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Primordial nucleosynthesis, majorons and heavy tau neutrinos

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Abstract

We determine the restrictions imposed by primordial nucleosynthesis upon a heavy tau neutrino, in the presence of ν_τ annihilations into majorons, as expected in a wide class of particle physics models of neutrino mass. We determine the equivalent number of light neutrino species $N_{\rm eq}$ as a function of m_{ν_τ} and the ν_τ–ν_τ–majoron coupling g. We show that for theoretically plausible g values $\gtrsim 10^{-4}$ present nucleosynthesis observations can not rule out ν_τ masses in the MeV range. Moreover, these models give $N_{\rm eq} \leqslant 3$ in the ν_τ mass region 1–10 MeV, for very reasonable values of $g \geqslant 3 \times 10^{-4}$. The evasion of the cosmological limits brings new interest to the improvement of the present laboratory limit on the ν_τ mass which can be achieved at a tau-charm factory.
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1. Introduction

Despite great experimental efforts, the tau neutrino still remains as the only one which can have mass in the MeV range. The present experimental limit on its mass is [1]

$$m_{\nu_\tau} < 23\,\text{MeV}. \tag{1}$$

Further progress will have to wait for the improvements expected at future tau-charm or B factories [2]. On the other hand, many particle physics models of massive neutrinos

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lead to a tau neutrino with mass in the MeV range [3]. Moreover such a neutrino may have interesting cosmological implications [4]. It is therefore interesting to critically examine the cosmological constraints.

The first comes from the critical density argument [5]. However, as has been widely illustrated with many particle physics models where neutrinos acquire their mass by the spontaneous violation of a global lepton number symmetry [6], this limit can be avoided due to the existence of fast ν_τ decays [7–9] and/or annihilations [10,8] into majorons. Although the majoron was first introduced in the context of the seesaw model [11], the spontaneous breaking of lepton number can be realized in many different models. There is only one important constraint on its properties following from the precision measurements of the invisible Z width at LEP, namely the majoron must be mostly singlet under the $SU(2) \otimes U(1)$ symmetry. It has been noted that in many models of this type the relic ν_τ number density can be depleted well below the required value for all masses obeying Eq. (1).

In order to demonstrate the cosmological viability of the MeV tau neutrino we must also consider the restrictions that follow from primordial nucleosynthesis considerations [12]. In the standard model, these rule out ν_τ masses in the range [13,14]

$$0.5\,\mathrm{MeV} < m_{\nu_\tau} < 35\,\mathrm{MeV} \tag{2}$$

This would imply that $m_{\nu_\tau} < 0.5$ MeV is the nucleosynthesis limit for the case of a Majorana tau neutrino. Here we will only assume that ν_τ is a Majorana particle, which is the most likely possibility. This assumes for the maximum allowed effective number of extra neutrino species ΔN_{eq} during nucleosynthesis either 0.4 or 0.6. Recent contradictory data on the primordial deuterium abundance [15,16] may cast some doubts on the validity of this assumption (for a recent analysis see Refs. [17,18]). In particular, if $\Delta N_{\mathrm{eq}} = 1$ is allowed [18], there may be an open window for neutrino mass somewhere near 20 MeV. However, it has been shown in Ref. [19] that this window actually does not exist, when one carefully takes into account the influence of non-equilibrium electronic neutrinos on the neutron-to-proton ratio. These neutrinos would come from massive ν_τ annihilations $\nu_\tau\nu_\tau \to \nu_e\nu_e$.

However, one knows that new interactions capable of depleting the MeV ν_τ density in the cosmic plasma are needed, at some level, in order to comply with the limit on the relic neutrino density. It is therefore reasonable to analyse their possible effect in relation with the primordial nucleosynthesis constraints [20].

In this paper we analyse the effect of neutrinos with large annihilation cross sections into majorons. In order to compute the relevant annihilation rates we must parametrize the majoron interactions. These arise from the diagrams shown in Fig. 1. The t-channel diagram is present in all majoron models, while the strength of the s-channel scalar exchange diagram is somewhat model dependent.

One way of writing the couplings of majorons to neutrinos is using the fact that the majorons are Nambu–Goldstone bosons and hence have derivative couplings. This is the so called *polar coordinate* method. The other method is to use a pseudoscalar interaction, sometimes referred to as the *cartesian method*. The two methods are equivalent, even

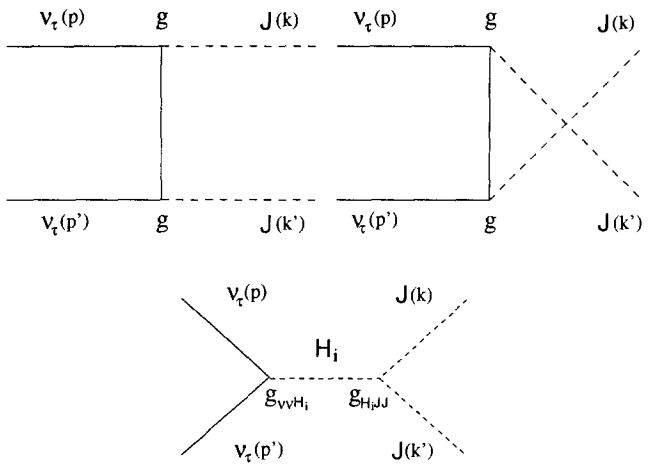

Fig. 1. Feynman diagrams for annihilations of tau neutrinos into majorons.

for second-order processes as we are considering here, if we include *all* the Feynman diagrams contributing at that order to the process of interest $\nu_\tau\nu_\tau \to JJ$.[4] In our calculations throughout this paper we will use the cartesian method of parametrizing the majoron interactions. Although we must in principle include also the s-channel diagram in Fig. 1, we will neglect this contribution. We explicitly show in Appendix A that it is justified in our case to use only the t-channel contribution in order to derive a *conservative* limit on neutrino mass m_{ν_τ} and majoron coupling g.

We have determined the restrictions imposed by primordial nucleosynthesis upon such a heavy tau neutrino in the presence of ν_τ annihilations into majorons. We show that if the $\nu_\tau\nu_\tau$ majoron coupling constant exceeds $g \gtrsim 10^{-4}$ or so, a large ν_τ mass in the MeV range is allowed by the present upper bounds on the extra number of neutrino species. As a result one cannot rule out any values of the ν_τ mass up to the present laboratory limit of Eq. (1).[5]

We also show how such g values are theoretically plausible in the context of the most attractive elementary particle physics models where MeV tau neutrinos arise, and which are based upon the spontaneous violation of lepton number.

2. Evolution of ν_τ number density in the presence of ν_τ annihilations

Massive tau neutrinos certainly interact with leptons via the standard weak interactions, $\nu_\tau\nu_\tau \leftrightarrow \nu_{e,\mu}\bar{\nu}_{e,\mu}$, e^+e^-, as assumed in Refs. [13,14]. Moreover, in many particle physics where neutrinos acquire mass from the spontaneous violation of a global lepton

[4] Although equivalent, for models with a large number of scalars and where the majoron is a linear combination of the imaginary parts of several fields, like the model of Ref. [23], the cartesian method is more convenient.

[5] In fact, with a larger coupling constant $g \gtrsim 10^{-3}$ it may be possible for a stable MeV ν_τ to obey the critical density limit, suggesting a possible role of ν_τ as dark matter.

number symmetry [6] heavy neutrinos, such as the ν_τ, annihilate to majorons $\nu_\tau\nu_\tau \to JJ$ via the diagonal coupling

$$\mathcal{L} = i\tfrac{1}{2}gJ\nu_\tau^T\sigma_2\nu_\tau + \text{H.c.}, \tag{3}$$

where ν_τ represents a two-component Majorana spinor, in the notation of Refs. [9,21,22]. This corresponds, in the usual four-component notation to

$$\mathcal{L} = i\tfrac{1}{2}gJ\overline{\nu}_\tau\gamma_5\nu_\tau. \tag{4}$$

The corresponding elastic processes do not change particle densities, but as long as they are effective they maintain all species with the same temperature.

We now comment on the cosmological bound provided by the critical density argument [5]. In order to be consistent with cosmological limits, the relic abundance of the heavy Majorana tau neutrinos must be suppressed over and above what is provided by the standard model charged and neutral current weak interactions, as well as those derived from Fig. 1. This happens automatically in many majoron models, where neutrinos decay with lifetimes shorter than required by the critical density constraint [6–9]. For example, in majoron models of the seesaw-type a massive ν_τ will typically decay with lifetimes shorter than the one required in order to obey the critical density bound, but longer than the relevant nucleosynthesis time, as illustrated in Fig. 18 of Ref. [6]. Another example is provided by the model of Ref. [23]. A ν_τ lifetime estimate was given for this model in Fig. 1 of Ref. [24]. It is seen explicitly that a ν_τ of mass in the MeV range of interest to us is expected to be stable on the nucleosynthesis time scale, but decays with lifetimes shorter than required by the critical density bound. This corresponds to a range of off-diagonal neutrino–majoron couplings $10^{-10} > g_{\text{off-diagonal}} > 10^{-13}$, which naturally occurs in many models.

For simplicity, we will assume from now in this paper that the massive ν_τ's decay with lifetimes shorter than required by the critical density bound, but are stable on the time scale relevant for nucleosynthesis considerations. The more general case where both decays and annihilations are simultaneously active on the nucleosynthesis time scale will be treated elsewhere [25].

2.1. *Before weak decoupling*

Let us assume first that all species are interacting so that they have the same temperature. The evolution of the ν_τ density can be found from the corresponding Boltzmann equation,

$$\dot{n}_{\nu_\tau} + 3Hn_{\nu_\tau} = -\sum_{i=J,e,\nu_{e,\mu}} \langle\sigma_i v\rangle \left(n_{\nu_\tau}^2 - (n_{\nu_\tau}^{\text{eq}})^2\frac{n_i^2}{(n_i^{\text{eq}})^2}\right). \tag{5}$$

In this expression $\langle\sigma_i v\rangle$ is the thermal average of the annihilation cross section times the ν_τ relative velocity v. Using the convention for the momenta as in Fig. 1, its value

for the process $\nu_\tau \nu'_\tau \leftrightarrow x_i x'_i$ is[6]

$$\langle \sigma_i v \rangle \equiv \frac{1}{(n^{\rm eq}_{\nu_\tau})^2} \int d\Pi_{\nu_\tau} d\Pi_{\nu'_\tau} d\Pi_{x_i} d\Pi_{x'_i} (2\pi)^4 \delta^4 (p + p' - k - k') \times |M|^2 e^{-E_p/T} e^{-E_{p'}/T}. \tag{6}$$

Here we have assumed kinetic equilibrium amongst the different species, as well as Boltzmann statistics. By $|M|^2$ we denote the invariant amplitude obtained with the usual Feynman rules for Majorana neutrinos [9,21,22], summed over all spins (and averaged over initial spins). Moreover we have set $d\Pi_A \equiv d^3 p_A / (2\pi)^3 2E_{p_A}$.

Following Ref. [26] we express $\langle \sigma_i v \rangle$ as a single integral using the dimensionless variable $x \equiv m_{\nu_\tau}/T$,

$$\langle \sigma_i v \rangle = \frac{x}{8 m^5_{\nu_\tau} K^2_2(x)} \int_{4m^2_{\nu_\tau}}^{\infty} ds \; (s - 4m^2_{\nu_\tau}) \sigma_i(s) \sqrt{s} K_1 \left(\frac{x\sqrt{s}}{m_{\nu_\tau}} \right), \tag{7}$$

where $K_i(x)$ are the modified Bessel functions of order i (see for instance Ref. [27]) and $s = (p + p')^2$ is the invariant of the process $\nu_\tau \nu'_\tau \leftrightarrow x_i x'_i$. Using the new variable $\eta \equiv 1 - 4m^2_{\nu_\tau}/s$ instead of s,

$$\langle \sigma_i v \rangle = \frac{4x}{K^2_2(x)} \int_0^1 d\eta \; \frac{\eta}{(1-\eta)^{7/2}} \sigma_i(\eta) K_1 \left(\frac{2x}{\sqrt{1-\eta}} \right). \tag{8}$$

The cross sections of the different annihilation processes are listed below. For annihilations to majorons we have[7]

$$\sigma_J(\eta) = \frac{g^4}{128\pi} \frac{1-\eta}{m^2_{\nu_\tau} \eta} \left[\ln \left(\frac{1+\sqrt{\eta}}{1-\sqrt{\eta}} \right) - 2\sqrt{\eta} \right], \tag{9}$$

where we have divided by 2! in order to account for identical majorons in the final state and divided by 4 in order to account the ν_τ spin factors. For the standard weak interaction-induced annihilations $\nu_\tau \bar{\nu}_\tau \leftrightarrow f_i \bar{f}_i$, in the limit of *massless* products we take

$$\sigma_i(\eta) = \frac{2G^2_F}{3\pi} \frac{m^2_{\nu_\tau} \sqrt{\eta}}{1-\eta} (b^2_{Li} + b^2_{Ri}) \; , \tag{10}$$

where $b^2_L + b^2_R = 1/2$ for $i = \nu_{e,\mu}$ and $b^2_L + b^2_R = 2((-1/2 + \sin^2\theta_W)^2 + (\sin^2\theta_W)^2) \simeq 0.25$ for $i = e$.

One may write evolution equations analogous to Eq. (5) for the other species present in the plasma, namely $\nu_{e,\mu}$ and $e^\pm$. However, we assume that the weak and electromagnetic interactions are effective enough to keep the $\nu_{e,\mu}$ and e densities in their equilibrium values, $n_k = n^{\rm eq}_k$ for $k = \nu_{e,\mu}, e$. Thus we are left with a system of just two coupled Boltzmann equations:

[6] Here $v = [(pp')^2 - m^4_{\nu_\tau}]^{1/2} / E_p E_{p'}$.

[7] The general formula is given in Appendix A, Eq. (A.1).

$$\dot{n}_{\nu_\tau} + 3Hn_{\nu_\tau} = -\sum_{i=e,\nu_e,\mu} \langle \sigma_i v \rangle \, (n_{\nu_\tau}^2 - (n_{\nu_\tau}^{\rm eq})^2) - \langle \sigma_J v \rangle \left(n_{\nu_\tau}^2 - (n_{\nu_\tau}^{\rm eq})^2 \frac{n_J^2}{(n_J^{\rm eq})^2} \right)$$

$$\equiv S_{\nu_\tau}, \tag{11}$$

$$\dot{n}_J + 3Hn_J = \langle \sigma_J v \rangle \left(n_{\nu_\tau}^2 - (n_{\nu_\tau}^{\rm eq})^2 \frac{n_J^2}{(n_J^{\rm eq})^2} \right) \equiv S_J. \tag{12}$$

Now let us briefly describe our calculations. First we normalized the number densities to the number density of a massless neutrino species, $n_0 \simeq 0.181T^3$, introducing the quantities $r_\alpha \equiv n_\alpha/n_0$, where $\alpha = \nu_\tau, J$, and the corresponding equilibrium functions $r_\alpha^{\rm eq}$. We then have for the time derivative of n_α

$$\dot{n}_\alpha = \dot{T}\frac{dn_\alpha}{dT} = S_\alpha - 3Hn_\alpha\,,$$

$$\frac{dn_\alpha}{dT} = n_0\frac{dr_\alpha}{dT} + r_\alpha\frac{3}{T}n_0$$

or, equivalently,

$$\frac{dr_\alpha}{dT} = \left(\frac{S_\alpha}{n_0} - 3Hr_\alpha\right)\frac{1}{\dot{T}} - \frac{3}{T}r_\alpha. \tag{13}$$

On the other hand, the time derivative of the temperature is obtained from the covariant energy conservation law

$$\dot{\rho} = -3H(\rho + P) \rightarrow \dot{T} = -3H(\rho + P)\frac{1}{d\rho/dT}, \tag{14}$$

where ρ is the total energy density and P is the pressure. Finally, as $\rho = \rho(T, r_J, r_{\nu_\tau})$ we can rewrite

$$\frac{d\rho}{dT} = \frac{\partial\rho}{\partial T} + \frac{\partial\rho}{\partial r_J}\frac{dr_J}{dT} + \frac{\partial\rho}{\partial r_{\nu_\tau}}\frac{dr_{\nu_\tau}}{dT},$$

and for the normalized particle densities one has

$$\frac{dr_{\nu_\tau}}{dT} = -\Sigma_{\nu_\tau}\left(\frac{\partial\rho}{\partial T} + \frac{\partial\rho}{\partial r_J}\frac{dr_J}{dT} + \frac{\partial\rho}{\partial r_{\nu_\tau}}\frac{dr_{\nu_\tau}}{dT}\right) - \frac{3}{T}r_{\nu_\tau}\,, \tag{15}$$

$$\frac{dr_J}{dT} = -\Sigma_J\left(\frac{\partial\rho}{\partial T} + \frac{\partial\rho}{\partial r_J}\frac{dr_J}{dT} + \frac{\partial\rho}{\partial r_{\nu_\tau}}\frac{dr_{\nu_\tau}}{dT}\right) - \frac{3}{T}r_J\,, \tag{16}$$

where, for $\alpha = \nu_\tau, J$, we have introduced

$$\Sigma_\alpha \equiv \frac{1}{\rho + P}\left(\frac{S_\alpha}{3Hn_0} - r_\alpha\right). \tag{17}$$

The final Boltzmann system for the normalized particle densities is obtained from Eqs. (15) and (16) introducing the dimensionless variable x previously defined. Denoting $r' \equiv dr/dx$, we have

$$r'_{\nu_\tau}\left(1+\Sigma_{\nu_\tau}\frac{\partial\rho}{\partial r_{\nu_\tau}}\right)+r'_J\Sigma_{\nu_\tau}\frac{\partial\rho}{\partial r_J}=\Sigma_{\nu_\tau}\frac{T}{x}\frac{\partial\rho}{\partial T}+\frac{3}{x}r_{\nu_\tau}, \tag{18}$$

$$r'_J\left(1+\Sigma_J\frac{\partial\rho}{\partial r_J}\right)+r'_{\nu_\tau}\Sigma_J\frac{\partial\rho}{\partial r_{\nu_\tau}}=\Sigma_J\frac{T}{x}\frac{\partial\rho}{\partial T}+\frac{3}{x}r_J. \tag{19}$$

This system is valid as long as the tau neutrinos are coupled to the weak interactions. The following is the complete set of entries in Eqs. (18) and (19) for the equilibrium quantities, total energy density and pressure, respectively:

$$r^{\rm eq}_{\nu_\tau}=\frac{1}{0.181\pi^2}x^3I_1(x),\qquad r^{\rm eq}_J=\frac{2}{3},$$

$$\rho=\rho_{\nu_0}+\rho_e+\rho_\gamma+\rho_J+\rho_{\nu_\tau}=\frac{3\pi^2}{10}T^4\left(1+\frac{1}{6}r_J+0.06x\frac{I_2(x)}{I_1(x)}r_{\nu_\tau}\right),$$

$$P=P_{\nu_0}+P_e+P_\gamma+P_J+P_{\nu_\tau}=\frac{\pi^2}{10}T^4\left(1+\frac{1}{6}r_J+0.06x\frac{I_3(x)}{I_1(x)}r_{\nu_\tau}\right). \tag{20}$$

In these expressions we have introduced the integral functions I_j, where $j=1,2,3$, defined as

$$I_1(x)=\int_0^\infty du\,u^2\exp(-x\sqrt{1+u^2}),$$

$$I_2(x)=\int_0^\infty du\,u^2\sqrt{1+u^2}\exp(-x\sqrt{1+u^2}),$$

$$I_3(x)=\int_0^\infty du\,\frac{u^4}{\sqrt{1+u^2}}\exp(-x\sqrt{1+u^2}). \tag{21}$$

2.2. Past weak decoupling

Once the ν_τ's decouple from the standard weak interactions, they remain in contact only with majorons. Then one has two different plasmas, one formed by ν_τ's and J's and the other by the rest of the particles, each one with its own temperature[8] (denoted as T and T_γ). Let us define now the variables

$$x=\frac{m_{\nu_\tau}}{T},\qquad y=\frac{m_{\nu_\tau}}{T_\gamma}.$$

We assume that the photon temperature evolves in the usual way, $\dot{y}=Hy$. The evolution equation of the ν_τ and J number densities are now simplified versions of (11) and (12), because $S_{\nu_\tau}\equiv -S_J$,

$$\dot{n}_{\nu_\tau}+3Hn_{\nu_\tau}=-S_J,\qquad \dot{n}_J+3Hn_J=S_J, \tag{22}$$

[8] Eventually the massless neutrinos will also decouple from the second plasma, while the e^+e^- pairs will annihilate to photons, thus generating the well-known $T_{\nu_0}-T_\gamma$ difference.

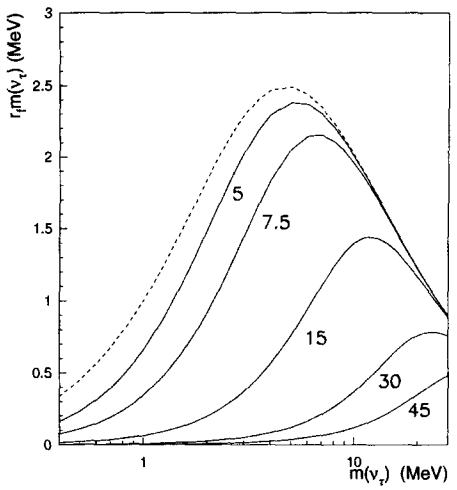

Fig. 2. Frozen values of $r_{\nu_\tau} m_{\nu_\tau}$ as a function of m_{ν_τ} for the standard model ($g = 0$) and for the majoron model with different g values in units of 10^{-5}.

or, in terms of r_α's,

$$r'_{\nu_\tau} = -\frac{S_J}{n_0 H y}\frac{dy}{dx}, \qquad r'_J = -r'_{\nu_\tau}. \tag{23}$$

Due to the second equation, the Boltzmann system reduces to a single evolution equation say, for r_{ν_τ}. However, one must still determine dy/dx which differs from unity because $T \neq T_\gamma$. An equation relating y and x is obtained using the energy balance condition for the $\nu_\tau + J$ plasma. If $\rho \equiv \rho_{\nu_\tau} + \rho_J$ and $P \equiv P_{\nu_\tau} + P_J$, we can write

$$\dot{\rho} = -3H(\rho + P), \qquad \text{where} \quad H = \sqrt{\frac{8\pi\rho_{\rm tot}}{3M_{pl}^2}}. \tag{24}$$

The expressions for $r_\alpha^{\rm eq}$, ρ and P given in Eqs. (20) need to be modified in order to take into account the fact that there are two distinct temperatures T and T_γ. This leads to the following equation:

$$\frac{dy}{dx} = \frac{y\left[\frac{\pi^2}{20}\frac{r_J}{x^2} + \left(\frac{I_4(x)}{I_1(x)} - \left(\frac{I_2(x)}{I_1(x)}\right)^2\right) r_{\nu_\tau}\right]}{3\left(0.06\frac{I_3(x)}{I_1(x)} r_{\nu_\tau} + \frac{\pi^2}{60}\frac{r_J}{x}\right) - \frac{r'_{\nu_\tau}}{H}\left(\frac{\pi^2}{20x} - 0.18\frac{I_2(x)}{I_1(x)}\right)}. \tag{25}$$

Here we defined $I_4(x) \equiv -dI_2(x)/dx$.

In order to determine the final frozen density of ν_τ, which will be relevant during nucleosynthesis, we have to solve numerically the corresponding set of differential equations. Before weak decoupling these are (18) and (19), while after decoupling one should combine Eqs. (23) and (25), with the initial conditions $r_\alpha = r_\alpha^{\rm eq}$, $\alpha = J, \nu_\tau$ valid at high temperatures.

In Fig. 2 we show the results of our calculations of the asymptotic (frozen) values of $r_{\nu_\tau} m_{\nu_\tau}$ as a function of m_{ν_τ} for the standard model ($g = 0$) and for the majoron model

with different g values. Note that in the standard $g = 0$ case we agree with the previous results of Ref. [14] but get somewhat larger values than those obtained in Ref. [13]. We ascribe this small discrepancy to the use, in Ref. [13], of an approximate expression for the ν_τ energies, rather than the exact ones.

3. Nucleosynthesis constraints on (m_{ν_τ}, g)

In this section we use the results obtained for the ν_τ number density in order to constrain its mass from nucleosynthesis arguments. The value of r_{ν_τ} as a function of (m_{ν_τ}, g) is used in order to estimate the variation of the total energy density $\rho_{\rm tot} = \rho_R + \rho_{\nu_\tau}$. In ρ_R all relativistic species are taken into account, including majorons and two massless neutrinos, whereas ρ_{ν_τ} is the energy density of the massive ν_τ's.

In order to compare with the standard model situation it is convenient for us to express the effect of the ν_τ mass and that of the presence of the majoron in terms of an effective number of massless neutrino species ($N_{\rm eq}$) which we calculate for each frozen value of $r_\tau(m_{\nu_\tau})$. In reality, the true value of $r_\tau(m_{\nu_\tau})$ is always larger than its frozen value, and we have taken this into account in order to obtain reliable limits in the low ν_τ mass region.

In order to derive the nucleosynthesis limits, first we developed a simple code for the numerical calculation of the neutron fraction r_n, as presented e.g. in Ref. [28], varying the value of $N_{\rm eq}$. Then we incorporated $\rho_{\rm tot}$ into this numerical code and performed the integration of the neutron–proton kinetic equations for each pair of (m_{ν_τ}, g) values, where g is the coupling constant which determines the strength of the ν_τ annihilation cross section. Comparing $r_n(m_{\nu_\tau}, g)$ with $r_n(N_{\rm eq})$ at $T_\gamma \simeq 0.075$ MeV (the moment when practically all neutrons are wound up in ^{4}He), we can relate (m_{ν_τ}, g) to $N_{\rm eq}$.

We repeated this calculation adapting Kawano's nucleosynthesis code [29] to the case of a massive tau neutrino, both in the standard model and the majoron extension. We have found that both methods are in good agreement. The results for the numerical calculations of the equivalent number of massless neutrinos during nucleosynthesis with the use of Kawano's numerical code are shown in Fig. 3. For comparison the case of $g = 0$ is shown (dashed line). From Fig. 3 one can see that, in the asymptotic limit of very large m_{ν_τ}, the annihilation into majorons is very inefficient (see Eq. (9)), so that the effective $N_{\rm eq}$ value is larger than in the standard $g = 0$ case precisely by a factor $4/7$, which corresponds to the extra majoron degree of freedom. Thus, if we take also g very large we get just $N_{\rm eq} = 2 + 4/7 \simeq 2.57$. Of course this asymptotic limit is already experimentally ruled out by the Aleph ν_τ mass limit [1] and thus is not displayed. For m_{ν_τ} values in the range from 10 to 23 MeV or so, $N_{\rm eq}$ can be made acceptable, provided g is raised sufficiently. For the intermediate ν_τ mass region, 1–10 MeV, and $g > 3 \times 10^{-4}$ the model may even give $N_{\rm eq} \leqslant 3$, which is possibly supported by some of the observational data.

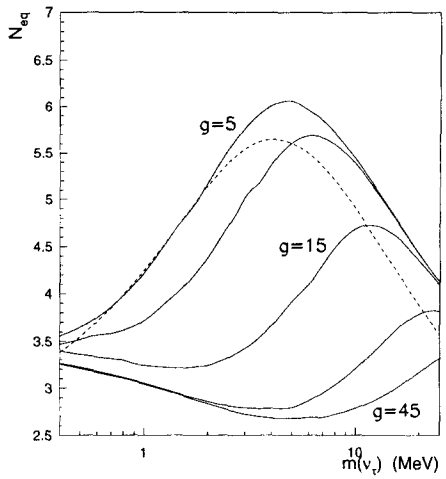

Fig. 3. Effective number of massless neutrinos equivalent to the contribution of heavy ν_τ's with different values of g in units of 10^{-5}. For comparison, the dashed line corresponds to the standard model case when $g = 0$.

Finally, in the small ν_τ mass limit the energy density of ν_τ is roughly the same as that of the massless ν_e or ν_μ, so that all g values shown in the figure lead to the same asymptotic value $N_{\rm eq} = 3 + 4/7 \simeq 3.57$, corresponding to the three massless neutrinos plus majoron (instead of $2 + 4/7$ for a very heavy ν_τ). However, it might be that observations eventually could lead to a tighter limit $N_{\rm eq}^{\rm max} \leqslant 3.57$. In such an event a simple way out is to have the majoron out-of-equilibrium, which would require a very small g value, $g < (2\text{–}3) \times 10^{-5}$, so that the production of majorons through annihilations of ν_τ's would be negligible.[9] Should the observations eventually lead to an even tighter limit $N_{\rm eq}^{\rm max} \leqslant 3$, the situation is qualitatively different, as it would raise a conflict with the standard model. A possible way to lower $N_{\rm eq}$ below three provided by our model is to have a massive ν_τ in the MeV range and with a relatively strong coupling with majorons. Indeed, one can see from Fig. 3 that, while it is not possible in the standard model to account for $N_{\rm eq}^{\rm max} \leqslant 3$, it is quite natural in our model to have $N_{\rm eq}^{\rm max} \leqslant 3$ for a wide range of intermediate tau neutrino masses and reasonable large values of the coupling constants g.

In summary, one sees that all ν_τ masses below 23 MeV are allowed by the nucleosynthesis condition $N_{\rm eq} \leqslant N_{\rm eq}^{\rm max}$ if $N_{\rm eq}^{\rm max} \geqslant 3.57$, provided that the coupling between ν_τ's and J's exceeds a value of a few times 10^{-4}. This situation seems at the moment compatible with the experimental data, at least the ^{4}He and ^{7}Li determinations [18].

It is instructive to express the above results in the m_{ν_τ}–g plane, as shown in Fig. 4. The region above each curve is allowed for the corresponding $N_{\rm eq}^{\rm max}$.

[9] Of course such m_{ν_τ} values are allowed by nucleosynthesis in the absence of ν_τ annihilations.

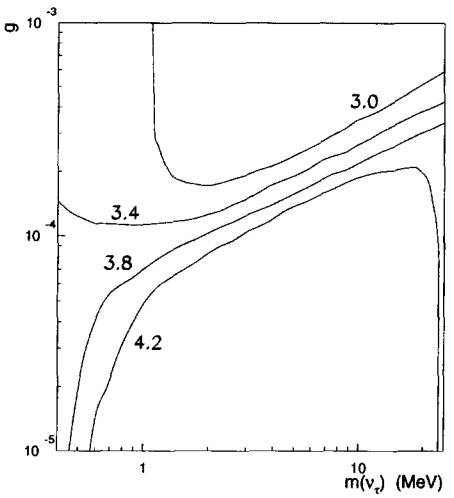

Fig. 4. The values of $g(m_{\nu_\tau})$ above each line would be allowed by nucleosynthesis if one adopts $N_{\rm eq}^{\rm max} = 3$, 3.4, 3.8, 4.2 (from top to bottom).

4. Significance of the nucleosynthesis limits

There has been a variety of majoron models proposed in the literature [6]. They are attractive extensions of the standard electroweak model where neutrinos acquire mass by virtue of the spontaneous violation of a global lepton number symmetry. Apart from their phenomenological interest as extensions of the lepton and/or Higgs sectors of the standard model [3], majoron models offer the possibility of loosening the cosmological limits on neutrino masses, either because neutrinos decay or because they annihilate to majorons. The first and most obvious of these is the limit that follows from the cosmological density argument [7,8]. As we saw in the previous section one can also place limits on a heavy tau neutrino with mass in the MeV range by using primordial element abundances. We have determined the restrictions imposed by primordial nucleosynthesis upon a heavy tau neutrino, in the presence of $\nu_\tau\nu_\tau$ annihilations into majorons. Our results are completely general and may be compared to any bound characterized by an allowed value of $N_{\rm eq}^{\rm max}$. Given any $N_{\rm eq}^{\rm max}$ value one can readily obtain the allowed regions of m_{ν_τ} and the majoron coupling constant g as shown in Fig. 4. As an example, a recent model-independent likelihood analysis of big bang nucleosynthesis based on ^{4}He and ^{7}Li determinations has claimed an upper limit $N_{\rm eq} < 4.0$ (at 95% C.L.) [18]. From Fig. 4 this would imply that all m_{ν_τ} masses are allowed, as long as g exceeds 10^{-4} or so. However, we believe that, in the present state of affairs, one should probably not assign a statistical confidence to nucleosynthesis results, to the extent that these are still dominated by systematic, rather than statistical errors. Strictly speaking, what Fig. 4 really displays is the equivalent neutrino number $N_{\rm eq}$ for various combinations of (m_{ν_τ}, g) parameters that give the same helium abundance, rather than real limits. Of course, from these contours containing the raw information an educated reader can judge which helium abundance should be considered plausible or not.

We now illustrate in concrete models the fact that such values of the $\nu_\tau\nu_\tau$ majoron coupling $\gtrsim 10^{-4}$ are theoretically plausible. Different models imply different expectations for the majoron coupling constants g and the relation they bear with the ν_τ mass m_{ν_τ}. Our discussion so far is applicable to the simplest seesaw majoron model of Ref. [11]. In this case one expects that [9]

$$g = \mathcal{O}\left(\frac{m_D^2}{M_R^2}\right), \tag{26}$$

where m_D is a typical Dirac neutrino mass and $M_R \propto \langle\sigma\rangle$ is the Majorana mass of the right-handed $SU(2)\otimes U(1)$ singlet neutrino. Clearly g values in the range required by nucleosynthesis are quite reasonable say, for $m_D \sim$ 1–100 GeV and $M_R \sim 10^4$–10^8 GeV. Moreover, it is a good approximation in this model to neglect the s-channel scalar exchange diagram of Fig. 1.

There is a wide class of alternative majoron models characterized by a low scale of lepton number violation [8,30,31]. These models are attractive because they lead to a wide variety of processes which may be experimentally accessible [3]. In this case one expects a simple direct correlation between the mass of the neutrinos and the magnitude of the diagonal couplings of neutrinos to majorons. The neutrino mass is simply the product of the Yukawa coupling g and the vacuum expectation value $\langle\sigma\rangle$ which characterizes the spontaneous violation of the global lepton number symmetry [6],

$$m = g\langle\sigma\rangle . \tag{27}$$

From this it follows that for $m_{\nu_\tau} \sim 10$ MeV and $\langle\sigma\rangle \sim 100$ GeV one obtains $g \sim 10^{-4}$. This situation is therefore characteristic of models where lepton number spontaneously breaks at the weak scale.

There are more complicated models where the degree of correlation between the ν_τ mass m_{ν_τ} and the lepton number violation scale may be different and may involve more free parameters. Just to give a concrete example of such models, let us consider the supersymmetric models with spontaneous violation of R-parity [23]. These models lead to

$$m \propto \frac{\langle\sigma\rangle^2}{M_{\rm SUSY}}, \tag{28}$$

where $\langle\sigma\rangle$ is identified with the vacuum expectation value of the right-handed $SU(2)\otimes U(1)$ singlet sneutrino and $M_{\rm SUSY}$ denotes a typical neutralino mass. The expected values of (g, m_{ν_τ}) are depicted in Fig. 5, obtained when one varies the other relevant free parameters over a theoretically reasonable range.

For all models with low-scale lepton number violation we have shown, by doing the full calculation, that the overall annihilation cross section for $\nu_\tau\nu_\tau$ annihilation into two majorons can be enhanced by an order of magnitude with respect to our above simplified calculation which neglected the s-channel scalar exchange diagram in Fig. 1. Although this would allow us to weaken our limits, the effect on g would only be a factor $10^{1/4} \lesssim 2$, so that the limits derived in Fig. 4 could be relaxed by a factor $\lesssim 2$ in this class of models.

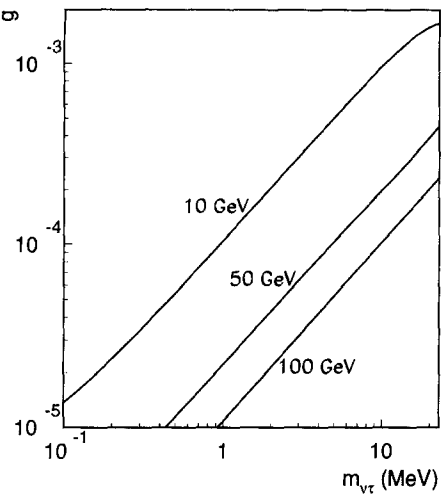

Fig. 5. Expected values of m_{ν_τ} and g in the model of Ref. [23].

As a last comment, we note that the limits obtained in our paper could also be tightened by including the influence of non-equilibrium electronic neutrinos (and anti-neutrinos) produced by $\nu_\tau\nu_\tau$ annihilations on the neutron-to-proton ratio [19] but, again, the effect is quite small on the bounds derived on g.

Last but not least, we must compare the limits obtained by primordial big bang nucleosynthesis with those derived from astrophysics. A new light particle, like the majoron, may have an important effect on stellar evolution and this allows one to place stringent limits on the strength of the interaction of such particles [32]. In the case we consider here, the majorons interact predominantly with a heavy ν_τ (with the mass in MeV range), so its influence may be noticeable in supernova explosions when the temperature reaches tens of MeV. The bounds on majoron properties which can be deduced from supernova physics have been widely discussed [33] and recently analysed in Ref. [32] (see also references therein). For example a majoron with Yukawa coupling to electronic neutrinos in the range 10^{-6}–10^{-3} could be important for supernova physics. However, in our model this coupling to ν_e is much smaller. A majoron coupling constant to tau neutrinos around 10^{-4} may be potentially interesting for supernova physics and will be discussed elsewhere. Here we only mention that g values larger than (a few) $\times 10^{-5}\sqrt{m/\mathrm{MeV}}$ may be dangerous because the coupling is strong enough for abundant production of majorons in high temperature regions in the supernova core and simultaneously small enough so that the mean free path of the produced majorons is larger than the central stellar core. Still the coupling $g > 10^{-4}$ seems to be allowed.

5. Conclusions

In this paper we have investigated the implications for primordial nucleosynthesis of a heavy tau neutrino in the MeV range, in the presence of sufficiently strong ν_τ

annihilations into majorons. We have determined the effective neutrino number $N_{\rm eq}$, or equivalently the primordial helium abundance, and studied the level of sensitivity that it exhibits when expressed in terms of the underlying ν_τ mass m_{ν_τ} and coupling parameter g, the relevant coupling constant determining the $\nu_\tau\nu_\tau$ annihilation cross section. Given the fact that present nucleosynthesis discussions are still plagued by systematics, it is useful to interpret our results this way, rather than as an actual limit in the statistical sense. For each m_{ν_τ} value, one can in principle identify the corresponding lower bounds on g for which the $\nu_\tau\nu_\tau$ annihilations to majorons are sufficiently efficient in order not to be in conflict with nucleosynthesis. Moreover, in contrast to the standard model, these models can account for a value of $N_{\rm eq} \leqslant 3$ if the ν_τ mass lies in the region 1–10 MeV, provided $\geqslant 3 \times 10^{-4}$.

We have been conservative in determining the nucleosynthesis limits to the extent that we have neglected model-dependent contributions from s-channel Higgs boson exchange, given in Fig. 1. This seems reasonable from the point of view of the relevant particle physics models [11,8,30,31,23].

We have also concluded that, indeed, the required choice of parameters can be naturally realized in majoron models both with weak and large-scale lepton number violation. As a result, for sufficiently large but plausible values of the $\nu_\tau\nu_\tau$ majoron coupling $\gtrsim 10^{-4}$ one can not rule out any values of the ν_τ mass up the present laboratory limit based on the cosmological argument. This highlights the importance of further experimental efforts in laboratory searches for the ν_τ mass. Improvements expected at a tau-charm factory are indeed necessary, since the primordial nucleosynthesis constraints on the ν_τ mass can be easily relaxed in a large class of extensions of the standard electroweak model.

Appendix A

Here we show why one can neglect the s-channel diagram of Fig. 1 in the determination of the nucleosynthesis bound on m_{ν_τ} and majoron coupling g.

The total cross section for the annihilation to majorons that corresponds to s-channel and t-channel diagrams of Fig. 1 is given by

$$\sigma_J(\epsilon,\eta) = \frac{1}{64\pi}\frac{g^4}{m_{\nu_\tau}^2}\left\{\epsilon^2\sqrt{\eta} + (1-2\epsilon)\frac{1-\eta}{2\eta}\left[\ln\left(\frac{1+\sqrt{\eta}}{1-\sqrt{\eta}}\right) - 2\sqrt{\eta}\right]\right\}, \quad \text{(A.1)}$$

where the parameter ϵ is defined by

$$\epsilon \equiv \frac{m_{\nu_\tau}}{g_{\nu\nu J}^2}\sum_i\left(\frac{g_{\nu\nu H_i}g_{H_iJJ}}{m_{H_i}^2}\right) \quad \text{(A.2)}$$

and $g_{\nu\nu H_i}$, g_{H_iJJ} are the couplings relevant for the s-channel diagram of Fig. 1. In Eq. (A.2) the sum is over all the CP-even scalars present in the model. From its definition one can see that ϵ is proportional to the couplings $\nu\nu H_i$ and H_iJJ. When $\epsilon \to 0$ the s-channel becomes zero.

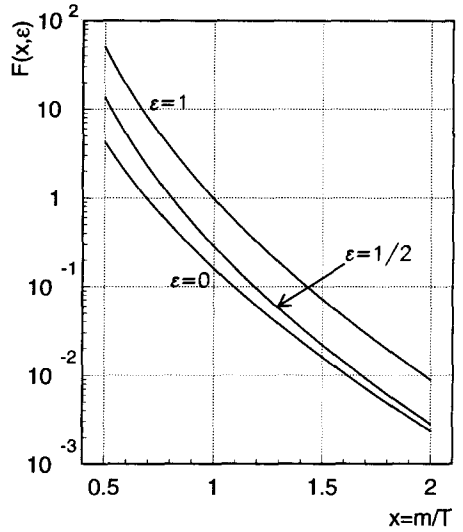

Fig. A.1. The function $F(x, \epsilon)$ for various values of ϵ.

The value of ϵ depends very much on the model. For the pure-singlet majoron models with low lepton number violation scale considered in Ref. [30] there is a strict correlation between the neutrino mass and the lepton number violation scale. In this case one has $\epsilon = 1$. For seesaw models, with lepton number violated at a large mass scale, one has $\epsilon \ll 1$. For the supersymmetric model with spontaneous breaking of R-parity [23] at the weak scale one can show that ϵ typically lies in a range around the value $1/2$. In our analysis we wanted to stay as much model independent as possible. In order to have an idea of the dependence of our results on ϵ we define

$$F(x,\epsilon) \equiv \int_0^1 d\eta \, \frac{\eta}{(1-\eta)^{7/2}} \sigma_J(\epsilon,\eta) K_1\left(\frac{2x}{\sqrt{1-\eta}}\right), \tag{A.3}$$

which is just the integrand of Eq. (8) in Section 2.1. In Fig. A.1 we plot the function $F(x, \epsilon)$ for $\epsilon = 0, 1/2$ and 1. We see that the value $\epsilon = 0$ represents a lower bound on that integral. For most models we would get a higher value. If we notice that the cross section is proportional to g^4, that difference in $F(x, \epsilon)$ would translate into a smaller value needed for g to satisfy the nucleosynthesis bounds. Therefore, one can obtain a model independent and conservative bound by taking the worst possible case, which corresponds to $\epsilon = 0$. Due to the dependence of F on g^4 the bounds on g would not be too sensitive to the value of ϵ in the range of interest. This justifies our simplified expression for σ_J used in Eq. (9).

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