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Unification of gauge couplings and the tau-neutrino mass in supergravity without R parity

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Abstract

Minimal R-parity violating supergravity predicts a value for $\alpha_s(\mathrm{M}_Z)$ smaller than in the case with conserved R-parity, and therefore closer to the experimental world average. We show that the R-parity violating effect on the α_s prediction comes from the larger two-loop b-quark Yukawa contribution to the renormalization group evolution of the gauge couplings which characterizes R-parity violating supergravity. The effect is related to the tau neutrino mass and is sensitive to the initial conditions on the soft supersymmetry breaking parameters at the unification scale. We show how a few percent effect on $\alpha_s(\mathrm{M}_Z)$ may occur even with ν_τ masses as small as indicated by the simplest neutrino oscillation interpretation of the atmospheric neutrino data from Super-Kamiokande. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

The prediction for the strong gauge coupling constant $\alpha_s(\mathrm{M}_Z)$ is one of the milestones of unification models [1–3]. Recent studies of gauge coupling unification in the context of minimal R-parity conserving supergravity [4–6] agree that using the experimental values for the electro-magnetic coupling and the weak mixing angle the prediction obtained for $\alpha_s(\mathrm{M}_Z) \approx 0.129 \pm 0.010$ [4] is about one σ larger than indicated by the most recent world average value $\alpha_s(\mathrm{M}_Z)^{\mathrm{W.A.}} = 0.1186 \pm 0.0013$ [7]. While this too small a discrepancy to be taken seriously it is hoped that the relatively large theoretical error may improve in the future. We are also encouraged by the smallness of the experimental error (for a detailed discussion of the reliability of the averaging procedures involved for α_s determined from

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different experiments at very different energies, see Ref. [8]) to reanalyse the α_s prediction in supersymmetric theories.

Here we reconsider the α_s prediction in supergravity (SUGRA). In addition to the standard MSUGRA we consider simplest supergravity version with a bilinear breaking of R-parity [9–21]. This model is theoretically motivated by the fact that it provides parameterization of many of the features of a class of models in which R-parity breaks spontaneously due to a sneutrino vacuum expectation value (VEV) [22–24]. Moreover, in the simplest case where R-parity violation lies only in the third generation, the model coincides with the most general explicit R-parity violating model and provides its simplest description.

One of the main features of R-parity violating models is the appearance of masses for the neutrinos [22–32]. As a result, these models have attracted a lot of attention [33–38] since the latest round of Super-Kamiokande results [39,40]. As shown in Ref. [41], irrespective of any assumption about Yukawa textures, one obtains a very predictive pattern of neutrino mass and mixing which leads naturally to the maximal mixing indicated by the atmospheric neutrino data.

In this paper we show that in the simplest SUGRA R-parity breaking model, with the same particle content as the MSSM and with no new interactions (such as trilinear R-parity breaking couplings), there appears an additional negative contribution to α_s, which can bring the theoretical prediction closer to the experimental world average. This additional contribution to α_s comes from two-loop b-quark Yukawa effects on the renormalization group equation (RGE) for α_s. Moreover, we show that this contribution is related to the tau-neutrino mass which is induced by R-parity breaking and which controls the R-parity violating effects. We also discuss this relation within different models for the initial conditions on the soft supersymmetry breaking parameters at the unification scale. We show how to obtain a sizable effect on $\alpha_s(\mathrm{M}_Z)$ even with ν_τ masses as small as indicated by the simplest neutrino oscillation interpretation of the atmospheric neutrino data from Super-Kamiokande.

2. The MSSM renormalization group equations

The two-loop renormalization group equations [42–46] for the gauge coupling constants in the MSSM have the form

$$\frac{\mathrm{d}g_i}{\mathrm{d}t} = \frac{g_i}{16\pi^2}\left(b_i g_i^2 + \frac{1}{16\pi^2}\left(\sum_{j=1}^{3} b_{ij}\, g_i^2 g_j^2 - \sum_{l=t,b,\tau} b'_{il}\, g_i^2 h_l^2\right)\right), \tag{1}$$

where g_i, $i = 1, 2, 3$, are the gauge couplings of the $U(1)$, $SU(2)$, and $SU(3)$ groups respectively, and h_l, $l = t, b, \tau$, are the quark and lepton Yukawa couplings of the third generation. The numerical coefficients b_i, b_{ij}, and b'_{il} are given in Refs. [42–46].

It is useful to obtain an approximate analytical solution to the gauge coupling constants from Eq. (1). This is done by neglecting the two-loop Yukawa contribution in first approximation. The result is [47]

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_{\mathrm{U}}(\mathrm{M_U})} + b_i t + \frac{1}{4\pi} \sum_{j=1,2,3} \frac{b_{ij}}{b_i} \ln\left[1 + b_j \alpha_{\mathrm{U}}(\mathrm{M_U}) t\right] - \Delta_i, \tag{2}$$

where $t = \frac{1}{2\pi} \ln(\mathrm{M_U}/\mu)$, α_{U} is the unified gauge coupling constant, M_{U} is the unification scale, μ is an arbitrary scale, and Δ_i are corrections due to several effects, mainly threshold corrections. Although GUT-type threshold corrections are potentially sizable, we neglect them here since they are in general model-dependent. For a discussion see Refs. [4,5,48]. Leading logarithms from supersymmetric spectra threshold corrections to $\alpha_S(\mathrm{M}_Z)$ can be summarized in the following formula [5,6]:

$$\Delta\alpha_s^{\mathrm{SUSY}} = -\frac{19\alpha_s^2}{28\pi} \ln\left(\frac{T_{\mathrm{SUSY}}}{\mathrm{M_t}}\right), \tag{3}$$

where T_{SUSY} is an effective mass scale given by

$$T_{\mathrm{SUSY}} = m_{\widetilde{H}} \left(\frac{m_{\widetilde{W}}}{m_{\tilde{g}}}\right)^{\frac{28}{19}} \left[\left(\frac{m_{\tilde{l}}}{m_{\tilde{q}}}\right)^{\frac{3}{19}} \left(\frac{m_H}{m_{\widetilde{H}}}\right)^{\frac{3}{19}} \left(\frac{m_{\widetilde{W}}}{m_{\widetilde{H}}}\right)^{\frac{4}{19}}\right]. \tag{4}$$

This scale is not simply an average of SUSY masses since it can be smaller than all the masses of the supersymmetric particles [5,6]. Large values of T_{SUSY} are experimentally preferred because in general they contribute negatively to $\Delta\alpha_s^{\mathrm{SUSY}}$, bringing $\alpha_s(M_Z)$ closer to the experimental average by an estimated $|\Delta\alpha_s^{\mathrm{SUSY}}| \leqslant 0.003$ [4]. There is in addition, a finite contribution from supersymmetric threshold corrections which may be important if the supersymmetric spectrum is light [49–51]. Moreover there is also a small conversion factor from $\overline{\mathrm{MS}}$ to $\overline{\mathrm{DR}}$ [52–55], as well as possible contributions coming from non renormalizable operators which can be induced from physics between the Planck to the GUT-unification scale [56,57].

Let us now turn to the important issue of the two-loop Yukawa contribution to the gauge coupling constants RGE. This contribution is not included in Eq. (2) and is crucial for our purposes, providing a correction which is negative and can be important if h_{t} or h_{b} are large ($t_\beta \approx 1$ or $t_\beta \approx 50$, respectively). Making a leading logarithm approximation we obtain the expression

$$\Delta\alpha_s^{\mathrm{YUK}} \approx -\frac{\alpha_s^2}{32\pi^3} \ln\left(\frac{\mathrm{M_U}}{\mathrm{M_t}}\right)\left\{b'_{3\mathrm{t}} h_{\mathrm{t}}^2 + b'_{3\mathrm{b}} h_{\mathrm{b}}^2\right\}. \tag{5}$$

In the small $\tan\beta$ region, the bottom Yukawa coupling is negligible compared to the top Yukawa, then we get $\Delta\alpha_s^{\mathrm{YUK}} \approx -0.1\alpha_s^2 h_{\mathrm{t}}^2$, giving us an estimate of the magnitude of this correction. Note that this correction is not bigger in the high $\tan\beta$ scenario, where both Yukawas are large, since they are not as large as the top Yukawa in the low $\tan\beta$ case.

In contrast, in the $\not{R}$–MSSM model, the bottom Yukawa coupling can be nonnegligible for any value of $\tan\beta$ [58]. As a result we cannot neglect the bottom-quark Yukawa coupling, since it can be as large as the top-quark Yukawa, especially if the R-parity violating parameters are large.

3. The $R\!\!\!/$-MSUGRA model

In order to illustrate the essential features of the model, it is enough to consider a one generation $R\!\!\!/$-MSSM [9,14–21], since it contains the main ingredients relevant for our present discussion. Of course, the correct prediction of neutrino masses and mixings require R-parity violation in the three generations [41]. Nevertheless, it is not the purpose of this paper to predict neutrino mixings. Furthermore, it is known that BRpV in three generations produce only one massive neutrino at tree level, whose mass is equivalent to the tree level tau neutrino mass in the case of BRpV only in the third generation. Therefore, the one generation BRpV approach considered in this paper is justified because it has been checked in [41] that BRpV parameters smaller than about 1 GeV in the three-generation model produce a tree level tau neutrino mass which dominates over the one-loop contributions. The tau neutrino mass calculated here, then, is a good approximation of the heaviest neutrino mass in the complete three-generation model.

The superpotential W is

$$W = W_{\mathrm{MSSM}} + W_{R\!\!\!/}, \tag{6}$$

where W_{MSSM} is the familiar superpotential of the MSSM:

$$W_{\mathrm{MSSM}} = \left[h_{\mathrm{t}} \widehat{Q}_3 \widehat{H}_u \widehat{U}_3 + \lambda_0^D \widehat{L}_0 \widehat{Q}_3 \widehat{D}_3 + h_\tau \widehat{L}_0 \widehat{L}_3 \widehat{R}_3 - \mu_0 \widehat{L}_0 \widehat{H}_u\right]. \tag{7}$$

Here we are using the notation $\widehat{L}_0 \equiv \widehat{H}_d$, $\mu_0 \equiv \mu$, and $\lambda_0^D \equiv h_{\mathrm{b}}$ in the superpotential, and $v_0 \equiv v_d$ for the $\widehat{H}_d$ vacuum expectation value. This notation is justified because $\widehat{H}_d$ and $\widehat{L}_3$ have the same quantum numbers. The piece of the superpotential which breaks R-parity is given by

$$W_{R\!\!\!/} = -\mu_3 \widehat{L}_3 \widehat{H}_u, \tag{8}$$

where μ_3 is the bilinear R-parity violating term (BRpV), denoted $-\epsilon_3$ in Refs. [9–13].

Notice that we do not generate a trilinear R-parity violating (TRpV) term in models that arise from spontaneous breaking of R-parity. In fact, even if explicit trilinear terms were present, for the simple one-generation case they can always be rotated away into the bilinear term given in Eq. (8). In other words, the most general one-generation explicit SUGRA R-parity violation model is characterized by a single parameter, which may be chosen either as μ_3, or as λ_3^D (defined below) or the sneutrino VEV. The converse is not true, BRpV cannot be rotated away in favour of TRpV.

Although the above presentation would be in some sense the simplest and sufficient for our purposes, it will be useful for us in what follows to keep a redundant parameterization in which the bilinear and trilinear R-parity violating terms coexist.

The scalar potential contains the following relevant soft terms:

$$\begin{aligned} V_{\mathrm{soft}} = & \begin{pmatrix} \widetilde{L}_0 \\ \widetilde{L}_3 \end{pmatrix}^\dagger \begin{pmatrix} M_{L_0}^2 & M_{L_{03}}^2 \\ M_{L_{30}}^2 & M_{L_3}^2 \end{pmatrix} \begin{pmatrix} \widetilde{L}_0 \\ \widetilde{L}_3 \end{pmatrix} \\ & - \left(\mu_\alpha B_\alpha \widetilde{L}_\alpha H_u - A_\alpha^D \lambda_\alpha^D \widetilde{L}_3 \widetilde{Q}_3 \widetilde{D}_3 + \mathrm{h.c.}\right), \end{aligned} \tag{9}$$

where $M_{L_i}^2$ are the soft-mass terms and mixing for the down type Higgs and slepton fields, B_α, $\alpha = 0, 3$, are the bilinear soft-mass parameters (B_0 corresponds to the usual B term in the MSSM), while A^D_α are the trilinear soft-mass parameters (A^D_0 is the usual A_D term in the MSSM).

The equality of the quantum numbers of the down-type Higgs and tau lepton $SU(2) \otimes U(1)$ superfields opens the possibility to work in different basis [17,59–62]. This field redefinition is

$$\begin{pmatrix} \widehat{L}'_0 \\ \widehat{L}'_3 \end{pmatrix} = \begin{pmatrix} \cos\alpha_L & \sin\alpha_L \\ -\sin\alpha_L & \cos\alpha_L \end{pmatrix} \begin{pmatrix} \widehat{L}_0 \\ \widehat{L}_3 \end{pmatrix}, \tag{10}$$

where α_L is the angle of rotation, which in turn induces a rotation of the μ-terms. Under this change of basis the Lagrangian parameters are redefined and, because of this, it is impossible to eliminate completely the effects of the bilinear terms [9–13,61,62]. Note that different basis may be convenient for different applications [60].

Here we are specially interested to express R-parity violating effects through basis independent parameters:

$$v_d \equiv \sqrt{v_0^2 + v_3^2}\,, \tag{11}$$

$$\mu \equiv \sqrt{\mu_0^2 + \mu_3^2}\,, \tag{12}$$

$$\lambda^D \equiv \sqrt{(\lambda^D_0)^2 + (\lambda^D_3)^2}. \tag{13}$$

From the above we can deduce that the natural generalization of the MSSM definition of $\tan\beta$ is given by

$$\tan\beta = \frac{v_u}{v_d}, \tag{14}$$

which is also a basis invariant. This definition differs from the one used in Refs. [9,58], namely $\tan\beta = v_u/v_0$. There are other invariants which turn out to be very useful [63–65] and are defined as

$$\cos\zeta = \frac{\mu_\alpha v_\alpha}{\mu v_d}\,, \tag{15}$$

$$\cos\gamma = \frac{\lambda^D_\alpha \mu_\alpha}{\lambda^D \mu}\,, \tag{16}$$

$$\cos\chi = \frac{\lambda^D_\alpha v_\alpha}{\lambda^D v_d}. \tag{17}$$

Note that these three parameters are not independent due to the trigonometric relation

$$\cos\chi = \cos(\gamma - \zeta). \tag{18}$$

The remaining R-parity violating variables $\sin\zeta$ and $\sin\gamma$ determine the ν_τ mass at tree level and the R-parity violating effects in general in the fermionic sector, while $\sin\chi$ characterizes the R-parity violating effects on α_s. As we will see below there is only one of these parameters which survives, owing to the minimization conditions of the theory.

In this model the top- and bottom-quark masses are given by

$$\mathrm{M_t} = \frac{h_\mathrm{t}}{\sqrt{2}} v_u = s_\beta h_\mathrm{t} \frac{\sqrt{2}\mathrm{M}_W}{g}\,, \tag{19}$$

$$\mathrm{M_b} = \frac{1}{\sqrt{2}}\left(\lambda_0^D v_0 + \lambda_3^D v_3\right) = c_\beta c_\chi \lambda^D \frac{\sqrt{2}\mathrm{M}_W}{g}. \tag{20}$$

This formula for the bottom-quark mass is specially interesting, since it is expressed in terms of basis-independent R-parity violating parameters.

As in the MSSM, to connect the phenomenology at the electroweak scale with the SUGRA parameter space we need to use the renormalization group equations. A question immediately arises as to the number of additional parameters necessary to characterize the model. For a one-generation model with universality of soft parameters at the unification scale only one additional parameter is needed in addition to the MSUGRA parameters [9]. We have, however, some freedom in this choice. To compute the Lagrangian parameters at the electroweak scale we can follow two different approaches [60]:

- the bilinear or μ_3-approach, in which the parameters which fix the model are:

  $$\left(\mathrm{A}_0, \mathrm{M}_0, \mathrm{M}_{1/2}, \mathrm{t}_\beta, \mu_3^{\mathrm{GUT}}\right).$$

  Because of the form of the RGE for λ_3^D: $\mathrm{d}\lambda_3^D/\mathrm{d}t \propto \lambda_3^D$, if λ_3^D is zero at the unification scale it will be zero at the electroweak scale;
- the second possibility is the λ_3^D-approach, in this case the fundamental parameters of the model are

  $$\left(\mathrm{A}_0, \mathrm{M}_0, \mathrm{M}_{1/2}, \mathrm{t}_\beta, \left(\lambda_3^D\right)^{\mathrm{GUT}}\right).$$

  In contrast to the previous case here one arrives at the electroweak scale to the coexistence of bilinear and trilinear R-parity breaking parameters.

It does not matter which approach we follow because both are equivalent. Notice that, while in the bilinear approach one can ignore trilinear terms without loss of generality, the converse is not true: one cannot neglect bilinear terms consistently due to the structure of the RGEs. One can change from one basis to another and thus compare calculations which have been performed in different basis. These results have to be the same.

Now we are ready to understand how R-parity violation can affect the gauge coupling unification through the two-loop Yukawa contribution to the RGES for α_s. One finds,

$$\Delta\alpha_s^{\mathrm{YUK}} \approx -\frac{\alpha_s^2}{32\pi^3}\ln\left(\frac{\mathrm{M_{GUT}}}{\mathrm{M_t}}\right)\left\{b'_{3\mathrm{t}}h_\mathrm{t}^2 + b'_{3\mathrm{b}}\left(\lambda_0^D\right)^2 + b'_{3\mathrm{b}}\left(\lambda_3^D\right)^2\right\}, \tag{21}$$

where one notes the appearance of the R-parity violating coupling λ_3^D. Clearly this term combines with λ_0^D to form the basis invariant λ^D as follows:

$$\Delta\alpha_s^{\mathrm{YUK}} \approx -\frac{\alpha_s^2}{32\pi^3}\ln\left(\frac{\mathrm{M_{GUT}}}{\mathrm{M_t}}\right)\left\{b'_{3\mathrm{t}}h_\mathrm{t}^2 + b'_{3\mathrm{b}}\left(\lambda^D\right)^2\right\}.$$

Using the formulas (20,19) for the top and bottom masses we obtain

$$\Delta\alpha_s^{\mathrm{YUK}} \approx -\frac{\alpha_s^2}{32\pi^3}\ln\left(\frac{\mathrm{M_{GUT}}}{\mathrm{M_t}}\right)\frac{g^2}{2\mathrm{M}_W^2}\left(1+t_\beta^2\right)\left\{b'_{3\mathrm{t}}\frac{\mathrm{M_t^2}}{t_\beta^2} + b'_{3\mathrm{b}}\frac{\mathrm{M_b^2}}{c_\chi^2}\right\}. \tag{22}$$

We are now set to demonstrate the relation between the last term in Eq. (22) and the magnitude of R-parity violation which, as already mentioned, is characterized by a unique parameter in this model. To see this we must make use of the three minimization equations of the scalar potential of the theory, the two analogous to the MSSM plus a third equation related to the tau sneutrino field. Using this equation one can find a relation between $\sin\zeta$ and $\sin 2\gamma$ which finally reduces the extra number of parameters to simply one, when compared with the R-parity conserving supergravity model. At first order in μ_3/μ it can be simplified to

$$\sin\zeta = \frac{\mu_0\mu_3}{\mu^2}(\delta_B t_\beta \pm \delta_M) = \frac{1}{2}\sin(2\gamma)(\delta_B t_\beta \pm \delta_M), \tag{23}$$

where

$$\delta_B = \frac{\mu\Delta B}{\left(M^2_{\tilde{\nu}_3} - \frac{\mu_3^2}{\mu^2}\Delta M^2\right)}, \qquad \delta_M = \frac{\Delta M^2}{\left(M^2_{\tilde{\nu}_3} - \frac{\mu_3^2}{\mu^2}\Delta M^2\right)},$$

and we have defined

$$\Delta B = B_3 - B_0, \qquad \Delta M^2 = M_3^2 - M_0^2,$$

with the parameters given at the weak scale.

We notice that the double sign in Eq. (23) is the result of the solution to a quadratic equation in the minimization conditions of the scalar potential. In models with universality of soft terms, δ_M is positive but δ_B can take either sign.

Thus Eq. (23) shows that, as anticipated, only one of the three parameters ζ, γ, χ is independent. This parameter, together with the rest of the SUGRA parameters, determines the Majorana mass for the tau neutrino. The latter is induced by the mixing of the original tau neutrino field with the neutralinos [22–32] and is determined mainly by the parameter $\sin\zeta$ through the approximate relation

$$\mathrm{M}_{\nu_\tau} = \frac{\mathrm{M}_Z^2 M_{\tilde{\gamma}}\mu s_\zeta^2 c_\beta^2}{\left(\mathrm{M}_Z^2 M_{\tilde{\gamma}} s_{2\beta} c_\zeta - M_1 M_2 \mu\right)} \tag{24}$$

valid at tree level, which depends on the SUGRA parameters, and where we have defined the parameter $M_{\tilde{\gamma}} \equiv c_W M_1 + s_W M_2$. From Eqs. (18), (23) and (24), it is evident that we can get an expression for $\cos\chi$ whose exact form is unimportant for our present argument, except for the property that

$$\cos\chi \to 1 \quad \text{as} \quad M_{\nu_\tau} \to 0.$$

Thus the maximum value $c_\chi = 1$ corresponds to the R-parity conserving case. From Eq. (17) we see that in the basis where there is no trilinear term, s_χ is proportional to the sneutrino vev v_3, thus it is clear that the larger the R-parity violating parameter v_3 the larger will be the additional contribution coming from the ratio $\mathrm{M_b}/c_\chi$ in Eq. (22). The above equation establishes a relationship that the basis-independent parameter c_χ bears with the tau neutrino mass.

We now turn to the implications of R-parity violation on the α_s predictions derived from Eq. (22) and to our numerical results. We have used the two-loop renormalization group equations for the gauge coupling constants and the Yukawa couplings and the one-loop

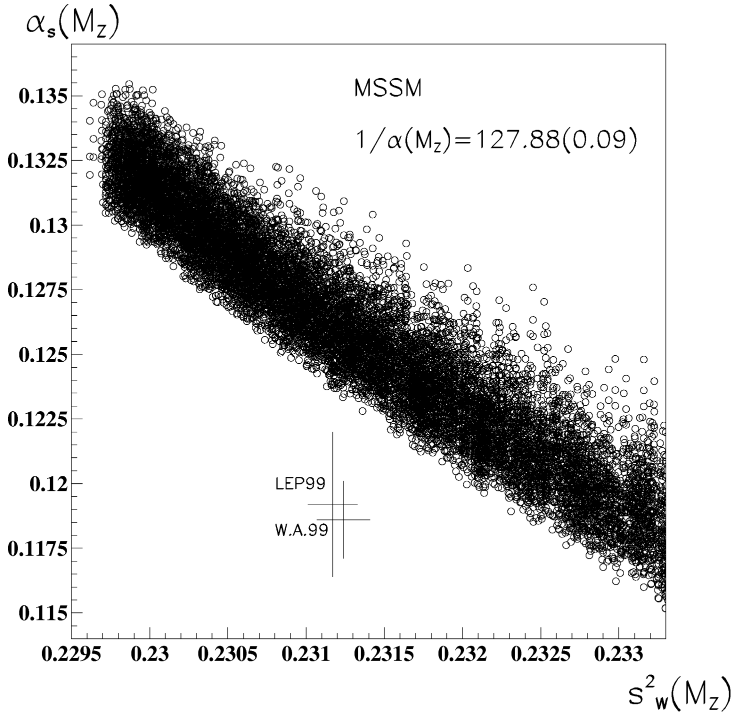

Fig. 1. $\alpha_s(\mathrm{M}_Z)$ versus $\hat{s}_Z$ for the MSSM

RGE for μ-terms and for the rest of the soft parameters [66–69]. We study the prediction for the gauge coupling constants at the M_Z scale in a model with universality of the soft terms at the unification scale. [1] We compare masses and couplings at the M_Z scale with their experimental values (see appendix for a detailed description of the method we have used for the running of the effective masses to their pole values and the running of the gauge couplings to their $\overline{\mathrm{MS}}$ values at the M_Z scale).

As a first step in our study of the supersymmetric $\alpha_s(\mathrm{M}_Z)$ and $\hat{s}_Z^2$ predictions we have updated the standard MSUGRA prediction taking into account the latest PDG experimental values for $\widehat{\alpha}(\mathrm{M}_Z)^{-1}$ [7]:

$$\widehat{\alpha}(\mathrm{M}_Z)^{-1} = 127.88 \pm 0.09.$$

On the other hand for the top, bottom and tau pole masses we have used [7] are:

$$\mathrm{M}_\mathrm{t}^{\mathrm{pol}} = 173 \pm 5.2\ \mathrm{GeV}, \quad \mathrm{M}_\mathrm{b}^{\mathrm{pol}} = 4.1\text{–}4.4\ \mathrm{GeV}, \quad \mathrm{M}_\tau^{\mathrm{pol}} = 1777.05\, {}^{+0.29}_{-0.26}\ \mathrm{MeV}.$$

In Fig. 1 we display a scatter plot with the updated MSUGRA prediction for $\alpha_s(\mathrm{M}_Z)$ and $\hat{s}_Z^2$, where each point corresponds to a different choice of SUGRA parameters, varying over a wide range:

[1] For the sake of generality and in order to simplify the discussion we will neglect possible GUT threshold contributions, as well as nonrenormalizable operator contributions.

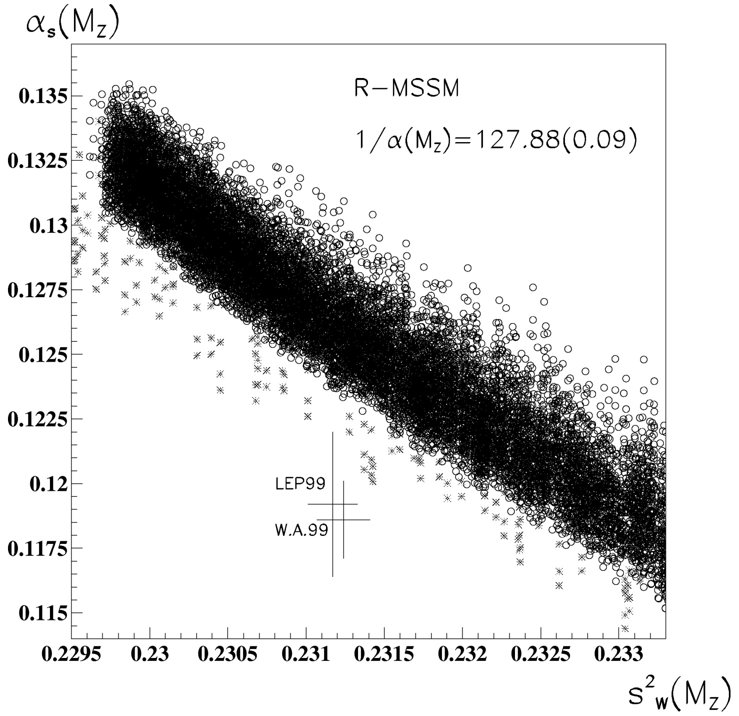

Fig. 2. $\alpha_s(M_Z)$ versus $\hat{s}_Z$ for the $R\!\!\!/$-MSSM model.

$$\begin{array}{ll} 0 < M_0 < 500\ \text{GeV}, & 0 < M_{1/2} < 500\ \text{GeV}, \\ -1000 < A_0 < 1000\ \text{GeV}, & 2 \lesssim t_\beta < 60. \end{array} \tag{25}$$

In this figure we display with error bars the present world average for $\alpha_s(M_Z)$:

$$\alpha_s(M_Z)^{\text{W.A.}} = 0.1186 \pm 0.0013,$$

and the 1999 average of the LEP measurements [7]:

$$\alpha_s(M_Z)^{\text{LEP98}} = 0.1192 \pm 0.0028.$$

For a discussion on the question of the average of values of α_s deduced at different energy scales, see Refs. [5,70–72]. We notice that if we fix $\hat{s}_Z^2$ inside its experimental range,

$$\left(\hat{s}_Z^2\right)^{\text{W.A.}} = 0.23124 \pm 0.00024,$$

the MSUGRA $\alpha_s(M_Z)$ prediction lies in the range $\alpha_s(M_Z) \approx 0.127 \pm 0.003$, which is a bit more than one σ higher than the most recent world average.

Now we turn to discuss the results we obtain in our bilinear R-parity breaking model, $R\!\!\!/$-MSSM for short, displayed in Fig. 2. The method we have used is similar to the previous procedure. In this case, additional complications appear because of the mixing between charginos and the tau lepton, and the necessity to ensure that the tau mass corresponds to the experimentally measured value. On the other hand, the mixing between the neutralinos and the neutrino generates a mass for the tau neutrino, as indicated by Eq. (24), which must lie below the experimental bound [73,74]. As we have already seen, the nonzero tau sneutrino VEV implies an additional constraint given by the third minimization equation. Once we satisfy these constraints, we find that $\not{R}$-MSUGRA predicts $\alpha_s(\mathrm{M}_Z)$ values closer to its experimental average, as compared to the R-parity conserving MSUGRA. This is due to the enhanced negative two-loop bottom-quark Yukawa contribution to the RGE's. Indeed, taking the world average experimental value of $\tilde{s}_Z^2$, one can move $\alpha_s(\mathrm{M}_Z)$ from a minimum value of approximately 0.125 in the MSUGRA case down to a minimum value of 0.122 or so in the $\not{R}$-MSSM model, bringing it closer to the W.A. and within one σ from the most recent average of LEP measurements given in Ref. [7]. These results can be clearly seen from Fig. 2, where each point represents a different parameter choice in the $\not{R}$-MSUGRA model. Notice that the $\not{R}$-MSUGRA model is totally fixed if we know the ν_τ mass, in addition to the other MSSM–SUGRA parameters. We have varied the tau neutrino mass below the laboratory bound $\mathrm{M}_{\nu_\tau} < 18.2$ MeV [73,74].

As it can be seen from Figs. 1 and 2, the net improvement of the α_s prediction in our BRpV model is 1σ compared to the MSSM. We note however that this is the result of a global scanning over the supersymmetric parameter space, and that the improvement on the α_s prediction at specified individual points in parameter space can be larger. In addition, it should be stressed out that the corrections to α_s are not directly governed by the parameter $\cos\zeta$ in Eq. (15), which controls the tree level neutrino mass, but by the parameter $\cos\chi$ in Eq. (17), which in the original basis is proportional to the tau-sneutrino VEV. In other words, large corrections to α_s are not necessarily associated to large ν_τ masses (see the next section).

4. Discussion: $\Delta\alpha_s$ versus m_{ν_τ}

Although ν_μ to ν_τ oscillations provide the preferred interpretation for the recent atmospheric neutrino data from underground detectors [75], other mechanisms such as conversion to sterile neutrinos [76–78], flavour-changing neutrino–matter interactions [79,80] or ν_μ decay [81,82] could also play an important role. For example, in the presence of a light sterile neutrino it is conceivable that even a very heavy tau-neutrino scenario (in which it decouples from the oscillations) would be acceptable by present underground data, which would be accounted for by oscillations among the three light neutrinos ν_e, ν_μ and the sterile neutrino. Oscillations amongst the latter would account for the conversions required to explain solar and atmospheric neutrino data. A non-supersymmetric scheme of this type has actually been suggested in Ref. [83]. Clearly in this case large negative corrections to $\alpha_s(\mathrm{M}_Z)$ naturally emerge, as the ones displayed in Fig. 2.

However, even in the preferred low neutrino-mass regime with ν_τ mass in the range close to few $\times 10^{-2}$ eV, indicated by the best fit of the oscillation hypothesis [76–78] (for previous analyses see, e.g. [77,78]), one can have a sizable decrease in $\alpha_s(\mathrm{M}_Z)$. This is possible provided there is some degree of cancellation as we explain below. We stress that the tree level ν_τ mass calculated in this paper is a good approximation of the heaviest neutrino mass calculated in the three generations case [41].

To better understand these statements we make a few approximations. Consider first Eq. (22). As we mentioned before, in BRpV the term proportional to $m_{\rm t}$ and the term proportional to $m_{\rm b}$ can be simultaneously large. In this case, with the two terms of similar magnitude, we have

$$\cos\chi \approx \frac{\mathrm{M_b}}{\mathrm{M_t}} \mathrm{t}_\beta \approx 0.017 \mathrm{t}_\beta \tag{26}$$

which is a necessary condition for large Yukawa contributions to α_s in BRpV. On the other hand, it is convenient to rewrite the formula for the neutrino mass in Eq. (24) by introducing the mass parameter Λ defined by the equation

$$\sin\zeta \equiv \frac{1}{c_\beta}\sqrt{\frac{\mathrm{M}_\nu}{\Lambda}} \tag{27}$$

where the neutrino mass M_ν is in Eq. (24) and $\Lambda = \mathcal{O}(\mathrm{M}_Z^2/\mathrm{M}_{1/2})$. Therefore, for a neutrino mass of the order of 0.1 eV we need $\sin\zeta \approx 10^{-5}/c_\beta\sqrt{\Lambda}$ with Λ in GeV, indicating that the parameter $\sin\zeta$ is very small. In this way, from Eq. (18) we see that small neutrino mass implies $\cos\chi \approx \cos\gamma$. Using this last relation in Eq. (23) we find a second expression for $\sin\zeta$:

$$\sin\zeta \approx s_\chi c_\chi (\delta_B \mathrm{t}_\beta \pm \delta_M) \tag{28}$$

where the δ's are defined below Eq. (23). The quantity in parenthesis is a good measure of the amount of cancellation necessary in order to have a sizable effect on α_s with small neutrino mass. The cancellation can occur with either sign since the sign of δ_B is not fixed. We have that

$$\delta \equiv (\delta_B \mathrm{t}_\beta \pm \delta_M) \approx \frac{1}{s_\chi c_\chi c_\beta}\sqrt{\frac{\mathrm{M}_\nu}{\Lambda}}. \tag{29}$$

We note that in SUGRA with universality of soft SUSY breaking parameters at unification δ_B is usually smaller than δ_M. As a result, the cancellation necessary in order to obtain small neutrino mass favours large $\tan\beta$ values. For example, for $\tan\beta = 40$, $c_\chi \approx s_\chi \approx 0.7$, and a 0.1 eV neutrino mass we have that for $\mathrm{M}_{1/2} = 200$ GeV the amount of cancellation is $\delta \approx 1 \times 10^{-4}$. If the gaugino mass parameter is increased to $\mathrm{M}_{1/2} = 1000$ GeV, the cancellation is $\delta \approx 3 \times 10^{-4}$. Our approximation is conservative since we have assumed δ_M of order 1. However, δ_M can be smaller because it is zero at the unification scale and arises only from the RGE evolution from unification to the weak scale. We do not think that this is a fine tuning. In fact we remind the reader that a similar amount of cancellation between VEV's is already present in the MSSM at high values of $\tan\beta$.

In short, while large negative corrections to $\alpha_s(\mathrm{M}_Z)$ are easier to find for large ν_τ masses, there is a range of parameters, motivated by universality of the soft breaking terms, where the effect is present even if the ν_τ mass is rather low. This guarantees also that the lightest neutralino would typically decay inside the detectors now under discussion, changing completely the phenomenology of supersymmetry from that expected in the MSSM.

5. Conclusion

In conclusion, we have shown how minimal R-parity violating supergravity can lower the $\alpha_s(\mathrm{M}_Z)$ prediction with respect to the case with conserved the R-parity, as suggested by the present experimental world average. We have identified the source of this effect on the α_s prediction as coming from the two-loop bottom Yukawa coupling contribution to the renormalization group evolution of the gauge couplings. We have also shown how this effect on the α_s prediction is related to the value of the tau neutrino mass which is generated by the mixing of neutralinos and neutrinos. We have also discussed to which extent this relation depends on the initial conditions for the soft supersymmetry breaking parameters at the unification scale. We showed how to obtain a sizable effect on $\alpha_s(\mathrm{M}_Z)$ even in the case that the ν_τ mass lies in the range indicated by the simplest neutrino oscillation interpretation of the atmospheric neutrino data.

Note added in proof

This paper has appeared in preprint form in June of 1999. Since then new experimental values of gauge coupling constants have appeared and been compiled in the last edition of the Particle Data Group 2000 (D.E. Groom et al., Eur. Phys. J. C15 (2000) 1). We have checked that the new values do not affect the conclusions of our paper.

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Appendix A. Numerical procedure

In this appendix we describe with some detail the method we follow to predict the strong gauge coupling constant at the M_Z scale. We have used the 2-loop RGE's for the gauge coupling constants and for the Yukawa couplings including R-parity violating couplings [66–69]. We neglect the Yukawa couplings of the first two generations. For the rest of the parameters of the $\not{R}$-MSSM model we have used 1-loop RGE's [66–69]. We have imposed universality of soft parameters and gauge coupling unification at a scale $\mathrm{M_U}$. We have explored between two values of the unification scale, $\mathrm{M_U}$, $1.2 \times 10^{16} < \mathrm{M_U} < 3.6 \times 10^{16}$ GeV, and the gauge coupling constant at the unification scale α_U, $23.5 < \alpha_\mathrm{U}^{-1} < 24.5$. Using the RGE's we have found the gauge coupling constants at $\mathrm{M_t}$ and then we have evolved down to M_Z scale as explain below. On the other hand we have computed the pole masses from the running masses at $\mathrm{M_t}$ following the same procedure as Ref. [84]. First of all we have to explain how we compute the Yukawa couplings at $\mathrm{M_t}$ at the SM side. We have to use the right matching conditions at $\mathrm{M_t}$ which are easy to compute from the formulas (19) and (20) for the h_t, λ^D, y h_τ Yukawas. In the $\not{R}$-MSSM model are [58]

$$\begin{aligned}
h_{\mathrm{t}}(\mathrm{M_t})^{\mathrm{SM}} &= s_\beta h_{\mathrm{t}}(\mathrm{M_t})^{\not{R}},\\
h_{\mathrm{b}}(\mathrm{M_t})^{\mathrm{SM}} &= c_\chi c_\beta \lambda^D(\mathrm{M_t}),\\
h_\tau(\mathrm{M_t})^{\mathrm{SM}} &= \frac{c_\beta}{\left(1-s_\zeta^2 f(\mathrm{M_2},\mathrm{t}_\beta,\mu,c_\zeta)\right)^{1/2}} h_\tau(\mathrm{M_t})^{\not{R}},
\end{aligned}$$

where the function f can be found in Ref. [58]. These conditions reduce to the MSSM matching conditions in the limit $c_\zeta, c_\chi \to 1$.

In order to run of masses and couplings to their experimental values we use known relations. First we have evolved α_1 and α_2 from scale $\mathrm{M_t}$ to scale M_Z to compute $\alpha(\mathrm{M}_Z)$ and $\hat{s}_Z^2$. For α_s, given the value $\alpha_s(\mathrm{M_t})$, which we get from the running of the RGE's from the unification to the $\mathrm{M_t}$ scale, we can compute Λ_{QCD} at $\mathrm{M_t}$ using the approximate solution for α_s in the SM [85–87] which includes 3-loop QCD contributions

$$\begin{aligned}
&\alpha_s(\mu) = \frac{\pi}{\beta_0 t}\left[1-\frac{\beta_1}{\beta_0^2}\frac{\ln(t)}{t}+\frac{\beta_1^2}{\beta_0^4 t^2}\left(\left(\ln(t)-\frac{1}{2}\right)^2+\frac{\beta_2\beta_0}{\beta_1^2}\frac{5}{4}\right)\right], \quad \text{where}\\
&\quad t=\ln\left(\frac{\mu^2}{\Lambda^2}\right), \qquad \beta_0=\left(11-\tfrac{2}{3}n_f\right)\frac{1}{4},\\
&\quad \beta_1=\left(51-\tfrac{19}{3}n_f\right)\tfrac{1}{8}, \qquad \beta_2=\left(2857-\tfrac{5033}{9}n_f-\tfrac{325}{27}n_f^2\right)\tfrac{1}{128}.
\end{aligned}$$

Later using the same formula we can extrapolate α_s at M_Z. To compute the top-quark pole mass we use [88]

$$\mathrm{M_t^{pol}} = \mathrm{M_t}(\mathrm{M_t})\left[1+\frac{4}{3\pi}\alpha_3(\mathrm{M_t})\right].$$

On the other hand to compute the bottom-quark pole mass we use the quark effective mass formula which includes 1-loop QED and 3-loop QCD contributions

$$\mathrm{M_b}(\mathrm{M_t}) = \mathrm{M_b}(\mathrm{M_b})\left(\frac{\alpha(\mathrm{M_t})}{\alpha(\mathrm{M_b}(\mathrm{M_b}))}\right)^{\gamma_0^{\mathrm{QED}}/b_0^{\mathrm{QED}}}\frac{F(\alpha_3(\mathrm{M_t}))}{F(\alpha_3(\mathrm{M_b}(\mathrm{M_b})))},$$

where the QED beta function and the anomalous dimension, γ_0^{QED} and b_0^{QED}, are given by[47]

$$\gamma_0^{\mathrm{QED}} = -3Q_f^2, \qquad b_0^{\mathrm{QED}} = \tfrac{4}{3}\left(3\sum Q_u^2 + 3\sum Q_d^2 + \sum Q_e^2\right),$$

and the sum runs over all the active fermions at the relevant scale. The formula $F(\alpha_s(\mu))$ is given by [85–87]

$$\begin{aligned}
F\left(\alpha_s(\mu)\right) &= \left(\frac{2\beta_0\alpha_s(\mu)}{\pi}\right)^{\gamma_0/\beta_0}\\
&\quad\times\left\{1+\left(\frac{\gamma_1}{\beta_0}-\frac{\gamma_0\beta_1}{\beta_0^2}\right)\frac{\alpha_s(\mu)}{\pi}\right.
\end{aligned}$$

$$+\frac{1}{2}\left[\left(\frac{\gamma_1}{\beta_0}-\frac{\gamma_0\beta_1}{\beta_0^2}\right)^2+\left(\frac{\gamma_2}{\beta_0}+\frac{\gamma_0\beta_1^2}{\beta_0^3}-\frac{\beta_1\gamma_1+\beta_2\gamma_0}{\beta_0^2}\right)\right]\left(\frac{\alpha_s(\mu)}{\pi}\right)^2$$
$$+\,\mathrm{O}\left(\alpha_s^3(\mu)\right)\Bigg\},$$

where

$$\gamma_0=1,\qquad \gamma_1=\left(\tfrac{202}{3}-\tfrac{20}{9}n_f\right)\tfrac{1}{16},$$
$$\gamma_2=\left(1249-\left(\tfrac{2216}{27}+\tfrac{160}{3}\zeta(3)\right)n_f-\tfrac{140}{81}n_f^2\right)\tfrac{1}{64}.$$

Finally to compute tau lepton pole mass from the tau running mass at $\mathrm{M_t}$ we use

$$\mathrm{m}_\tau^{\mathrm{pol}}=\mathrm{m}_\tau(\mu)\left[1+\frac{\alpha(\mu)}{\pi}\left(1+\frac{3}{4}\ln\left(\frac{\mu^2}{\mathrm{m}_\tau^2(\mu)}\right)\right)\right].$$

In summary, starting with the basic parameters M_0, A_0, $\mathrm{M}_{1/2}$, t_β, μ_3, $\mathrm{M_U}$ and α_G we have required that $\alpha(\mathrm{M}_Z)$ as well as the top, bottom and tau pole masses τ were inside their experimental measurements in order to obtain a prediction for the variables $\hat{s}_Z^2$ and $\alpha_s(\mathrm{M}_Z)$ which can be seen in figures.

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