On the interdependence of the structure of string effective actions at different orders in α'

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We discuss the interplay between field redefinition ambiguities at orders α' and α'^2 in the gravitational sector of closed string theory effective actions for the case of a non-constant dilaton field. We show that, as a consequence, apparently different effective actions for this sector existing in the literature are, in fact, equivalent and discuss the implications of our results for the conjectured relationship between σ-model β-functions and string equations of motion.

The structure of the gravitational sector of low-energy closed string effective actions (EA), namely the bosonic, heterotic and type II superstrings, has been extensively examined in the literature [1]; in this letter, we shall be particularly interested in the case where there is a non-constant dilaton field. The structure of the EA for this sector has been studied up to order α'^2, both in the string S-matrix and non-linear σ-model approaches. These approaches are connected through the assumption that the equations of motion for the massless string modes are equivalent to the σ-model Weyl invariance conditions, which is expected to hold at all orders in α' [2]. The explicit verification of this equivalence at order α'^2, when the dilaton field in included, was first analysed by the authors of ref. [3]. However, these authors, while using the relevant σ-model β-functions of ref. [4], assume a form for the EA at order α'^2 which does not seem to be corroborated by the explicit calculation of ref. [5], in the context of the S-matrix approach, which seems to put their proof into question. More explicitly, the authors of ref. [3] discard terms involving derivatives of the dilaton in the order α'^2 part of the EA, whereas the authors of ref. [5] find such terms essential to match the amplitudes generated by such an action with the string amplitudes.

In this letter we show that this discrepancy is due to the fact that these references start with different actions at order α' and that, as far as can be indicated by the four-point amplitudes (there may be an extra term at order α'^2 whose coefficient remains undetermined by the four-point amplitudes) the EA found in ref. [5], hereafter referred to as scheme I, can be brought to the form assumed in ref. [3] (scheme II) through field redefinitions. This result is a consequence of the interplay between field redefinition ambiguities at different orders in α' and, in particular, of the fact that the way one chooses to fix them at order α' affects the order α'^2 results.

We would like to stress that, although it is possible to remove terms with derivative of the dilaton at order α'^2, this is at the expense of introducing them at order α' and therefore it is not possible to remove this type of terms altogether i.e. at order α' and α'^2 simultaneously.

Explicitly, ref. [3] assumes the only modification necessary to incorporate the effect of a non-constant dilaton in relation to the case of a purely gravitational background, studied in ref. [6], is the insertion of the exponential of the dilaton:

$$S_2^{(\mathrm{II})} = -\frac{2}{\kappa^2}\int \mathrm{d}^D x\,\sqrt{g}\,\alpha'^2\,\mathrm{e}^{-2\phi}\,(\tfrac{1}{48}I_1 + \tfrac{1}{24}G_3)\,, \tag{1}$$

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where $\kappa^2=32\pi G$, G_3 is related to the curvature-cubed Euler invariant Ω_3:

$$\Omega_3=G_3+\text{Ricci terms},\quad G_3=I_1-2I_2\,,$$
$$I_1=R_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta}{}^{\gamma\sigma}R_{\gamma\sigma}{}^{\mu\nu},\quad I_2=R_{\mu\nu}{}^{\alpha\beta}R^{\nu}{}_{\gamma\beta\sigma}R^{\gamma\mu\sigma}{}_{\alpha}\,, \tag{2}$$

and the notation $S_2^{(\mathrm{II})}$ refers to the order α'^2 part of the EA in scheme II.

On the order hand, in ref. [5] it is found that there are extra terms involving derivatives of the dilaton [#1] (scheme I):

$$S_2^{(\mathrm{I})}=-\frac{2}{\kappa^2}\int \mathrm{d}^Dx\,\sqrt{g}\,\alpha'^2\,\mathrm{e}^{-2\phi}[\rho_1I_1+\rho_2G_3+\rho_3(\mathrm{D}_\mu\partial_\nu\phi)^2(\partial\phi)^2+\rho_4R(\partial\phi)^4$$
$$+\rho_5R_{\alpha\beta\mu\nu}\,\mathrm{D}^\alpha\partial^\mu\phi\,\partial^\beta\phi\,\partial^\nu\phi+\rho_6R^2_{\mu\nu\rho\sigma}(\partial\phi)^2+\rho_7R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma}\,\partial_\mu\phi\,\partial^\nu\phi]\,, \tag{3}$$

with

$$\rho_1^{(\mathrm{B})}=\tfrac{1}{48},\quad \rho_2^{(\mathrm{B})}=\tfrac{1}{24},\qquad \rho_1^{(\mathrm{H})}=0,\quad \rho_2^{(\mathrm{H})}=0\,,$$
$$\rho_3^{(\mathrm{B})}=-8\frac{(D-3)(D-6)}{(D-2)^4},\quad \rho_5^{(\mathrm{B})}=0,\qquad \rho_3^{(\mathrm{H})}=-2\frac{(D-3)(D-6)}{(D-2)^4},\quad \rho_5^{(\mathrm{H})}=0\,, \tag{4}$$
$$\rho_6^{(\mathrm{B})}=\frac{1}{2}\frac{D-6}{(D-2)^2},\quad \rho_7^{(\mathrm{B})}=-\frac{2(D-4)}{(D-2)^2},\quad \rho_6^{(\mathrm{H})}=\frac{1}{8}\frac{D-6}{(D-2)^2},\quad \rho_7^{(\mathrm{H})}=-\frac{1}{2}\frac{D-4}{(D-2)^2},$$

and $\rho_i{}^{(\mathrm{S})}=0$, where (B), (H) and (S) refer to the bosonic, heterotic and supersymmetric strings respectively.

The point we wish to make in this letter is that one cannot directly compare (1) and (3) because these were established assuming different forms for the EA at order α', namely

$$S_1^{(\mathrm{I})}=-\frac{2}{\kappa^2}\int \mathrm{d}^Dx\,\sqrt{g}\,\mathrm{e}^{-2\phi}\,\lambda_0\alpha'\,(R^2_{\mu\nu\alpha\beta}-4R^2_{\mu\nu}+R^2)\,, \tag{5}$$

and

$$S_1^{(\mathrm{II})}=-\frac{2}{\kappa^2}\int \mathrm{d}^Dx\,\sqrt{g}\,\alpha'\,\mathrm{e}^{-2\phi}\,\lambda_0[G_2-16R_{\mu\nu}\,\partial^\mu\phi\,\partial^\nu\phi+8R(\partial\phi)^2+16(\mathrm{D}^2\phi)(\partial\phi)^2-16(\partial\phi)^4] \tag{6}$$

where $G_2=R^2_{\mu\nu\alpha\beta}-4R^2_{\mu\nu}+R^2$ and λ_0 is fixed by the three-point amplitudes to be $\frac{1}{4}$, $\frac{1}{8}$ and 0 for the bosonic, heterotic and supersymmetric strings respectively.

In the following, we make a field redefinition analysis to determine whether the two schemes are equivalent or not. We start with the EA in its most general form [7–9]:

[#1] The term with coefficient ρ_4 chosen by the authors of ref. [5] is $\mathrm{D}_\mu\partial_\nu\phi\,\mathrm{D}^\mu\partial^\rho\phi\,\mathrm{D}_\rho\partial^\nu\phi$; however, this term should not be chosen since it is not independent from the others and therefore should not even be in the action they start with [see their eq. (4)]. On the other hand, our choice of a term that can only be determined by higher-point amplitudes [$R(\partial\phi)^4$] is related with the fact that it seems that the coefficients of any of the other possible choices of terms that would contribute to four-point amplitudes cannot, however, be determined by the four-point string amplitudes due to tricky cancellations among the various contributions involved in each case (see ref. [7] for details).

$$S=-\frac{2}{\kappa^2}\int \mathrm{d}^Dx\,\sqrt{g}\,\mathrm{e}^{-2\phi}\{R+4(\partial\phi)^2+\alpha'\lambda_0[R^2_{\lambda\mu\nu\rho}+a_1R^2_{\mu\nu}+a_2R^2$$
$$+b_1R_{\mu\nu}\,\partial^\mu\phi\,\partial^\nu\phi+b_2R(\partial\phi)^2+b_3R(\mathrm{D}^2\phi)+c_1(\mathrm{D}^2\phi)^2+c_2(\mathrm{D}^2\phi)(\partial\phi)^2+c_3(\partial\phi)^4]$$
$$+\alpha'^2[d_1I_1+d_2G_3+d_3R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma}R^{\mu\nu}+d_4R_{\mu\nu\rho\lambda}R^{\nu\lambda}R^{\mu\rho}+d_5R_{\mu\nu}R^{\nu\lambda}R^\mu{}_\lambda+d_6R_{\mu\nu}\,\mathrm{D}^2R^{\mu\nu}+d_7R^2_{\lambda\mu\nu\rho}R$$
$$+d_8R^2_{\mu\nu}R+d_9R^3+d_{10}R\,\mathrm{D}^2R+e_1(\mathrm{D}^2\phi)^3+e_2(\mathrm{D}^2\phi)^2(\partial\phi)^2+e_3(\mathrm{D}^2\phi)(\partial\phi)^4+e_4(\partial\phi)^6+e_5\mathrm{D}^2(\mathrm{D}^2\phi)\mathrm{D}^2\phi$$
$$+e_6\mathrm{D}^2(\mathrm{D}^2\phi)(\partial\phi)^2+e_7(\mathrm{D}_\mu\partial_\nu\phi)^2\mathrm{D}^2\phi+e_8(\mathrm{D}_\mu\partial_\nu\phi)^2(\partial\phi)^2+f_1R(\partial\phi)^4+f_2R(\partial\phi)^2\mathrm{D}^2\phi+f_3R(\mathrm{D}^2\phi)^2$$
$$+f_4R\,\mathrm{D}^2(\mathrm{D}^2\phi)+f_5R(\mathrm{D}_\mu\partial_\nu\phi)^2+f_6R\,\mathrm{D}_\mu\partial_\nu\phi\,\partial^\mu\phi\,\partial^\nu\phi+f_7R\,\mathrm{D}^2(\partial_\mu\phi)\partial^\mu\phi+f_8R_{\alpha\beta}\,\partial^\alpha\phi\,\partial^\beta\phi\,\mathrm{D}^2\phi$$
$$+f_9R_{\alpha\beta}\,\partial^\alpha\phi\,\partial^\beta\phi(\partial\phi)^2+f_{10}R_{\alpha\beta}\,\mathrm{D}^\alpha\partial^\beta\phi\,\mathrm{D}^2\phi+f_{11}R_{\alpha\beta}\,\mathrm{D}^\alpha\partial^\beta\phi\,(\partial\phi)^2+f_{12}R_{\alpha\beta}\,\mathrm{D}_\mu\partial^\alpha\phi\,\mathrm{D}^\mu\partial^\beta\phi$$
$$+f_{13}R_{\alpha\beta\mu\nu}\,\mathrm{D}^\alpha\partial^\mu\phi\,\partial^\beta\phi\,\partial^\nu\phi+f_{14}R^2(\partial\phi)^2+f_{15}R^2\,\mathrm{D}^2\phi+f_{16}R^2_{\mu\nu}(\partial\phi)^2+f_{17}R^2_{\mu\nu}\,\mathrm{D}^2\phi+f_{18}RR_{\mu\nu}\,\partial^\mu\phi\,\partial^\nu\phi$$
$$+f_{19}RR_{\mu\nu}\,\mathrm{D}^\mu\partial^\nu\phi+f_{20}R_{\mu\alpha}R^\alpha{}_\nu\,\partial^\mu\phi\,\partial^\nu\phi+f_{21}R_{\mu\alpha}R^\alpha{}_\nu\,\mathrm{D}^\mu\partial^\nu\phi+f_{22}R_{\mu\alpha\beta\nu}R^{\alpha\beta}\,\partial^\mu\phi\,\partial^\nu\phi$$
$$+f_{23}R_{\mu\alpha\beta\nu}R^{\alpha\beta}\,\mathrm{D}^\mu\partial^\nu\phi+f_{24}R^2_{\lambda\mu\nu\rho}(\partial\phi)^2+f_{25}R^2_{\lambda\mu\nu\rho}\,\mathrm{D}^2\phi+f_{26}R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma}\,\partial^\mu\phi\,\partial^\nu\phi]+\mathrm{O}(\alpha'^3)\}\,. \quad (7)$$

Notice that, in eq. (7), we have made a particular choice of the independent dimension four and six invariants. This is the same choice as in refs. [8,9] respectively for the order α' and α'^2 parts of the action, except for the $\mathrm{D}_\mu\partial_\nu\phi\,\mathrm{D}^\mu\partial^\rho\phi\,\mathrm{D}_\rho\partial^\nu\phi$ term (see footnote 1).

Next, we need to compute the variation of the EA at order α'^2 under the field redefinitions i.e.

$$\delta S_2=\frac{\delta S_0}{\delta g_{\mu\nu}}\,\delta g^{(2)}_{\mu\nu}+\frac{\delta S_0}{\delta\phi}\,\delta\phi^{(2)}+S_0(\delta g^{(1)}_{\mu\nu},\delta\phi^{(1)})+\frac{\delta S_1}{\delta g_{\mu\nu}}\,\delta g^{(1)}_{\mu\nu}+\frac{\delta S_1}{\delta\phi}\,\delta\phi^{(1)}\,, \quad (8)$$

where S_0 and S_1 are, respectively, the order α'^0 and α' parts of the action (7) and $\delta g^{(2)}_{\mu\nu}$, $\delta\phi^{(2)}$ $(\delta g^{(1)}_{\mu\nu}, \delta\phi^{(1)})$ are the order $\alpha'^2(\alpha')$ parts of the field variations (for explicit expressions see ref. [7]).

If, as in refs. [5,9], we consider that the freedom contained in $\delta g^{(1)}_{\mu\nu}$ and $\delta\phi^{(1)}$ has already been used to put S_1 in the form exhibited in (5), we only have to calculate the contributions of the first two terms in (8) and we eventually arrive at the form (3) for S_2 (ref. [9]). However, since we want to examine the trade between order α' and α'^2 terms, we clearly have to relax this assumption, and take the contribution of all terms in (8). We then find that all coefficients in S_2 change under the field redefinitions, except for d_1 and d_2; however, this does not mean that all the remaining coefficients can be transformed away because there are five relations among their variations:

$$-4\delta d_7+\delta f_{24}+2\delta f_{25}+\tfrac{1}{2}\lambda_0^2(\delta b_1-6\delta b_2+\delta c_2)=0\,,$$
$$2\delta d_4-4\delta d_6+\delta f_{23}+\delta f_{26}-2\lambda_0^2\,\delta b_1=0\,,$$
$$-8\delta d_5-2\delta f_{12}-\delta f_{13}+4\delta f_{21}-2\delta f_{22}=0\,,$$
$$64\delta d_9-8\delta e_1-4\delta e_2-2\delta e_3-\delta e_4+4\delta f_1+8\delta f_2+16\delta f_3-16\delta f_{14}-32\delta f_{15}$$
$$+\tfrac{1}{2}\lambda_0^2[(2c_2+c_3-4b_2+16)\delta c_2+(-6c_2-3c_3+12b_2-48)\delta b_1+(-12c_2-6c_3+24b_2-96)\delta b_2]=0\,,$$
$$8\delta d_6-16\delta d_8+32\delta d_{10}+8\delta e_5+4\delta e_6+2\delta e_7+\delta e_8-16\delta f_4-4\delta f_5-4\delta f_{10}-2\delta f_{11}+4\delta f_{16}+8\delta f_{17}+8\delta f_{19}-2\delta f_{20}$$
$$+\tfrac{1}{8}\lambda 20\{[-8(D-2)\delta b2+2(D-4)\delta c2+2(D-4)c2$$
$$-8(D-2)b_2+2(D-10)b_1+(D-10)\delta b_1+64(D-5)]\delta b_1$$
$$+[-8(D-2)b_1-8(D-1)\delta c_2-8(D-1)c_2+32(D-1)b_2+16(D-1)\delta b_2-128(2D-3)]\delta b_2$$
$$+[2(D-4)b_1+(D-2)\delta c_2+2(D-2)c_2-8(D-1)b_2+64(D-2)]\delta c_2\}=0\,. \quad (9)$$

Hence, we can set to zero all the ambiguous coefficients but five, which the authors of ref. [5] choose to be e_8, f_1, f_{24} and f_{26} [see eq. (3)]. Notice that relations (9) involve the order α' as well as the order α'^2 coefficient variations, implying that there is an interdependence between the structure of the EA at orders α' and α'^2. We now assume that the above field redefinition is the one that transforms the order α' part of the EA from scheme I to scheme II and then determined whether the corresponding change in the order α'^2 part of the action leads to scheme II as well. Notice that, in fact, we should take for $S_1^{(1)}$ the action as it was before the field redefinition ambiguities were fixed at order α'; this is given by [7]

$$S_1^{(1)} = -\frac{2}{\kappa^2}\int \mathrm{d}^D x\,\sqrt{g}\,\alpha'\,\mathrm{e}^{-2\phi}\lambda_0\bigg(G_2 - 16D\frac{D-3}{(D-2)^2}R_{\mu\nu}\,\partial^\mu\phi\,\partial^\nu\phi + 8D\frac{D-3}{(D-2)^2}R(\partial\phi)^2$$
$$+16\frac{D^2-D-6}{(D-2)^2}\mathrm{D}^2\phi(\partial\phi)^2 + 16\frac{-D^3+2D^2+11D-28}{(D-2)^3}(\partial\phi)^4\bigg). \tag{10}$$

In fact, in (9), we have already used the constraints that, in scheme I

$$a_1=-4,\quad a_2=1,\quad b_3=c_1=0\,, \tag{11}$$

and that, in changing from scheme I to scheme II: $\delta a_1=\delta a_2=\delta b_3=\delta c_1=0$. We now enforce the remaining conditions, namely:

$$\delta b_1 = b_{1(\mathrm{II})}-b_{1(\mathrm{I})} = 16\frac{D-4}{(D-2)^2},\quad \delta b_2 = b_{2(\mathrm{II})}-b_{2(\mathrm{I})} = -8\frac{D-4}{(D-2)^2},$$
$$\delta c_2 = c_{2(\mathrm{II})}-c_{2(\mathrm{I})} = -16\frac{3D-10}{(D-2)^2}, \tag{12}$$

and find the values of the order α'^2 coefficients entailed by these field redefinitions. For instance

$$\rho_7' = \rho_{7(\mathrm{I})} + \delta\rho_7\,, \tag{13}$$

where $\rho_{7(\mathrm{I})}$ is given by eq. (4) and $\delta\rho_7=\delta f_{26}$ can be found from the second relation in eq. (9):

$$\delta\rho_7 = \delta f_{26} = -2\delta d_4 + 4\delta d_6 - \delta f_{23} + 2\lambda_0^2\,\delta b_1\,. \tag{14}$$

Since $\delta d_4=\delta d_6=\delta f_{23}=0$, we get

$$\delta\rho_7 = 2\frac{D-4}{(D-2)^2} \;\Rightarrow\; \rho_7'=0\,, \tag{15}$$

which is indeed the value of ρ_7 in scheme II. Repeating the same procedure for the remaining coefficients, we obtain

$$6\rho_3' = \rho_5' = \rho_6' = \rho_7' = 0\,, \tag{16}$$

which are also the values of these coefficients in scheme II [cf. eq. (1)]. Notice, however, that since ρ_4 remains undetermined we cannot completely guarantee that the form (1) for the order EA is correct; a complete check can only be made when higher-point amplitudes become available and ρ_4 is determined. However, we can show that the consistency of the equivalence conjecture with the constraints emerging from the study of field redefinition ambiguities requires the vanishing of this coefficient of this coefficient. Indeed, we have seen that the comparison with the four-point string amplitudes leaves us with the action

$$S_2 = -\frac{2}{\kappa^2}\int \mathrm{d}^D x\,\sqrt{g}\,\alpha'^2\,\mathrm{e}^{-2\phi}\left[\tfrac{1}{48}I_1 + \tfrac{1}{24}G_3 + \rho_4 R(\partial\phi)^4\right], \tag{17}$$

the four-point amplitudes being unable to fix the coefficient ρ_4. We want to see if it is possible to find K-matrices, which relate the string EA and the σ-model β-functions:

$$\frac{\delta S}{\delta g_i}=K_{ij}\beta_j\,, \tag{18}$$

for an action equivalent to (17) upon field redefinitions. After some algebra one finds that such an action is [7]

$$S_2=-\frac{2}{\kappa^2}\int \mathrm{d}^D x\,\sqrt{g}\,\alpha'^2\,\mathrm{e}^{-2\phi}\,[\tfrac{1}{48}I_1+\tfrac{1}{24}G_3+\bar{\rho}_4 R(\partial\phi)^4+f_2 R(\partial\phi)^2 \mathrm{D}^2\phi+f_3 R(\mathrm{D}^2\phi)^2$$
$$+e_2(\mathrm{D}^2\phi)^2(\partial\phi)^2+e_3\mathrm{D}^2\phi(\partial\phi)^4+e_4(\partial\phi)^6]\,, \tag{19}$$

where $\bar{\rho}_4$ is fixed by the order α' invariant relations (see eq. (17) of ref. [7]) to be

$$\bar{\rho}_4=\rho_4+\delta\rho_4=\rho_4+\tfrac{1}{4}(-8\delta f_2-16\delta f_3+4\delta e_2+2\delta e_3+\delta e_4)$$
$$=\rho_4+\tfrac{1}{4}(8f_2+16f_3-4e_2-2e_3-e_4)\,. \tag{20}$$

Furthermore we find that it is possible to obtain K-matrices satisfying eq. (18) if and only if the coefficients f_2, ..., e_4 assume the following values:

$$f_2=-\bar{\rho}_4,\quad f_3=\tfrac{1}{4}\bar{\rho}_4,\quad e_2=\bar{\rho}_4,\quad e_3=-4\bar{\rho}_4,\quad e_4=4\bar{\rho}_4\,. \tag{21}$$

Substituting (21) into (20), we obtain

$$\bar{\rho}_4=\rho_4+\bar{\rho}_4\,, \tag{22}$$

which in turn implies that

$$\rho_4=0\,. \tag{23}$$

Hence the result that the validity of the equivalence conjecture, eq. (1), requires that $\rho_4=0$.

As to the conclusion of ref. [3] regarding the K-matrices, namely that these necessarily involve derivative operators acting on the β-functions, we point out that these authors start their analysis from an action in which terms involving derivatives of the dilaton field were ignored. Since, at order α', it is the inclusion of this type of terms which leads to derivative-free K-matrices at this order, it seems reasonable to admit that this may also occur at order α'^2. On the other hand, although some general arguments have been advanced in ref. [10] to relate the equivalence conjecture with the absence of derivative operators in the K-matrices, there is already evidence that this may not be the case – see Ellwanger et al. [1]. Regarding the order α'^2 analysis, we have obtained a set of K-matrices, derived from an action containing terms with derivatives of the dilaton, but again we find that these necessarily involve derivative operators. We cannot, however, exclude the possibility that, by considering an increasing number of independent terms in the string action [see eq. (7)], derivative operators acting on the β-functions can be eliminated from the K-matrices.

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