ELSEVIER

Nuclear Physics B (Proc. Suppl.) 81 (2000) 231–241

PROCEEDINGS SUPPLEMENTS

www.elsevier.nl/locate/npe

Neutrino Masses from Broken R-Parity

J. C. Romão[a] *

[a]Instituto Superior Técnico, Departamento de Física
A. Rovisco Pais 1, 1049-001 Lisboa, Portugal

We review models where R–parity is broken, either spontaneously or explicitly. In this last case we consider the situation where R–parity is broken via bilinear terms in the superpotential. We show that although at tree level only one neutrino gets mass, at one–loop level all three neutrinos became massive. We study the conditions under which bimaximal mixing can be achieved and show that the masses can be in the correct ranges needed for solving the atmospheric and solar neutrino problems.

1. Introduction

In the past most discussions of supersymmetric (SUSY)[1,2] phenomenology assumed R–parity (R_P) conservation where,

$$R_P = (-1)^{2J+3B+L} \tag{1}$$

This implies that SUSY particles are pair produced, every SUSY particle decays into another SUSY particle and that there is a *LSP* that it is stable. But this is just an *ad hoc* assumption without a deep justification. In this talk we will review how R_P can be broken, either spontaneously or explicitly, and discuss the most important features of these models [3]. We will also describe recent results [4] on one–loop generated masses and mixings in the context of a model that is a minimal extension of the minimal extension of the MSSM–GUT [5] in which R_P Violation (RPV) is introduced via a bilinear term in the MSSM superpotential [6,7].

2. Spontaneously Broken R-Parity

2.1. The Original Proposal

In the original proposal [8] the content was just the MSSM and the breaking was induced by

$$\langle \tilde{\nu}_\tau \rangle = v_L \tag{2}$$

The problem with this model was that the Majoron J coupled to Z^0 with gauge strength and therefore the decay $Z^0 \to \rho_L J$ contributed to the invisible Z width the equivalent of half a (light) neutrino family. After LEP I this was excluded.

Table 1
Lepton number assignments.

Field	L	e^c	ν^c	S	others
Lepton #	1	−1	−1	1	0

2.2. A Viable Model for SBRP

The way to avoid the previous difficulty is to enlarge the model and make J mostly out of *isosinglets*. This was proposed by Masiero and Valle [9]. The content is the MSSM plus a few Isosinglet Superfields that carry lepton number,

$$\nu_i^c \equiv (1,0,-1)\ ;\ S_i \equiv (1,0,1)\ ;\ \Phi \equiv (1,0,0) \tag{3}$$

The model is defined by the superpotential [9,10],

$$\begin{aligned} W &= h_u u^c Q H_u + h_d d^c Q H_d + h_e e^c L H_d \\ &\quad + (h_0 H_u H_d - \mu^2)\Phi \\ &\quad + h_\nu \nu^c L H_u + h \Phi \nu^c S \end{aligned}$$

where the lepton number assignments are shown in Table 1. The spontaneous breaking of R parity and lepton number is driven by [10]

$$v_R = \langle \tilde{\nu}_{R\tau} \rangle \quad v_S = \left\langle \tilde{S}_\tau \right\rangle \quad v_L = \langle \tilde{\nu}_\tau \rangle \tag{4}$$

The electroweak breaking and fermion masses arise from

$$\langle H_u \rangle = v_u \qquad \langle H_d \rangle = v_d \tag{5}$$

*This work was supported by the TMR network grant ERBFMRXCT960090 of the European Union.

0920-5632/00/$ – see front matter
PII S0920-5632(99)00881-6

with $v^2 = v_u^2 + v_d^2$ fixed by the W mass. The Majoron is given by the imaginary part of

$$\frac{v_L^2}{Vv^2}(v_u H_u - v_d H_d) + \frac{v_L}{V}\tilde{\nu_\tau} - \frac{v_R}{V}\tilde{\nu^c}_\tau + \frac{v_S}{V}\tilde{S}_\tau \quad (6)$$

where $V = \sqrt{v_R^2 + v_S^2}$. Since the Majoron is mainly an $SU(2) \otimes U(1)$ singlet it does not contribute to the invisible Z^0 decay width.

2.3. Some Results on SBRP

The SBRP model has been extensively studied. The implications for accelerator [11] and non–accelerator [12] physics have been presented before and we will not discuss them here [3]. As in this talk we are concerned with the neutrino properties in the context of R_P models we will only review here the neutrino results.

- *Neutrinos have mass*

  Neutrinos are massless at Lagrangian level but get mass from the mixing with neutralinos[13,14]. In the SBRP model it is possible to have non zero masses for two neutrinos [14].

- *Neutrinos mix*

  The mixing is related to the the coupling matrix $h_{\nu_{ij}}$. This matrix has to be non diagonal in generation space to allow

  $$\nu_\tau \to \nu_\mu + J \quad (7)$$

  and therefore evading [14] the *Critical Density Argument* against $\nu's$ in the MeV range.

- *Avoiding BBN constraints on the* m_{ν_τ}

  In the *SM* BBN arguments [15] rule out ν_τ masses in the range

  $$0.5\ MeV < m_{\nu_\tau} < 35 MeV \quad (8)$$

  We have shown [16] that *SBRP* models can evade that constraint due to new annihilation channels

  $$\nu_\tau \nu_\tau \to JJ \quad (9)$$

3. Explicitly Broken R-Parity

The most general superpotential W with the particle content of the MSSM is given by [6,7]

$$W = W_{MSSM} + W_{\not R} \quad (10)$$

where

$$W_{MSSM} = \varepsilon_{ab}\left[h_U^{ij}\hat{Q}_i^a\hat{U}_j\hat{H}_u^b + h_D^{ij}\hat{Q}_i^b\hat{D}_j\hat{H}_d^a + h_E^{ij}\hat{L}_i^b\hat{R}_j\hat{H}_d^a - \mu\hat{H}_d^a\hat{H}_u^b\right] \quad (11)$$

and

$$W_{\not R} = \varepsilon_{ab}\left[\lambda_{ijk}\hat{L}_i^a\hat{L}_j^b\hat{R}_k + \lambda'_{ijk}\hat{D}_i\hat{L}_j^a\hat{Q}_k^b\right] + \lambda''_{ijk}\hat{D}_i\hat{D}_j\hat{U}_k + \varepsilon_{ab}\,\epsilon_i\hat{L}_i^a\hat{H}_u^b \quad (12)$$

where $i,j = 1,2,3$ are generation indices, $a,b = 1,2$ are $SU(2)$ indices.
The set of soft supersymmetry breaking terms are

$$V^{soft} = V_{MSSM}^{soft} + V_{\not R}^{soft} \quad (13)$$

$$\begin{aligned} V_{\text{MSSM}}^{soft} = {} & M_Q^{ij2}\tilde{Q}_i^{a*}\tilde{Q}_j^a + M_U^{ij2}\tilde{U}_i^*\tilde{U}_j + M_D^{ij2}\tilde{D}_i^*\tilde{D}_j \\ & + M_L^{ij2}\tilde{L}_i^{a*}\tilde{L}_j^a + M_R^{ij2}\tilde{R}_i^*\tilde{R}_j \\ & + m_{H_d}^2 H_d^{a*}H_d^a + m_{H_u}^2 H_u^{a*}H_u^a \\ & - \left[\tfrac{1}{2}\sum_{i=1}^{3} M_i\lambda_i\lambda_i + h.c.\right] \\ & + \varepsilon_{ab}\left[A_U^{ij}\tilde{Q}_i^a\tilde{U}_j H_u^b + A_D^{ij}\tilde{Q}_i^b\tilde{D}_j H_d^a \right. \\ & \left. + A_E^{ij}\tilde{L}_i^b\tilde{R}_j H_d^a - B\mu H_d^a H_u^b + h.c.\right], \quad (14) \end{aligned}$$

and

$$V_{\not R}^{soft} = \varepsilon_{ab}\left[A_\lambda^{ij}\tilde{L}_i^a\tilde{L}_j^b\tilde{R}_k + A_{\lambda'}^{ijk}\tilde{D}_i\tilde{L}_j^a\tilde{Q}_k^b\right] + A_{\lambda''}^{ij}\tilde{D}_i\tilde{D}_j\tilde{U}_k + \varepsilon_{ab}\,B_i\epsilon_i\tilde{L}_i^a H_u^b + h.c. \quad (15)$$

The bilinear R_P violating term *cannot* be eliminated by superfield redefinition as sometimes it is claimed. To show this we consider the case [17] where all the trilinear couplings in Eq. (12) are zero and for simplicity we take $\epsilon_1 = \epsilon_2 = 0$. Then the superpotential is

$$W = \varepsilon_{ab}\left[h_t\hat{Q}_3^a\hat{U}_3\hat{H}_u^b + h_b\hat{Q}_3^b\hat{D}_3\hat{H}_d^a + h_\tau\hat{L}_3^b\hat{R}_3\hat{H}_d^a\right.$$

$$-\mu\widehat{H}_d^a\widehat{H}_u^b + \epsilon_3\widehat{L}_3^a\widehat{H}_u^b\Big] \qquad (16)$$

Consider now the rotation

$$\widehat{H}'_d = \frac{\mu\widehat{H}_d - \epsilon_3\widehat{L}_3}{\sqrt{\mu^2+\epsilon_3^2}}, \qquad \widehat{L}'_3 = \frac{\epsilon_3\widehat{H}_d + \mu\widehat{L}_3}{\sqrt{\mu^2+\epsilon_3^2}} \qquad (17)$$

In the new basis

$$W = h_t\widehat{Q}_3\widehat{U}_3\widehat{H}_u + h_b\frac{\mu}{\mu'}\widehat{Q}_3\widehat{D}_3\widehat{H}'_d + h_\tau\widehat{L}'_3\widehat{R}_3\widehat{H}'_d$$
$$-\mu'\widehat{H}'_d\widehat{H}_u + h_b\frac{\epsilon_3}{\mu'}\widehat{Q}_3\widehat{D}_3\widehat{L}'_3 \qquad (18)$$

where

$$\mu'^2 = \mu^2 + \epsilon_3^2 \qquad (19)$$

But the soft terms,

$$V_{soft} = m_{H_d}^2|H_d|^2 + M_{L_3}^2|\widetilde{L}_3|^2 + \Big[B\mu H_d H_u - B_2\epsilon_3\widetilde{L}_3 H_u + h.c.\Big] + \cdots \qquad (20)$$

in the rotated basis is

$$V_{soft} = \frac{m_{H_d}^2\mu^2 + M_{L_3}^2\epsilon_3^2}{\mu'^2}|H'_d|^2 + \frac{m_{H_d}^2\epsilon_3^2 + M_{L_3}^2\mu^2}{\mu'^2}|\widetilde{L}'_3|^2 - \Big[\frac{B\mu^2 + B_2\epsilon_3^2}{\mu'}H'_d H_u - \frac{\epsilon_3\mu}{\mu'^2}(m_{H_d}^2 - M_{L_3}^2)\widetilde{L}'_3 H'_d - \frac{\epsilon_3\mu}{\mu'}(B_2 - B)\widetilde{L}'_3 H_u + h.c.\Big] + \cdots \qquad (21)$$

The last two terms violate R_P and induce a non-zero VEV for the τ sneutrino field in the rotated basis $\langle\tilde{\nu}'_\tau\rangle = v'_3/\sqrt{2}$, where

$$v'_3 \approx -\frac{\epsilon_3\mu}{\mu'^2 m^2_{\tilde{\nu}^0_\tau}}\left(v'_1\Delta m^2 + \mu' v_2\Delta B\right) \qquad (22)$$

and

$$\Delta m^2 \equiv m_{H_1}^2 - M_{L_3}^2 \approx -\frac{3h_b^2}{8\pi^2}\left(m_{H_1}^2 + M_Q^2 + M_D^2 + A_D^2\right)\ln\frac{M_{GUT}}{m_Z}$$

$$\Delta B \equiv B_2 - B \approx \frac{3h_b^2}{8\pi^2}A_D\ln\frac{M_{GUT}}{m_Z} \qquad (23)$$

4. Bilinear R-Parity Violation

4.1. The Model

The superpotential W for the bilinear R_P violation model is given by [6,7]

$$W = \varepsilon_{ab}\Big[h_U^{ij}\widehat{Q}_i^a\widehat{U}_j\widehat{H}_u^b + h_D^{ij}\widehat{Q}_i^b\widehat{D}_j\widehat{H}_d^a + h_E^{ij}\widehat{L}_i^b\widehat{R}_j\widehat{H}_d^a - \mu\widehat{H}_d^a\widehat{H}_u^b + \epsilon_i\widehat{L}_i^a\widehat{H}_u^b\Big] \qquad (24)$$

where $i,j = 1,2,3$ are generation indices, $a,b = 1,2$ are $SU(2)$ indices. The set of soft supersymmetry breaking terms are

$$V_{soft} = M_Q^{ij2}\widetilde{Q}_i^{a*}\widetilde{Q}_j^a + M_U^{ij2}\widetilde{U}_i^*\widetilde{U}_j + M_D^{ij2}\widetilde{D}_i^*\widetilde{D}_j + M_L^{ij2}\widetilde{L}_i^{a*}\widetilde{L}_j^a + M_R^{ij2}\widetilde{R}_i^*\widetilde{R}_j + m_{H_d}^2 H_d^{a*}H_d^a + m_{H_u}^2 H_u^{a*}H_u^a - \Big[\tfrac{1}{2}\sum M_i\lambda_i\lambda_i + h.c.\Big] + \varepsilon_{ab}\Big[A_U^{ij}\widetilde{Q}_i^a\widetilde{U}_j H_u^b + A_D^{ij}\widetilde{Q}_i^b\widetilde{D}_j H_d^a + A_E^{ij}\widetilde{L}_i^b\widetilde{R}_j H_d^a - B\mu H_d^a H_u^b + B_i\epsilon_i\widetilde{L}_i^a H_u^b\Big]. \qquad (25)$$

The electroweak symmetry is broken when the VEVS of the two Higgs doublets H_d and H_u, and the sneutrinos.

$$H_d = \begin{pmatrix}\frac{1}{\sqrt{2}}[\chi_d^0 + v_d + i\varphi_d^0]\\ H_d^-\end{pmatrix} \qquad (26)$$

$$H_u = \begin{pmatrix}H_u^+\\ \frac{1}{\sqrt{2}}[\chi_u^0 + v_u + i\varphi_u^0]\end{pmatrix} \qquad (27)$$

$$L_i = \begin{pmatrix}\frac{1}{\sqrt{2}}[\tilde{\nu}_i^R + v_i + i\tilde{\nu}_i^I]\\ \tilde{\ell}^i\end{pmatrix} \qquad (28)$$

The gauge bosons W and Z acquire masses

$$m_W^2 = \tfrac{1}{4}g^2v^2 \quad ; \quad m_Z^2 = \tfrac{1}{4}(g^2+g'^2)v^2 \qquad (29)$$

where

$$v^2 \equiv v_d^2 + v_u^2 + v_1^2 + v_2^2 + v_3^2 = (246\ \text{GeV})^2 \qquad (30)$$

We introduce the following notation in spherical coordinates:

$$v_d = v\sin\theta_1\sin\theta_2\sin\theta_3\cos\beta$$
$$v_u = v\sin\theta_1\sin\theta_2\sin\theta_3\sin\beta$$
$$v_1 = v\sin\theta_1\sin\theta_2\cos\theta_3$$

$$v_2 = v \sin\theta_1 \cos\theta_2$$
$$v_3 = v \cos\theta_1$$

which preserves the MSSM definition $\tan\beta = v_u/v_d$. The angles θ_i are equal to $\pi/2$ in the MSSM limit.
The full scalar potential may be written as

$$V_{total} = \sum_i \left|\frac{\partial W}{\partial z_i}\right|^2 + V_D + V_{soft} + V_{RC} \qquad (31)$$

where z_i denotes any one of the scalar fields in the theory, V_D are the usual D-terms, V_{soft} the SUSY soft breaking terms, and V_{RC} are the one-loop radiative corrections.
In writing V_{RC} we use the diagrammatic method and find the minimization conditions by correcting to one–loop the tadpole equations. This method has advantages with respect to the effective potential when we calculate the one–loop corrected scalar masses. The scalar potential contains linear terms

$$V_{linear} = t_d\sigma_d^0 + t_u\sigma_u^0 + t_i\tilde{\nu}_i^R \equiv t_\alpha\sigma_\alpha^0\,, \qquad (32)$$

where we have introduced the notation

$$\sigma_\alpha^0 = (\sigma_d^0, \sigma_u^0, \nu_1^R, \nu_2^R, \nu_3^R) \qquad (33)$$

and $\alpha = d, u, 1, 2, 3$. The one loop tadpoles are

$$t_\alpha = t_\alpha^0 - \delta t_\alpha^{\overline{MS}} + T_\alpha(Q)$$
$$= t_\alpha^0 + T_\alpha^{\overline{MS}}(Q) \qquad (34)$$

where $T_\alpha^{\overline{MS}}(Q) \equiv -\delta t_\alpha^{\overline{MS}} + T_\alpha(Q)$ are the finite one–loop tadpoles.

4.2. Main Features

The ϵ–model is a one (three) parameter(s) generalization of the MSSM. It can be thought as an effective model showing the more important features of the SBRP–model [10] at the weak scale. The mass matrices, charged and neutral currents, are similar to the SBRP–model if we identify

$$\epsilon \equiv v_R h_\nu \qquad (35)$$

The R_P violating parameters ϵ_i and v_i violate lepton number, inducing a non-zero mass for only one neutrino, which could be considered to be the

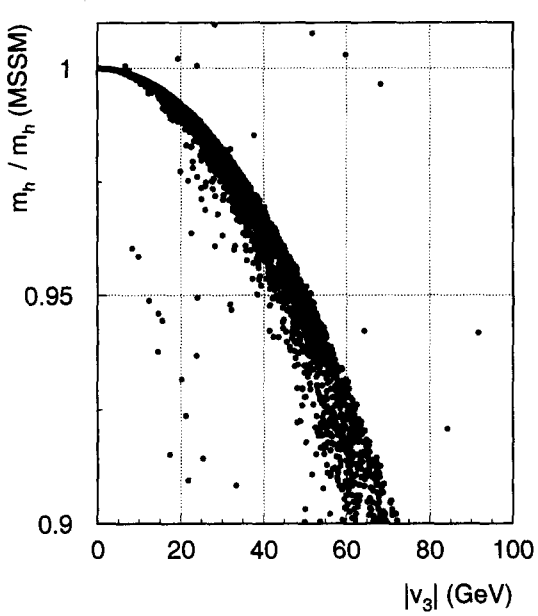


Figure 1. Ratio of the lightest CP-even Higgs boson mass m_h in the ϵ–model and in the MSSM as a function of v_3.

the ν_τ. The ν_e and ν_μ remain massless in first approximation. As we will explain below, they acquire masses from supersymmetric loops [4,18] that are typically smaller than the tree level mass.

The model has the MSSM as a limit. This can be illustrated in Figure 1 where we show the ratio of the lightest CP-even Higgs boson mass m_h in the ϵ–model and in the MSSM as a function of v_3. Many other results concerning this model and the implications for physics at the accelerators can be found in ref. [6,7].

5. Radiative Breaking

5.1. Radiative Breaking in the ϵ model: The minimal case

At $Q = M_{GUT}$ we assume the standard minimal supergravity unifications assumptions,

$$A_t = A_b = A_\tau \equiv A\,,$$
$$B = B_2 = A - 1\,,$$
$$m_{H_d}^2 = m_{H_u}^2 = M_L^2 = M_R^2 = m_0^2\,,$$

$$M_Q^2 = M_U^2 = M_D^2 = m_0^2\,,$$

$$M_3 = M_2 = M_1 = M_{1/2}$$

In order to determine the values of the Yukawa couplings and of the soft breaking scalar masses at low energies we first run the RGE's from the unification scale $M_{GUT} \sim 10^{16}$ GeV down to the weak scale. We randomly give values at the unification scale for the parameters of the theory.

$$\begin{array}{rcccl} 10^{-2} & \leq & h_{t\,GUT}^2/4\pi & \leq & 1 \\ 10^{-5} & \leq & h_{b\,GUT}^2/4\pi & \leq & 1 \\ -3 & \leq & A/m_0 & \leq & 3 \\ 0 & \leq & \mu_{GUT}^2/m_0^2 & \leq & 10 \\ 0 & \leq & M_{1/2}/m_0 & \leq & 5 \\ 10^{-2} & \leq & \epsilon_{i\,GUT}^2/m_0^2 & \leq & 10 \end{array} \qquad (36)$$

The values of $h_e^{GUT}, h_\mu^{GUT}, h_\tau^{GUT}$ are defined in such a way that we get the charged lepton masses correctly. As the charginos mix with the leptons, through a mass matrix given by

$$\mathcal{M}_C = \begin{bmatrix} M_C & A \\ B & M_L \end{bmatrix} \qquad (37)$$

where M_C is the usual MSSM chargino mass matrix,

$$M_C = \begin{bmatrix} M & \frac{1}{\sqrt{2}} g v_u \\ \frac{1}{\sqrt{2}} g v_d & \mu \end{bmatrix} \qquad (38)$$

M_L is the lepton mass matrix, that we consider diagonal,

$$M_L = \begin{bmatrix} \frac{1}{\sqrt{2}} h_{E_{11}} v_d & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} h_{E_{22}} v_d & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} h_{E_{33}} v_d \end{bmatrix} \qquad (39)$$

and A and B are matrices that are non zero due to the violation of R_P and are given by

$$A^T = \begin{bmatrix} -\frac{1}{2} h_{E_{11}} v_1 & 0 \\ -\frac{1}{2} h_{E_{22}} v_1 & 0 \\ -\frac{1}{2} h_{E_{33}} v_3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{2} g v_3 & -\epsilon_1 \\ \frac{1}{2} g v_3 & -\epsilon_2 \\ \frac{1}{2} g v_3 & -\epsilon_3 \end{bmatrix} \qquad (40)$$

We used an iterative procedure to accomplish that the three lightest eigenvalues of $\mathcal{M}_C$ are in agreement with the experimental masses of the leptons. After running the RGE we have a complete set of parameters, Yukawa couplings and soft-breaking masses $m_i^2(RGE)$ to study the minimization. This is done by the following method: we solve the minimization equations for the soft masses squared. This is easy because those equations are linear on the soft masses squared. The values obtained in this way, that we call m_i^2 are not equal to the values $m_i^2(RGE)$ that we got via RGE. To achieve equality we define a function

$$\eta = max\left(\frac{m_i^2}{m_i^2(RGE)}, \frac{m_i^2(RGE)}{m_i^2}\right) \quad \forall i \qquad (41)$$

with the obvious property that

$$\eta \geq 1 \qquad (42)$$

Then we adjust the parameters to minimize η.

Before we end this section let us discuss the counting of free parameters in this model and in the minimal N=1 supergravity unified version of the MSSM. In Table 2 we show this counting for the MSSM and in Table 3 for the ϵ–model. Finally, we note that in either case, the sign of the mixing parameter μ is physical and has to be taken into account.

Table 2
Counting of free parameters in MSSM

Parameters	Conditions	Free Parameters
h_t, h_b, h_τ	m_W, m_t	$\tan\beta$
v_d, v_u,$M_{1/2}$	m_b, m_τ	2 Extra
m_0, A, μ	$t_i = 0$, $i = 1,2$	(*e.g.* m_h, m_A)
Total = 9	Total = 6	Total = 3

Table 3
Counting of free parameters in our model

Parameters	Conditions	Free Parameters
h_t, h_b, h_τ	m_W, m_t	$\tan\beta$, ϵ_i
v_d, v_u, $M_{1/2}$	m_b, m_τ	
m_0,A, μ	$t_i = 0$	2 Extra
v_i, ϵ_i	$(i = 1,\ldots,5)$	(*e.g.* m_h, m_A)
Total = 15	Total = 9	Total = 6

5.2. Gauge and Yukawa Unification in the ϵ model

There is a strong motivation to consider GUT theories where *both* gauge and Yukawa unification can achieved. This is because besides achieving gauge coupling unification, GUT theories also reduce the number of free parameters in the Yukawa sector and this is normally a desirable feature. The situation with respect to the MSSM can be summarized as follows:

- In $SU(5)$ models, $h_b = h_\tau$ at M_{GUT}. The predicted ratio m_b/m_τ at M_{WEAK} agrees with experiments.
- A relation between m_{top} and $\tan\beta$ is predicted. Two solutions are possible: low and high $\tan\beta$.
- In $SO(10)$ and E_6 models $h_t = h_b = h_\tau$ at M_{GUT}. In this case, only the large $\tan\beta$ solution survives.

We have shown [19] that the ϵ–model allows $b-\tau$ Yukawa unification for any value of $\tan\beta$ and satisfying perturbativity of the couplings. We also find the $t-b-\tau$ Yukawa unification easier to achieve than in the MSSM, occurring in a wider high $\tan\beta$ region.

6. Tree Level Neutrino Masses and Mixings

6.1. Neutral fermion mass matrix

In the basis

$$\psi^{0T} = (-i\lambda', -i\lambda^3, \tilde{H}_d^1, \tilde{H}_u^2, \nu_e, \nu_\mu, \nu_\tau) \tag{43}$$

the neutral fermions mass terms in the Lagrangian are given by

$$\mathcal{L}_m = -\frac{1}{2}(\psi^0)^T \boldsymbol{M}_N \psi^0 + h.c. \tag{44}$$

where the neutralino/neutrino mass matrix is

$$\boldsymbol{M}_N = \begin{bmatrix} \mathcal{M}_{\chi^0} & m^T \\ m & 0 \end{bmatrix} \tag{45}$$

with

$$\mathcal{M}_{\chi^0} = \begin{bmatrix} M_1 & 0 & -\frac{1}{2}g'v_d & \frac{1}{2}g'v_u \\ 0 & M_2 & \frac{1}{2}gv_d & -\frac{1}{2}gv_u \\ -\frac{1}{2}g'v_d & \frac{1}{2}gv_d & 0 & -\mu \\ \frac{1}{2}g'v_u & -\frac{1}{2}gv_u & -\mu & 0 \end{bmatrix} \tag{46}$$

$$m = \begin{bmatrix} -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & \epsilon_1 \\ -\frac{1}{2}g'v_2 & \frac{1}{2}gv_2 & 0 & \epsilon_2 \\ -\frac{1}{2}g'v_3 & \frac{1}{2}gv_3 & 0 & \epsilon_3 \end{bmatrix} \tag{47}$$

The mass matrix $\boldsymbol{M}_N$ is diagonalized by

$$\mathcal{N}^* \boldsymbol{M}_N \mathcal{N}^{-1} = \mathrm{diag}(m_{\chi_i^0}, m_{\nu_j}) \tag{48}$$

where $(i = 1, \cdots, 4)$ for the neutralinos, and $(j = 1, \cdots, 3)$ for the neutrinos.

6.2. Approximate diagonalization of mass matrices

If the $\not{R}_p$ parameters are small it is convenient to define [20] the matrix

$$\xi = m \cdot \mathcal{M}_{\chi^0}^{-1} \tag{49}$$

If the elements of this matrix satisfy

$$\forall \xi_{ij} \ll 1 \tag{50}$$

then one can find an approximate solution for mixing matrix $\mathcal{N}$. Explicitly we have

$$\begin{aligned} \xi_{i1} &= \frac{g' M_2 \mu}{2det(\mathcal{M}_{\chi^0})}\Lambda_i \\ \xi_{i2} &= -\frac{g M_1 \mu}{2det(\mathcal{M}_{\chi^0})}\Lambda_i \\ \xi_{i3} &= -\frac{\epsilon_i}{\mu} + \frac{(g^2 M_1 + g'^2 M_2)v_2}{4det(\mathcal{M}_{\chi^0})}\Lambda_i \\ \xi_{i4} &= -\frac{(g^2 M_1 + g'^2 M_2)v_1}{4det(\mathcal{M}_{\chi^0})}\Lambda_i \end{aligned} \tag{51}$$

where

$$\Lambda_i = \mu v_i + v_d \epsilon_i \tag{52}$$

From Eq. (51) and Eq. (52) one can see that $\xi = 0$ in the MSSM limit where $\epsilon_i = 0$, $v_i = 0$. In leading order in ξ the mixing matrix $\mathcal{N}$ is given by,

$$\mathcal{N}^* = \begin{pmatrix} N^* & 0 \\ 0 & V_\nu^T \end{pmatrix} \begin{pmatrix} 1 - \frac{1}{2}\xi^\dagger\xi & \xi^\dagger \\ -\xi & 1 - \frac{1}{2}\xi\xi^\dagger \end{pmatrix} \tag{53}$$

The second matrix above block-diagonalizes $\boldsymbol{M}_N$ approximately to the form $\mathrm{diag}(m_{eff}, \mathcal{M}_{\chi^0})$

$$m_{eff} = -m \cdot \mathcal{M}_{\chi^0}^{-1} m^T$$

$$= \frac{M_1 g^2 + M_2 g'^2}{4\, det(\mathcal{M}_{\chi^0})} \begin{pmatrix} \Lambda_e^2 & \Lambda_e\Lambda_\mu & \Lambda_e\Lambda_\tau \\ \Lambda_e\Lambda_\mu & \Lambda_\mu^2 & \Lambda_\mu\Lambda_\tau \\ \Lambda_e\Lambda_\tau & \Lambda_\mu\Lambda_\tau & \Lambda_\tau^2 \end{pmatrix} \quad (54)$$

The sub-matrices N and V_ν in eq. diagonalize $\mathcal{M}_{\chi^0}$ and m_{eff}

$$N^* \mathcal{M}_{\chi^0} N^\dagger = \mathrm{diag}(m_{\chi_i^0}), \quad (55)$$

$$V_\nu^T m_{eff} V_\nu = \mathrm{diag}(0, 0, m_\nu), \quad (56)$$

where

$$m_\nu = Tr(m_{eff}) = \frac{M_1 g^2 + M_2 g'^2}{4\, det(\mathcal{M}_{\chi^0})} |\vec{\Lambda}|^2. \quad (57)$$

For V_ν we have (we can rotate away one angle)

$$V_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & -\sin\theta_{23} \\ 0 & \sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \times \begin{pmatrix} \cos\theta_{13} & 0 & -\sin\theta_{13} \\ 0 & 1 & 0 \\ \sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}, \quad (58)$$

where the mixing angles can be expressed in terms of the *alignment vector* $\vec{\Lambda}$ as follows:

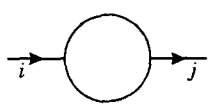
$$\tan\theta_{13} = -\frac{\Lambda_e}{(\Lambda_\mu^2 + \Lambda_\tau^2)^{\frac{1}{2}}}, \quad (59)$$

$$\tan\theta_{23} = \frac{\Lambda_\mu}{\Lambda_\tau}. \quad (60)$$

7. One Loop Neutrino Masses and Mixings

7.1. Definition

The Self–Energy for the neutralino/neutrino is



$$\equiv i \left\{ /p \left[P_L \Sigma_{ij}^L + P_R \Sigma_{ij}^R \right] - \left[P_L \Pi_{ij}^L + P_R \Pi_{ij}^R \right] \right\} \quad (61)$$

Then

$$M_{ij}^{\text{pole}} = M_{ij}^{\overline{\text{DR}}}(\mu_R) + \Delta M_{ij} \quad (62)$$

where

$$\Delta M_{ij} = \left[\tfrac{1}{2} \left(\Pi_{ij}^V(m_i^2) + \Pi_{ij}^V(m_j^2) \right) \right.$$

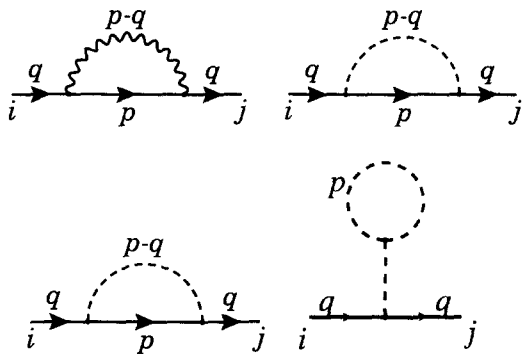


Figure 2. Diagrams contributing

$$\left. - \tfrac{1}{2} \left(m_{\chi_i^0} \Sigma_{ij}^V(m_i^2) + m_{\chi_j^0} \Sigma_{ij}^V(m_j^2) \right) \right]_{\Delta=0} \quad (63)$$

where

$$\Sigma^V = \tfrac{1}{2} \left(\Sigma^L + \Sigma^R \right) \qquad \Pi^V = \tfrac{1}{2} \left(\Pi^L + \Pi^R \right) \quad (64)$$

and

$$\Delta = \frac{2}{4-d} - \gamma_E + \ln 4\pi \quad (65)$$

7.2. Diagrams Contributing

In Fig. (2) are shown the classes of diagrams contributing to the self–energy at one loop. These diagrams can be calculated in a straightforward way. For instance the W diagram in the $\xi = 1$ gauge gives

$$\Sigma_{ij}^V = -\frac{1}{16\pi^2} \sum_{k=1}^{5} 2 \left(O_{Ljk}^{\text{ncw}} O_{Lki}^{\text{cnw}} + O_{Rjk}^{\text{ncw}} O_{Rki}^{\text{cnw}} \right) B_1(p^2, m_k^2, m_W^2)$$

$$\Pi_{ij}^V = -\frac{1}{16\pi^2} \sum_{k=1}^{5} (-4) \left(O_{Ljk}^{\text{ncw}} O_{Rki}^{\text{cnw}} + O_{Rjk}^{\text{ncw}} O_{Lki}^{\text{cnw}} \right) m_k\, B_0(p^2, m_k^2, m_W^2) \quad (66)$$

where B_0 and B_1 are the Passarino-Veltman functions, and the coupling matrices appear in the vertices in the following way

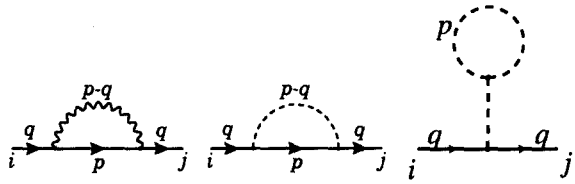


Figure 3. Set 1

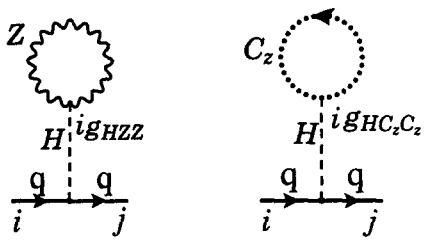
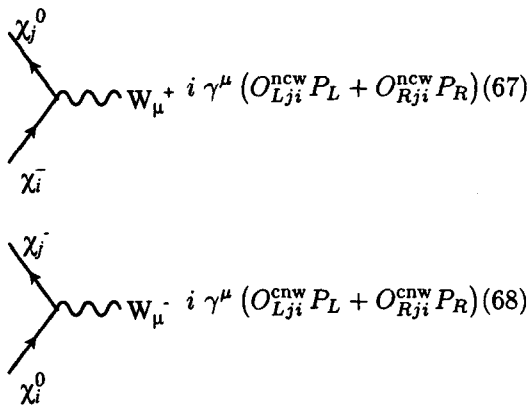


Figure 4. Set 2



7.3. Gauge Invariance

When calculating the self–energies the question of gauge invariance arises. In the R_ξ gauge the sets of diagrams of Figs. 3-5 depend on ξ. We have shown that the gauge dependence cancels among the diagrams in each set. So in the actual calculations we considered the tadpoles needed in those sets to ensure gauge invariance. The other tadpoles were included in the minimization procedure of Eq. (34). This is a gauge invariant splitting.

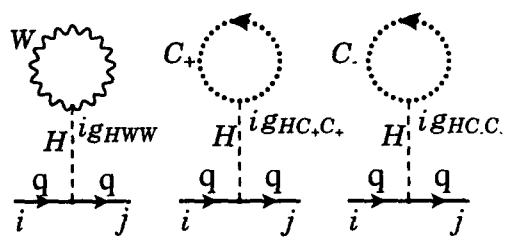


Figure 5. Set 3

7.4. The One–Loop Mass Matrix

The one–loop corrected mass matrix is given by

$$M^{1L} = M_{diag}^{0L} + \Delta M^{1L} \tag{69}$$

where

$$M_{diag}^{0L} = \mathcal{N} M_N \mathcal{N}^T \tag{70}$$

Now we diagonalize the 1–loop mass matrix

$$M_{diag}^{1L} = \mathcal{N}' M^{1L} \mathcal{N}'^T \tag{71}$$

Then the mass eigenstates are related to the weak basis states by

$$\chi_0^{mass} = \mathcal{N}_{i\alpha}^{1L}\, \chi_0^{weak} \tag{72}$$

with

$$\mathcal{N}^{1L} = \mathcal{N}'\, \mathcal{N} \tag{73}$$

The usual convention in neutrino physics

$$\nu_\alpha = U_{\alpha k}\, \nu_k \tag{74}$$

is recovered in our notation as

$$U_{\alpha k} = \mathcal{N}_{4+k,4+\alpha}^{1L} \tag{75}$$

7.5. Solar and Atmospheric Neutrino Parameters

Assuming hierarchy in the masses m_{ν_2} and m_{ν_3} the survival probabilities for the solar and atmospheric neutrinos are

$$P_e = 1 - 4U_{e1}^2 U_{e2}^2 \sin^2\left(\frac{\Delta m_{21}^2 t}{4E}\right) - 2U_{e3}^2(1 - U_{e3}^2)$$

$$P_\mu = 1 - 4U_{\mu 3}^2(1 - U_{\mu 3}^2) \sin^2\left(\frac{\Delta m_{21}^2 t}{4E}\right) \tag{76}$$

As U_{e3} has to be small we neglect it and write the usual two neutrino mixing angle as

$$\sin^2(2\theta_{12}) = 4\, U_{e1}^2 U_{e2}^2 \tag{77}$$

and

$$\sin^2(2\theta_{13}) = 4\, U_{\mu 3}^2(1 - U_{\mu 3}^2) \tag{78}$$

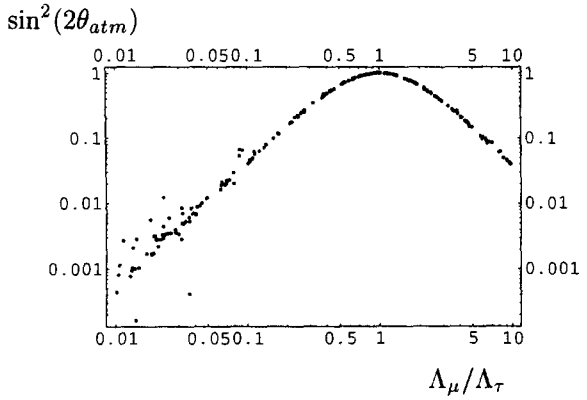


Figure 6. The atmospheric angle as function of Λ_μ/Λ_τ for $|\epsilon_i| = \epsilon$ and $\Lambda_e = 0.1\Lambda_\tau$. ϵ^2/Λ has an upper cut of $\epsilon^2/\Lambda \leq 0.1$ in this plot, since larger values lead to larger scatter for very small Λ_μ/Λ_τ

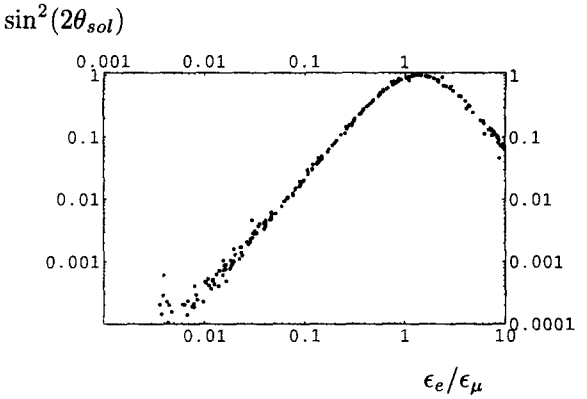


Figure 7. The solar angle as function of ϵ_e/ϵ_μ for $\epsilon_\mu = \epsilon_\tau$ and $\Lambda_\mu = \Lambda_\tau$ applying the sign condition.

7.6. Our Preliminary Results

We have found [4], that if $\epsilon^2/\Lambda \ll 100$ then the approximate formulas hold

$$\begin{aligned} U_{e3} &\approx \sin\left(\tan^{-1}\left(\Lambda_1/\sqrt{\Lambda_2^2+\Lambda_3^2}\right)\right) \\ U_{\mu 3} &\approx \sin\left(\tan^{-1}\left(\Lambda_2/\sqrt{\Lambda_1^2+\Lambda_3^2}\right)\right) \\ U_{\tau 3} &\approx \sin\left(\tan^{-1}\left(\Lambda_3/\sqrt{\Lambda_1^2+\Lambda_2^2}\right)\right) \end{aligned} \quad (79)$$

Then if we take

$$\Lambda_1 \ll \Lambda_2 \simeq \Lambda_3 \quad (80)$$

we immediately get maximal mixing for the atmospheric neutrinos. This is shown in Figure 6 where we see that maximality of the mixing is only possible for $\Lambda_\mu = \Lambda_\tau$.

To get bimaximality we have to fix the solar angle. We have discovered that if $\epsilon_e \simeq \epsilon_\mu \simeq \epsilon_\tau$ and $\Lambda_e \ll \Lambda_\mu \simeq \Lambda_\tau$ we get bimaximality if the following sign condition applies

$$(\epsilon_\mu/\epsilon_\tau) \times (\Lambda_\mu/\Lambda_\tau) \leq 0 \quad (81)$$

This is illustrated in Figure 7. In practice we do not need perfect maximality. We took

$$4/5 \leq |\Lambda_\mu/\Lambda_\tau| \leq 5/4 \quad \text{and} \quad |\Lambda_e/\Lambda_\mu| \ll 1 \quad (82)$$

to fix the atmospheric angle and

$$(\epsilon_\mu/\epsilon_\tau) \times (\Lambda_\mu/\Lambda_\tau) \leq 0 \text{ and } 0.6 \leq \epsilon_e/\epsilon_\mu \leq 1.2 (83)$$

to fix the solar angle. Next we have to fix the masses to solve the atmospheric and solar neutrino problems. We found [4,20] that the range

$$0.03\text{GeV}^2 \leq |\Lambda| \leq 0.25\text{GeV}^2 \quad (84)$$

fixes the tree level mass to reproduce the atmospheric neutrino problem. This is illustrated in Figure 8 where, besides the conditions in Eqs.(82) and (83) to fix the angles and condition Eq. (84), all the other parameters were chosen randomly. Consistency of the parameters was required in the sense that minimization the scalar potential including the tadpoles was performed as well as the matching with the RGE solutions with universality at GUT scale.

Finally we have to check if it is possible to have masses in the range to solve the solar neutrino problem. We found [4] that the relevant parameter for this purpose is

$$(\epsilon_1^2+\epsilon_1^2+\epsilon_1^2)/|\Lambda| \equiv \epsilon^2/|\Lambda| \quad (85)$$

Depending on this quantity in the range $0.01 \leq \epsilon^2/|\Lambda| \leq 1$ the solar neutrino problem can be solved: Low values give *just-so* solutions, high values tend to give large angle MSW (LA-MSW). This is illustrated in Figure 9 and in Figure 10. Another question of relevance that we addressed was the study of the decay length of the lightest

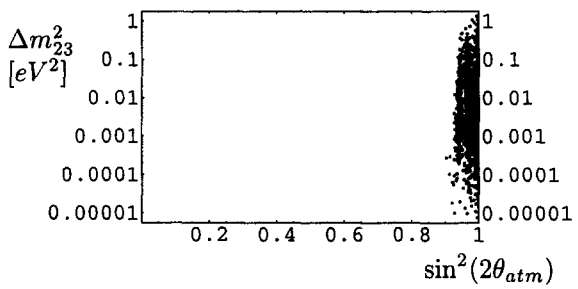


Figure 8. Δm^2_{23} versus $\sin^2(2\theta_{atm})$. All points obey Eqs. (82), (83 and (84) no further cut applied except that $0.3 \leq \epsilon^2/|\Lambda| \leq 1$.

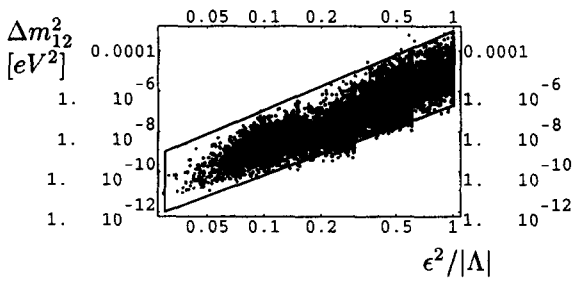


Figure 9. Δm^2_{12} as a function of $\epsilon^2/|\Lambda|$. A box is drawn to guide the eye.

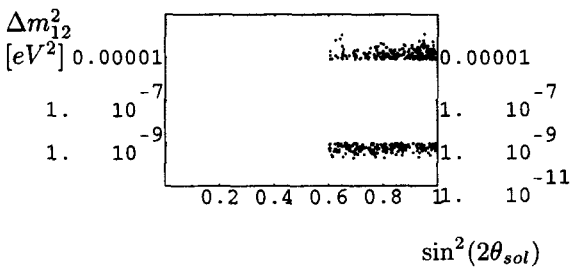


Figure 10. Δm^2_{12} versus $\sin^2(2\theta_{sol})$ for those points which have Δm^2_{23} and $\sin^2(2\theta_{atm})$ correct and at the same time fit either the vacuum or LA-MSW solutions.

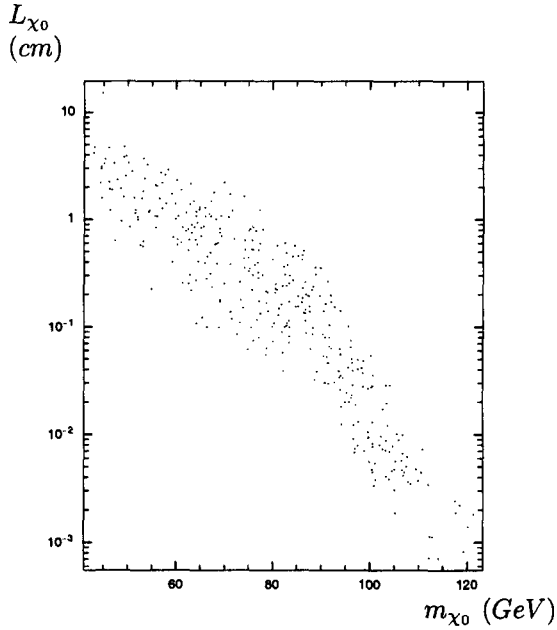


Figure 11. Neutralino decay length in cm

neutralino. This is important, because if the decay length is greater than the detector, then in practice it will be invisible like in the MSSM. As we can see in Figure 11, that is not the case, the neutralino decays well inside the detector leading to novel signatures.

8. Conclusions

There is a viable model for SBRP that leads to a very rich phenomenology, both at laboratory experiments, and at present (LEP) and future (LHC, LNC) accelerators. We have shown that the radiative breaking of both the Gauge Symmetry and R_P can be achieved. In these type of models neutrinos have mass and can decay thus avoiding the critical density argument. They also can evade the BBN limits on a ν_τ on the MeV scale. Most of these phenomenology can be described by an effective model with bilinear explicit R_P violation. We have calculated the *one-loop* corrected masses and mixings for the neutrinos in a completely consistent way, including the RG equations and correctly minimizing the potential. We have shown that it is possible to

get bimaximal solutions for both the atmospheric and solar neutrino problems. We emphasize that the lightest neutralino decays inside the detectors, thus leading to a very different phenomenology than the MSSM.

REFERENCES

1. Yu. A. Gol'fand and E. P. Likhtman, *Sov. Phys. JETP Lett.* **13** (1971) 323; D.V. Volkov and V.P. Akulov, *Sov. Phys. JETP Lett.* **16** (1972) 438; J. Wess and B. Zumino, *Nucl. Phys.* **B 70** (1974) 39.
2. H.P. Nilles, *Phys. Rep.* **110** (1984) 1; H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75; R. Barbieri, *Riv. Nuovo Cimento* **11** (1988) 1.
3. For a review see *e.g* J. C. Romão, Lectures given at the *5th Gleb Wataghin School on High-Energy Phenomenology*, Campinas, Brazil, 13-17 Jul 1998. e-Print Archive: hep-ph/9811454.
4. M.A. Díaz, M. Hirsch, W. Porod, J.C. Romão and J.W.F. Valle in preparation.
5. M.A. Díaz, J.C. Romão, and J.W.F. Valle, *Nucl. Phys.* **B 524** (1998) 23; M.A. Díaz, talk given at *International Europhysics Conference on High Energy Physics*, Jerusalem, Israel, 19-26 Aug 1997, hep-ph/9712213; J.C. Romão, talk given at *International Workshop on Physics Beyond the Standard Model: From Theory to Experiment* (Valencia 97), Valencia, Spain, 13-17 Oct 1997, hep-ph/9712362; J.W.F. Valle, review talk given at the *Workshop on Physics Beyond the Standard Model: Beyond the Desert: Accelerator and Nonaccelerator Approaches*, Tegernsee, Germany, 8-14 Jun 1997, hep-ph/9712277.
6. F. de Campos, M.A. García-Jareño, A.S. Joshipura, J. Rosiek, and J. W. F. Valle, *Nucl. Phys.* **B 451** (1995) 3;T. Banks, Y. Grossman, E. Nardi, and Y. Nir, *Phys. Rev.* **D 52** (1995) 5319; A. S. Joshipura and M.Nowakowski, *Phys. Rev.* **D 51** (1995) 2421; R. Hempfling, *Nucl. Phys.* **B 478** (1996) 3; F. Vissani and A.Yu. Smirnov, *Nucl. Phys.* **B 460** (1996) 37; H. P. Nilles and N. Polonsky, *Nucl. Phys.* **B 484** (1997) 33; B. de Carlos, P. L. White, *Phys. Rev.* **D 55** (1997) 4222; S. Roy and B. Mukhopadhyaya, *Phys. Rev.* **D 55** (1997) 7020.
7. A. Akeroyd, M.A. Díaz, J. Ferrandis, M.A. Garcia-Jareño, and Jose W.F. Valle, *Nucl. Phys.* **B 529** (1998) 3.
8. C Aulakh, R Mohapatra, *Phys. Lett.* **B 119** (1983) 136; A Santamaria, J W F Valle, *Phys. Lett.* **B 195** (1987) 423; *Phys. Rev. Lett.* **60** (1988) 397; *Phys. Rev.* **D 39** (1989) 1780.
9. A Masiero, J W F Valle, *Phys. Lett.* **B 251** (1990) 273.
10. A. Masiero and J.W.F. Valle, *Phys. Lett.* **B 251** (1990) 273; J.C. Romão, C.A. Santos, J.W.F. Valle, *Phys. Lett.* **B 288** (1992) 311;
11. A Lopez-Fernandez, J. Romão, F. de Campos and J. W. F. Valle, *Phys. Lett.* **B 312** (1993) 240; *ibidem*, Proceedings of Moriond '94. pag.81-86, edited by J. Tran Thanh Van, Éditions Frontiéres, 1994. J. C. Romão, F. de Campos, M. A. Garcia-Jareno, M. B. Magro and J. W. F. Valle, *Nucl. Phys.* **B 482** (1996) 3.
12. J C Romão, N Rius and J W F Valle, *Nucl. Phys.* **B 363** (1991) 369.
13. P Nogueira, J C Romão, J W F Valle, *Phys. Lett.* **B 251** (1990) 142.
14. J. C. Romão, J. W. F. Valle, *Nucl. Phys.* **B 381** (1992) 87.
15. S. Bertolini and G. Steigman, *Nucl. Phys.* **B 387** (1990) 193; M. Kawasaki et al, *Nucl. Phys.* **B 402** (1993) 323; *Nucl. Phys.* **B 419** (1994) 105; S. Dodelson, G. Gyuk and M.S. Turner, *Phys. Rev.* **D 49** (1994) 5068.
16. A. D. Dolgov, S. Pastor, J.C. Romão and J. W. F. Valle, *Nucl. Phys.* **B 496** (1997) 24.
17. M.A. Díaz, talk given at the *International Workshop on Physics Beyond the Standard Model: From Theory to Experiment* (Valencia 97), Valencia, Spain, 13-17 Oct 1997, hep-ph/9802407.
18. R. Hempfling, *Nucl. Phys.* **B 478** (1996) 3.
19. M.A. Díaz, J. Ferrandis, J.C. Romão, and J.W.F. Valle, *Phys. Lett.* **B 453** (1999) 263.
20. M. Hirsch, J.W.F. Valle, e-Print Archive: hep-ph/9812463.