A{}^2-\tfrac{1}{2}\mathscr{F}_Z{}^2-|\mathscr{F}_+|^2 \qquad (1)$$

with

$$\mathscr{F}_A=\partial_\mu A^\mu , \qquad (2a)$$

$$\mathscr{F}_Z=\partial_\mu Z^\mu - M_Z\phi_Z , \qquad (2b)$$

$$\mathscr{F}_+=(\partial^\mu+ieA^\mu)W_\mu^+ - iM_W\phi^+ . \qquad (2c)$$

Notice that the same set of equations but without the term proportional to the photon in Eq. (2c) defines the usual (e.g., Ref. 3) linear gauge. From Eq. (1) the FP prescription leads to the following Lagrangian for the ghosts fields $(C_A, C_Z, C^\pm)$:

$$\mathscr{L}_G = -\bar{C}_A \mathfrak{s}\mathscr{F}_A - \bar{C}_Z \mathfrak{s}\mathscr{F}_Z - \bar{C}^+ \mathfrak{s}\mathscr{F}_+ - \bar{C}^- \mathfrak{s}\mathscr{F}_- , \quad (3)$$

where $\mathfrak{s}$ is the Slavnov operator which is related to the BRS transformation by

$$\delta_{\text{BRS}}(\text{any field}) = \mathfrak{s}(\text{any field})\theta ,$$

where θ is a space-time-independent anticommuting parameter. In the Appendix we summarize some of the properties of $\mathfrak{s}$ and give the results of operating with $\mathfrak{s}$ on each field of the GWS model.

A. Structure of the counterterms: an example

The renormalization of the GWS theory has been studied by different authors using several alternative schemes (see Ref. 12 for recent reviews). Here we follow an on-shell renormalization procedure where the parameters that characterize the theory are the electric charge e, the gauge-boson masses M_W and M_Z, and the Higgs-boson and fermion masses M_H and m_f, respectively. In the linear gauge the global $SU(2)\times U(1)$ symmetry of $\mathscr{L}_{\text{gf}}$ and the fact that $\mathscr{L}_{\text{gf}}$ does not need to be renormalized reduces the number of renormalization constants and simplifies the structure of the ghost counterterms. Even so, the complete list of counterterms is somewhat lengthy.[13] Hence, it should be clear that, to present this list in the case of $\mathscr{L}_{\text{gf}}^{\text{nl}}$ is beyond the scope of the present work. Nevertheless, let us give an example that will be sufficient to illustrate one of our conclusions. The simplification of not having the vertice $A^\mu W_\mu^\mp \phi^\pm$ is clearly overbalanced by the complication of the renormalization program.

Let us consider the kinetic terms for A_μ and Z_μ in the classical Lagrangian for the GWS theory. Introducing the relation between bare and renormalized fields, i.e.,

$$\begin{bmatrix} A_\mu \\ Z_\mu \end{bmatrix}_0 = \begin{bmatrix} Z_A{}^{1/2} & \frac{1}{2}\delta Z_{AZ} \\ \frac{1}{2}\delta Z_{ZA} & Z_Z{}^{1/2} \end{bmatrix} \begin{bmatrix} A_\mu \\ Z_\mu \end{bmatrix} , \quad (4)$$

it is easy to obtain the counterterms

$$\begin{aligned} \mathscr{L}^c = & -\tfrac{1}{4}\delta Z_A(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ & -\tfrac{1}{4}\delta Z_Z(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) \\ & -\tfrac{1}{4}(\delta Z_{ZA} + \delta Z_{AZ})(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) \\ & +\tfrac{1}{2}(\delta Z_M + \delta Z_Z) M_Z{}^2 Z_\mu Z^\mu + \tfrac{1}{2}\delta Z_{ZA} M_Z{}^2 Z_\mu A^\mu . \end{aligned} \quad (5)$$

Notice that the corresponding expression for the linear gauge is simpler[13] since in this second case there are only two wave-function renormalization constants for the fields A_μ, Z_μ, and $W_\mu^\pm$. This is an example of the global $SU(2)\times U(1)$ symmetry that we have already referred to.

To proceed we have to evaluate the gauge-boson self-energies $\Sigma_{ij}^{\mu\nu}$ with $i,j = A$ or Z, and impose the on-shell conditions. Most of the results are not important for what follows and will not be given here, except the Z-boson—photon renormalization constant which is $\delta Z_{ZA} = 0$. Instead, let us examine the ghost sector corresponding to $\mathscr{F}_A$ and $\mathscr{F}_Z$: namely,

$$\mathscr{L} = -\tfrac{1}{2}\mathscr{F}_A{}^2 - \tfrac{1}{2}\mathscr{F}_Z{}^2 - \bar{C}_A \mathfrak{s}\mathscr{F}_A - \bar{C}_Z \mathfrak{s}\mathscr{F}_Z , \quad (6)$$

where $\mathfrak{s}$ should be understood as the renormalized Slavnov operator and the renormalized fields and constants are

$$\begin{bmatrix} C_A \\ C_Z \end{bmatrix}_0 = \begin{bmatrix} \tilde{Z}_A & \delta\tilde{Z}_{AZ} \\ \delta\tilde{Z}_{ZA} & \tilde{Z}_Z \end{bmatrix} \begin{bmatrix} C_A \\ C_Z \end{bmatrix} , \quad (7)$$

$$\begin{aligned} & (\phi_Z)_0 = Z_\phi{}^{1/2}\phi_Z, \quad H_0 = Z_H{}^{1/2} H , \\ & \phi_0^\pm = Z_{\phi^\pm}{}^{1/2}\phi^\pm, \quad C_0^\pm = \tilde{Z}_{C^\pm} C^\pm , \\ & g_0 = Z_g g, \quad g_0' = Z_{g'} g' . \end{aligned} \quad (8)$$

Using these equations and the Slavnov relations given in the Appendix it is straightforward to obtain the counterterms corresponding to the piece of the effective Lagrangian given by Eq. (6).

These are

$$\begin{aligned} \mathscr{L}^c = & -(\delta\tilde{Z}_A - \tfrac{1}{2}\delta Z_A)\bar{C}_A \Box C_A - (\delta\tilde{Z}_{AZ} - \tfrac{1}{2}\delta Z_{AZ})\bar{C}_A \Box C_Z \\ & - ie(-\tfrac{1}{2}\delta Z_A + \tfrac{1}{2}\cot\theta_W \delta Z_{AZ} + \delta Z_g \sin^2\theta_W + \delta Z_{g'}\cos^2\theta_W + \tfrac{1}{2}\delta Z_{\phi^\pm} + \delta\tilde{Z}_{C^\pm})\bar{C}_A \partial^\mu (C^- W_\mu^+ - C^+ W_\mu^-) \\ & -(\delta\tilde{Z}_Z - \tfrac{1}{2}\delta Z_Z)\bar{C}_Z \Box C_Z - (\delta\tilde{Z}_{ZA} - \tfrac{1}{2}\delta Z_{ZA})\bar{C}_Z \Box C_A \\ & + ig\cos\theta_W[-\tfrac{1}{2}\delta Z_Z + (1+\sin^2\theta_W)\delta Z_g - \sin^2\theta_W \delta Z_{g'} + \tfrac{1}{2}\tan\theta_W \delta Z_{ZA} + \tfrac{1}{2}\delta Z_\phi + \delta\tilde{Z}_{C^\pm}]\bar{C}_Z \partial^\mu (C^- W_\mu^+ - C^+ W_\mu^-) \\ & - \delta\tilde{Z}_{ZA} M_Z{}^2 \bar{C}_Z C_A - \frac{g}{2\cos\theta_W} M_Z \delta\tilde{Z}_{ZA} \bar{C}_Z C_A H - (\tfrac{1}{2}\delta Z_M - \tfrac{1}{2}\delta Z_{\phi_Z} + \delta\tilde{Z}_Z - \tfrac{1}{2}\delta Z_Z) M_Z{}^2 \bar{C}_Z C_Z \\ & - \frac{g}{2\cos\theta_W}(\tfrac{1}{2}\delta Z_H - \tfrac{1}{2}\delta Z_{\phi_Z} + \delta\tilde{Z}_Z - \tfrac{1}{2}\delta Z_Z + \cos^2\theta_W \delta Z_g + \sin^2\theta_W \delta Z_{g'})\bar{C}_Z C_Z H \\ & + \tfrac{1}{2}\delta Z_{AZ}\bar{C}_A C_Z M_Z{}^2 + \tfrac{1}{2}\delta Z_{AZ}\frac{g}{2\cos\theta_W}\bar{C}_A C_Z H - \tfrac{1}{2}\delta Z_{AZ}\frac{g}{2}\bar{C}_A (C^-\phi^+ + C^+\phi^-) \\ & + \frac{g}{2} M_Z(\delta Z_g + \delta\tilde{Z}_{C^\pm} - \tfrac{1}{2}\delta Z_Z + \tfrac{1}{2}\delta Z_{\phi^\pm} - \tfrac{1}{2}\delta Z_{\phi_Z})\bar{C}_Z (C^-\phi^+ + C^+\phi^-) . \end{aligned} \quad (9)$$

After such a long equation we believe we have convinced our readers that the nonlinear gauge is not so simple. On the other hand, for those that might wonder why the ghost counterterms could be important let us add one last comment. Consider the $ZH\gamma$ one-particle-irreducible Green's function $T^{\rho\mu}$. In a previous paper[14] we have shown that, in the nonlinear gauge, $T^{\rho\mu}$ obeys the Ward identity

$$k_\mu T^{\rho\mu}=0 \; , \tag{10}$$

where k_μ is the photon momentum. On physical grounds this is to be expected since $\mathscr{L}_{\rm gf}^{\rm nl}$ is invariant under the electromagnetic U(1). However, to prove Eq. (10) we need to know the $\bar{C}_Z C_A H$ counterterm. Evaluating the $C_Z C_A$ self-energy Σ_{ZA} we obtain

$$\Sigma_{ZA}^R(p^2)=(p^2-M_Z{}^2)\frac{eg\cos\theta_W}{16\pi^2}\left[\Gamma(\epsilon/2)-\int_0^1 dx \ln\left[1-\frac{p^2}{M_W{}^2}x(1-x)\right]\right]-\delta\tilde{Z}_{ZA}(p^2-M_Z{}^2)+\tfrac{1}{2}\delta Z_{ZA}p^2 \; . \tag{11}$$

Imposing the on-shell conditions

$$\Sigma_{ZA}^R(0)=\Sigma_{ZA}^R(M_Z{}^2)=0 \tag{12}$$

and recalling the previous result $\delta Z_{ZA}=0$ we get

$$\delta\tilde{Z}_{ZA}=\frac{eg\cos\theta_W}{16\pi^2}\Gamma(\epsilon/2) \; . \tag{13}$$

Finally, inserting this equation into Eq. (9) one obtains the $\bar{C}_Z C_A H$ counterterm used before.[14]

B. Failure of the FP prescription

From Eq. (3) and using the results of the Appendix it is easy to write the FP Lagrangian corresponding to the nonlinear gauge-fixing term. Rather than doing this we show in Fig. 1 the vertices that are different from those in the linear gauge. For completeness, and having in mind future users, we also list the vertices resulting from $\mathscr{L}_{\rm gf}^{\rm nl}$. Notice that the ghost vertices of type (b) do not exist in the linear gauge.

We are now prepared to show the second difficulty associated with $\mathscr{L}_{\rm gf}^{\rm nl}$. Let us consider the Green's function $T=\langle 0 | TC^+C^-\bar{C}^+\bar{C}^- | 0\rangle$. In Fig. 2 we display the divergent diagrams which contribute to this Green's function in lowest order. A straightforward application of the Feynman rules given previously leads to

$$T=\frac{e^2}{16\pi^2}(5e^2+3g^2\cos^2\theta_W)\Gamma(\epsilon/2)+\text{finite terms} \; , \tag{14}$$

i.e., T is divergent. Hence one needs a quartic ghost interaction which is not present in the original Lagrangian. It is interesting to see why a similar problem does not arise in the linear gauge. From a technical point of view the reason can be traced back to the fact that in the linear gauge the vertices of type (a) are independent of the incoming momentum. A simple calculation will show that this is sufficient to make T finite. On more general grounds our example illustrates the failure of the FP prescription. In fact, one can prove[11] that the FP method is stable under renormalization if and only if the gauge-fixing condition is linear. So, the study of the GWS theory using $\mathscr{L}_{\rm gf}^{\rm nl}$ immediately implies an alternative way to deal with perturbative gauge theories.

III. EFFECTIVE LAGRANGIAN WITH A NONLINEAR GAUGE

After pointing out the inconsistency of the FP method we address ourselves to the problem of finding an alterna-

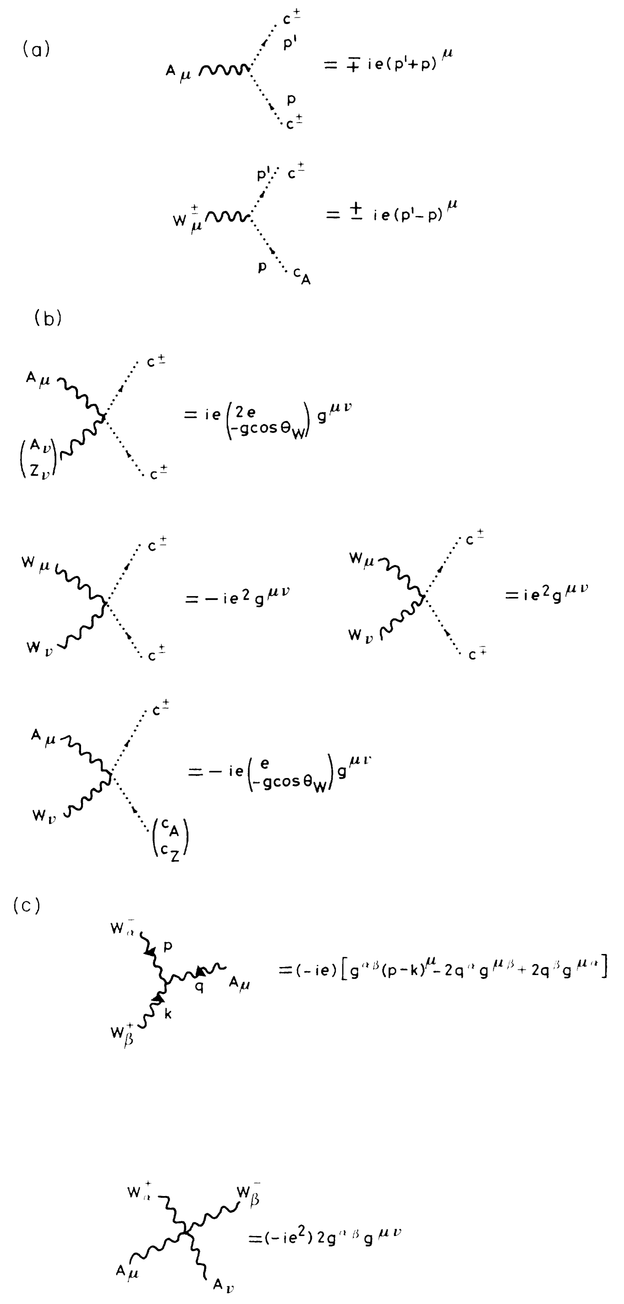


FIG. 1. Feynman rules for the vertices contained in $\Delta\mathscr{L}_0$ [cf. Eqs. (1), (3), and (35)].

tive one. For this we follow the work of Baulieu[11] and since the full complexity of the GWS model is not crucial for the argument we consider a non-Abelian gauge theory based on a simple group G. The effective quantum action is

$$S_{\rm eff}=S_{\rm cl}+\Delta S\ , \tag{15}$$

where $S_{\rm cl}$ is the classical action and ΔS the extra terms needed to have a well-defined perturbative expansion. The usual path to obtain ΔS is widely known and, as we have seen, does not work in our case. Instead, we assume that the BRS invariance is the quantum version of the classical gauge symmetry. So, we find ΔS by imposing the condition

$$s S_{\rm eff}=0\ . \tag{16}$$

In terms of the gauge fields A^a_μ, with $a=1,\dots,n$ where n is the number of generators of G, ghost C^a and $\bar C^a$ and auxiliary fields b^a the Slavnov operation is

$$\begin{aligned} s A^a_\mu &= \partial_\mu C^a - g f^{abc} C^b A^c_\mu\ ,\\ s C^a &= \tfrac{1}{2} g f^{abc} C^b C^c\ ,\\ s \bar C^a &= b^a,\quad s b^a = 0\ , \end{aligned} \tag{17}$$

where f^{abc} are the structure constants of G and g is the coupling constant. Recalling that s is nilpotent we write

$$\Delta S=\int d^4x\, s\bar K_3(x)\ , \tag{18}$$

where $\bar K_3$ is the sum of all monomials invariant under G with mass dimension 3 and ghost number $N_g=-1$. For the case of a pure gauge theory one can easily see that the most general form of $\bar K_3$ is

$$\begin{aligned} \bar K_3 = -\bar C^a(&-\tfrac{1}{2}b^a+\partial^\mu A^a_\mu+\alpha f^{abc}\bar C^b C^c\\ &+\beta d^{abc}A^b_\mu A^{\mu c})\ , \end{aligned} \tag{19}$$

where d^{abc} is the group symmetric tensor and α and β are arbitrary constants.

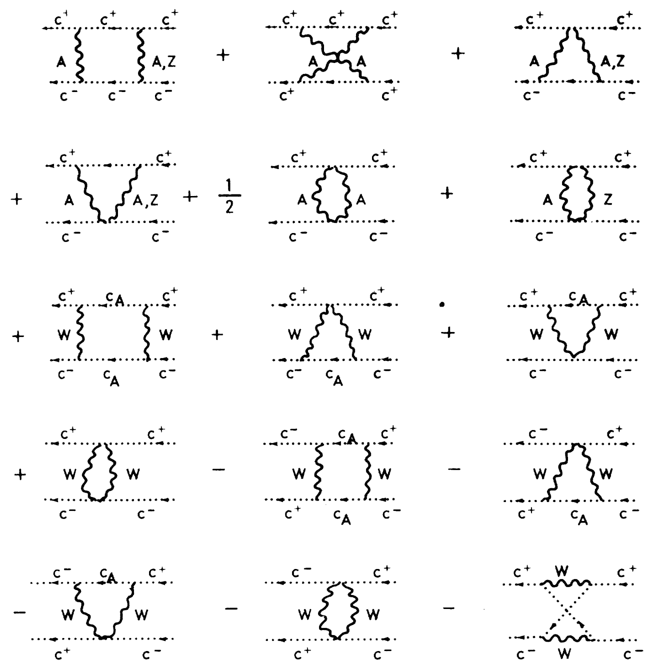
For simplicity let us take $G=\mathrm{SU}(2)$ and define G^a as

$$G^a=f^{abc}\bar C^b C^c\ . \tag{20}$$

Then, using the Jacobi identity, the effective Lagrangian can be written

$$\begin{aligned} \mathcal{L}_{\rm eff} &= \mathcal{L}_{\rm cl}+s\bar K_3\\ &= \mathcal{L}_{\rm cl}+\tfrac{1}{2}b^a b^a - b^a(\partial^\mu A^a_\mu+2\alpha G^a)\\ &\quad -\bar C^a s(\partial^\mu A^a_\mu)+\alpha g G^a G^a\ . \end{aligned} \tag{21}$$

Finally, using the equation of motion of the b field, namely,



+ Finite diagrams

FIG. 2. Divergent diagrams that contribute to the Green's function $\langle 0|\,TC^+\bar C\,\bar C^+\bar C^-|0\rangle$.

$$b^a=\partial^\mu A^a_\mu+2\alpha G^a\equiv\mathcal{F}^a\ , \tag{22}$$

we obtain

$$\mathcal{L}_{\rm eff}=\mathcal{L}_{\rm cl}-\tfrac{1}{2}\mathcal{F}^a\mathcal{F}^a-\bar C^a s\mathcal{F}^a+2\alpha\mathcal{F}^a G^a-\alpha g G^a G^a\ . \tag{23}$$

This equation can be interpreted as a quantization of the SU(2) theory using the nonlinear gauge-fixing condition specified by Eq. (22). If we had followed the usual FP prescription the last two terms in Eq. (23) would not be present. Clearly this effective Lagrangian includes quartic ghosts couplings. When α goes to zero the gauge fixing is linear and Eq. (23) coincides with the FP result. In this case the gauge independence of the renormalized S matrix can be proved (e.g., Ref. 15). However, this proof assumes that the gauge fixing is independent of the ghost fields. So we generalize it here for $\mathcal{L}_{\rm eff}$ given by Eq. (23).

Consider the generating functional for Green's functions,

$$Z_\alpha[J^a_\mu,\eta^a,\bar\eta^a]=\int\mathcal{D}(A^a_\mu,C^a,\bar C^a)\exp\left[iS_{\rm eff}+i\int d^4x(\bar J\cdot A+\bar\eta C+\bar C\eta)\right]\ , \tag{24}$$

where J^a_μ, η^a, and $\bar\eta^a$ are the sources for the fields A^a_μ, $\bar C^a$, and C^a, respectively, and the subscript α reminds us of this α dependence. The BRS invariance of the integration measure and of $S_{\rm eff}$ gives

$$\int\mathcal{D}(A^a_\mu,C^a,\bar C^a)\int d^4y(J\cdot sA+\bar\eta^a sC^a-\mathcal{F}^a\eta^a)\exp[i(S_{\rm eff}+\text{source terms})]=0 \tag{25}$$

and taking a functional derivative with respect to $\eta^a(x)$ we obtain the Ward identity (we are assuming an anomaly-free theory)

$$\int \mathscr{D}(A_\mu^a,C^a,\bar{C}^a)\left[\mathscr{F}^a+i\bar{C}^a\int d^4y(J\cdot \mathscr{s}A+\bar{\eta}\cdot \mathscr{s}C-\mathscr{F}\eta)\right]\exp[i(S_{\text{eff}}+\text{source terms})]=0\ . \tag{26}$$

From Eq. (24) and for $\Delta\alpha$ infinitesimal we write

$$Z_{\alpha+\Delta\alpha}-Z_\alpha=i\Delta\alpha\int \mathscr{D}(A_\mu^a,C^a,\bar{C}^a)\int d^4x(-2\mathscr{F}^aG^a+gG^aG^a)\exp[i(S_{\text{eff}}+\text{source terms})]\ , \tag{27}$$

which, using Eq. (20), becomes

$$Z_{\alpha+\Delta\alpha}-Z_\alpha=i\Delta\alpha\int d^4x\, f^{abc}\frac{\delta}{i\delta\eta^b}\frac{\delta}{i\delta\bar{\eta}^c}\int \mathscr{D}(A,C,\bar{C})2\mathscr{F}^A\exp[i(S_{\text{eff}}+\text{sources})]$$
$$+i\Delta\alpha\int \mathscr{D}(A,C,\bar{C})\int d^4x\, gG^aG^a\exp[i(S_{\text{eff}}+\text{sources})]\ . \tag{28}$$

Applying Eq. (26) to the first term on the right-hand ride of Eq. (28) it is possible, after some algebra, to obtain

$$Z_{\alpha+\Delta\alpha}=\int \mathscr{D}(A,C,\bar{C})\exp\left[iS_{\text{eff}}+i\int d^4y(J^A\mathscr{A}_\mu^a+\bar{\eta}^a\mathscr{C}^a+\bar{\mathscr{C}}^a\eta^a)\right]\ , \tag{29}$$

with

$$\mathscr{A}_\mu^a(y)=A_\mu^a(y)-i\Delta\alpha\int d^4x\, \mathscr{s}A_\mu^a(y)\bar{C}^b(x)G^b(x)\ ,$$
$$\mathscr{C}^a(y)=C^a(y)-i\Delta\alpha\int d^4x\, \mathscr{s}C^a(y)\bar{C}^b(x)G^b(x)\ , \tag{30}$$
$$\bar{\mathscr{C}}^a(y)=\bar{C}(y)+i\Delta\alpha\int d^4x\, \mathscr{F}^a(y)\bar{C}^b(x)G^b(x)\ .$$

Equations (29) and (30) show that a change in α corresponds to a change in the source terms. Recalling the equivalence theorem[15] this is sufficient to prove that the renormalized S matrices calculated with Z_α and $Z_{\alpha+\Delta\alpha}$ are equal. Hence, the amplitude of any physical process is independent of α.

IV. $\mathscr{L}_{\text{eff}}$ FOR THE GWS MODEL

After the discussion of the previous section it should be clear how to proceed in the case of the GWS theory. Writing

$$\mathscr{L}_{\text{eff}}=\mathscr{L}_{\text{cl}}+\Delta\mathscr{L}\ , \tag{31}$$

it is easy to verify that Eqs. (1)–(3) are equivalent to

$$\Delta\mathscr{L}=\mathscr{s}\bar{K}_3 \tag{32}$$

with

$$\bar{K}_3=-\bar{C}_A(-\tfrac{1}{2}b_A+\mathscr{F}_A)-\bar{C}_Z(-\tfrac{1}{2}b_Z+\mathscr{F}_Z)$$
$$-\bar{C}^+(-\tfrac{1}{2}b^++\mathscr{F}_+)-\bar{C}^-(-\tfrac{1}{2}b^-+\mathscr{F}_-)\ . \tag{33}$$

This is the $\Delta\mathscr{L}$ obtained with the FP methods which, clearly, does not give the most general BRS-invariant action. Rather than writing a general $\bar{K}_3$ we would like to propose a minimal modification of Eq. (33) that will lead to a consistent theory. We base our result on the observation that for a linear condition no quartic ghost interactions are generated in perturbation theory. So, there is no need to modify the terms involving $\mathscr{F}_A$ and $\mathscr{F}_Z$. On the contrary, such modification is necessary for the other two terms, and instead of K_3 we write

$$\bar{K}'_3=-\bar{C}_A(-\tfrac{1}{2}b_A+\mathscr{F}_A)-\bar{C}_Z(-\tfrac{1}{2}b_Z+\mathscr{F}_Z)$$
$$-\bar{C}^+(-\tfrac{1}{2}b^++\mathscr{F}_++i\alpha\bar{C}^-C_A+i\beta\bar{C}^-C_Z)$$
$$-\bar{C}^-(-\tfrac{1}{2}b^-+\mathscr{F}_--i\alpha\bar{C}^+C_A-i\beta\bar{C}^+C_Z)\ . \tag{34}$$

This, in turn, gives

$$\Delta\mathscr{L}=\Delta\mathscr{L}_0+2i\partial^\mu W_\mu^+(\alpha\bar{C}^+C_A+\beta\bar{C}^+C_Z)-2i\partial^\mu W_\mu^-(\alpha\bar{C}^-C_A+\beta\bar{C}^-C_Z)+2M_W\phi^+(\alpha\bar{C}^+C_A+\beta\bar{C}^+C_Z)$$
$$+2M_W\phi^-(\alpha\bar{C}^-C_A+\beta\bar{C}^-C_Z)-2eA^\mu W_\mu^+(\alpha\bar{C}^+C_A+\beta\bar{C}^+C_Z)-2eA^\mu W_\mu^-(\alpha\bar{C}^-C_A+\beta\bar{C}^-C_Z)$$
$$+2g(\alpha\sin\theta_W-\beta\cos\theta_W)\bar{C}^+\bar{C}^-C^+C^-\ , \tag{35}$$

where $\Delta\mathscr{L}_0$ contains the couplings already shown in Fig. 1, and the new ones, depending on α or β, are explicitly given. In particular, notice the quartic coupling which is indispensable to absorb the divergency of T discussed in the second paragraph. In Fig. 3 we display the new vertices of the GWS model in the nonlinear gauge.

Obviously, our previous proof of the gauge independence of the S matrix is applicable to the present theory. Hence, physical quantities are independent of α and β. Nevertheless, if, from the beginning, we set $\alpha=\beta=0$ the renormalization program cannot be carried out. It is also fair to say that for physical processes, i.e., with physical particles in the external lines, at the one-loop level one can use $\Delta\mathscr{L}=\Delta\mathscr{L}_0$; i.e., one can effectively ignore the α and β terms. In fact the divergent quartic ghost box diagrams contribute to the ghost self-energies at two loops, which means that they are relevant for physical processes if the calculation is taken beyond two loops.

V. CONCLUSIONS

In this article we have considered in detail the perturbative expansion of the GWS model using a nonlinear

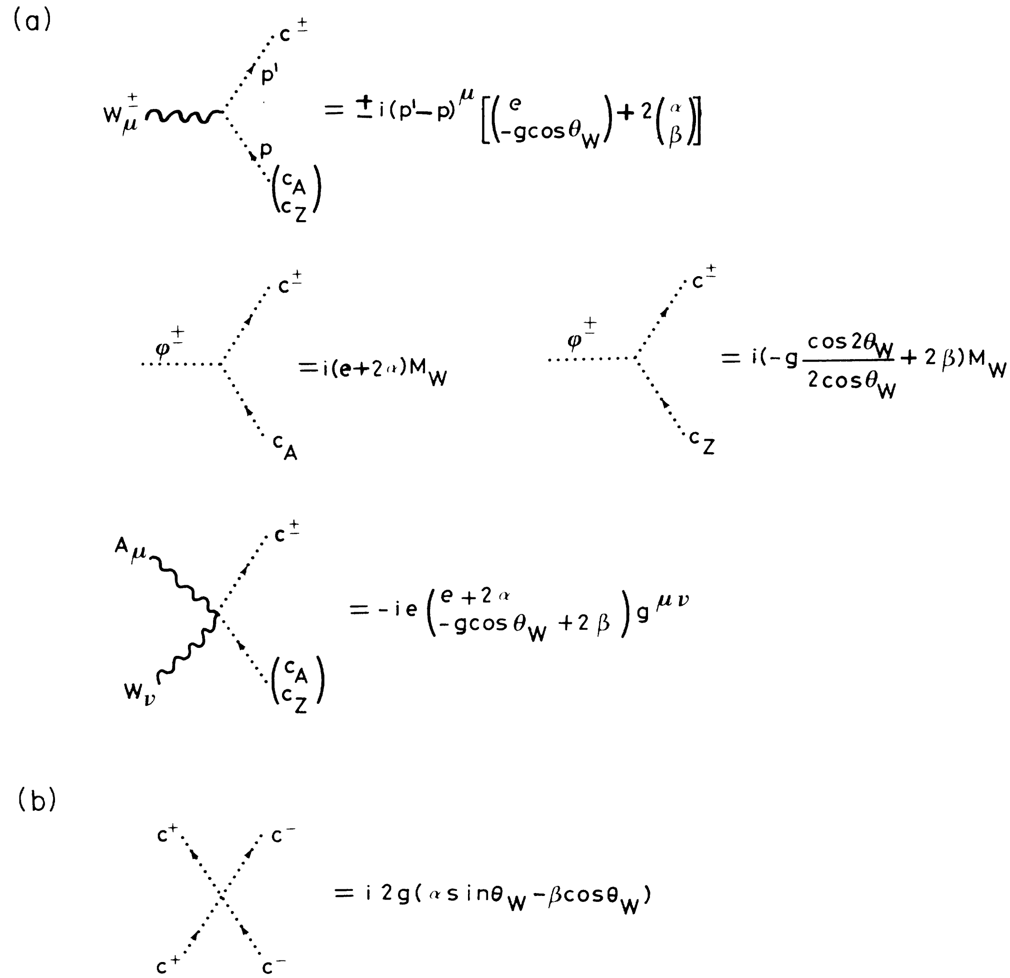


FIG. 3. Ghost vertices depending on the parameters α and β. Vertices of type (a) existed already but are now modified and vertex (b) is new.

gauge-fixing term.

At first sight it seems that this gauge simplifies most calculations because the vertices $A^{\mu}W_{\mu}^{\pm}\phi^{\pm}$ are not present. However, we have shown that the extra difficulties associated with the renormalization program and the existence of a more complicated ghost sector derogates some of the merits of $\mathscr{L}_{\text{gf}}^{\text{nl}}$.

On the other hand, the study of this problem reveals the need to generalize the usual FP approach. This is done promoting the BRS invariance to the level of guiding principle to be followed in order to establish the quantum action. We hope that our example will give further support to this Feynman[16]-Baulieu[11] method.

APPENDIX: THE SLAVNOV OPERATION

1. Action on the fields

The Slavnov operation has the following action on the fields:

$$
\begin{aligned}
&s A_{\mu}=\partial_{\mu}C_A+ie(C^{-}W_{\mu}^{+}-C^{+}W_{\mu}^{-})\ ,\\
&s Z_{\mu}=\partial_{\mu}C_Z-ig\cos\theta_W(C^{-}W_{\mu}^{+}-C^{+}W_{\mu}^{-})\ ,\\
&s W_{\mu}^{\pm}=\partial_{\mu}C^{\pm}\mp igC^{\pm}(\cos\theta_W Z_{\mu}-\sin\theta_W A_{\mu})\\
&\qquad\pm igW_{\mu}^{\pm}(\cos\theta_W C_Z-\sin\theta_W C_A)\ ,\\
&s\phi^{\pm}=ig\frac{\cos2\theta_W}{2\cos\theta_W}\phi^{+}C_Z-ie\phi^{+}C_A+iM_W C^{+}+i\frac{g}{2}HC^{+}\\
&\qquad-\frac{g}{2}\phi_Z C^{+}\ ,\\
&s\phi_Z=-\frac{g}{2\cos\theta_W}HC_Z-M_Z C_Z+\frac{g}{2}(C^{-}\phi^{+}+C^{+}\phi^{-})\ ,\\
&s H=\frac{g}{2\cos\theta_W}\phi_Z C_Z+i\frac{g}{2}(C^{-}\phi^{+}-C^{+}\phi^{-})\ ,\\
&s C_A=ieC^{+}C^{-},\quad s C_Z=-ig\cos\theta_W C^{+}C^{-}\ ,\\
&s C^{\pm}=\pm igC^{\pm}(\cos\theta_W C_Z-\sin\theta_W C_A)\ ,\\
&s\bar{C}_A=b_A,\quad s\bar{C}_Z=b_Z\ ,\\
&s\bar{C}^{\pm}=b^{\mp},\quad s b^{\pm}=s b_A=s b_Z=0\ .
\end{aligned}
\tag{A1}
$$

2. Properties

(a) If B and F are bosonic and fermionic fields the following properties hold:

$$\begin{aligned} &s(B_1B_2)=sB_1B_2+B_1sB_2\ ,\\ &s(BF)=-sBF+BsF\ ,\\ &s(FB)=sFB+FsB\ ,\\ &s(F_1F_2)=-sF_1F_2+F_1sF_2\ . \end{aligned} \tag{A2}$$

(b) The operator s is nilpotent. Using Eqs. (A1) and (A2) it is easy to verify that

$$s^2(\text{any field})=0\ . \tag{A3}$$

Notice that this is only true if we include the auxiliary b fields. After substituting the b fields by their equations of motion we have

$$s^2(\text{ghosts } \bar{C})\neq 0\ . \tag{A4}$$

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