ORDER α'^2 TERMS IN THE GRAVITATIONAL SECTOR OF STRING EFFECTIVE ACTIONS WITH THE INCLUSION OF THE DILATON FIELD

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We discuss field redefinition ambiguities in string effective actions at order α'^2, in the parameterization which is most suitable for comparison with the string amplitudes and the σ-model parameterization. We find that, when the dilaton field is included, the effective action can be most simply described by seven terms, whose coefficients can be fixed from the string S-matrix. We derive relations among the coefficients that appear in each of the above mentioned parameterizations.

1. Many attempts to understand string low-energy physics start from a low-energy effective action for the massless modes of the string [1–13]. In this letter, we investigate the gravitational sector of the effective action of closed string theories i.e. bosonic, heterotic and type II superstring.

The effective action is non-unique since local field redefinitions do not affect the S-matrix [8,10,13,14]. Hence a proper understanding of the structure of the effective action at any order requires necessarily a systematic study of the ambiguities generated by local redefinitions. This is already apparent at order α'; in particular, the coefficients of the $R^2_{\mu\nu}$ and R^2 terms are ambiguous and the fact that they are usually written in the Gauss–Bonnet combination is just a choice which renders the theory manifestly ghost-free. Actually, it has been shown in ref. [13] that the terms which lead to ghosts in the graviton propagator can always be removed by appropriate field redefinitions, to all orders in α'. This result has recently been extended to the case of general backgrounds ($g_{\mu\nu}$, $H_{\mu\nu\rho}$, ϕ) [15].

The study of ambiguities at order α', when the antisymmetric tensor and dilaton fields are included, has been done in refs. [14,16]. At order α'^2, the structure of the curvature cubed terms has already been discussed in ref. [17], with the result that there are only two independent R^3-invariants whose coefficients can be unambiguously fixed from the string S-matrix: $I_1=R^{\alpha\beta}{}_{\lambda\rho}R^{\lambda\rho}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta}$ and the "Gauss–Bonnet" combination $G_3\equiv I_1-2I_2$, with $I_2=R^{\mu}{}_{\nu\alpha\beta}R^{\nu\lambda\beta\gamma}R_{\lambda\mu\gamma}{}^{\alpha}$.

In this letter, we extend the analysis of the structure of the effective action at order α'^2 by including the dilaton field. The dependence of the action on the dilaton is non-unique; there is, in particular, a class of effective actions which have the following dependence on ϕ:

$$S=\int \mathrm{d}^D x\sqrt{g}\,\mathrm{e}^{-2\phi}\,\mathscr{L}(g_{\mu\nu}, R^{\lambda}{}_{\rho\mu\nu}, \mathrm{D}_\mu, \partial_\nu\phi)\ . \tag{1}$$

Here and throughout this paper, we use euclidean signature and the conventions

$$R^{\lambda}{}_{\mu\nu\rho}=\partial_\nu\Gamma^{\lambda}_{\mu\rho}-...,\quad R_{\mu\nu}=R^{\lambda}{}_{\mu\lambda\nu},\quad \mathrm{D}_\mu A^\lambda=\partial_\mu A^\lambda+\Gamma^{\lambda}_{\mu\nu}A^\nu\ . \tag{2}$$

The above form for S is most suitable for comparison with the σ-model and, following ref. [16], we call it hereafter the "σ-parameterization" of the effective action. This form is, however, not convenient for comparison

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with the string S-matrix since $g_{\mu\nu}$ and ϕ mix in the propagator. That does not occur in the "s-parameterization" of the action

$$S=\int \mathrm{d}^D x\sqrt{g}\,\mathrm{e}^{-\gamma\phi}\,\mathscr{L}(\mathrm{e}^{-\gamma\phi}g_{\mu\nu}, R^{\lambda}{}_{\mu\nu\rho}, \mathrm{D}_\mu, \partial_\nu\phi), \quad \gamma=-4/(D-2)\,, \tag{3}$$

which is obtained from the σ-parameterization by making a Weyl transformation: $g_{\mu\nu}\to \mathrm{e}^{-\gamma\phi}g_{\mu\nu}$.

We seek the simplest form of the action, after field redefinition ambiguities have been taken into account, in both these parameterizations. We find that in either case, the action can be written with a minimum of seven terms at order α'^2, whose coefficients can be fixed from the string amplitudes. Furthermore, these terms can be chosen such that the resulting theory is manifestly ghost-free. Finally, we find the relation between the two sets of coefficients i.e. the ones in the σ-parameterization and the ones in the s-parameterization.

2. Consider the most general action containing all possible independent dimension-six invariants involving curvatures and derivatives of the dilaton field. In the s-parameterization, this can be written as

$$\begin{aligned}
S^{(s)}=-\frac{2}{\kappa^2}\int \mathrm{d}^D x\sqrt{g}\{&R+\gamma(\partial\phi)^2+\alpha'\,\mathrm{e}^{\gamma\phi}\lambda_0\{R^2_{\lambda\mu\nu\rho}-4R^2_{\mu\nu}+R^2+[\gamma^2(D-4)/(D-2)](\partial\phi)^4\}\\
&+\alpha'^2\,\mathrm{e}^{2\gamma\phi}[a_1 I_1+a_2 G_3+a_3 R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma}R^{\mu\nu}+a_4 R_{\mu\nu\rho\lambda}R^{\nu\lambda}R^{\mu\rho}+a_5 R_{\mu\nu}R^{\nu\lambda}R^{\mu}{}_{\lambda}\\
&+a_6 R_{\mu\nu}\mathrm{D}^2R^{\mu\nu}+a_7 R^2_{\mu\nu\rho\sigma}R+a_8 R^2_{\mu\nu}R+a_9 R^3+a_{10}R\mathrm{D}^2R+b_1(\mathrm{D}^2\phi)^3\\
&+b_2(\mathrm{D}^2\phi)^2(\partial\phi)^2+b_3(\mathrm{D}^2\phi)(\partial\phi)^4+b_4(\partial\phi)^6+b_5\mathrm{D}^2(\mathrm{D}^2\phi)\mathrm{D}^2\phi+b_6\mathrm{D}^2(\mathrm{D}^2\phi)(\partial\phi)^2+b_7(\mathrm{D}_\mu\partial_\nu\phi)^2\mathrm{D}^2\phi\\
&+b_8(\mathrm{D}_\mu\partial_\nu\phi)^2(\partial\phi)^2+b_9(\mathrm{D}_\mu\partial_\nu\phi\mathrm{D}^\mu\partial^\rho\phi\mathrm{D}_\rho\partial^\nu\phi)+c_1R(\partial\phi)^4+c_2R(\partial\phi)^2\mathrm{D}^2\phi+c_3R(\mathrm{D}^2\phi)^2\\
&+c_4R\mathrm{D}^2(\mathrm{D}^2\phi)+c_5R(\mathrm{D}_\mu\partial_\nu\phi)^2+c_6R\mathrm{D}_\mu\partial_\nu\phi\partial^\mu\phi\partial^\nu\phi+c_7R\mathrm{D}^2(\partial_\mu\phi)\partial^\mu\phi+c_8R_{\alpha\beta}\partial^\alpha\phi\partial^\beta\phi\mathrm{D}^2\phi\\
&+c_9R_{\alpha\beta}\partial^\alpha\phi\partial^\beta\phi(\partial\phi)^2+c_{10}R_{\alpha\beta}\mathrm{D}^\alpha\partial^\beta\phi\mathrm{D}^2\phi+c_{11}R_{\alpha\beta}\mathrm{D}^\alpha\partial^\beta\phi(\partial\phi)^2+c_{12}R_{\alpha\beta}\mathrm{D}_\mu\partial^\alpha\phi\mathrm{D}^\mu\partial^\beta\phi\\
&+c_{13}R_{\alpha\beta\mu\nu}\mathrm{D}^\alpha\partial^\mu\phi\partial^\beta\phi\partial^\nu\phi+c_{14}R^2(\partial\phi)^2+c_{15}R^2\mathrm{D}^2\phi+c_{16}R^2_{\mu\nu}(\partial\phi)^2+c_{17}R^2_{\mu\nu}\mathrm{D}^2\phi+c_{18}RR_{\mu\nu}\partial^\mu\phi\partial^\nu\phi\\
&+c_{19}RR_{\mu\nu}\mathrm{D}^\mu\partial^\nu\phi+c_{20}R_{\mu\alpha}R^\alpha{}_\nu\partial^\mu\phi\partial^\nu\phi+c_{21}R_{\mu\alpha}R^\alpha{}_\nu\mathrm{D}^\mu\partial^\nu\phi+c_{22}R_{\mu\alpha\beta\nu}R^{\alpha\beta}\partial^\mu\phi\partial^\nu\phi\\
&+c_{23}R_{\mu\alpha\beta\nu}R^{\alpha\beta}\mathrm{D}^\mu\partial^\nu\phi+c_{24}R^2_{\mu\nu\alpha\beta}(\partial\phi)^2+c_{25}R^2_{\mu\nu\alpha\beta}\mathrm{D}^2\phi+c_{26}R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma}\partial^\mu\phi\partial^\nu\phi]+\mathrm{O}(\alpha'^3)\}\,,
\end{aligned} \tag{4}$$

where I_2 and G_3 have been defined in section 1 and $\kappa^2=32\pi G$.

The order α' part of the action has been found in ref. [16] by comparison with the string amplitudes and $\lambda_{0(\mathrm{B})}=\frac{1}{4}$, $\lambda_{0(\mathrm{H})}=\frac{1}{8}$, $\lambda_{0(\mathrm{S})}=0$, where the subscripts (B), (H) and (S) refer to the bosonic, heterotic and supersymmetric string, respectively.

The most general local field redefinitions are

$$\begin{aligned}
\delta g_{\mu\nu}=\alpha'(\dots)+\alpha'^2\,\mathrm{e}^{2\gamma\phi}\{&A_1R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma}+A_2R_{\mu\alpha\beta\nu}R^{\alpha\beta}+A_3R_{\mu\lambda}R^\lambda{}_\nu+A_4\mathrm{D}^2R_{\mu\nu}+A_5g_{\mu\nu}R^2_{\lambda\rho\alpha\beta}\\
&+A_6g_{\mu\nu}R^2_{\alpha\beta}+A_7g_{\mu\nu}R^2+A_8g_{\mu\nu}\mathrm{D}^2R+A_9RR_{\mu\nu}+B_1\mathrm{D}_\mu\partial_\nu\phi\mathrm{D}^2\phi+B_2\mathrm{D}_\alpha\partial_\mu\phi\mathrm{D}^\alpha\partial_\nu\phi\\
&+B_3\mathrm{D}_\mu\partial_\nu\phi(\partial\phi)^2+B_4\partial_\mu\phi\partial_\nu\phi\mathrm{D}^2\phi+B_5\partial_\mu\phi\partial_\nu\phi(\partial\phi)^2+B_6g_{\mu\nu}\mathrm{D}^2(\mathrm{D}^2\phi)+B_7g_{\mu\nu}\mathrm{D}^2(\partial_\alpha\phi)\partial^\alpha\phi\\
&+B_8g_{\mu\nu}(\mathrm{D}^2\phi)^2+B_9g_{\mu\nu}(\mathrm{D}_\alpha\partial_\beta\phi)^2+B_{10}g_{\mu\nu}\mathrm{D}^2\phi(\partial\phi)^2+B_{11}g_{\mu\nu}\mathrm{D}_\alpha\partial_\beta\phi\partial^\alpha\phi\partial^\beta\phi+B_{12}g_{\mu\nu}(\partial\phi)^4\\
&+C_1R_{\mu\alpha\beta\nu}\partial^\alpha\phi\partial^\beta\phi+C_2R_{\mu\alpha\beta\nu}\mathrm{D}^\alpha\partial^\beta\phi+C_3[R_{\mu\alpha}\mathrm{D}^\alpha\partial_\nu\phi+(\mu\leftrightarrow\nu)]+C_4[R_{\mu\alpha}\partial^\alpha\phi\partial_\nu\phi+(\mu\leftrightarrow\nu)]\\
&+C_5R_{\mu\nu}\mathrm{D}^2\phi+C_6R_{\mu\nu}(\partial\phi)^2+C_7\mathrm{D}_\alpha R_{\mu\nu}\partial^\alpha\phi+C_8g_{\mu\nu}R_{\alpha\beta}\mathrm{D}^\alpha\partial^\beta\phi+C_9g_{\mu\nu}R_{\alpha\beta}\partial^\alpha\phi\partial^\beta\phi\\
&+C_{10}g_{\mu\nu}R\mathrm{D}^2\phi+C_{11}g_{\mu\nu}R(\partial\phi)^2+C_{12}g_{\mu\nu}\partial_\alpha R\partial^\alpha\phi+C_{13}R\mathrm{D}_\mu\partial_\nu\phi+C_{14}R\partial_\mu\phi\partial_\nu\phi\}+\mathrm{O}(\alpha'^3)
\end{aligned} \tag{5}$$

and

$$\delta\phi=\alpha'(...)+\alpha'^2\,e^{2\gamma\phi}[A_{10}R^2_{\mu\alpha\beta\gamma}+A_{11}R^2_{\mu\nu}+A_{12}R^2+A_{13}D^2R+B_{13}D^2(D^2\phi)+B_{14}D^2(\partial_\mu\phi)\partial^\mu\phi$$
$$+B_{15}(D^2\phi)^2+B_{16}(D_\mu\partial_\nu\phi)^2+B_{17}(D^2\phi)(\partial\phi)^2+B_{18}D_\mu\partial_\nu\phi\partial^\mu\phi\partial^\nu\phi+B_{19}(\partial\phi)^4+C_{15}R_{\mu\nu}D^\mu\partial^\nu\phi$$
$$+C_{16}R_{\mu\nu}\partial^\mu\phi\partial^\nu\phi+C_{17}RD^2\phi+C_{18}R(\partial\phi)^2+C_{19}\partial_\mu R\partial^\mu\phi]+O(\alpha'^3)\,. \quad (6)$$

We have not included terms like $D^2(D_\mu\partial_\nu\phi)$ in (5) because $\delta g_{\mu\nu}=2BD^2(D_\mu\partial_\nu\phi)$, $\delta\phi=BD^2(\partial_\mu\phi)\partial^\mu\phi$ would correspond to a general coordinate transformation, with parameter $\xi_\mu=BD^2(\partial_\mu\phi)$ in this particular case, under which (4) is, of course, invariant. Other terms of this type are $D^2(\partial_\mu\phi)\partial_\nu\phi$, $D_\mu D_\nu\partial_\rho\phi\partial^\rho\phi$, $D_\mu R_{\nu\alpha}\partial^\alpha\phi+(\mu\leftrightarrow\nu)$, etc ...

In order to find the change in the action (4), at order α'^2, under the field redefinitions (5) and (6), we use

$$\delta S_2=(\delta S_0/\delta g_{\mu\nu})\,\delta g^{(2)}_{\mu\nu}+(\delta S_0/\delta\phi)\delta\phi^{(2)}\,, \quad (7)$$

where S_0 and S_2 are, respectively, the order α'^0 and α'^2 parts of the action (4) and $\delta g^{(2)}_{\mu\nu}$, $\delta\phi^{(2)}$ the order α'^2 parts of the field variations. Notice that we do not include in (7) the $(\delta S_1/\delta g_{\mu\nu})\delta g^{(1)}_{\mu\nu}+(\delta S_1/\delta\phi)\delta\phi^{(1)}$ variation; this is because we consider that the freedom contained in $\delta g^{(1)}_{\mu\nu}$ and $\delta\phi^{(1)}$ has already been used to put the action S_1 in the form exhibited in (4).

Using (7), and after integrations by parts, we find that, under the transformations (5) and (6), all the coefficients that appear in (4) at order α'^2 change, except a_1 and a_2. All coefficients but a_1 and a_2 are therefore ambiguous. This does not mean that they can all be set to zero by a proper choice of the parameters in (5), (6) since there are five combinations of the ambiguous coefficients that remain invariant under the field redefinitions; these are

$$-\gamma\delta a_3+\delta c_{26}=0\,,$$
$$-\gamma\delta a_7+\delta c_{24}=0\,,$$
$$-\delta b_9+\delta c_{13}-\gamma\delta c_{23}=0\,,$$
$$\delta b_8+2\gamma\delta b_9-\gamma\delta c_5+\tfrac{1}{2}\gamma\delta c_{12}+2\gamma^2\delta a_{10}-\gamma^2\delta a_6=0\,,$$
$$-\gamma^3\delta a_5+\gamma^3(\gamma-1)\delta a_6-\gamma^3\delta a_8-\gamma^3\delta a_9-2\gamma^3\delta a_{10}+\delta b_4-\gamma^2(\gamma+\tfrac{3}{2})\delta b_9-\gamma\delta c_1+\tfrac{1}{2}\gamma^2\delta c_6+\gamma^2\delta c_7-\gamma\delta c_9$$
$$+\tfrac{1}{2}\gamma^2\delta c_{11}-\tfrac{1}{2}\gamma^2(1+\gamma)\delta c_{12}+\gamma^2\delta c_{14}+\gamma^2\delta c_{16}+\gamma^2\delta c_{18}-\tfrac{1}{2}\gamma^3\delta c_{19}+\gamma^2\delta c_{20}-\tfrac{1}{2}\gamma^3\delta c_{21}=0\,, \quad (8)$$

where $\delta a_i=a'_i-a_i$, $\delta b_j=b'_j-b_j$, $\delta c_k=c'_k-c_k$ are the variations of a_i, b_j, c_k under the transformations (5), (6). Hence, only 38 of the 43 ambiguous coefficients in (4) can be transformed away by a proper choice of the 50 parameters in (5), (6). We choose the five coefficients that parameterize the effective action together with a_1 and a_2, to be b_8, b_9, c_{13}, c_{24} and c_{26}. We can then give arbitrary values to the remaining coefficients; in particular, we can set them to zero, thus getting the simplest representation of the effective action, which we call hereafter the "minimal scheme". Notice that there is a class of actions in the minimal scheme, depending on which coefficients in (8) we choose to parameterize the action. The choice we have made is based on the criterion that one may be able to calculate these coefficients by comparison with the four-point string amplitudes [18].

Hence, the effective action in the s-parameterization, at order α'^2, can be written, in the minimal scheme, as

$$S^{(s)}_2=-\frac{2}{\kappa^2}\int d^Dx\,\sqrt{g}\alpha'^2\,e^{2\gamma\phi}[\lambda_1I_1+\lambda_2G_3+\lambda_3(D_\mu\partial_\nu\phi)^2(\partial\phi)^2+\lambda_4D_\mu\partial_\nu\phi D^\mu\partial^\rho\phi D_\rho\partial^\nu\phi$$
$$+\lambda_5R_{\alpha\beta\mu\nu}D^\alpha\partial^\mu\phi\partial^\beta\phi\partial^\nu\phi+\lambda_6R^2_{\mu\nu\alpha\beta}(\partial\phi)^2+\lambda_7R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma}\partial^\mu\phi\partial^\nu\phi] \quad (9)$$

where $\lambda_1=a'_1$, $\lambda_2=a'_2$, $\lambda_3=b'_8$, $\lambda_4=b'_9$, $\lambda_5=c'_{13}$, $\lambda_6=c'_{24}$, $\lambda_7=c'_{26}$ can be uniquely fixed by comparison with the string S-matrix; λ_1 and λ_2 have already been found to be [17]

$$\lambda_{1(B)}=\tfrac{1}{48},\quad \lambda_{1(H)}=\lambda_{1(S)}=0,\quad \lambda_{2(B)}=\tfrac{1}{24},\quad \lambda_{2(H)}=\lambda_{2(S)}=0\,. \quad (10)$$

3. The above analysis can be repeated in the σ-parameterization, where

$$S^{(\sigma)}=-\frac{2}{\kappa^2}\int d^Dx\sqrt{g}\,e^{-2\phi}[R+4(\partial\phi)^2+\lambda_0\alpha'(R^2_{\mu\nu\alpha\beta}-4R^2_{\mu\nu}+R^2)$$
$$+\alpha'^2(\hat{a}_1I_1+\hat{a}_2G_3+...+\hat{c}_{26}R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma}\partial^\mu\phi\partial^\nu\phi)+O(\alpha'^3)]\,. \quad (11)$$

The order α' part of the action, S_1, has been found in ref. [16] (notice that there is no $(\partial\phi)^4$ term in this parameterization) and S_2 has the same structure as in (4).

Taking the field redefinitions (5), (6) and substituting in (7), we find that once again only $\hat{a}_1$ and $\hat{a}_2$ remain invariant and all other coefficients change; furthermore, there are also five combinations of the ambiguous coefficients that remain invariant:

$$-4\delta\hat{a}_7+\delta\hat{c}_{24}+2\delta\hat{c}_{25}=0\,,$$
$$2\delta\hat{a}_4-4\delta\hat{a}_6+\delta\hat{c}_{23}+\delta\hat{c}_{26}=0\,,$$
$$-8\delta\hat{a}_5+\delta\hat{b}_9-2\delta\hat{c}_{12}-\delta\hat{c}_{13}+4\delta\hat{c}_{21}-2\delta\hat{c}_{22}=0\,,$$
$$64\delta\hat{a}_9-8\delta\hat{b}_1-4\delta\hat{b}_2-2\delta\hat{b}_3-\delta\hat{b}_4+4\delta\hat{c}_1+8\delta\hat{c}_2+16\delta\hat{c}_3-16\delta\hat{c}_{14}-32\delta\hat{c}_{15}=0\,,$$
$$8\delta\hat{a}_6-16\delta\hat{a}_8+32\delta\hat{a}_{10}+8\delta\hat{b}_5+4\delta\hat{b}_6+2\delta\hat{b}_7+\delta\hat{b}_8-16\delta\hat{c}_4-4\delta\hat{c}_5-4\delta\hat{c}_{10}-2\delta\hat{c}_{11}$$
$$+4\delta\hat{c}_{16}+8\delta\hat{c}_{17}+8\delta\hat{c}_{19}-2\delta\hat{c}_{20}=0\,. \quad (12)$$

We choose $\hat{b}_8$, $\hat{b}_9$, $\hat{c}_1$, $\hat{c}_{24}$ and $\hat{c}_{26}$ to parameterize the effective action and set the remaining coefficients to zero. Except for $\hat{c}_1$, this is the very choice that we have made in the previous section for the s-parameterization; $\hat{c}_1$ is replacing c_{13}, which does not appear in (12) and therefore cannot be chosen.

Hence, the effective action at order α'^2, in the σ-parameterization, can be written, in the minimal scheme, as

$$S_2^{(\sigma)}=-\frac{2}{\kappa^2}\int d^Dx\sqrt{g}\alpha'^2\,e^{-2\phi}[\rho_1I_1+\rho_2G_3+\rho_3(D_\mu\partial_\nu\phi)^2(\partial\phi)^2+\rho_4D_\mu\partial_\nu\phi D^\mu\partial^\rho\phi D_\rho\partial^\nu\phi$$
$$+\rho_5R(\partial\phi)^4+\rho_6R^2_{\mu\nu\rho\sigma}(\partial\phi)^2+\rho_7R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma}\partial_\mu\phi\partial^\nu\phi]\,, \quad (13)$$

where $\rho_1=\hat{a}'_1$, $\rho_2=\hat{a}'_2$, $\rho_3=\hat{b}'_8$, $\rho_4=\hat{b}'_9$, $\rho_5=\hat{c}'_1$, $\rho_6=\hat{c}'_{24}$, $\rho_7=\hat{c}'_{26}$.

4. To find the relation between the coefficients $(\lambda_1,...,\lambda_7)$ and $(\rho_1,...,\rho_7)$, we perform a Weyl transformation [19]

$$g^{(s)}_{\mu\nu}=e^{-2\tau\phi}g^{(\sigma)}_{\mu\nu}\,,$$
$$\Gamma^{\lambda(s)}_{\mu\nu}=\Gamma^{\lambda(\sigma)}_{\mu\nu}-2\delta^\lambda{}_{(\mu}\xi_{\nu)}+g_{\mu\nu}\xi^\lambda\,,$$
$$R^{\lambda(s)}{}_{\mu\nu\rho}=R^{\lambda(\sigma)}{}_{\mu\nu\rho}+2\delta^\lambda{}_{[\nu}\sigma_{\rho]\mu}+2\sigma^\lambda{}_{[\nu}g_{\rho]\mu}-2\xi^2\delta^\lambda{}_{[\nu}g_{\rho]\mu}\,,$$
$$\xi_\mu=\tau\partial_\mu\phi,\quad \sigma_{\mu\nu}=D_\mu\xi_\nu+\xi_\mu\xi_\nu,\quad \tau=2/(D-2)\,, \quad (14)$$

in order to change the parameterization and then the scheme to finally obtain from the results of section 3, the minimal scheme of eq. (13).

Performing the Weyl transformation on (9) and integrating by parts, we find, after a considerable amount of algebra, that the action can indeed be written in the form (11), with the corresponding coefficients given by

$$\hat{a}_1 = \lambda_1 ,$$

$$\hat{a}_2 = \lambda_2 ,$$

$$\hat{a}_3 = \hat{a}_4 = \hat{a}_5 = \hat{a}_6 = \hat{a}_7 = \hat{a}_8 = \hat{a}_9 = \hat{a}_{10} = 0 ,$$

$$\hat{b}_1 = -4\tau^3\lambda_2 ,$$

$$\hat{b}_2 = -48\tau^4\lambda_1 + 6\tau^4(5D-22)\lambda_2 - \tfrac{1}{2}\tau\lambda_5 + 4\tau^2\lambda_6 - 2\tau^2\lambda_7 ,$$

$$\hat{b}_3 = 3\tau^4[10\tau(D-2)+22-D]\lambda_1 + 3\tau^4[-5\tau(D^2-7D-10) - \tfrac{23}{2}D + 53]\lambda_2 - 3\tau\lambda_3 + \tfrac{3}{4}\tau(4\tau-1)\lambda_4$$
$$+ \tfrac{1}{8}[2\tau(D-6)+11]\tau\lambda_5 + 10\tau^3(2-D)\lambda_6 + \tfrac{1}{4}\tau^2(22-D)\lambda_7 ,$$

$$\hat{b}_4 = 4\tau^4[\tau^2(-D^2+3D-2) - 3\tau(D-2) + \tfrac{3}{2}(D-6)]\lambda_1$$
$$+ \tau^4[2\tau^2(D^3-8D^2+17D-10) + 6\tau(D^2-7D+10) + 3(7D-26)]\lambda_2 + \tau(\tau D+2)\lambda_3$$
$$-\tau[\tau^2(D-2) - \tfrac{3}{2}]\lambda_4$$
$$+ \tfrac{1}{4}\tau[-2\tau(D-2)-3]\lambda_5 + 2\tau^3[\tau(D^2-3D+2) + 2(D-2)]\lambda_6 + \tfrac{1}{2}\tau^2(D-6)\lambda_7 ,$$

$$\hat{b}_5 = 0 ,$$

$$\hat{b}_6 = 3\tau^4(2-D)\lambda_1 - \tfrac{9}{2}\tau^4(D-2)\lambda_2 - \tfrac{3}{4}\tau\lambda_4 + \tfrac{1}{8}\tau\lambda_5 + \tfrac{1}{4}\tau^2(2-D)\lambda_7 ,$$

$$\hat{b}_7 = 3\tau^3[\tau(2-D)+8]\lambda_1 + \tfrac{3}{2}\tau^3[-3\tau(D-2) - 8(D-5)]\lambda_2 - \tfrac{3}{4}\tau\lambda_4 + \tfrac{1}{8}\tau\lambda_5 - \tfrac{1}{4}\tau^2(D-2)\lambda_7 ,$$

$$\hat{b}_8 = 24\tau^4(4-D)\lambda_1 + 6\tau^4(D^2-14D+36)\lambda_2 + \lambda_3 - 3\tau\lambda_4 + \tfrac{3}{2}\tau\lambda_5 + 4\tau^2(D-2)\lambda_6 + 4\tau^2\lambda_7 ,$$

$$\hat{b}_9 = 2\tau^3[3\tau(D-2) + 4(D-4)]\lambda_1 + \tau^3[9\tau(D-2) + 4(5D-16)]\lambda_2 + (1+\tfrac{3}{2}\tau)\lambda_4 - \tfrac{1}{4}\tau\lambda_5 + \tfrac{1}{2}\tau^2(D-2)\lambda_7 ,$$

$$\hat{c}_1 = -3\tau^2[\tau^2(D-6) + 8\tau + 16]\lambda_1 + \tfrac{3}{2}\tau^2[\tau^2(26-7D) - 32\tau + 8(D-8)]\lambda_2 - \tfrac{3}{4}\tau\lambda_4 + \tfrac{1}{8}\tau\lambda_5$$
$$-4\tau^2\lambda_6 + \tfrac{1}{4}\tau[\tau(-D+2) - 8]\lambda_7 ,$$

$$\hat{c}_2 = \tfrac{3}{2}\tau^2[(D-2)\tau^2 + 8\tau + 32]\lambda_1 + \tfrac{3}{8}\tau^2[6\tau^2(D-2) + 96\tau + 32(8-D)]\lambda_2 + \tfrac{3}{8}\tau\lambda_4 - \tfrac{1}{16}\tau\lambda_5 + \tfrac{1}{8}\tau[\tau(D-2)+8]\lambda_7 ,$$

$$\hat{c}_3 = -12\tau^2\lambda_1 + 3\tau^2(D-8)\lambda_2 ,$$

$$\hat{c}_4 = 0 ,$$

$$\hat{c}_5 = 12\tau^2\lambda_1 + 3\tau^2(6-D)\lambda_2 ,$$

$$\hat{c}_6 = 3\tau^2[\tau^2(D-2) + 8\tau + 8]\lambda_1 + \tfrac{3}{2}\tau^2[3(D-2)\tau^2 + 24\tau + 4(8-D)]\lambda_2 + \tfrac{3}{4}\tau\lambda_4 - \tfrac{1}{8}\tau\lambda_5 + \tfrac{1}{4}\tau[\tau(D-2)+8]\lambda_7 ,$$

$$\hat{c}_7 = 0 ,$$

$$\hat{c}_8 = -48\tau^2\lambda_1 - 12\tau^2(\tau - D + 8)\lambda_2 ,$$

$$\hat{c}_9 = 48\tau^2(\tau+2)\lambda_1 + 24\tau^2(4\tau - D + 8)\lambda_2 - \tfrac{1}{2}\tau\lambda_5 + 8\tau^2\lambda_6 + 2\tau(\tau+2)\lambda_7 ,$$

$$\hat{c}_{10} = 48\tau^2\lambda_1 + 12\tau^2(7-D)\lambda_2 ,$$

$$\hat{c}_{11} = 3\tau^2[-\tau^2(D-2) - 24\tau - 32]\lambda_1 + \tfrac{3}{2}\tau^2[-3\tau^2(D-2) + 8\tau(D-13) + 16(D-8)]\lambda_2 - \tfrac{3}{4}\tau\lambda_4$$
$$+ \tfrac{1}{8}\tau\lambda_5 + 8\tau\lambda_6 + \tfrac{1}{4}\tau[(2-D)\tau - 8]\lambda_7 ,$$

$$\hat{c}_{12} = -24\tau^2\lambda_1 + 12\tau^2(D-4)\lambda_2 ,$$

$$\hat{c}_{13} = 6\tau^2[\tau^2(D-2) + 8(\tau+1)]\lambda_1 + 3\tau^2[3\tau^2(D-2) - 4\tau(D-8) - 4(D-8)]\lambda_2 + \tfrac{3}{2}\tau\lambda_4 + (1 - \tfrac{1}{4}\tau)\lambda_5$$
$$+ \tfrac{1}{2}\tau[\tau(D-2)+8]\lambda_7 , \qquad (15)$$

$$\hat{c}_{14}=-6\tau\lambda_1-9\tau\lambda_2\,,$$

$$\hat{c}_{15}=3\tau\lambda_1+\tfrac{9}{2}\tau\lambda_2\,,$$

$$\hat{c}_{16}=24\tau\lambda_1+6\tau(\tau+6)\lambda_2\,,$$

$$\hat{c}_{17}=-12\tau\lambda_1-18\tau\lambda_2\,,$$

$$\hat{c}_{18}=12\tau(\tau+2)\lambda_1+3\tau[-\tau(D-8)+12]\lambda_2\,,$$

$$\hat{c}_{19}=-12\tau\lambda_1-18\tau\lambda_2\,,$$

$$\hat{c}_{20}=-24\tau(\tau+2)\lambda_1+6\tau[\tau(D-8)-12]\lambda_2\,,$$

$$\hat{c}_{21}=24\tau\lambda_1+36\tau\lambda_2\,,$$

$$\hat{c}_{22}=24\tau(\tau+2)\lambda_1+6\tau[-\tau(D-10)+12]\lambda_2\,,$$

$$\hat{c}_{23}=-24\tau(\lambda_1+\lambda_2)\,,$$

$$\hat{c}_{24}=-6\tau(\tau+1)\lambda_1-9\tau(\tau+1)\lambda_2+\lambda_6\,,$$

$$\hat{c}_{25}=3\tau\lambda_1+\tfrac{9}{2}\tau\lambda_2\,,$$

$$\hat{c}_{26}=24\tau(\tau+1)\lambda_1+[(42-3D)\tau^2+36\tau]\lambda_2+\lambda_7\,. \qquad (15\text{cont'd})$$

Finally, we change to the minimal scheme of eq. (13). To fix their variations we use the fact that, in this scheme, all coefficients (except those appearing in (13)) are transformed away by field redefinitions:

$$\delta\hat{a}_i=\hat{a}'_i-\hat{a}_i=-\hat{a}_i \quad (i=3,...,10), \quad \delta\hat{b}_j=\hat{b}'_j-\hat{b}_j=-\hat{b}_j \quad (j=1,...,7),$$

$$\delta\hat{c}_k=\hat{c}'_k-\hat{c}_k=-\hat{c}_k \quad (k=2,...,23,25) \qquad (16)$$

with the $\hat{a}_i$, $\hat{b}_j$, $\hat{c}_k$ given by eqs. (15).

Regarding the coefficients which appear in (13), ρ_1 and ρ_2 can immediately be found using (15) and the fact that $\hat{a}_1$ and $\hat{a}_2$ remain invariant under the field redefinitions

$$\rho_1=\delta\hat{a}_1+\hat{a}_1=\lambda_1\,, \quad \rho_2=\delta\hat{a}_2+\hat{a}_2=\lambda_2\,. \qquad (17)$$

As to the remaining coefficients, their variation is fixed by (12) and (16) and they can be found with the help of (15). We get:

$$\rho_3=12\tau^3[\tau(10-3D)+16]\lambda_1+6\tau^2[\tau^2(D^2-17D+42)+8\tau(9-D)+16]\lambda_2+\lambda_3-6\tau\lambda_4+2\tau\lambda_5$$
$$+4\tau[\tau(D-2)-4]\lambda_6+\tau[\tau(6-D)+4]\lambda_7\,,$$

$$\rho_4=8\tau^2[\tau(D-10)-6]\lambda_1+8\tau^2[4\tau(D-5)-15]\lambda_2+\lambda_4-\lambda_5-4\tau\lambda_7\,,$$

$$\rho_5=\tau^4[\tau^2(D^2-3D+2)-12\tau(D-2)+36]\lambda_1$$
$$+\tfrac{1}{2}\tau^3[\tau^3(-D^3+8D^2-17D+10)+12\tau^2(D^2-7D+10)-12\tau(4D-17)+64]\lambda_2$$
$$-\tfrac{1}{4}\tau(\tau D-4)\lambda_3+\tfrac{1}{4}\tau^2[\tau(D-2)-6]\lambda_4+\tfrac{1}{2}\tau^2\lambda_5+\tfrac{1}{2}\tau^2[\tau^2(-D^2+3D-2)+8\tau(D-2)-16]\lambda_6\,,$$

$$\rho_6=-6\tau^2\lambda_1-9\tau^2\lambda_2+\lambda_6\,,$$

$$\rho_7=24\tau^2\lambda_1+[(42-3D)\tau^2+12\tau]\lambda_2+\lambda_7\,. \qquad (18)$$

5. Hence, we have shown that, in both the s- and σ-parameterizations, the effective action, at order α'^2, can be described with a minimum of seven terms, when the dilaton field is included. Furthermore, it is possible to

choose these terms such that there are no corrections to the standard propagators and therefore the theory is manifestly unitary, in agreement with the general result of ref. [15]. We have derived the relations between the coefficients of these terms in both parameterizations so that, once the s-parameterization coefficients are found by comparison of the amplitudes generated by (9) with the string amplitudes [18], it is straightforward to get the σ-parameterization coefficients using eq. (18). This result would allow a direct comparison with the corresponding σ-model computations [20] and thereby examine the equivalence of the string equations of motion and the σ-model Weyl invariance conditions at order α'^2 [21].

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