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Supersymmetric solution to the solar and atmospheric neutrino problems

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The simplest unified extension of the minimal supersymmetric standard model with bi-linear *R*-parity violation provides a predictive scheme for neutrino masses which can account for the observed atmospheric and solar neutrino anomalies in terms of bi-maximal neutrino mixing. The maximality of the atmospheric mixing angle arises dynamically, by minimizing the scalar potential, while the solar neutrino problem can be accounted for either by large or by small mixing oscillations. One neutrino picks up mass by mixing with neutralinos, while the degeneracy and masslessness of the other two is lifted only by loop corrections. Despite the smallness of neutrino masses *R*-parity violation is observable at present and future high-energy colliders, providing an unambiguous cross check of the model.

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The pattern of fermion masses and mixings constitutes one of the most important issues in modern physics. Here we propose a model for the structure of lepton mixing which accounts for the atmospheric and solar neutrino anomalies. It is based on the simplest one-parameter extension of minimal supergravity with bi-linear *R*-parity violation [1] as would arise, perhaps, from gravitation.

The recent announcement of high statistics atmospheric neutrino data by the Super-Kamiokande Collaboration [2] has confirmed the deficit of muon neutrinos, especially at small zenith angles, opening a new era in neutrino physics. Although there may be alternative solutions of the atmospheric neutrino anomaly [3], it is fair to say that the simplest interpretation of the data is in terms of ν_μ to ν_τ flavor oscillations with maximal mixing. This excludes a large mixing among ν_τ and ν_e [2], in agreement also with the CHOOZ reactor data. On the other hand the persistent disagreement between solar neutrino data and theoretical expectations [4] has been a long-standing problem in physics. Recent solar neutrino data [5] are consistent with both vacuum oscillations and Mikheyev-Smirnov-Wolfenstein (MSW) conversions. In the latter case one can have either the large or the small mixing angle solutions, with a slight trend towards the latter [6]. The situation might become clearer in the near future when rate-independent observables such as spectrum, day-night and seasonal variations are better measured. In summary one sees that while quarks are weakly mixed, there is now the intriguing possibility that neutrino mixing is (close to) bi-maximal [7].

Our model breaks lepton number and therefore necessarily generates nonzero Majorana neutrino masses [8]. It has strong predictive power and allows for a dynamical determination of the atmospheric neutrino angle. Moreover it leads, under certain circumstances, to bi-maximal neutrino mixing. At tree level only one of the neutrinos picks up a mass by mixing with neutralinos [9], leaving the other two neutrinos massless [10]. While this can explain the atmospheric neutrino problem, to reconcile it with the solar neutrino data requires going beyond the tree-level approximation. This is the purpose of the present paper. For an analysis including only the atmospheric neutrino problem in the tree-level approximation see Ref. [11].

We have performed a full one-loop calculation of the neutralino-neutrino mass matrix in the bi-linear $\not{R}_p$ minimal supersymmetric standard model (MSSM). As is shown below, in order to explain the solar and atmospheric neutrino data it is both necessary and sufficient to work at one-loop level. In contrast to other papers [12,13] we have taken special care to achieve gauge invariance of the calculation. Moreover we have performed the renormalization of the heaviest neutrino, thus refining the approximate approaches used, for example, in Ref. [13]. For estimates in the approximation where loop neutrino masses arise just from tri-linear *R*-parity breaking see Ref. [14].

Bi-linear *R*-parity breaking supersymmetry has been extensively discussed in the literature [1]. It is motivated on the one hand by the fact that it provides an effective truncation of models where *R* parity breaks spontaneously [15] around the weak scale. Moreover, they allow for the radiative breaking of *R* parity, opening also new ways to unify gauge and Yukawa couplings [16] and with a potentially slightly lower prediction for α_s [17]. If present at the fundamental level, tri-linear breaking of *R* parity will always imply bi-linear breaking at some level, as a result of the renormalization group evolution. In contrast, bi-linear breaking may exist in the absence of tri-linear, as would be the case if it arises spontaneously.

Here, we concentrate only on those features of the model which are related to neutrino masses. Our model consists of the MSSM particle spectrum and superpotential except for the addition of the following ϵ_i terms:

$$W = W_{MSSM} + \epsilon_i \hat{L}_i^a \hat{H}_u^b . \tag{1}$$

Should supersymmetry not be broken, the above bi-linear terms would be superfluous since a suitable redefinition of the lepton and Higgs superfields [18] would convert them

into trilinear *R*-parity violating terms. However, since supersymmetry must be broken, they give rise to a second source for *R*-parity violation:

$$V_{soft} = V_{soft,MSSM} + B_i \epsilon_i \tilde{L}_i^a H_u^b . \tag{2}$$

In the presence of soft supersymmetry breaking terms, the bi-linear *R*-parity violating terms can not be rotated away, except in the particular case when $B = B_i$ and $m_{H_d}^2 = m_{L_i}^2$, $i = 1,2,3$ which is untypical. Thus we prefer to work in the original basis, containing no tri-linear $\not{R}_p$ vertices.

The presence of the bi-linear terms in Eq. (2) implies that the tadpole equations for the sneutrinos are nontrivial, i.e., lead to finite vacuum expectation value (VEV) for the scalar neutrinos. As a consequence the neutrinos and neutralinos, charged leptons and charginos, as well as the Higgses and sleptons of the MSSM, mix with each other. Detailed mass matrices are found in [19,20]. For the neutrino masses the most important aspect is, of course, the neutrino-neutralino mixing. It generates the following (7×7) mass matrix,

$$\mathcal{M}_0 = \begin{pmatrix} 0 & m \\ m^T & \mathcal{M}_{\chi^0} \end{pmatrix}, \tag{3}$$

where $\mathcal{M}_{\chi^0}$ is the usual MSSM neutralino mass matrix and the sub-matrix m contains entries from the bi-linear $\not{R}_p$ parameters,

$$m = \begin{pmatrix} -\frac{1}{2} g' \langle \tilde{\nu}_e \rangle & \frac{1}{2} g \langle \tilde{\nu}_e \rangle & 0 & \epsilon_e \\ -\frac{1}{2} g' \langle \tilde{\nu}_\mu \rangle & \frac{1}{2} g \langle \tilde{\nu}_\mu \rangle & 0 & \epsilon_\mu \\ -\frac{1}{2} g' \langle \tilde{\nu}_\tau \rangle & \frac{1}{2} g \langle \tilde{\nu}_\tau \rangle & 0 & \epsilon_\tau \end{pmatrix}, \tag{4}$$

and $\langle \tilde{\nu}_e \rangle$, $\langle \tilde{\nu}_\mu \rangle$, and $\langle \tilde{\nu}_\tau \rangle$ are the VEVs of the scalar neutrinos and g, g' are electroweak gauge couplings.

It is easy to show that this mass matrix (3) has such a structure that only one combination of ν_e, ν_μ, ν_τ picks up a mass, while the remaining two states remain massless. This structure is reminiscent of that found in Ref. [10]. If the *R*-parity violating (RPV) parameters are smaller than the typical size of the MSSM parameters, there exists a simple approximation formula for the nonzero mass of the neutrino,

$$m_\nu \simeq \frac{M_1 g^2 + M_2 g'^2}{4 det(\mathcal{M}_{\chi^0})} |\vec{\Lambda}|^2, \tag{5}$$

where

$$\Lambda_i = \mu \langle \tilde{\nu}_i \rangle + v_d \epsilon_i , \tag{6}$$

and M_1, M_2 are supersymmetry breaking electroweak gaugino masses. This ''alignment'' vector plays a prominent role in all of the discussion below since it will fix both the overall neutrino mass scale as well as the atmospheric neutrino mixing. With two neutrinos being massless one of the angles describing the mixing between them can be rotated away [21]. However, in the presence of loops this angle, which will characterize the solar neutrino conversions, will acquire a meaning, together with a (Dirac-type) CP phase.

There are three simple topologies of relevant Feynman diagrams contributing to the neutrino-neutralino mass matrix [20]. With these the one-loop corrected mass matrix is calculated as

$$M_{ij}^{pole} = M_{ij}^{\overline{DR}}(\mu_R) + \frac{1}{2}(\Pi_{ij}(p_i^2) + \Pi_{ij}(p_j^2) - m_{\chi_i^0}\Sigma_{ij}(p_i^2) - m_{\chi_j^0}\Sigma_{ij}(p_j^2)), \tag{7}$$

where Σ_{ij} and Π_{ij} are self energies. For a complete description see [20]. Here, $\overline{DR}$ signifies the minimal dimensional reduction subtraction scheme and μ_R is the renormalization scale. In order to check for gauge invariance in calculating Π_{ij} and Σ_{ij} we have used the general R_ξ gauges. As demonstrated in Ref. [20] gauge invariance requires the inclusion of the tadpole diagrams for the Goldstone bosons associated with the Z^0 and $W^\pm$ into the self energies. Moreover, in minimizing the scalar potential, for consistency reasons it is necessary to also include tadpole diagrams, when solving the tadpole equations, but to exclude the Goldstone tadpole graphs which have been already included into the self energies [20]. On the other hand, if Goldstone tadpole graphs are kept in the tadpole equations rather than in the self energies, gauge dependent VEVs would be generated. This problem has been ignored so far in all previous descriptions [13].

The scalar potential contains terms linear in the real part of the neutral scalar fields $\sigma_\alpha \equiv (\sigma_d^0, \sigma_u^0, \mathrm{Re}(\tilde{\nu}_1), \mathrm{Re}(\tilde{\nu}_2), \mathrm{Re}(\tilde{\nu}_3))$:

$$V_{linear} = t_d \sigma_d^0 + t_u \sigma_u^0 + t_i \mathrm{Re}(\tilde{\nu}_i) \equiv t_\alpha \sigma_\alpha^0 . \tag{8}$$

The coefficients of these terms are the tadpoles. Including the one-loop contribution we write

$$t_\alpha = t_\alpha^0 - \delta t_\alpha^{\overline{DR}} + T_\alpha(Q) = t_\alpha^0 + T_\alpha^{\overline{DR}}(Q) \tag{9}$$

where $T_\alpha^{\overline{DR}}(Q) \equiv -\delta t_\alpha^{\overline{DR}} + T_\alpha(Q)$ are the finite one-loop tadpoles. The minimization of the scalar potential corresponds then to solve $t_\alpha = 0$. This is done by solving these equations for the soft masses squared. This is easy because those equations are linear on the soft masses squared. However the values obtained in this way, which we call m_i^2, are not equal to the values $m_i^2(RGE)$ that we got via the renormalization group equations (RGE) starting from universal soft masses at the unification scale. To achieve equality we define a function

$$\eta = \max\left(\frac{m_i^2}{m_i^2(RGE)}, \frac{m_i^2(RGE)}{m_i^2} \right) \quad \forall i \tag{10}$$

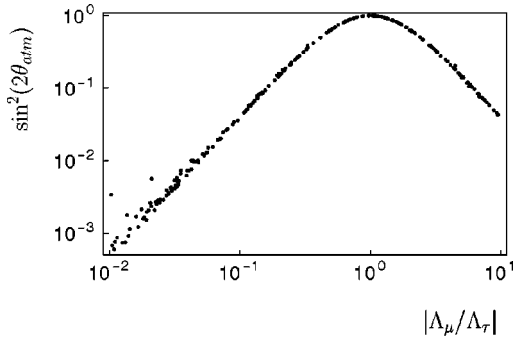

FIG. 1. The atmospheric angle as function of $|\Lambda_\mu/\Lambda_\tau|$, for $|\epsilon_i|=\epsilon$ and $\Lambda_e=0.1\Lambda_\tau$. Here $\epsilon^2/\Lambda\lesssim 0.1$, since larger values lead to larger scatter for very small $|\Lambda_\mu/\Lambda_\tau|$. Maximality of atmospheric mixing is only possible for $|\Lambda_\mu|\simeq|\Lambda_\tau|$.

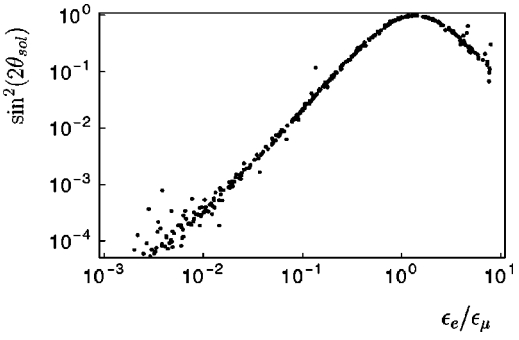

FIG. 2. The solar angle as function of ϵ_e/ϵ_μ, for $\epsilon_\mu=\epsilon_\tau$ and $\Lambda_\mu=\Lambda_\tau$, but $\Lambda_e=0.1\Lambda_\mu$, applying the condition: $(\Lambda_\mu/\Lambda_\tau)\times(\epsilon_\mu/\epsilon_\tau)\leq 0$. Maximality of solar mixing is only possible for $\epsilon_\mu\simeq\epsilon_e$.

with the obvious property that $\eta\geqslant 1$. Then we adjust the parameters at unification scale to minimize η.

We have performed a complete scan of the $\not{R}_p$MSSM parameter space, following the procedure outlined above. As an example we allow the MSSM parameters to vary within the range $M_2, |\mu|$ up to 500 GeV, m_0 up to 1 TeV, and assumed $|A_0/m_0|\leqslant 3$, which helps avoiding charge breaking minima. Moreover we assumed $\tan\beta\lesssim 10$. The latter is needed in order to obtain a nearly maximal atmospheric angle, since otherwise the sizeable loop involving down quarks and squarks would distort this feature due to its very strong $\tan\beta$ dependence. Note also that the bound on $\tan\beta$ implies that the lightest CP even Higgs boson mass lies below 115 GeV or so. As we will see below we can find simultaneous solutions to the atmospheric and solar neutrino problems only in those parts of parameter space where the one-loop contributions to the *neutrino* mass are smaller than the tree-level contribution. In this case it is possible to give a simple approximate formula for the composition of the third neutrino mass eigenstate,

$$U_{\alpha,3}=\sin\left(\arctan\left(\frac{\Lambda_\alpha}{\sqrt{\sum_{\beta\neq\alpha}\Lambda_\beta^2}}\right)\right). \qquad (11)$$

Accounting for the atmospheric neutrino anomaly requires that the $\nu_\mu-\nu_\tau$ mixing be large, with little effect of ν_e in the atmospheric neutrino oscillations. Fitting for the atmospheric neutrino data then fixes $|\Lambda_\mu/\Lambda_\tau|$ through this simple equation. These parameters are dynamically determined since they involve Higgs bosons and sneutrino VEVs obtained from the scalar potential. In fact, the $|\Lambda_\alpha|$ parameters are simply proportional to the sneutrino vacuum expectation values in the basis where the bilinear term in the superpotential is ''rotated away'' in favor of a tri-linear one [1]. As the illustration in Fig. 1 shows, the $\nu_\mu-\nu_\tau$ angle is a function of $|\Lambda_\mu/\Lambda_\tau|$ for $\Lambda_e\simeq 0.1\Lambda_\mu$ for an otherwise random variation of parameters. Clearly the condition $|\Lambda_\mu|=|\Lambda_\tau|$ is sufficient to ensure near maximal mixing, as long as Λ_e is somewhat suppressed.

One immediate consequence of the smallness of loop with respect to tree contributions is that the absolute $\not{R}_p$scale is then fixed by the atmospheric neutrino mass scale. For the above choice of sampling one has $|\Lambda|\simeq 0.03-0.25\ \mathrm{GeV}^2$. While this value is surely smaller than the weak scale, it may arise naturally in models where the sneutrino VEVs are generated radiatively [1].

With the magnitude of $\not{R}_p$parameters fixed by the atmospheric neutrino problem, the question arises, whether the loop-induced oscillation parameters, mass splitting and angle, are in the right range for either the vacuum or the MSW solution to the solar neutrino problem. Since the ratio of the loop masses to the tree-level mass depends on the relative size of the bi-linear $\not{R}_p$parameters with respect to the alignment vector Λ, this can not in general be predicted in the bi-linear $\not{R}_p$model. We have found, however, that with our assumption of generation-independent bi-linear parameters ϵ_i there should be a relative sign between the dynamically determined $\not{R}_p\Lambda$ parameters, i.e., $\Lambda_\mu\simeq-\Lambda_\tau$. Figure 2 shows how, having fixed the Λ_i by the atmospheric neutrino problem, the solar angle is determined under the above sign assumption as a function of ϵ_e/ϵ_μ.

As for the solar neutrino scale we show in Fig. 3 Δm^2_{12} versus $\epsilon^2/|\Lambda|$, where $\epsilon^2=\Sigma_i\epsilon_i^2$. As is seen from the figure, for fixed tree-level mass the loop masses depend strongly on this quantity. Large values give Δm^2_{12} in the MSW range, while low values could give vacuum solutions to the solar neutrino problem.

While a certain amount of ''alignment'' is needed, for

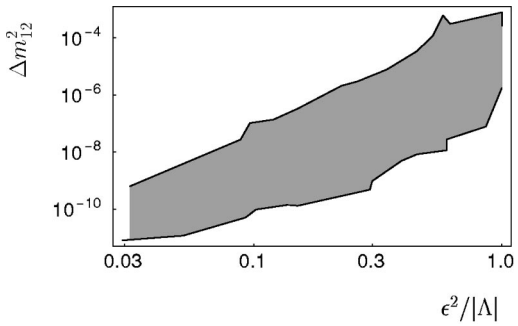

FIG. 3. Δm^2_{12} as function of $\epsilon^2/|\Lambda|$. Low values lead to neutrino masses in the just-so range, whereas high values give Δm^2_{12} in the range of the MSW solution.

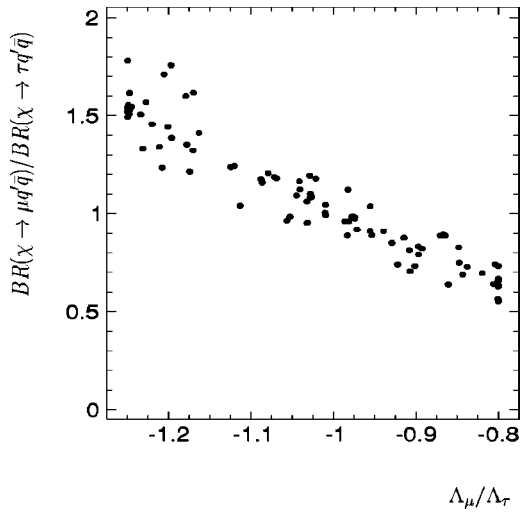

FIG. 4. Ratio of branching ratios for semileptonic LSP decays into muons and taus: $BR(\chi \to \mu q' \bar{q})/BR(\chi \to \tau q' \bar{q})$ as function of Λ_μ/Λ_τ.

masses in the MSW range, the model by itself does not prefer one solution over the other. It is also clear that bimaximal neutrino mixing is generated in the bilinear $\not{R}_p$MSSM—independent of the actual values of SUSY parameters—if (i) the $\not{R}_p$bi-linear terms are (nearly) generation blind, (ii) $\tan\beta \lesssim 10$ implying that the lightest CP even Higgs boson mass lies below 115 GeV or so, and (iii) $\Lambda_\mu \simeq -\Lambda_\tau$, as long as Λ_e is somewhat smaller than the Λ_μ and Λ_τ. We have checked explicitly that (iii) arises dynamically by minimizing the scalar potential of the theory.

Finally, apart from the possible detection of lightest CP even Higgs boson (mass below 115 GeV or so), we would like to point out that, despite the smallness of neutrino masses R-parity violation is observable at accelerators through the observation of the decay of the lightest supersymmetric particle (LSP), typically a neutralino. For example for a LSP mass of about 50 to 60 GeV the decay will occur inside typical CERN e^+e^- collider LEP, Fermilab Tevatron and CERN Large Hadron Collider (LHC) detectors (the neutralino decay length can be of the order of one meter or so), diluting the missing momentum signal to a maximum of 15% of the MSSM expectations. The LSP decay will give rise to high multiplicity events, providing an unambiguous test of the model. This is specially so if branching ratios are measured. In order to demonstrate this correlation we have calculated the ratio of semi-leptonic branching ratios of the LSP into muons and taus. We found the striking result that the correlation depicted in Fig. 1, which is required by the atmospheric neutrino anomaly, is mapped into a well-defined correlation for the ratio of semi-leptonic LSP branching ratios into muons and taus (see Fig. 4). Note that this correlation holds for the semi-leptonic decays, despite the fact that there are many scalar boson exchanges contributing to the LSP decay. The case where the LSP is heavier than the W and has two-body decays mainly to W and Z was considered by the authors of Ref. [22] who found a similar correlation in the approximation where the LSP decay into lightest neutral supersymmetric Higgs boson was neglected. The result we show in Fig. 4 is general, independent of the neutralino mass, and provides a powerful way to probe the solution of the atmospheric neutrino anomaly and open the potential to measure the related neutrino angles at high energy accelerators.

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