Neutrino masses and mixings from supersymmetry with bilinear *R*-parity violation: A theory for solar and atmospheric neutrino oscillations

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The simplest unified extension of the minimal supersymmetric standard model with bilinear *R*-parity violation naturally predicts a hierarchical neutrino mass spectrum, in which one neutrino acquires mass by mixing with neutralinos, while the other two get mass radiatively. We have performed a full one-loop calculation of the neutralino-neutrino mass matrix in the bilinear $\not R_p$ minimal supersymmetric standard model, taking special care to achieve a manifestly gauge invariant calculation. Moreover we have performed the renormalization of the heaviest neutrino, needed in order to get meaningful results. The atmospheric mass scale and maximal mixing angle arise from tree-level physics, while solar neutrino scale and oscillations follow from calculable one-loop corrections. If universal supergravity assumptions are made on the soft-supersymmetry breaking terms then the atmospheric scale is calculable as a function of a single $\not R_p$ violating parameter by the renormalization group evolution due to the nonzero bottom quark Yukawa coupling. The solar neutrino problem must be accounted for by the small mixing angle Mikheyev-Smirnov-Wolfenstein (MSW) solution. If these assumptions are relaxed then one can implement large mixing angle solutions. The theory predicts the lightest supersymmetic particle decay to be observable at high-energy colliders, despite the smallness of neutrino masses indicated by experiment. This provides an independent way to test this solution of the atmospheric and solar neutrino anomalies.

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I. INTRODUCTION

The high statistics data by the SuperKamiokande Collaboration [1] has confirmed the deficit of atmospheric muon neutrinos, especially at small zenith angles, opening a new era in neutrino physics. On the other hand the persistent disagreement between solar neutrino data and theoretical expectations has been a long-standing problem in physics [2]. Altogether these constitute the only solid evidence we now have in favor of physics beyond the present standard model, providing a strong hint for neutrino conversion. Although massless neutrino conversions [3] can be sizable in matter, and may even provide alternative solutions of the neutrino anomalies [4], it is fair to say that the simplest interpretation of the present data is in terms of massive neutrino oscillations. Taking for granted such an interpretation, the present data do provide an important clue on the pattern of neutrino masses and mixing. The atmospheric data indicate ν_μ to ν_τ flavor oscillations with maximal mixing [5], while the solar data can be accounted for in terms of either small mixing angle (SMA) and large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solutions [6], as well as through vacuum or just-so solutions [7]. A large mixing among ν_τ and ν_e is excluded both by the atmospheric data and by reactor data on neutrino oscillations [8]. There has indeed been an avalanche [9] of papers trying to address this issue in the framework of unified models adopting *ad hoc* texture structures for the Yukawa couplings.

Here we propose an alternative approach to describe the structure of lepton mixing which accounts for the atmospheric and solar neutrino anomalies [10] based on the simplest extension of minimal supergravity with bilinear *R*-parity violation (BRPV) [11]. The particles underlying the mechanism of neutrino mass generation are the neutral supersymmetric partners of the standard model gauge and Higgs bosons which have mass at the weak scale and are thus accessible to accelerators.

Our model breaks lepton number and therefore necessarily generates nonzero Majorana neutrino masses [12]. At the tree level only one of the neutrinos picks up a mass by mixing with neutralinos [13], leaving the other two neutrinos massless [14]. While this can explain the atmospheric neutrino problem, to reconcile it with the solar neutrino data requires going beyond the tree-level approximation. This is the purpose of the present paper. Here we improve the work of Ref. [15] by performing a full one-loop calculation of the neutrino mass matrix and also update the discussion in the light of the recent global fits of solar and atmospheric neutrino data. This can also be used to improve the discussion given in Ref. [16] where the tree approximation was assumed. For simplified analyses including only the atmospheric neutrino problem in the tree-level approximation see Ref. [17] and a number of papers in Ref. [18].

We have performed a full one-loop calculation of the neutralino-neutrino mass matrix in the bilinear $\not R_p$ minimal supersymmetric standard model (MSSM), showing that, in order to explain the solar and atmospheric neutrino data, it is necessary and sufficient to work at the one-loop level, provided one performs the renormalization of the heaviest neutrino. In contrast with all existing papers [15,18], we have taken special care to verify the gauge invariance of the calculation, thus refining the approximate approaches so far used in the literature. We find that if the soft-supersymmetry breaking terms are universal at the unification scale then only the small mixing angle MSW solution to the solar neutrino problem exists. On the other hand if these assumptions are relaxed then one can implement large mixing angle solutions, either MSW or just so.

Bilinear R-parity breaking supersymmetry has been extensively discussed in the literature [10]. It is motivated on the one hand by the fact that it provides an effective truncation of models where R parity breaks *spontaneously* by singlet sneutrino vevs around the weak scale [19]. Moreover, they allow for the radiative breaking of R parity, opening also new ways to unify gauge and Yukawa couplings [20] and with a potentially slightly lower prediction for α_s [21]. For recent papers on phenomenological implications of these models see Refs. [22–24]. If present at the fundamental level trilinear breaking of R parity will always imply bilinear breaking at some level, as a result of the renormalization group evolution. In contrast, bilinear breaking may exist in the absence of trilinear breaking, as would be the case if it arises spontaneously.

This paper is organized as follows. In Secs. II–IV we describe the model, the minimization of the scalar potential, and the radiative breaking of the electroweak symmetry. In Sec. V the tree level masses and mixings are described, while the contributions to the one loop mass matrix and the gauge invariance issue are studied in Sec. VI. Finally the neutrino masses and mixings are discussed in Sec. VII where we show our results for solar and atmospheric oscillation parameters. The more technical questions regarding the mass matrices, couplings, and one loop results as well as further details of gauge invariance are given in the appendixes. We also briefly discuss how, despite the smallness of neutrino masses indicated by experiment, the theory can lead to observable $R\!\!\!/_p$ phenomena at high-energy accelerators.

II. THE SUPERPOTENTIAL AND THE SOFT BREAKING TERMS

Using the conventions of Refs. [23,25] we introduce the model by specifying the superpotential, which includes BRPV [10] in three generations. It is given by

$$W=\varepsilon_{ab}[h_U^{ij}\hat{Q}_i^a\hat{U}_j\hat{H}_u^b+h_D^{ij}\hat{Q}_i^b\hat{D}_j\hat{H}_d^a+h_E^{ij}\hat{L}_i^b\hat{R}_j\hat{H}_d^a-\mu\hat{H}_d^a\hat{H}_u^b +\epsilon_i\hat{L}_i^a\hat{H}_u^b], \tag{1}$$

where the couplings h_U, h_D, and h_E are 3×3 Yukawa matrices and μ and ϵ_i are parameters with units of mass. The bilinear term in Eq. (1) violates lepton number in addition to R parity.

Supersymmetry breaking is parametrized with a set of soft supersymmetry breaking terms. In the MSSM these are given by

$$\begin{aligned}\mathcal{L}_{\rm soft}^{\rm MSSM}={}&M_Q^{ij2}\tilde{Q}_i^{a*}\tilde{Q}_j^a+M_U^{ij2}\tilde{U}_i\tilde{U}_j^*+M_D^{ij2}\tilde{D}_i\tilde{D}_j^*+M_L^{ij2}\tilde{L}_i^{a*}\tilde{L}_j^a\\&+M_R^{ij2}\tilde{R}_i\tilde{R}_j^*+m_{H_d}^2H_d^{a*}H_d^a+m_{H_u}^2H_u^{a*}H_u^a\\&-\left[\frac{1}{2}M_s\lambda_s\lambda_s+\frac{1}{2}M\lambda\lambda+\frac{1}{2}M'\lambda'\lambda'+\text{H.c.}\right]\\&+\varepsilon_{ab}[A_U^{ij}\tilde{Q}_i^a\tilde{U}_jH_u^b+A_D^{ij}\tilde{Q}_i^b\tilde{D}_jH_d^a+A_E^{ij}\tilde{L}_i^b\tilde{R}_jH_d^a\\&-B\mu H_d^aH_u^b].\end{aligned} \tag{2}$$

In addition to the MSSM soft supersymmetry (SUSY) breaking terms in $\mathcal{L}_{\rm soft}^{\rm MSSM}$ the BRPV model contains the following extra term:

$$V_{\rm soft}^{\rm BRPV}=-B_i\epsilon_i\varepsilon_{ab}\tilde{L}_i^aH_u^b, \tag{3}$$

where the B_i have units of mass. In what follows, we neglect intergenerational mixing in the soft terms in Eq. (2).

The electroweak symmetry is broken when the two Higgs doublets H_d and H_u, and the neutral component of the slepton doublets $\tilde{L}_i^1$ acquire vacuum expectation values (VEVs). We introduce the notation

$$H_d=\begin{pmatrix}H_d^0\\H_d^-\end{pmatrix},\quad H_u=\begin{pmatrix}H_u^+\\H_u^0\end{pmatrix},\quad \tilde{L}_i=\begin{pmatrix}\tilde{L}_i^0\\\tilde{l}_i^-\end{pmatrix}, \tag{4}$$

where we shift the neutral fields with nonzero VEVs as

$$\begin{aligned}&H_d^0\equiv\frac{1}{\sqrt{2}}[\sigma_d^0+v_d+i\varphi_d^0],\quad H_u^0\equiv\frac{1}{\sqrt{2}}[\sigma_u^0+v_u+i\varphi_u^0],\\&\tilde{L}_i^0\equiv\frac{1}{\sqrt{2}}[\tilde{\nu}_i^R+v_i+i\tilde{\nu}_i^I].\end{aligned} \tag{5}$$

Note that the W boson acquires a mass $m_W^2=\frac{1}{4}g^2v^2$, where $v^2\equiv v_d^2+v_u^2+v_1^2+v_2^2+v_3^2\simeq(246\,\text{GeV})^2$. We introduce the following notation in spherical coordinates for the vacuum expectation values:

$$\begin{aligned}v_d&=v\sin\theta_1\sin\theta_2\sin\theta_3\cos\beta,\\v_u&=v\sin\theta_1\sin\theta_2\sin\theta_3\sin\beta,\\v_3&=v\sin\theta_1\sin\theta_2\cos\theta_3,\\v_2&=v\sin\theta_1\cos\theta_2,\\v_1&=v\cos\theta_1,\end{aligned} \tag{6}$$

which preserves the MSSM definition $\tan\beta=v_u/v_d$. In the MSSM limit, where $\epsilon_i=v_i=0$, the angles θ_i are equal to $\pi/2$. In addition to the above MSSM parameters, our model contains nine new parameters ϵ_i, v_i, and B_i. The three VEVs are determined by the one-loop tadpole equations, and we will assume universality of the B terms, $B=B_i$ at the unification scale. Therefore, the only new and free parameters can be chosen as the ϵ_i.

III. THE SCALAR POTENTIAL

The electroweak symmetry is broken when the Higgs and lepton fields acquire nonzero VEVs. These are calculated via the minimization of the effective potential or, in the diagramatic method, via the tadpole equations. The full scalar potential at the tree level is

$$V^0_{\rm total} = \sum_i \left| \frac{\partial W}{\partial z_i} \right|^2 + V_D + V^{\rm MSSM}_{\rm soft} + V^{\rm BRPV}_{\rm soft}, \tag{7}$$

where z_i is any one of the scalar fields in the superpotential in Eq. (1), V_D are the D terms, and $V^{\rm BRPV}_{\rm soft}$ is given in Eq. (3).

The tree level scalar potential contains the following linear terms:

$$V^0_{\rm linear} = t^0_d \sigma^0_d + t^0_u \sigma^0_u + t^0_1 \tilde{\nu}^R_1 + t^0_2 \tilde{\nu}^R_2 + t^0_3 \tilde{\nu}^R_3, \tag{8}$$

where the different t^0 are the tadpoles at tree level. They are given by

$$\begin{aligned}
t^0_d &= (m^2_{H_d} + \mu^2) v_d + v_d D - \mu(B v_u + v_i \epsilon_i),\\
t^0_u &= -B\mu v_d + (m^2_{H_u} + \mu^2) v_u - v_u D + v_i B_i \epsilon_i + v_u \epsilon^2,\\
t^0_1 &= v_1 D + \epsilon_1(-\mu v_d + v_u B_1 + v_i \epsilon_i) + \frac{1}{2}(v_i M^2_{Li1} + M^2_{L1i} v_i),\\
t^0_2 &= v_2 D + \epsilon_2(-\mu v_d + v_u B_2 + v_i \epsilon_i) + \frac{1}{2}(v_i M^2_{Li2} + M^2_{L2i} v_i),\\
t^0_3 &= v_3 D + \epsilon_3(-\mu v_d + v_u B_3 + v_i \epsilon_i) + \frac{1}{2}(v_i M^2_{Li3} + M^2_{L3i} v_i),
\end{aligned} \tag{9}$$

where we have defined $D = \frac{1}{8}(g^2 + g'^2)(v_1^2 + v_2^2 + v_3^2 + v_d^2 - v_u^2)$ and $\epsilon^2 = \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2$. A repeated index i in Eq. (9) implies summation over $i = 1,2,3$. The five tree level tadpoles t^0_α are equal to zero at the minimum of the tree level potential, and from there one can determine the five tree level vacuum expectation values.

It is well known that in order to find reliable results for the electroweak symmetry breaking it is necessary to include the one-loop radiative corrections. The full scalar potential at one loop level, called effective potential, is

$$V_{\rm total} = V^0_{\rm total} + V_{\rm RC}, \tag{10}$$

where $V_{\rm RC}$ include the quantum corrections. In this paper we use the diagramatic method, which incorporates the radiative corrections through the one-loop corrected tadpole equations. The one loop tadpoles are

$$t_\alpha = t^0_\alpha - \delta t^{\overline{\rm DR}}_\alpha + T_\alpha(Q) = t^0_\alpha + \tilde{T}^{\overline{\rm DR}}_\alpha(Q), \tag{11}$$

where $\alpha = d,u,1,2,3$ and $\tilde{T}^{\overline{\rm DR}}_\alpha(Q) \equiv -\delta t^{\overline{\rm MS}}_\alpha + T_\alpha(Q)$ are the finite one loop tadpoles. At the minimum of the potential we have $t_\alpha = 0$, and the vevs calculated from these equations are the renormalized VEVs.

Neglecting intergenerational mixing in the soft masses, the five tadpole equations can be conveniently written in matrix form as

$$[t^0_u, t^0_d, t^0_1, t^0_2, t^0_3]^T = \mathbf{M}^2_{\rm tad}[v_u, v_d, v_1, v_2, v_3]^T, \tag{12}$$

where the matrix $\mathbf{M}^2_{\rm tad}$ is given by

$$\mathbf{M}^2_{\rm tad} = \begin{bmatrix} m^2_{H_d} + \mu^2 + D & -B\mu & -\mu\epsilon_1 & -\mu\epsilon_2 & -\mu\epsilon_3 \\ -B\mu & m^2_{H_u} + \mu^2 + \epsilon^2 - D & B_1\epsilon_1 & B_2\epsilon_2 & B_3\epsilon_3 \\ -\mu\epsilon_1 & B_1\epsilon_1 & M^2_{L_1} + \epsilon_1^2 + D & \epsilon_1\epsilon_2 & \epsilon_1\epsilon_3 \\ -\mu\epsilon_2 & B_2\epsilon_2 & \epsilon_1\epsilon_2 & M^2_{L_2} + \epsilon_2^2 + D & \epsilon_2\epsilon_3 \\ -\mu\epsilon_3 & B_3\epsilon_3 & \epsilon_1\epsilon_3 & \epsilon_2\epsilon_3 & M^2_{L_3} + \epsilon_3^2 + D \end{bmatrix} \tag{13}$$

and depends on the VEVs only through the D term defined above.

In order to have approximate solutions for the tree level vevs, consider the following rotation among the H_d and lepton superfields:

$$\mathbf{M}'^2_{\rm tad} = \mathbf{R}\mathbf{M}^2_{\rm tad}\mathbf{R}^{-1}, \tag{14}$$

where the rotation $\mathbf{R}$ can be split as

$$\mathbf{R}=\begin{bmatrix} c_3 & 0 & 0 & 0 & -s_3 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ s_3 & 0 & 0 & 0 & c_3 \end{bmatrix} \times \begin{bmatrix} c_2 & 0 & 0 & -s_2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ s_2 & 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_1 & 0 & -s_1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ s_1 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{15}$$

where the three angles are defined as

$$\begin{aligned} c_1&=\frac{\mu}{\mu'}, \quad s_1=\frac{\epsilon_1}{\mu'}, \quad \mu'=\sqrt{\mu^2+\epsilon_1^2},\\ c_2&=\frac{\mu'}{\mu''}, \quad s_2=\frac{\epsilon_2}{\mu''}, \quad \mu''=\sqrt{\mu'^2+\epsilon_2^2}.\\ c_3&=\frac{\mu''}{\mu'''}, \quad s_3=\frac{\epsilon_3}{\mu'''}, \quad \mu'''=\sqrt{\mu''^2+\epsilon_3^2}. \end{aligned} \tag{16}$$

It is clear that this rotation $\mathbf{R}$ leaves the D term invariant. The rotated VEVs are given by

$$[v'_u, v'_d, v'_1, v'_2, v'_3]^T=\mathbf{R}[v_u, v_d, v_1, v_2, v_3]^T, \tag{17}$$

and under the assumption that $v'_1, v'_2, v'_3 \ll v$, these three small VEVs have the approximate solution

$$\begin{aligned} v'_1&\approx-\frac{\mu\epsilon_1}{M'^2_{L_1}+D}\left[\frac{m^2_{H_d}-M^2_{L_1}}{\mu'\mu'''}v'_d+\frac{B_1-B}{\mu'}v'_u\right],\\ v'_2&\approx-\frac{\mu'\epsilon_2}{M'^2_{L_2}+D}\left[\frac{m'^2_{H_d}-M^2_{L_2}}{\mu''\mu'''}v'_d+\frac{B_2-B'}{\mu''}v'_u\right],\\ v'_3&\approx-\frac{\mu''\epsilon_3}{M'^2_{L_3}+D}\left[\frac{m''^2_{H_d}-M^2_{L_3}}{\mu'''^2}v'_d+\frac{B_3-B''}{\mu'''}v'_u\right], \end{aligned} \tag{18}$$

where we have defined the following rotated soft terms:

$$\begin{aligned} m'^2_{H_d}&=\frac{m^2_{H_d}\mu^2+M^2_{L_1}\epsilon_1^2}{\mu'^2}, \quad m''^2_{H_d}=\frac{m'^2_{H_d}\mu'^2+M^2_{L_2}\epsilon_2^2}{\mu''^2},\\ m'''^2_{H_d}&=\frac{m''^2_{H_d}\mu''^2+M^2_{L_3}\epsilon_3^2}{\mu'''^2}, \quad B'=\frac{B\mu^2+B_1\epsilon_1^2}{\mu'^2},\\ B''&=\frac{B'\mu'^2+B_2\epsilon_2^2}{\mu''^2}, \quad B'''=\frac{B''\mu''^2+B_3\epsilon_3^2}{\mu'''^2},\\ M'^2_{L_1}&=\frac{m^2_{H_d}\epsilon_1^2+M^2_{L_1}\mu^2}{\mu'^2}, \quad M'^2_{L_2}=\frac{m'^2_{H_d}\epsilon_2^2+M^2_{L_2}\mu'^2}{\mu''^2},\\ M'^2_{L_3}&=\frac{m''^2_{H_d}\epsilon_3^2+M^2_{L_3}\mu''^2}{\mu'''^2}. \end{aligned} \tag{19}$$

The approximation $v'_1, v'_2, v'_3 \ll v$ is justified in supergravity (SUGRA) models with universality of soft masses at the weak scale, as shown in the next section.

IV. RADIATIVE BREAKING OF THE ELECTROWEAK SYMMETRY

It was demonstrated in Ref. [10] that BRPV can be succesfully embedded into SUGRA with universal boundary conditions at the unification scale, and with a radiatively broken electroweak symmetry. At $Q=M_U$ we assume the standard minimal supergravity unification assumptions:

$$\begin{aligned} &A_t=A_b=A_\tau=A,\\ &B=B_i=A-1,\\ &m^2_{H_d}=m^2_{H_u}=M^2_{L_i}=M^2_{R_i}=M^2_{Q_i}=M^2_{U_i}=M^2_{D_i}=m_0^2,\\ &M_3=M_2=M_1=M_{1/2}. \end{aligned} \tag{20}$$

We run the renormalization group equations (RGEs) from the unification scale $M_U \sim 2\times10^{16}$ GeV down to the weak scale, giving random values to the fundamental parameters at the unification scale:

$$\begin{aligned} 10^{-2}&\leq h^2_{tU}/4\pi\leq 1,\\ 10^{-5}&\leq h^2_{bU}/4\pi\leq 1,\\ -3&\leq a_0\equiv A/m_0\leq 3,\\ 0&\leq \mu^2_U/m_0^2\leq 10,\\ 0&\leq M_{1/2}/m_0\leq 5. \end{aligned} \tag{21}$$

The Yukawa couplings are determined by requiring that three eigenvalues of the chargino/charged-lepton mass matrix correspond to the experimentally measured tau, muon, and electron masses.[1]

As in the MSSM, the electroweak symmetry is broken because the large value of the top-quark mass drives the Higgs boson mass parameter $m^2_{H_U}$ to negative values at the

[1]For the case of large tree-level neutrino mass one must note that the lepton Yukawa couplings are no longer related to the lepton masses via the simple relations valid in the standard model. Since charginos mix with charged leptons, the Yukawa couplings depend also on the parameters of the chargino sector. For the case of interest here (light ν_τ mass fixed by the atmospheric scale) this correction is less important.

weak scale via its RGE [26]. In the rotated basis, the parameter μ'''^2 is determined at one loop by

$$\mu'''^2 = -\frac{1}{2}[m_Z^2 - \tilde{A}_{ZZ}(m_Z^2)] + \frac{(m_{H_d}'''^2 + \tilde{T}_{v_d'}^{\overline{\mathrm{DR}}}) - (m_{H_u}^2 + \tilde{T}_{v_u'}^{\overline{\mathrm{DR}}})t_\beta'^2}{t_\beta'^2 - 1}, \tag{22}$$

where $t'_\beta = v'_u/v'_d$ is defined in the rotated basis and is analogous to $\tan\beta$ in Eq. (6) defined in the original basis. The finite dimensional reduction ($\overline{DR}$) Z-boson self-energy is $\tilde{A}_{ZZ}(m_Z^2)$, and the one-loop tadpoles $T_{v_d'}^{\overline{\mathrm{DR}}}$ and $T_{v_u'}^{\overline{\mathrm{DR}}}$ are obtained by applying to the original tadpoles in Eq. (11) the rotation $\mathbf{R}$ defined in Eq. (15). The radiative breaking of the electroweak symmetry is valid in the BRPV model in the usual way: the large value of the top quark Yukawa coupling drives the parameter $m_{H_U}^2$ to negative values, breaking the symmetry of the scalar potential.

As we will see a radiative mechanism is also responsible for the smallness of the neutrino masses in models with universality of soft mass parameters at the unification scale. The relevant parameters are the bilinear mass parameters B and B_i, the Higgs boson mass parameter $m_{H_d}^2$, and the slepton mass parameters $M_{L_i}^2$.

The RGEs for the B parameters are

$$\frac{dB}{dt} = \frac{1}{8\pi^2}\left(3h_t^2A_t + 3h_b^2A_b + h_\tau^2A_\tau + 3g_2^2M_2 + \frac{3}{5}g_1^2M_1\right),$$

$$\frac{dB_3}{dt} = \frac{1}{8\pi^2}\left(3h_t^2A_t + h_\tau^2A_\tau + 3g_2^2M_2 + \frac{3}{5}g_1^2M_1\right), \tag{23}$$

$$\frac{dB_2}{dt} = \frac{dB_1}{dt} = \frac{1}{8\pi^2}\left(3h_t^2A_t + 3g_2^2M_2 + \frac{3}{5}g_1^2M_1\right),$$

where we do not write the effect of Yukawa couplings of the first two generations. Similarly, the RGE for the down-type Higgs boson mass is

$$\frac{dm_{H_d}^2}{dt} = \frac{1}{8\pi^2}\left(3h_b^2X_b + h_\tau^2X_\tau - 3g_2^2M_2^2 - \frac{3}{5}g_1^2M_1^2\right), \tag{24}$$

and the RGEs for the slepton mass parameters are

$$\frac{dM_{L_3}^2}{dt} = \frac{1}{8\pi^2}\left(h_\tau^2X_\tau - 3g_2^2M_2^2 - \frac{3}{5}g_1^2M_1^2\right),$$

$$\frac{dM_{L_2}^2}{dt} = \frac{dM_{L_1}^2}{dt} = -\frac{1}{8\pi^2}\left(3g_2^2M_2^2 + \frac{3}{5}g_1^2M_1^2\right), \tag{25}$$

where $X_b = m_{H_d}^2 + M_{Q_3}^2 + M_{D_3}^2 + A_b^2$ and $X_\tau = m_{H_d}^2 + M_{L_3}^2 + M_{R_3}^2 + A_\tau^2$.

With the aid of these RGEs we can find an approximate expression for the slepton VEVs in the rotated basis v'_i, given in Eq. (18). The relevant soft term differences, defined as $\Delta B_i \equiv B_i - B$ and $\Delta m_i^2 \equiv M_{L_i}^2 - m_{H_d}^2$, are approximated by

$$\Delta B_3 = \frac{1}{8\pi^2}(3h_b^2A_b)\ln\frac{M_U}{m_{\mathrm{weak}}},$$

$$\Delta B_2 = \Delta B_1 = \frac{1}{8\pi^2}(3h_b^2A_b + h_\tau^2A_\tau)\ln\frac{M_U}{m_{\mathrm{weak}}} \tag{26}$$

for the B terms, and by

$$\Delta m_3^2 = \frac{1}{8\pi^2}(3h_b^2X_b)\ln\frac{M_U}{m_{\mathrm{weak}}},$$

$$\Delta m_2^2 = \Delta m_1^2 = \frac{1}{8\pi^2}(3h_b^2X_b + h_\tau^2X_\tau)\ln\frac{M_U}{m_{\mathrm{weak}}} \tag{27}$$

for the mass squared terms. This way, if we assume that $\epsilon_i \ll \mu$ we can neglect the rotations in Eq. (18) and we find

$$v'_i \approx \frac{v_d\epsilon_i/\mu}{M_{L_i}^2 + D}(\Delta m_i^2 - t_\beta\mu\Delta B_i) \tag{28}$$

which give us an approximate expression for the sneutrino VEVs v'_i in the basis where the ϵ_i terms are absent from the superpotential. In a model with unified universal boundary conditions on the soft SUSY breaking terms (SUGRA case, for short) the v'_i are calculable in terms of the renormalization group evolution due to the nonzero bottom quark Yukawa coupling. We should stress here that for our subsequent numerical calculation we solve the tadpole equations *exactly*.

The symmetry of the neutralino/neutrino mass matrix implies that only one neutrino acquires a tree level mass, and the other two remain massless [14] (see next section). The massive neutrino will have the largest component along τ, μ, or e if the largest VEV is v'_3, v'_2, or v'_1, respectively. On the other hand, the most obvious difference between the third generation sneutrino VEV and the first two generations is in the extra contribution from h_τ to ΔB_i in Eq. (26) and to Δm_i^2 in Eq. (27) for the first two generations. Due to the tau lepton contribution, ΔB_1 and ΔB_2 are larger than ΔB_3, and similarly for the Δm_i^2, specially if $\tan\beta \gg 1$. However, we have checked that it is possible without fine-tuning the parameters in an unnatural way to arrange for the heaviest of the neutrinos to be an equal mixture of ν_μ and ν_τ as needed in order to obtain an explanation of the atmospheric neutrino anomaly. That this is possible can be understood by noticing that there can be a cancellation between the ΔB and Δm^2 terms in Eq. (28) for v'_1 and v'_2.

V. TREE LEVEL NEUTRINO MASSES AND MIXINGS

Here we discuss the tree level structure of neutrino masses and mixings. For a complete discussion of the fermion mass matrices in this model see Appendix A.[2] In the basis $\psi^{0T}=(-i\lambda',-i\lambda^3,\widetilde{H}_d^1,\widetilde{H}_u^2,\nu_e,\nu_\mu,\nu_\tau)$ the neutral fermion mass matrix $\mathbf{M}_N$ is given by

$$\mathbf{M}_N=\begin{bmatrix}\mathcal{M}_{\chi^0} & m^T\\ m & 0\end{bmatrix}, \tag{29}$$

where

$$\mathcal{M}_{\chi^0}=\begin{bmatrix} M_1 & 0 & -\frac{1}{2}g'v_d & \frac{1}{2}g'v_u\\ 0 & M_2 & \frac{1}{2}gv_d & -\frac{1}{2}gv_u\\ -\frac{1}{2}g'v_d & \frac{1}{2}gv_d & 0 & -\mu\\ \frac{1}{2}g'v_u & -\frac{1}{2}gv_u & -\mu & 0\end{bmatrix} \tag{30}$$

is the standard MSSM neutralino mass matrix and

$$m=\begin{bmatrix} -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & \epsilon_1\\ -\frac{1}{2}g'v_2 & \frac{1}{2}gv_2 & 0 & \epsilon_2\\ -\frac{1}{2}g'v_3 & \frac{1}{2}gv_3 & 0 & \epsilon_3\end{bmatrix} \tag{31}$$

characterizes the breaking of R parity. The mass matrix $\mathbf{M}_N$ is diagonalized by (see Appendix A)

$$\mathcal{N}^*\mathbf{M}_N\mathcal{N}^{-1}=\mathrm{diag}(m_{\chi_i^0},m_{\nu_j}), \tag{32}$$

where $(i=1,...,4)$ for the neutralinos and $(j=1,...,3)$ for the neutrinos.

We are interested in the case where the neutrino mass which is determined at the tree level is small, since it will be determined in order to account for the atmospheric neutrino anomaly. The above form for $\mathbf{M}_N$ is especially convenient in this case in order to provide an approximate analytical discussion valid in the limit of small $R\!\!\!/_p$ violation parameters. Indeed in this case we perform a perturbative diagonalization of the neutral mass matrix, using the method of Ref. [27], by defining [24]

$$\xi=m\cdot\mathcal{M}_{\chi^0}^{-1}. \tag{33}$$

If the elements of this matrix satisfy

$$\forall\,\xi_{ij}\ll 1, \tag{34}$$

then one can use it as expansion parameter in order to find an approximate solution for the mixing matrix $\mathcal{N}$. Explicitly we have

$$\xi_{i1}=\frac{g'M_2\mu}{2\det(\mathcal{M}_{\chi^0})}\Lambda_i,$$

$$\xi_{i2}=-\frac{gM_1\mu}{2\det(\mathcal{M}_{\chi^0})}\Lambda_i,$$

$$\xi_{i3}=-\frac{\epsilon_i}{\mu}+\frac{(g^2M_1+g'^2M_2)v_u}{4\det(\mathcal{M}_{\chi^0})}\Lambda_i,$$

$$\xi_{i4}=-\frac{(g^2M_1+g'^2M_2)v_d}{4\det(\mathcal{M}_{\chi^0})}\Lambda_i, \tag{35}$$

where

$$\Lambda_i=\mu v_i+v_d\epsilon_i\propto v_i' \tag{36}$$

are the alignment parameters. From Eqs. (35) and (36) one can see that $\xi=0$ in the MSSM limit where $\epsilon_i=0$, $v_i=0$. In leading order in ξ the mixing matrix $\mathcal{N}$ is given by

$$\mathcal{N}^*=\begin{pmatrix}N^* & 0\\ 0 & V_\nu^T\end{pmatrix}\begin{pmatrix}1-\frac{1}{2}\xi^\dagger\xi & \xi^\dagger\\ -\xi & 1-\frac{1}{2}\xi\xi^\dagger\end{pmatrix}. \tag{37}$$

The second matrix above block diagonalizes the mass matrix $\mathbf{M}_N$ approximately to the form diag$(\mathcal{M}_{\chi^0},m_{\mathrm{eff}})$, where

$$\begin{aligned}m_{\mathrm{eff}}&=-m\cdot\mathcal{M}_{\chi^0}^{-1}m^T\\ &=\frac{M_1g^2+M_2g'^2}{4\det(\mathcal{M}_{\chi^0})}\begin{pmatrix}\Lambda_e^2 & \Lambda_e\Lambda_\mu & \Lambda_e\Lambda_\tau\\ \Lambda_e\Lambda_\mu & \Lambda_\mu^2 & \Lambda_\mu\Lambda_\tau\\ \Lambda_e\Lambda_\tau & \Lambda_\mu\Lambda_\tau & \Lambda_\tau^2\end{pmatrix}.\end{aligned} \tag{38}$$

The submatrices N and V_ν diagonalize $\mathcal{M}_{\chi^0}$ and m_{eff}:

$$N^*\mathcal{M}_{\chi^0}N^\dagger=\mathrm{diag}(m_{\chi_i^0}), \tag{39}$$

$$V_\nu^T m_{\mathrm{eff}}V_\nu=\mathrm{diag}(0,0,m_\nu), \tag{40}$$

where

$$m_\nu=Tr(m_{\mathrm{eff}})=\frac{M_1g^2+M_2g'^2}{4\det(\mathcal{M}_{\chi^0})}|\vec{\Lambda}|^2. \tag{41}$$

Clearly, one neutrino acquires mass due to the projective nature of the effective neutrino mass matrix m_{eff}, a feature often encountered in $R\!\!\!/_p$ models [14]. As a result one can rotate away one of the three angles [12] in the matrix V_ν, leading to [28]

$$\begin{aligned}V_\nu=&\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta_{23} & -\sin\theta_{23}\\ 0 & \sin\theta_{23} & \cos\theta_{23}\end{pmatrix}\\ &\times\begin{pmatrix}\cos\theta_{13} & 0 & -\sin\theta_{13}\\ 0 & 1 & 0\\ \sin\theta_{13} & 0 & \cos\theta_{13}\end{pmatrix},\end{aligned} \tag{42}$$

where the mixing angles can be expressed in terms of the *alignment vector* $\vec{\Lambda}$ as follows:

[2]In our notation the four component Majorana neutral fermions are obtained from the two component via the relation $\chi_i^0=\binom{F_i^0}{\bar{F}_i^0}$.

$$\tan\theta_{13} = -\frac{\Lambda_e}{(\Lambda_\mu^2+\Lambda_\tau^2)^{1/2}}, \tag{43}$$

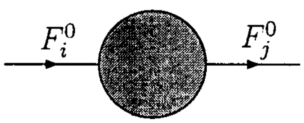
$$\tan\theta_{23} = -\frac{\Lambda_\mu}{\Lambda_\tau}. \tag{44}$$

VI. ONE-LOOP NEUTRINO MASS MATRIX

One-loop radiative corrections to the neutralino-neutrino mass matrix in the BRPV model were calculated first in Ref. [15], working in the 't Hooft–Feynman gauge ($\xi=1$). Our analysis improves the previous work in that we check explicitly the gauge invariance using the R_ξ gauge. We use dimensional reduction to regularize the divergences [29] and include all possible MSSM particles with consistently determined mass spectra and couplings in the relevant loops.

A. Two-point function renormalization

We denote the sum of all one-loop graphs contributing to the two-point function as



$$\equiv i\,\Sigma^{ij}_{FF}(p).$$

The most general expression for the one-loop contribution to the unrenormalized neutralino-neutrino two-point function is

$$i\Sigma^{ij}_{FF}(p) \equiv i\{\not{p}[P_L\Sigma^L_{ij}(p^2)+P_R\Sigma^R_{ij}(p^2)]-[P_L\Pi^L_{ij}(p^2) + P_R\Pi^R_{ij}(p^2)]\}, \tag{45}$$

where the indices i and j run from 1 to 7, $P_R=\frac{1}{2}(1+\gamma_5)$ and $P_L=\frac{1}{2}(1-\gamma_5)$ are the right and left projection operators, and p is the external four momenta. The functions Σ and Π are unrenormalized self-energies and depend on the external momenta squared p^2. The neutral fermions F_i^0 are a mixture of weak eigenstate neutralinos and neutrinos and given by

$$F_i^0 = \mathcal{N}_{ij}\psi_j^0, \tag{46}$$

where $\mathcal{N}$ is the 7×7 matrix that diagonalizes the neutralino-neutrino mass matrix according to Eq. (32).

The inverse propagator at one loop is obtained by adding to the tree level propagator, this self-energy previously renormalized with the dimensional reduction $\overline{\mathrm{DR}}$ scheme and denoted as $\tilde{\Sigma}$ and $\tilde{\Pi}$. In the $\overline{\mathrm{DR}}$ scheme, the counterterms cancel only the divergent pieces of the self-energies. In this way, they become finite and dependent on the arbitrary scale Q. The tree level masses are promoted to running masses in order to cancel the explicit scale dependence of the self-energies. Thus, the inverse propagator of the neutral fermion F_i^0 is

$$\Gamma^{(2)}_{FF}(p) = p_\mu\gamma^\mu - m_{F_i}(Q) + \tilde{\Sigma}^{ii}_{FF}(p,Q). \tag{47}$$

The physical pole mass is given by the zero of the inverse propagator, in the limit where $p_\mu\gamma^\mu \to m_{F_i}$, and may be found using

$$\tilde{Z}^{-1}_{F_i}\overline{u}(p)[p_\mu\gamma^\mu - m_{F_i}]u(p) = \overline{u}(p)[p_\mu\gamma^\mu - m_{F_i}(Q) + \tilde{\Sigma}^{ii}_{FF}(p,Q)]u(p), \tag{48}$$

where u and $\overline{u}$ are two on-shell spinors, m_{F_i} and $m_{F_i}(Q)$ are the neutral fermion pole and running masses, respectively, and $\tilde{\Sigma}^{ii}_{FF}(p,Q)$ is the renormalized two-point function in the $\overline{\mathrm{DR}}$ scheme. The quantity $\tilde{Z}^{-1}_{F_i}$ corresponds to the finite ratio of the infinite wave function renormalization constants in the $\overline{\mathrm{DR}}$ scheme and the on-shell scheme, and it accounts for the fact that the residue of the $\overline{\mathrm{DR}}$ propagator at the pole is not 1 [30]. The renormalized function $\tilde{\Sigma}^{ii}_{FF}(p,Q)$ is calculated by subtracting the pole terms proportional to the regulator of dimensional reduction

$$\Delta = \frac{2}{4-d} - \gamma_E + \ln 4\pi, \tag{49}$$

where γ_E is the Euler's constant and d is the number of space-time dimensions. In practice we have

$$\tilde{\Sigma}^{ii}_{FF}(p,Q) = [\Sigma^{ii}_{FF}(p)]_{\Delta=0}. \tag{50}$$

Since $\overline{u}(p)\gamma_5 u(p)=0$, the terms proportional to γ_5 in $\tilde{\Sigma}^{ii}_{FF}$ do not contribute. From Eq. (48) we find

$$\Delta m_{F_i} \equiv m_{F_i} - m_{F_i}(Q) = \tilde{\Pi}^V_{ii}(m^2_{F_i}) - m_{F_i}\tilde{\Sigma}^V_{ii}(m^2_{F_i}), \tag{51}$$

where

$$\tilde{\Sigma}^V = \tfrac{1}{2}(\tilde{\Sigma}^L+\tilde{\Sigma}^R), \quad \tilde{\Sigma}^V = \tfrac{1}{2}(\tilde{\Pi}^L+\tilde{\Pi}^R), \tag{52}$$

and the tilde implies renormalized self-energies. A given set of input parameters in the neutralino-neutrino mass matrix defines the set of tree level running masses $m_{F_i}(Q)$, among them two massless and degenerate neutrinos. The one-loop renormalized masses m_{F_i} are then found through Eq. (51), and the masslessness and degeneracy of the two lightest neutrinos is lifted.

The tree level masslessness of the lightest neutrinos implies an indetermination of the corresponding eigenvectors. In order to find the correct neutrino mixing angles we diagonalize the one-loop corrected neutralino-neutrino mass matrix. We define

$$M^{\mathrm{pole}}_{ij} = M^{\overline{\mathrm{DR}}}_{ij}(Q) + \Delta M_{ij} \tag{53}$$

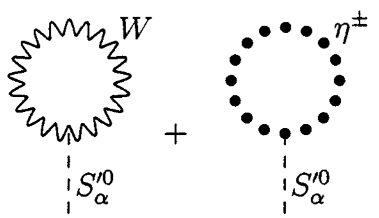
with

$$\Delta M_{ij} = \tfrac{1}{2}[\tilde{\Pi}^V_{ij}(m_i^2)+\tilde{\Pi}^V_{ij}(m_j^2)] - \tfrac{1}{2}[m_{\chi_i^0}\tilde{\Sigma}^V_{ij}(m_i^2) + m_{\chi_j^0}\tilde{\Sigma}^V_{ij}(m_j^2)], \tag{54}$$

where the symmetrization is necessary to achieve gauge invariance. Of course, the diagonal elements of ΔM_{ij} correspond to the difference between the pole and running masses defined in Eq. (51).

B. Gauge invariance

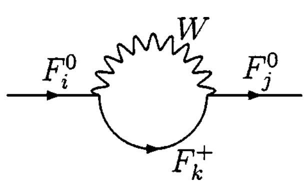
As explained in Sec. II B, the one-loop corrected vacuum expectation values are found by solving the one-loop corrected tadpole equations in Eq. (11). Of course, it is desirable to work with gauge invariant VEVs. In order to achieve the gauge invariance of the v_α's, the one-loop tadpole $\widetilde{T}_\alpha^{\overline{\rm DR}}(Q)$ must be independent of the gauge parameter ξ. As it is shown in Appendix C the following set of tadpoles is gauge invariant:



$$= i\,[T_\alpha(Q)]^{W,\eta^\pm}$$

where S'^0_α denote neutral scalar bosons in the weak basis (see Appendix A) η's are the Fadeev-Popov ghosts. A similar set for the Z gauge boson exists. Nevertheless, the tadpole with a charged Goldstone boson in the loop introduces a gauge dependence that cannot be canceled. For this reason, the Goldstone boson loops are removed from the tadpoles $T_\alpha(Q)$ and introduced into the self-energies. This in turn allows us to achieve the gauge invariance for the two point functions, as well as for the VEVs, as explained below.

Among the loops contributing to the self-energies, consider for example the W-boson loop, which in the general R_ξ gauge is



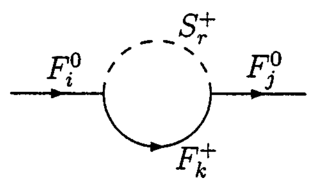
$$= i\,(\not p\, \Sigma^V_{ij} - \Pi^V_{ij})^W + \dots$$

where ellipses indicate terms proportional to γ_5 which are irrelevant for us, F^+_k are charged fermions resulting from mixing between charginos and charged leptons, and

$$(\Sigma^V_{ij})^W = -\frac{1}{16\pi^2}\sum_{k=1}^{5}(O^{\rm ncw}_{Ljk}O^{\rm cnw}_{Lki}+O^{\rm ncw}_{Rjk}O^{\rm cnw}_{Rki})$$
$$\times\Bigg\{2B_1(p^2,m_k^2,m_W^2)+B_0(p^2,m_k^2,m_W^2)$$
$$-\xi B_0(p^2,m_k^2,\xi m_W^2)-\frac{m_k^2-p^2}{m_W^2}[B_1(p^2,m_W^2,m_k^2)$$
$$-B_1(p^2,\xi m_W^2,m_k^2)]\Bigg\},$$

$$(\Pi^V_{ij})^W = \frac{1}{16\pi^2}\sum_{k=1}^{5}(O^{\rm ncw}_{Ljk}O^{\rm cnw}_{Rki}+O^{\rm ncw}_{Rjk}O^{\rm cnw}_{Lki})$$
$$\times m_k[3B_0(p^2,m_k^2,m_W^2)+\xi B_0(p^2,m_k^2,\xi m_W^2)]. \tag{55}$$

This graph introduces an explicit dependence on the gauge parameter ξ. The other self-energy graph with ξ dependence is the one that includes the charged Goldstone boson. The charged Goldstone boson is one of the eight charged scalars S^+_k resulting from mixing between the two charged Higgs fields and the six charged sleptons. This contribution is

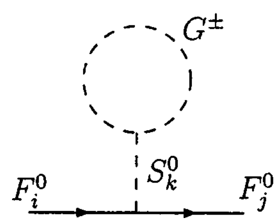


$$= i\,(\not p\, \Sigma^V_{ij} - \Pi^V_{ij})^{S^+} + \dots$$

where again, ellipses indicate terms proportional to γ_5, and

$$(\Sigma^V_{ij})^{S^+} = -\frac{1}{16\pi^2}\sum_{r=1}^{8}\sum_{k=1}^{5}(O^{\rm ncs}_{Rjkr}O^{\rm cns}_{Lkir}$$
$$+O^{\rm ncs}_{Ljkr}O^{\rm cns}_{Rkir})B_1(p^2,m_k^2,m_r^2),$$
$$(\Pi^V_{ij})^{S^+} = -\frac{1}{16\pi^2}\sum_{r=1}^{8}\sum_{k=1}^{5}(O^{\rm ncs}_{Ljkr}O^{\rm cns}_{Lkir}$$
$$+O^{\rm ncs}_{Rjkr}O^{\rm cns}_{Rkir})m_k B_0(p^2,m_k^2,m_r^2) \tag{56}$$

with the couplings given in Appendix B. Nevertheless, gauge dependence is not canceled after combining Eqs. (55) and (56). In order to achieve it the inclusion of the Goldstone boson tadpole graphs, left over from the tadpole equations, is necessary:



$$= i\,(\not p\, \Sigma^V_{ij} - \Pi^V_{ij})^{Tad} + \dots$$

where $(\Sigma^V_{ij})^{\rm tad}=0$ and

$$(\Pi^V_{ij})^{\rm tad} = -\frac{1}{32\pi^2}\sum_{k=1}^{5}(O^{\rm nns}_{Ljik}+O^{\rm nns}_{Rjik})$$
$$\times\frac{1}{m_{S^0_k}}g^{S^0S^+S^-}_{kG^+G^-}A_0(\xi m_W^2). \tag{57}$$

A_0, B_0, and B_1 appearing above are Passarino-Veltman functions [31], $g^{S^0S^+S^-}_{kG^+G^-}$ being the neutral scalar coupling to a pair of charged Goldstone bosons, and O^{nns} the neutral scalar couplings to a pair of neutral fermions (neutralino-

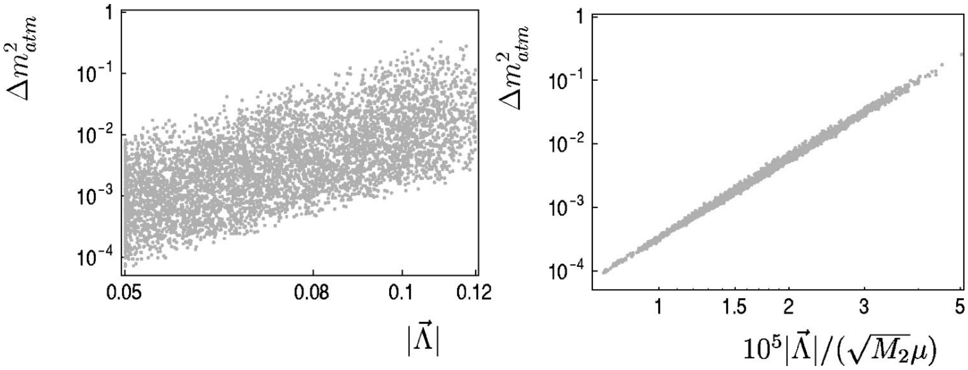


FIG. 1. Example of calculated $\Delta m^2_{\rm atm}$ as a function of (left) the alignment parameter $\vec{\Lambda}$ and (right), as function of $|\vec{\Lambda}|/(\sqrt{M_2}\mu)$, all of these expressed in GeV. The figure shows that Eq. (41) can be used to fix the relative size of R-parity breaking parameters to obtain the correct $\Delta m^2_{\rm atm}$.

neutrinos). Numerically, we have checked that, by adding the Goldstone tadpoles to the self-energies our results do not change by varying the gauge parameter from $\xi=1$ to $\xi=10^9$, thus establishing the gauge invariance of the calculation. Similarly we have also checked that the corresponding set of diagrams involving the neutral gauge boson tadpole (Z) plus neutral ghost tadpoles is gauge invariant and, similarly, the contribution to the self-energies due to Z exchange plus neutral pseudoscalars and neutral Goldstones is also gauge invariant.

Before we close this section we would like to add a short discussion on the basic structure of the loops which will be useful in the following. It is useful to do this in the approximation where the $R\!\!\!/_p$ parameters are small, as discussed above. As seen from the expression for $m_{\rm eff}$, at tree-level the effective neutrino mass matrix in this limit has the structure $m_{ij}\sim\Lambda_i\Lambda_j$, and at this level of sophistication neutrino angles are simple functions of ratios of Λ_i/Λ_j. The one-loop corrections, however, in general destroy this simple picture. This can be seen as follows. The one-loop corrections have the general form

$$(\Sigma_{ij},\Pi_{ij})\sim\sum\,(\mathcal{O}_{i,a}\mathcal{O}_{j,b}+\mathcal{O}_{i,c}\mathcal{O}_{j,d})(B_1,mB_0),\quad(58)$$

where the $\mathcal{O}$ stand symbolically for the various couplings. Now, since the expansion matrix ξ, defined in Eq. (33) can be written as $\xi_{i\alpha}\sim f_\alpha\epsilon_i+g_\alpha\Lambda_i$ [see Eq. (35)] a product of two couplings involving neutrino-neutralino mixing has the general structure

$$\mathcal{O}_{i,a}\mathcal{O}_{j,b}\sim(f_\alpha\epsilon_i+g_\alpha\Lambda_i)\times(f'_\alpha\epsilon_j+g'_\alpha\Lambda_j)\times F(\cdots),\quad(59)$$

where all other dependence on the SUSY parameters has been hidden symbolically in $F(\cdots)$. The one-loop corrections therefore also carry a certain index structure, which can be written as

$$m_{ij}^{\text{one-loop}}\sim a\epsilon_i\epsilon_j+b(\epsilon_i\Lambda_j+\Lambda_i\epsilon_j)+c\Lambda_i\Lambda_j,\quad(60)$$

where a, b, c are again complicated functions of SUSY parameters involving couplings, the Passarino-Veltman functions, etc. Clearly, the terms proportional to c in Eq. (60) above will lead only to a renormalization of the heaviest neutrino mass eigenstate. On the other hand the terms proportional to a in Eq. (60) are genuine loop corrections. Consider the simple case where all $\Lambda_i\equiv0$. Clearly in this case the tree-level neutrino mass is absent, but the one-loop effective neutrino mass has the same index structure as before, but now in terms of $\epsilon_{i,j}$'s instead of $\Lambda_{i,j}$'s. In this idealized case angles are given as simple functions of ϵ_i ratios. For nonzero Λ_i the terms proportional to b in Eq. (60), however, destroy this simple picture. Any mismatch between ϵ_i/ϵ_j and Λ_i/Λ_j will lead, in general, to a very complex parameter dependence of the neutrino angles.

VII. NUMERICAL RESULTS ON NEUTRINO MASSES AND MIXINGS

Here we collect our numerical results on neutrino masses and mixings. As we have seen, a characteristic of the BRPV model is the appearance of vacuum expectation values for the sneutrino fields, v'_i which imply a tree-level mass for one of the neutrinos given by Eq. (41). The one-loop-corrected neutrino mass matrix gives important contributions to the heaviest neutrino mass which we have determined through

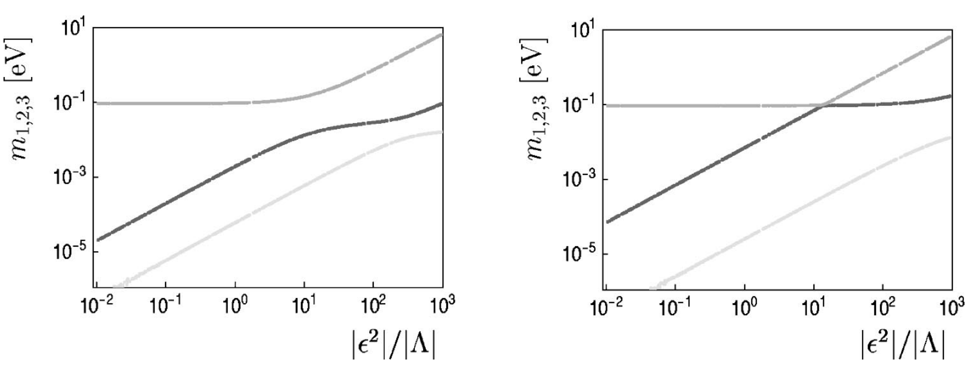


FIG. 2. Example of calculated neutrino masses in units of eV as a function of $|\epsilon^2|/|\Lambda|$, for a particular though typical choice for the other parameters (see text), illustrating the relative importance of tree versus loop-induced neutrino masses.

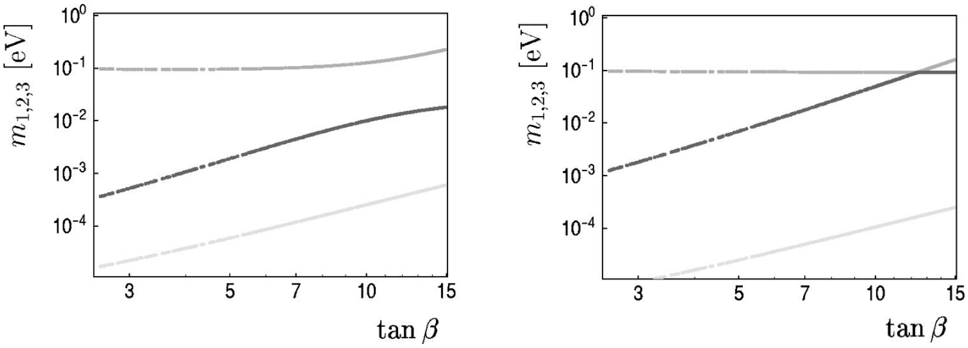


FIG. 3. The neutrino mass spectrum versus tan β, for parameters otherwise chosen as in Fig. 2. The importance of loops increases strongly with tan β.

the renormalization procedure sketched above.

First we note that in order to solve the atmospheric *and* solar neutrino problem one requires $m^{\text{one-loop}} \ll m^{\text{tree}}$. If this is satisfied it is essentially trivial within our model to solve the atmospheric neutrino problem. It is simply equivalent to choosing an adequate size of the alignment vector $|\vec{\Lambda}|^2$, as can be seen from Eq. (41) and is also demonstrated in Fig. 1. However there are regions of parameters where the one-loop contributions are comparable to the tree-level neutrino mass. This is discussed in quite some detail below, where we give an illustrative parameter study in order to isolate the main features of the dependence on the underlying parameters. First we get a rough idea of the magnitude of the neutrino masses including the one-loop corrections by displaying in Fig. 2 the three lightest eigenvalues of the neutrino-neutralino mass matrix as a function of the parameter $|\epsilon^2|/|\Lambda|$. Other parameters are fixed as follows: (a) MSSM parameters $m_0 = \mu = 500$ GeV, $M_2 = 200$ GeV, $\tan\beta = 5$, $B = -A = m_0$. (b) RPV parameters $|\Lambda| = 0.16\,\text{GeV}^2$, $10\Lambda_e = \Lambda_\mu = \Lambda_\tau$ and $\epsilon_1 = \epsilon_2 = \epsilon_3$. In the left panel we give the predicted masses in the general case, while in the one on the right we apply the sign condition

$$(\epsilon_\mu/\epsilon_\tau) \times (\Lambda_\mu/\Lambda_\tau) \leqslant 0 \tag{61}$$

to be discussed in more detail below.

One notices that the parameter $|\epsilon^2|/|\Lambda|$ determines the importance of the loop contribution relative to the tree-level-induced masses. For example, from the right panel one sees that, below $|\epsilon^2|/|\Lambda| \ll 10$ the heaviest neutrino mass m_3 is mainly a tree-level mass, while for $|\epsilon^2|/|\Lambda| \gtrsim 10$ the loop-induced masses are important relative to the tree-level one. Similar results are obtained for other choices of MSSM parameters.

It is also interesting to analyze the dependence of the neutrino mass spectrum obtained in this model as a function of other supersymmetric parameters. In Fig. 3 we show the three lightest eigenvalues of the neutrino neutralino mass matrix as a function of tan β, keeping the other parameters fixed as in Fig. 2, fixing $\epsilon^2/|\Lambda| = 1$. Again in the right figure we have applied the sign condition discussed in more details below. Loop contributions are very strongly correlated with tan β. Similarly one can compute the three lightest eigenvalues of the neutrino-neutralino mass matrix as a function of m_0, as shown in Fig. 4. Larger m_0 leads to smaller loop masses, as expected. From Figs. 2–4 one sees that, as expected, the pattern of neutrino masses obtained in the bilinear $\not\!R_p$ scenario, for almost all choices of parameters, is a *hierarchical* one.

In the above we have not paid attention to whether or not the parameter values used in the evaluation of the neutrino mass spectrum are indeed solutions of the minimization tadpole conditions of the Higgs potential. We now move to a more careful study of the magnitude of the neutrino mass spectrum derived in the $\not\!R_p$ scenario.

In order to proceed further with the discussion of the solutions to the solar neutrino anomalies in this model we must distinguish two cases: (1) unified universal boundary conditions on the soft SUSY breaking terms (SUGRA case, for short); (2) nonuniversal boundary conditions on the soft SUSY breaking terms (MSSM case, for short). In what follows we refer to these two possibilities as SUGRA and MSSM cases, accordingly.

For the analysis of the neutrino *masses* these two scenarios are very similar so we focus on the case where the low-scale parameters are derivable from a universal supergravity scheme. In Fig. 5 we show the mass squared difference Δm^2_{12} which is relevant for the analysis of neutrino

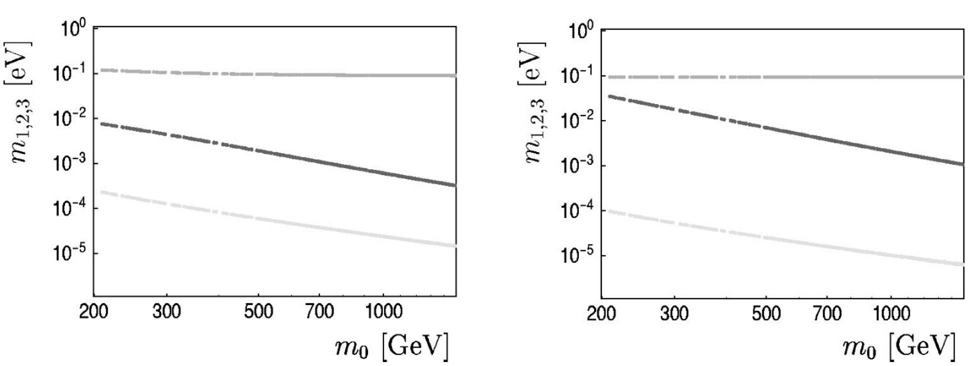


FIG. 4. The neutrino mass spectrum versus m_0, for parameters otherwise chosen as in Fig. 2. The importance of loops decreases with increasing m_0.

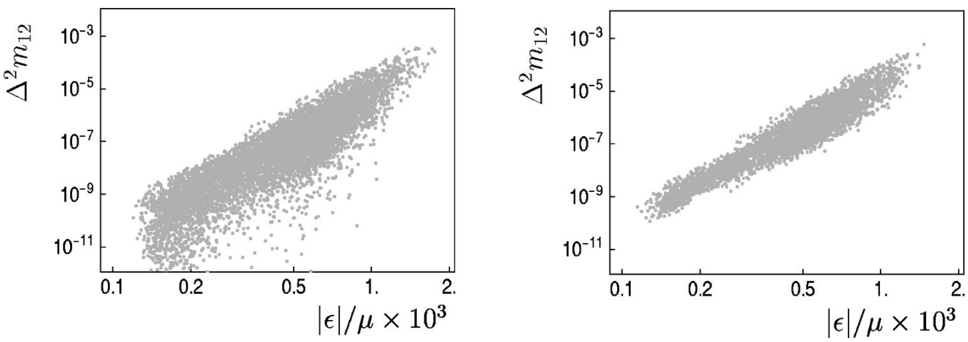


FIG. 5. $\Delta m^2_{\rm sol}$ versus $|\epsilon|/\mu$ for $\mu \leqslant 0$ (left) and $\mu > 0$ (right).

oscillations and therefore relevant to the interpretation of solar data, as a function of the parameter $|\epsilon|/\mu$. In the left panel we display $\mu \leqslant 0$ while on the right panel $\mu > 0$. Small values prefer Δm^2_{12} in the range of the vacuum solution to the solar neutrino problem, while large values give masses in the range of the MSW solutions.

Points shown in the following figures were obtained scanning the relevant parameters randomly over the region: M_2 and $|\mu|$ from 0 to 500 GeV, m_0 [0.2,1.0 TeV], a_0 and $b_0[-3,3]$ and $\tan\beta$ [2.5,10], and for the $R\!\!\!/_p$ parameters, $|\Lambda_\mu/\Lambda_\tau| = 0.8 - 1.25$, $\epsilon_\mu/\epsilon_\tau = 0.8 - 1.25$, $|\Lambda_e/\Lambda_\tau| = 0.05 - 0.1$, $\epsilon_e/\epsilon_\tau = 0.6 - 1.25$, and $|\Lambda| = 0.05 - 0.12\,\mathrm{GeV}^2$. They were subsequently tested for consistency with the minimization (tadpole) conditions of the Higgs potential and for phenomenological constraints from supersymmetric particle searches.

One can also explicitly determine the attainable range of Δm^2_{12} for which the corresponding Δm^2_{23} (see below) lies in the range required for the correct interpretation of the atmospheric neutrino data. The result obtained is displayed in Fig. 6 in which we show Δm^2_{12} as function of $\tan\beta$ for those points which solve the atmospheric neutrino problem.

We now turn to the discussion of the three neutrino mixing angles and of how they must be identified in terms our our underlying parameters. Following the usual convention the relation

$$\mathcal{N}^{1L} = \mathcal{N}'\mathcal{N}, \tag{62}$$

$$\nu_\alpha = U_{\alpha k}\nu_k \tag{63}$$

connecting mass-eigenstate and weak-eigenstate neutrinos are recovered in our notation as

$$U_{\alpha k} = \mathcal{N}^{1L}_{4+k,4+\alpha}, \tag{64}$$

where the mixing coefficients $\mathcal{N}$ are determined numerically by diagonalizing the neutral fermion mass matrix. Note that, without loss of generality, in the bilinear model one can always choose as basis the one in which the charged lepton mass matrix is already diagonal. The neutrino mixing angles relevant in the interpretation of solar and atmospheric data are identified as (if $U_{e3} \ll 1$, as indicated by the atmospheric data and the reactor neutrino constraints):

$$\sin^2(2\theta_{13}) = 4U^2_{\mu 3}(1 - U^2_{\mu 3}), \tag{65}$$

$$\sin^2(2\theta_{12}) = 4U^2_{e1}U^2_{e2}. \tag{66}$$

Note that the maximality of the atmospheric angle is achieved for $\Lambda_\mu = \Lambda_\tau$ (see Fig. 7) and Λ_e is smaller than the other two, as required by the Chooz data (see below). In fact we have found [32] that if $\epsilon^2/\Lambda \ll 10$ then the approximate formula holds

$$U_{\alpha 3} \approx \Lambda_\alpha/|\vec{\Lambda}|. \tag{67}$$

In Fig. 8 we show the expected magnitude of U^2_{e3} versus the relevant ratio of $R\!\!\!/_p$ parameters. In order to comply with the reactor data from the Chooz experiment one should have U^2_{e3} below 0.05. This implies a bound on Λ_e which can be read off from the figure.

The discussion on the solar mixing angle is more involved. First note that it has no meaning before adding the one-loop corrections to the neutrino mass, since in that limit the two low-lying neutrinos would be degenerate in mass.

In order to proceed further with the discussion of the solutions to the solar neutrino problem in this model we must analyze carefully the implications of Eq. (28). Here it is important to distinguish between case 1 (SUGRA) and case 2 (MSSM) discussed above.

In the SUGRA case by taking the ratio of the first two equations in Eq. (28),

$$\frac{\epsilon_e}{\epsilon_\mu}\,\frac{\Delta m^2_e - \tan\beta\mu\Delta B_e}{\Delta m^2_\mu - \tan\beta\mu\Delta B_\mu} \simeq \frac{\Lambda_e}{\Lambda_\mu}, \tag{68}$$

we conclude that, since $\Lambda_e < \Lambda_\mu$ and, since the relevant ratio of SUSY soft-breaking terms is close to 1, it follows that

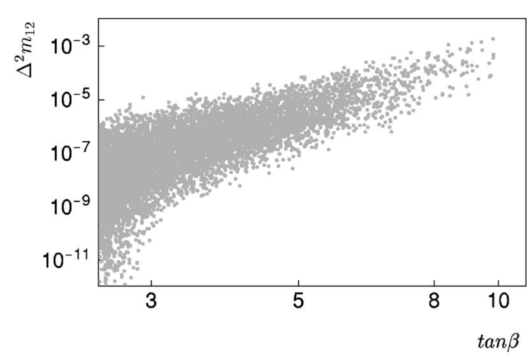


FIG. 6. Δm^2_{12} versus $\tan\beta$ for points which solve the atmospheric neutrino problem.

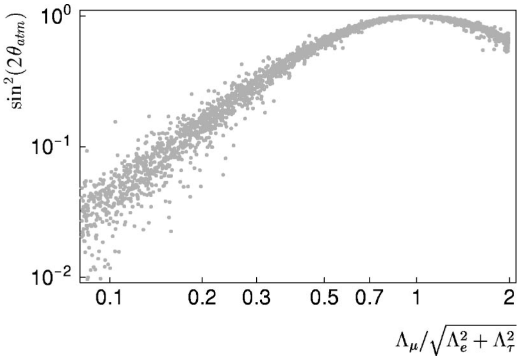


FIG. 7. Atmospheric angle versus $\Lambda_\mu/\sqrt{\Lambda_e^2+\Lambda_\tau^2}$. Maximality is obtained for $\Lambda_\mu \simeq \Lambda_\tau$ if Λ_e is smaller than the other two (see Fig. 8).

$\sin^2(2\theta_\odot)$ is small. The predictions for the solar angle as a function of the $\not R_p$ breaking parameters is indicated in Fig. 9.

More precisely, the interpretation of the solar data [6] in terms of the small angle MSW solution indicates that

$$\sin^2(2\theta_\odot) \lesssim 10^{-3} - 10^{-2} \tag{69}$$

and this in turn selects the required ratio of Λ_e to Λ_μ and Λ_τ. Therefore in this case the large angle solutions, including the vacuum or just-so solutions do not fit in the scheme.

We now move to the general MSSM case. In this case the ratio of SUSY soft-breaking terms appearing in Eq. (68) is in general arbitrary and thus the ratios of Λ_i/Λ_j are no longer tied up to the ratios of ϵ_i/ϵ_j's. This opens up the possibility for large angle solutions to the solar neutrino problem. At first sight it would seem that all predictivity of the solar angle is lost in this case, as seen in left panel of Fig. 10.

The ability of our model to determine the solar neutrino angle may be understood in terms of Eq. (60). For example, in the SUGRA case we see from Eq. (68) that the ϵ and Λ ratios are fixed within a narrow range, leading to the small mixing angle prediction for the solution to the solar neutrino problems. There is, however, another way to obtain predictivity for the general MSSM case, namely, by applying Eq. (61).

The possibility of our model predicting the solar angle even in the general MSSM case by assuming Eq. (61) can be understood as follows. Consider first the simplified limit

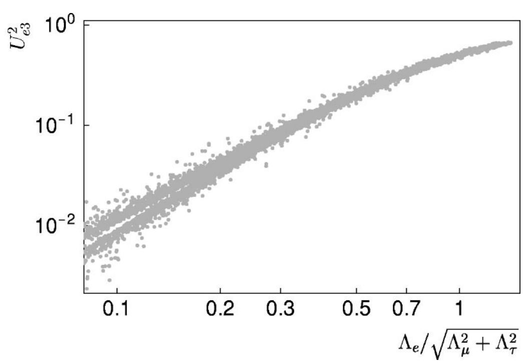


FIG. 8. U_{e3}^2 versus $\Lambda_e/\sqrt{\Lambda_\mu^2+\Lambda_\tau^2}$. To obey the experimental bound $U_{e3}^2 \leqslant 0.05$, Λ_e must be smaller than Λ_μ,Λ_τ.

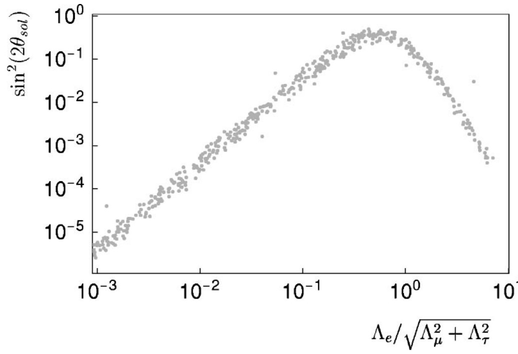


FIG. 9. $\sin^2(2\theta_{\rm sol})$ versus $\Lambda_e/\sqrt{\Lambda_\mu^2+\Lambda_\tau^2}$ for the SUGRA case, for a discussion see text.

$\Lambda_e \equiv 0$. In this case $\nu_1 \equiv \nu_e$ at tree level and there is no mixing at all between the electron neutrino and the other two states, but a finite mixing exists at one loop, due to the terms proportional to ϵ_e. In this case the sign condition, defined in Eq. (61) introduces two more zeros into the matrix proportional to b in Eq. (60) above, if $|\epsilon_\mu| \equiv |\epsilon_\tau|$ and $|\Lambda_\mu| \equiv |\Lambda_\tau|$. This fact simplifies the calculation of the solar angle very much, since one of the neutrino eigenvectors (the one for ν_e) has no dependence on the Λ_i ratios but only on the ϵ_i ratios. For a nonzero Λ_e (and small departures from equality of ϵ_μ, ϵ_τ, or $|\Lambda_\mu|$, $|\Lambda_\tau|$) this feature is destroyed and a Λ_e dependence reintroduced in the solar angle. However, as long as the one-loop contributions are smaller than the tree-level one and as long as $\Lambda_e \ll \Lambda_{\mu,\tau}$, the "cross talk" between the Λ_e and ϵ_e pieces is sufficiently small, such that some predictivity of the solar angle is retained, as illustrated in Fig. 10 (right panel).

The discussion on mixing angles may be summarized as follows. In the case that one-loop corrections are not larger than the tree-level contributions, the approximate formula

$$U_{\alpha 3} \approx \Lambda_\alpha/|\Lambda| \tag{70}$$

holds. This allows one to fix the atmospheric angle and at the same time obey the CHOOZ constraint. For the solar angle, however, the results depend on whether one wants to work in a SUGRA motivated scenario or not. For the SUGRA scenario we have found that our model allows only the small mixing angle MSW solution (SMA), while for the general case also LMA and vacuum oscillation solutions are possible.

VIII. CONCLUSIONS

We have shown that the simplest unified extension of the minimal supersymmetric standard model with bilinear R-parity violation typically predicts a hierarchical neutrino mass spectrum, offering a natural theory for the solar and atmospheric neutrino anomalies. In this model only one neutrino acquires mass due to mixing with neutralinos, while the other two get mass only as a result of radiative corrections. We have performed a full one-loop calculation of the effective neutrino mass matrix in the bilinear $\not R_p$ MSSM, taking special care to achieve a manifestly gauge invariant calcula-

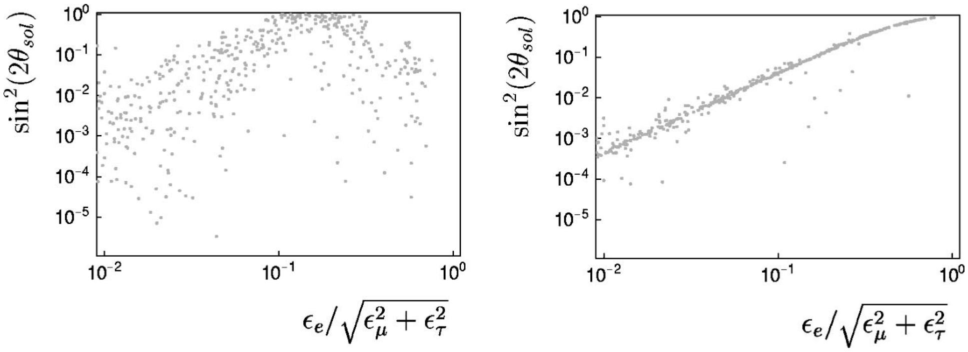


FIG. 10. $\sin^2(2\theta_{\rm sol})$ versus $\epsilon_e/\sqrt{\epsilon_\mu^2+\epsilon_\tau^2}$. The left panel corresponds to the case without the sign condition and the the right panel assumes the sign condition.

tion and performing the renormalization of the heaviest neutrino, needed in order to get reliable results. The atmospheric mass scale and maximal mixing angle arise from tree-level physics, while the solar neutrino scale and oscillations follow from calculable one-loop corrections.

Under the assumption of universal boundary conditions for the soft-supersymmetry breaking terms at the unification scale we find that the atmospheric scale is calculable by the renormalization group evolution due to the nonzero bottom quark Yukawa coupling. In this case one predicts the small mixing angle (SMA) MSW solution to be the only viable solution to the solar neutrino problem.

In contrast, for the general MSSM model, where the above assumptions are relaxed, one can implement a *bimaximal* [33] neutrino mixing scheme, in which the solar neutrino problem is accounted for through large mixing angle solutions. A great advantage of our approach is that the parameters required in order to solve the neutrino anomalies can be independently tested at high-energy accelerators, as originally proposed in Ref. [14]. In fact, as shown in Refs. [32,34] the bilinear $R\!\!\!/_p$ model predicts the lightest supersymmetric particle (LSP) decay to be observable at high-energy colliders, since the expected decay path can easily be shorter the typical detector sizes. This happens despite the smallness of neutrino masses indicated by the SuperKamiokande data. This provides a way to test this solution of the atmospheric and solar neutrino anomalies and potentially discriminate between the large and small mixing solutions to the solar neutrino problem.

ACKNOWLEDGMENTS

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APPENDIX A: MASS MATRICES

1. Scalar mass matrices

a. Charged scalars

The mass matrix of the charged scalar sector follows from the quadratic terms in the scalar potential

$$V_{\rm quadratic}=S'^{-}\mathbf{M}^2_{S^\pm}S'^{+}, \tag{A1}$$

where the unrotated charged scalars are $S'^{+}=(H_d^+,H_u^+,\tilde e_L^+,\tilde\mu_L^+,\tilde\tau_L^+,\tilde e_R^+,\tilde\mu_R^+,\tilde\tau_R^+)$. For convenience we will divide this (8×8) matrix into blocks in the following way:

$$\mathbf{M}^2_{S^\pm}=\begin{bmatrix}\mathbf{M}^2_{HH} & \mathbf{M}^{2T}_{H\tilde l}\\ \mathbf{M}^2_{H\tilde l} & \mathbf{M}^2_{\tilde l\tilde l}\end{bmatrix}+\xi m_W^2\begin{bmatrix}\mathbf{M}^2_A & \mathbf{M}^{2T}_B\\ \mathbf{M}^2_B & \mathbf{M}^2_C\end{bmatrix}, \tag{A2}$$

where the charged Higgs block is

$$\mathbf{M}^2_{HH}=\begin{bmatrix} B\mu\dfrac{v_u}{v_d}+\dfrac14 g^2\left(v_2^2-\Sigma_{i=1}^3 v_i^2\right)+\dfrac{t_d}{v_d} & B\mu+\dfrac14 g^2 v_d v_u \\ +\mu\Sigma_{i=1}^3\epsilon_i\dfrac{v_i}{v_d}+\dfrac12\Sigma_{i,j=1}^3 v_i(h_E h_E^\dagger)_{ij}v_j & \\ B\mu+\dfrac14 g^2 v_d v_u & B\mu\dfrac{v_d}{v_u}+\dfrac14 g^2\left(v_d^2+\Sigma_{i=1}^3 v_i^2\right) \\ & -\Sigma_{i=1}^3 B_i\epsilon_i\dfrac{v_i}{v_u}+\dfrac{t_u}{v_u}\end{bmatrix}. \tag{A3}$$

This matrix reduces to the usual charged Higgs mass matrix in the MSSM when we set $v_i=\epsilon_i=0$ and we call $m_{12}^2=B\mu$. The slepton block is given by

$$\mathbf{M}^2_{\tilde{l}\tilde{l}}=\begin{bmatrix}\mathbf{M}^2_{LL} & \mathbf{M}^2_{LR}\\ \mathbf{M}^2_{RL} & \mathbf{M}^2_{RR}\end{bmatrix}, \tag{A4}$$

where

$$(\mathbf{M}^2_{LL})_{ij}=\frac{1}{2}v_d^2(h_E^* h_E^T)_{ij}+\frac{1}{4}g^2\left(-\sum_{k=1}^{3} v_k^2-v_d^2+v_u^2\right)\delta_{ij}+\frac{1}{4}g^2 v_i v_j-\frac{v_u}{v_i}B_i\epsilon_i\delta_{ij}+\frac{t_i}{v_i}\delta_{ij}$$
$$+\mu\frac{v_d}{v_i}\epsilon_i\delta_{ij}-\epsilon_i\left(\sum_{k=1}^{3}\frac{v_k}{v_i}\epsilon_k\right)\delta_{ij}+\epsilon_i\epsilon_j+M^2_{Lji}-\frac{1}{2}\sum_{k=1}^{3}\frac{v_k}{v_i}(M^2_{Lik}+M^2_{Lki})\delta_{ij}, \tag{A5}$$

$$\mathbf{M}^2_{LR}=\frac{1}{\sqrt{2}}(v_d A_E^*-\mu v_u h_E^*), \tag{A6}$$

$$\mathbf{M}^2_{RL}=(\mathbf{M}^2_{LR})^{\dagger}, \tag{A7}$$

$$(\mathbf{M}^2_{RR})_{ij}=\frac{1}{4}g'^2\left(-\sum_{k=1}^{3} v_k^2-v_d^2+v_u^2\right)\delta_{ij}+\frac{1}{2}v_d^2(h_E^T h_E^*)_{ij}+\left(\sum_{k=1}^{3}(h_E^T)_{ik}v_k\right)\left(\sum_{s=1}^{3}(h_E^*)_{sj}v_s\right)+M^2_{Rji}. \tag{A8}$$

We recover the usual stau mass matrix again by replacing $v_i=\epsilon_i=0$ [note that we need to replace the expression of the tadpole t_i in Eq. (9) before taking the limit]. The mixing between the charged Higgs sector and the slepton sector is given by the following 6×2 block (repeated indices are not summed unless an explicit sum appears):

$$\mathbf{M}^2_{H\tilde{l}}=\begin{bmatrix}-\mu\epsilon_i-\frac{1}{2}v_d\sum_{k=1}^{3}(h_E^* h_E^T)_{ik}v_k+\frac{1}{4}g^2 v_d v_i & -B_i\epsilon_i+\frac{1}{4}g^2 v_u v_i\\ -\frac{1}{\sqrt{2}}v_u\sum_{k=1}^{3}(h_E^T)_{ik}\epsilon_k-\frac{1}{\sqrt{2}}\sum_{k=1}^{3}(A_E^T)_{ik}v_k & -\frac{1}{\sqrt{2}}\sum_{k=1}^{3}(h_E^T)_{ik}(\mu v_k+\epsilon_k v_d)\end{bmatrix} \tag{A9}$$

and as expected, this mixing vanishes in the limit $v_i=\epsilon_i=0$. The charged scalar mass matrix in Eq. (A2), after setting $t_u=t_d=t_i=0$, has determinant equal to zero for $\xi=0$, since one of the eigenvectors corresponds to the charged Goldstone boson with zero eigenvalue.

For our one loop calculations one has to had the gauge fixing. The part of the mass matrix in Eq. (A2) that comes from the gauge fixing reads for the $(2\times 2)A$ block

$$\mathbf{M}^2_A=\begin{bmatrix}\frac{v_d^2}{v^2} & \frac{-v_u v_d}{v^2}\\ \frac{-v_u v_d}{v^2} & \frac{v_u^2}{v^2}\end{bmatrix} \tag{A10}$$

for the (6×2) B and the (6×6) C blocks

$$\mathbf{M}^2_B=\begin{bmatrix}\frac{v_i v_d}{v^2} & \frac{-v_i v_u}{v^2}\\ 0 & 0\end{bmatrix},\quad \mathbf{M}^2_C=\begin{bmatrix}\mathbf{M}^2_D & 0\\ 0 & 0\end{bmatrix}, \tag{A11}$$

where the $(3\times 3)D$ block is

$$\mathbf{M}^2_D=\begin{bmatrix}\frac{v_1^2}{v^2} & \frac{v_1 v_2}{v^2} & \frac{v_1 v_3}{v^2}\\ \frac{v_2 v_1}{v^2} & \frac{v_2^2}{v^2} & \frac{v_2 v_3}{v^2}\\ \frac{v_3 v_1}{v^2} & \frac{v_2 v_3}{v^2} & \frac{v_3^2}{v^2}\end{bmatrix}. \tag{A12}$$

The charged scalar mass matrices are diagonalized by the following rotation matrices:

$$S_i^{\pm}=\mathbf{R}_{ij}^{S^{\pm}}S_j^{\pm\prime} \tag{A13}$$

with the eigenvalues $\mathrm{diag}(m_{S_1}^2,\ldots,m_{S_8}^2)=\mathbf{R}^{S\pm}\mathbf{M}_{S\pm}^2(\mathbf{R}^{S\pm})^T$.

b. CP-even neutral scalars

The quadratic scalar potential includes

$$V_{\text{quadratic}}=\frac{1}{2}[\sigma_d^0,\sigma_u^0,\tilde{\nu}_i^R]\mathbf{M}_{S_0}^2\begin{bmatrix}\sigma_d^0\\ \sigma_u^0\\ \tilde{\nu}_i^R\end{bmatrix}+\cdots, \tag{A14}$$

where the neutral *CP*-even scalar sector mass matrix in Eq. (A14) is given by

$$\mathbf{M}_{S^0}^2=\begin{bmatrix}\mathbf{M}_{SS}^2 & \mathbf{M}_{S\tilde{\nu}_R}^2\\ \mathbf{M}_{S\tilde{\nu}_R}^2 & \mathbf{M}_{\tilde{\nu}_R\tilde{\nu}_R}^2\end{bmatrix} \tag{A15}$$

where

$$\mathbf{M}_{SS}^2=\begin{bmatrix} B\mu\frac{v_u}{v_d}+\frac{1}{4}g_Z^2v_d^2+\mu\sum_{k=1}^{3}\epsilon_k\frac{v_k}{v_d}+\frac{t_d}{v_d} & -B\mu-\frac{1}{4}g_Z^2v_dv_u\\ -B\mu-\frac{1}{4}g_Z^2v_dv_u & B\mu\frac{v_d}{v_\mu}+\frac{1}{4}g_Z^2v_u^2-\sum_{k=1}^{3}B_k\epsilon_k\frac{v_k}{v_u}+\frac{t_u}{v_u}\end{bmatrix}, \tag{A16}$$

$$\mathbf{M}_{S\tilde{\nu}_R}^2=\begin{bmatrix}-\mu\epsilon_i+\frac{1}{4}g_Z^2v_dv_i\\ B_i\epsilon_i-\frac{1}{4}g_Z^2v_uv_i\end{bmatrix}, \tag{A17}$$

and

$$(\mathbf{M}_{\tilde{\nu}_R\tilde{\nu}_R}^2)_{ij}=\left(\mu\epsilon_i\frac{v_d}{v_i}-B_i\epsilon_i\frac{v_u}{v_i}-\epsilon_i\sum_{k=1}^{3}\epsilon_k\frac{v_k}{v_i}-\frac{1}{2}\sum_{k=1}^{3}\frac{v_k}{v_i}(M_{Lik}^2+M_{Lki}^2)+\frac{t_i}{v_i}\right)\delta_{ij}+\frac{1}{4}g_Z^2v_iv_j+\epsilon_i\epsilon_j+\frac{1}{2}(M_{Lij}^2+M_{Lji}^2), \tag{A18}$$

where we have defined $g_Z^2\equiv g^2+g'^2$. In the upper-left 2×2 block, in the limit $v_i=\epsilon_i=0$, the reader can recognize the MSSM mass matrix corresponding to the *CP*-even neutral Higgs sector. To define the rotation matrices let us define the unrotated fields by

$$S'^0=(\sigma_d^0,\sigma_u^0\tilde{\nu}_1^R,\tilde{\nu}_2^R,\tilde{\nu}_2^R). \tag{A19}$$

Then the mass eigenstates are S_i^0 given by

$$S_i^0=\mathbf{R}_{ij}^{S^0}S_j'^0 \tag{A20}$$

with the eigenvalues $\mathrm{diag}(m_{S_1}^2,\ldots,m_{S_5}^2)=\mathbf{R}^{S^0}\mathbf{M}_{S^0}^2(\mathbf{R}^{S^0})^T$.

c. CP-odd neutral scalars

The quadratic scalar potential includes

$$V_{\text{guadratic}}=\frac{1}{2}[\varphi_1^0,\varphi_2^0,\tilde{\nu}_i^I]\mathbf{M}_{P^0}^2\begin{bmatrix}\varphi_1^0\\ \varphi_2^0\\ \tilde{\nu}_i^I\end{bmatrix}+\cdots, \tag{A21}$$

where the *CP*-odd neutral scalar mass matrix is

$$\mathbf{M}^2_{P^0}=\begin{bmatrix}\mathbf{M}^2_{PP} & \mathbf{M}^2_{P\tilde{\nu}_I}\\ \mathbf{M}^{2T}_{P\tilde{\nu}_I} & \mathbf{M}^2_{\tilde{\nu}_I\tilde{\nu}_I}\end{bmatrix}+\xi m_Z^2\begin{bmatrix}\mathbf{M}^2_E & \mathbf{M}^{2T}_F\\ \mathbf{M}^2_F & \mathbf{M}^2_G\end{bmatrix}, \tag{A22}$$

where

$$\mathbf{M}^2_{PP}=\begin{bmatrix} B\mu\dfrac{v_u}{v_d}+\mu\displaystyle\sum_{k=1}^{3}\epsilon_k\frac{v_k}{v_d}+\frac{t_d}{v_d} & B\mu\\ B\mu & B\mu\dfrac{v_d}{v_u}-\displaystyle\sum_{k=1}^{3}B_k\epsilon_k\frac{v_k}{v_u}+\frac{t_u}{v_u}\end{bmatrix}, \tag{A23}$$

$$\mathbf{M}^2_{P\tilde{\nu}_I}=\begin{bmatrix}-\mu\epsilon_i\\ -B_i\epsilon_i\end{bmatrix}, \tag{A24}$$

and

$$(\mathbf{M}^2_{\tilde{\nu}_I\tilde{\nu}_I})_{ij}=\left(\mu\epsilon_i\frac{v_d}{v_i}-B_i\epsilon_i\frac{v_u}{v_i}-\epsilon_i\sum_{k=1}^{3}\epsilon_k\frac{v_k}{v_i}-\frac{1}{2}\sum_{k=1}^{3}\frac{v_k}{v_i}(M^2_{Lik}+M^2_{Lki})+\frac{t_i}{v_i}\right)\delta_{ij}+\epsilon_i\epsilon_j+\frac{1}{2}(M^2_{Lij}+M^2_{Lji}). \tag{A25}$$

Finally the part of the mass matrix in Eq. (A22) that comes from the gauge fixing reads for the (2×2) E block

$$\mathbf{M}^2_E=\begin{bmatrix}\dfrac{v_d^2}{v^2} & \dfrac{-v_u v_d}{v^2}\\ \dfrac{-v_u v_d}{v^2} & \dfrac{v_u^2}{v^2}\end{bmatrix} \tag{A26}$$

for the (3×2) F block

$$\mathbf{M}^2_F=\begin{bmatrix}\dfrac{v_i v_d}{v^2} & \dfrac{-v_i v_u}{v^2}\end{bmatrix}, \tag{A27}$$

and for the $(3\times3)G$ block

$$\mathbf{M}^2_G=\begin{bmatrix}\dfrac{v_1^2}{v^2} & \dfrac{v_1v_2}{v^2} & \dfrac{v_1v_3}{v^2}\\ \dfrac{v_2v_1}{v^2} & \dfrac{v_2^2}{v^2} & \dfrac{v_2v_3}{v^2}\\ \dfrac{v_3v_1}{v^2} & \dfrac{v_3v_2}{v^2} & \dfrac{v_3^2}{v^2}\end{bmatrix}. \tag{A28}$$

The charged pseudoscalar mass matrices are diagonalized by the following rotation matrices:

$$P_i=\mathbf{R}^{P^0}_{ij}P'_j \tag{A29}$$

with the eigenvalues $\mathrm{diag}(m^2_{A_1},...,m^2_{A_5})=\mathbf{R}^{P^0}\mathbf{M}^2_{P^0}(\mathbf{R}^{P^0})^T$, where the unrotated fields are

$$P'^0=(\varphi^0_d,\varphi^0_u\tilde{\nu}^I_1,\tilde{\nu}^I_2,\tilde{\nu}^I_3). \tag{A30}$$

d. Squark mass matrices

In the unrotated basis $\tilde{u}'_i=(\tilde{u}_{Li},\tilde{u}^*_{Ri})$ and $\tilde{d}'_i=(\tilde{d}_{Li},\tilde{d}^*_{Ri})$ we get

$$V_{\text{quadratic}}=\frac{1}{2}\tilde{u}'^\dagger\mathbf{M}^2_{\tilde{u}}\tilde{u}'+\frac{1}{2}\tilde{d}'^\dagger\mathbf{M}^2_{\tilde{d}}\tilde{d}', \tag{A31}$$

where

$$\mathbf{M}^2_{\tilde{q}}=\begin{pmatrix}\mathbf{M}^2_{\tilde{q}LL} & \mathbf{M}^2_{\tilde{q}LR}\\ M^2_{\tilde{q}RL} & \mathbf{M}^2_{\tilde{q}RR}\end{pmatrix} \tag{A32}$$

with $\tilde{q}=(\tilde{u},\tilde{d})$. The blocks are different for up and down type squarks. We have

$$\begin{aligned}
\mathbf{M}^2_{\tilde{u}LL}&=\frac{1}{2}v_u^2h_U^*h_U^T+M_Q^2+\tfrac{1}{6}(4m_W^2-m_Z^2)\cos2\beta,\\
\mathbf{M}^2_{\tilde{u}RR}&=\frac{1}{2}v_u^2h_U^Th_U^*+M_U^2+\tfrac{2}{3}(m_Z^2-m_W^2)\cos2\beta,\\
\mathbf{M}^2_{\tilde{u}LR}&=\frac{v_u}{\sqrt{2}}A_U^*-\mu\frac{v_d}{\sqrt{2}}h_U^*+\sum_{i=1}^{3}\frac{v_i}{\sqrt{2}}\epsilon_ih_U^*,\\
\mathbf{M}^2_{\tilde{u}RL}&=\mathbf{M}^{2\,\dagger}_{\tilde{u}LR},
\end{aligned} \tag{A33}$$

and

$$\mathbf{M}^2_{\tilde{d}LL}=\frac{1}{2}v_d^2h_D^*h_D^T+M_Q^2-\tfrac{1}{6}(2m_W^2+m_Z^2)\cos2\beta,$$

$$\mathbf{M}^2_{\tilde{d}RR}=\frac{1}{2}v_d^2 h_D^T h_D^* + M_D^2 - \tfrac{1}{3}(m_Z^2 - m_W^2)\cos 2\beta,$$

$$\mathbf{M}^2_{\tilde{d}LR}=\frac{v_d}{\sqrt{2}}A_D^* - \mu\frac{v_u}{\sqrt{2}}h_D^*,$$

$$\mathbf{M}^2_{\tilde{d}RL}=\mathbf{M}^{2\,\dagger}_{\tilde{d}LR}. \tag{A34}$$

We define the mass eigenstates

$$\tilde{q}=\mathbf{R}^{\tilde{q}}\tilde{q}', \tag{A35}$$

which implies

$$\tilde{q}'_i=\mathbf{R}^{\tilde{q}*}_{ji}\tilde{q}_j. \tag{A36}$$

The rotation matrices are obtained from

$$\mathbf{R}^{\tilde{q}\dagger}(\mathbf{M}^{\rm diag}_{\tilde{q}})^2\mathbf{R}^{\tilde{q}}=\mathbf{M}^2_{\tilde{q}}. \tag{A37}$$

In our case the matrices in Eq. (A32) are real and therefore the rotation matrices $\mathbf{R}^{\tilde{q}}$ are orthogonal matrices.

2. Chargino mass matrix

The charginos mix with the charged leptons forming a set of five charged fermions $F_i^{\pm}$, $i=1,...,5$ in two component spinor notation. In a basis where $\psi^{+T}=(-i\lambda^+,\widetilde{H}_u^+,e_R^+,\mu_R^+,\tau_R^+)$ and $\psi^{-T}=(-i\lambda^-,\widetilde{H}_d^-,e_L^-,\mu_L^-,\tau_L^-)$, the charged fermion mass terms in the Lagrangian are

$$\mathcal{L}_m=-\frac{1}{2}(\psi^{+T},\psi^{-T})\begin{pmatrix}\mathbf{0} & \mathbf{M}_C^T\\ \mathbf{M}_C & \mathbf{0}\end{pmatrix}\begin{pmatrix}\psi^+\\ \psi^-\end{pmatrix}+\text{H.c.} \tag{A38}$$

where the chargino/lepton mass matrix is given by

$$\mathbf{M}_C=\begin{bmatrix} M & \frac{1}{\sqrt{2}}gv_u & 0 & 0 & 0\\ \frac{1}{\sqrt{2}}gv_d & \mu & -\frac{1}{\sqrt{2}}(h_E)_{11}v_1 & -\frac{1}{\sqrt{2}}(h_E)_{22}v_2 & -\frac{1}{\sqrt{2}}(h_E)_{33}v_3\\ \frac{1}{\sqrt{2}}gv_1 & -\epsilon_1 & \frac{1}{\sqrt{2}}(h_E)_{11}v_d & 0 & 0\\ \frac{1}{\sqrt{2}}gv_2 & -\epsilon_2 & 0 & \frac{1}{\sqrt{2}}(h_E)_{22}v_d & 0\\ \frac{1}{\sqrt{2}}gv_3 & -\epsilon_3 & 0 & 0 & \frac{1}{\sqrt{2}}(h_E)_{33}v_d \end{bmatrix} \tag{A39}$$

and M is the SU(2) gaugino soft mass. We note that chargino sector decouples from the lepton sector in the limit $\epsilon_i=v_i=0$. As in the MSSM, the chargino mass matrix is diagonalized by two rotation matrices $\mathbf{U}$ and $\mathbf{V}$ defined by

$$F_i^-=U_{ij}\psi_j^-,\, F_i^+=V_{ij}\psi_j^+. \tag{A40}$$

Then

$$\mathbf{U}^*\mathbf{M}_C\mathbf{V}^{-1}=\mathbf{M}_{CD}, \tag{A41}$$

where $\mathbf{M}_{CD}$ is the diagonal charged fermion mass matrix. To determine $\mathbf{U}$ and $\mathbf{V}$ we note that

$$\mathbf{M}^2_{CD}=\mathbf{V}\mathbf{M}_C^\dagger\mathbf{M}_C\mathbf{V}^{-1}=\mathbf{U^*}\mathbf{M}_C\mathbf{M}_C^\dagger(U^*)^{-1} \tag{A42}$$

implying that $\mathbf{V}$ diagonalizes $\mathbf{M}_C^\dagger\mathbf{M}_C$ and $\mathbf{U}^*$ diagonalizes $\mathbf{M}_C\mathbf{M}_C^\dagger$. For future reference we note that

$$\psi_j^-=\mathbf{U}^*_{kj}F_k^-,\quad \psi_j^+=\mathbf{V}^*_{kj}F_k^+. \tag{A43}$$

In the previous expressions the $F_i^{\pm}$ are two component spinors. We construct the four component Dirac spinors out of the two component spinors with the conventions,[3]

[3]Here we depart from the conventions of Ref. [25] because we want the e^-, μ^-, and τ^- to be the particles and not the antiparticles.

$$\chi_i^- = \begin{pmatrix} F_i^- \\ \overline{F_i^+} \end{pmatrix}. \tag{A44}$$

APPENDIX B: THE COUPLINGS

1. The neutralino couplings

Using four component spinor notation the relevant part of the Lagrangian can be written as

$$\begin{aligned}\mathcal{L} = &\overline{\chi_i^-}\gamma^\mu(O^{\rm cnw}_{Lij}P_L + O^{\rm cnw}_{Rij}P_R)\chi_j^0 W_\mu^- + \overline{\chi_i^0}\gamma^\mu(O^{\rm cnw}_{Lij}P_L + O^{\rm ncw}_{Lij}P_R)\chi_j^- W_\mu^+ + \overline{\chi_i^-}(O^{\rm cns}_{Lijk}P_L + O^{\rm cns}_{Rijk}P_R)\chi_j^0 S_k^- \\ &+ \overline{\chi_i^0}(O^{\rm cns}_{Ljik}P_L + O^{\rm cns}_{Lijk}P_R)\chi_j^- S_k^+ + \frac{1}{2}\overline{\chi_i^0}\gamma^\mu(O^{\rm nnz}_{Lij}P_L + O^{\rm nnz}_{Rij}P_R)\chi_j^0 Z_\mu^0 + \frac{1}{2}\overline{\chi_i^0}(O^{\rm nnh}_{Lijk}P_L + O^{\rm nnh}_{Rijk}P_R)\chi_j^0 H_k^0 \\ &+ i\frac{1}{2}\overline{\chi_i^0}(O^{\rm nna}_{Lijk}P_L + O^{\rm nna}_{Rijk}P_R)\chi_j^0 A_k^0 + \overline{q_i}(O^{\rm qns}_{Lijk}P_L + O^{\rm qns}_{Rijk}P_R)\chi_j^0\tilde{q}_k + \overline{\chi_i^0}(O^{\rm nqs}_{Lijk}P_L + O^{\rm nqs}_{Rijk}P_R)q_j\tilde{q}_k^*,\end{aligned} \tag{B1}$$

where q can be either d or u. The various couplings are as follows.

a. Chargino-neutralino-W

$$\begin{aligned} O^{cnw}_{Lij} &= g\,\eta_i\eta_j\left[-\mathbf{N}^*_{j2}\mathbf{U}_{i1} - \frac{1}{\sqrt{2}}\left(\mathbf{N}^*_{j3}\mathbf{U}_{i2} + \sum_{k=1}^{3}\mathbf{N}^*_{j,4+k}\mathbf{U}_{i,2+k}\right)\right], \\ O^{cnw}_{Rij} &= g\left(-\mathbf{N}_{j2}\mathbf{V}^*_{i1} + \frac{1}{\sqrt{2}}\mathbf{N}_{j4}\mathbf{V}^*_{i2}\right), \\ O^{ncw}_{Lij} &= (O^{cnw}_{Lji})^*, O^{ncw}_{Rij} = (O^{cnw}_{Rji})^*. \end{aligned} \tag{B2}$$

b. Neutralino-neutralino-Z

$$\begin{aligned} O^{nnz}_{Lij} &= \frac{g}{\cos\theta_w}\,\eta_i\eta_j\frac{1}{2}\left(\mathbf{N}_{i4}\mathbf{N}^*_{j4} - \mathbf{N}_{i3}\mathbf{N}^*_{j3} - \sum_{k=1}^{3}\mathbf{N}_{i,4+k}\mathbf{N}^*_{j,4+k}\right), \\ O^{nnz}_{Rij} &= -\frac{g}{\cos\theta_w}\frac{1}{2}(\mathbf{N}^*_{i4}\mathbf{N}_{j4} - \mathbf{N}^*_{i3}\mathbf{N}_{j3} - \Sigma_{k=1}^{3}\mathbf{N}^*_{i,4+k}\mathbf{N}_{j,4+k}). \end{aligned} \tag{B3}$$

c. Chargino-neutralino-charged scalar

$$\begin{aligned} O^{cns}_{Lijk} = \eta_j\Bigg[&\mathbf{R}^{S^\pm}_{k1}(h_{E11}\mathbf{N}^*_{j5}\mathbf{V}^*_{i3} + h_{E22}\mathbf{N}^*_{j6}\mathbf{V}^*_{i4} + h_{E33}\mathbf{N}^*_{j7}\mathbf{V}^*_{i5}) + \mathbf{R}^{S^\pm}_{k2}\left(-\frac{g}{\sqrt{2}}\mathbf{N}^*_{j2}\mathbf{V}^*_{i2} - \frac{g'}{\sqrt{2}}\mathbf{N}^*_{j1}\mathbf{V}^*_{i2} - g\mathbf{N}^*_{j4}\mathbf{V}^*_{i1}\right) - \mathbf{R}^{S^\pm}_{k3}h_{E11}\mathbf{N}^*_{j3}\mathbf{V}^*_{i3} \\ &- \mathbf{R}^{S^\pm}_{k4}h_{E22}\mathbf{N}^*_{j3}\mathbf{V}^*_{i4} - \mathbf{R}^{S^\pm}_{k5}h_{E33}\mathbf{N}^*_{j3}\mathbf{V}^*_{i5} - \mathbf{R}^{S^\pm}_{k6}g'\sqrt{2}\mathbf{N}^*_{j1}\mathbf{V}^*_{i3} - \mathbf{R}^{S^\pm}_{k7}g'\sqrt{2}\mathbf{N}^*_{j1}\mathbf{V}^*_{i4} - \mathbf{R}^{S^\pm}_{k8}g'\sqrt{2}\mathbf{N}^*_{j1}\mathbf{V}^*_{i5}\Bigg], \end{aligned} \tag{B4}$$

$$\begin{aligned} O^{cns}_{Rijk} = \eta_i\Bigg[&\mathbf{R}^{S^\pm}_{k1}\left(\frac{g}{\sqrt{2}}\mathbf{N}_{j2}\mathbf{U}_{i2} + \frac{g'}{\sqrt{2}}\mathbf{N}_{j1}\mathbf{U}_{i2} - g\mathbf{N}_{j3}\mathbf{U}_{i1}\right) + \mathbf{R}^{S^\pm}_{k3}\left(\frac{g}{\sqrt{2}}\mathbf{N}_{j2}\mathbf{U}_{i3} + \frac{g'}{\sqrt{2}}\mathbf{N}_{j1}\mathbf{U}_{i3} - g\mathbf{N}_{j5}\mathbf{U}_{i1}\right) \\ &+ \mathbf{R}^{S^\pm}_{k4}\left(\frac{g}{\sqrt{2}}\mathbf{N}_{j2}\mathbf{U}_{i4} + \frac{g'}{\sqrt{2}}\mathbf{N}_{j1}\mathbf{U}_{i4} - g\mathbf{N}_{j6}\mathbf{U}_{i1}\right) + \mathbf{R}^{S^\pm}_{k5}\left(\frac{g}{\sqrt{2}}\mathbf{N}_{j2}\mathbf{U}_{i5} + \frac{g'}{\sqrt{2}}\mathbf{N}_{j1}\mathbf{U}_{i5} - g\mathbf{N}_{j7}\mathbf{U}_{i1}\right) \\ &+ \mathbf{R}^{S^\pm}_{k6}h_{E11}(\mathbf{N}_{j5}\mathbf{U}_{i2} - \mathbf{N}_{j3}\mathbf{U}_{i3}) + \mathbf{R}^{S^\pm}_{k7}h_{E22}(\mathbf{N}_{j6}\mathbf{U}_{i2} - \mathbf{N}_{j3}\mathbf{U}_{i4}) + \mathbf{R}^{S^\pm}_{k8}h_{E33}(\mathbf{N}_{j7}\mathbf{U}_{i2} - \mathbf{N}_{j3}\mathbf{U}_{i5})\Bigg], \end{aligned} \tag{B5}$$

$$O^{cns}_{Lijk} = (O^{ncs}_{Rjik})^*, \quad O^{cns}_{Rijk} = (O^{ncs}_{Ljik})^*. \tag{B6}$$

d. Neutralino-neutralino scalar

$$O^{nnh}_{Lijk}=\eta_j\frac{1}{2}[\mathbf{R}^{S^0}_{k1}(-g\mathbf{N}^*_{i2}\mathbf{N}^*_{j3}+g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j3}-g\mathbf{N}^*_{j2}\mathbf{N}^*_{i3}+g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i3})+\mathbf{R}^{S^0}_{k2}(+g\mathbf{N}^*_{i2}\mathbf{N}^*_{j4}-g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j4}+g\mathbf{N}^*_{j2}\mathbf{N}^*_{i4}-g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i4})$$
$$+\mathbf{R}^{S^0}_{k3}(-g\mathbf{N}^*_{i2}\mathbf{N}^*_{j5}+g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j5}-g\mathbf{N}^*_{j2}\mathbf{N}^*_{i5}+g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i5})+\mathbf{R}^{S^0}_{k4}(-g\mathbf{N}^*_{i2}\mathbf{N}^*_{j6}+g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j6}-g\mathbf{N}^*_{j2}\mathbf{N}^*_{i6}+g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i6})$$
$$+\mathbf{R}^{S^0}_{k5}(-g\mathbf{N}^*_{i2}\mathbf{N}^*_{j7}+g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j7}-g\mathbf{N}^*_{j2}\mathbf{N}^*_{i7}+g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i7})], \tag{B7}$$
$$O^{nnh}_{Rijk}=(O^{nnh}_{Lijk})^*.$$

e. Neutralino-neutralino pseudoscalar

$$O^{nna}_{Lijk}=-\eta_j\frac{1}{2}[\mathbf{R}^{P^0}_{k1}(-g\mathbf{N}^*_{i2}\mathbf{N}^*_{j3}+g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j3}-g\mathbf{N}^*_{j2}\mathbf{N}^*_{i3}+g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i3})+\mathbf{R}^{P^0}_{k2}(+g\mathbf{N}^*_{i2}\mathbf{N}^*_{j4}-g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j4}+g\mathbf{N}^*_{j2}\mathbf{N}^*_{i4}-g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i4})$$
$$+\mathbf{R}^{P^0}_{k3}(-g\mathbf{N}^*_{i2}\mathbf{N}^*_{j5}+g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j5}-g\mathbf{N}^*_{j2}\mathbf{N}^*_{i5}+g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i5})+\mathbf{R}^{P^0}_{k4}(-g\mathbf{N}^*_{i2}\mathbf{N}^*_{j6}+g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j6}-g\mathbf{N}^*_{j2}\mathbf{N}^*_{i6}+g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i6})$$
$$+\mathbf{R}^{P^0}_{k5}(-g\mathbf{N}^*_{i2}\mathbf{N}^*_{j7}+g'\mathbf{N}^*_{i1}\mathbf{N}^*_{j7}-g\mathbf{N}^*_{j2}\mathbf{N}^*_{i7}+g'\mathbf{N}^*_{j1}\mathbf{N}^*_{i7})], \tag{B8}$$
$$O^{nna}_{Rijk}=-(O^{nna}_{Ljik})^*.$$

The factors η_i are the signs one has to include if we consider **N**, **U**, and **V**, as real matrices and the mass of the fermion i is negative.

f. Neutralino–up-quark–up-squark

$$O^{uns}_{Lijk}=\frac{4}{3}\left(\frac{g}{\sqrt{2}}\right)\tan\theta_W\mathbf{N}^*_{j1}\mathbf{R}^{\tilde{u}*}_{k,m+3}\mathbf{R}^{u}_{\mathbf{R}i,m}-(h_u)_{ml}\mathbf{R}^{\tilde{u}*}_{k,m}\mathbf{R}^{u}_{\mathbf{R}i,l}\mathbf{N}^*_{j4}, \tag{B9}$$

$$O^{uns}_{Rijk}=-\left(\frac{g}{\sqrt{2}}\right)\left(\mathbf{N}_{j2}+\frac{1}{3}\tan\theta_W\mathbf{N}_{j1}\right)\mathbf{R}^{\tilde{u}*}_{k,m}\mathbf{R}^{*u}_{\mathbf{L}m,i}-(h^*_u)_{ml}\mathbf{R}^{\tilde{u}*}_{k+3,l}\mathbf{R}^{*u}_{\mathbf{L}m,i}\mathbf{N}_{j4} \tag{B10}$$

and

$$O^{nus}_{Lijk}=(O^{uns}_{Rjik})^*,\quad O^{nus}_{Rijk}=(O^{uns}_{Ljik})^*. \tag{B11}$$

g. Neutralino–down-quark–down-squark

$$O^{dns}_{Lijk}=-\frac{2}{3}\left(\frac{g}{\sqrt{2}}\right)\tan\theta_W\mathbf{N}^*_{j1}\mathbf{R}^{\tilde{d}*}_{k,m+3}\mathbf{R}^{d}_{\mathbf{R}i,m}-(h_d)_{ml}\mathbf{R}^{\tilde{d}*}_{k,m}\mathbf{R}^{d}_{\mathbf{R}i,l}\mathbf{N}^*_{j3}, \tag{B12}$$

$$O^{dns}_{Rijk}=\left(\frac{g}{\sqrt{2}}\right)\left(\mathbf{N}_{j2}-\frac{1}{3}\tan\theta_W\mathbf{N}_{j1}\right)\mathbf{R}^{\tilde{d}*}_{k,m}\mathbf{R}^{*d}_{\mathbf{L}m,i}-(h^*_d)_{ml}\mathbf{R}^{\tilde{d}*}_{k,l+3}\mathbf{R}^{*d}_{\mathbf{L}m,i}\mathbf{N}_{j3} \tag{B13}$$

and

$$O^{nds}_{Lijk}=(O^{dns}_{Rjik})^*_i O^{nds}_{Rijk}=(O^{dns}_{Ljik})^*. \tag{B14}$$

2. The neutral scalar couplings

To evaluate the tadpoles we need the couplings of the neutral scalars with all the fields in the model. These couplings are easier to write in the unrotated basis. The couplings for the mass eigenstates can always be obtained by appropriate multiplication by the rotation matrices. As an example, and to fix the notation (repeated indices are understood to be summed unless otherwise stated), the couplings of three neutral scalars in the two basis will be related by

$$g^{S^0S^0S^0}_{ijk}=\mathbf{R}^{S^0}_{ip}\mathbf{R}^{S^0}_{jq}\mathbf{R}^{S^0}_{kr}g^{S'^0S'^0S'^0}_{pqr}. \tag{B15}$$

Sometimes we will also use partially rotated couplings, for instance,

$$g_{ijk}^{S'^0S^0S^0}=\mathbf{R}^{S^0}_{jq}\mathbf{R}^{S^0}_{kr}g_{iqr}^{S'^0S'^0S'^0} \tag{B16}$$

in an obvious notation. These couplings are defined as follows:

$$g_{ijk}^{S^0S^0S^0}=\frac{\partial^3\mathcal{L}}{\partial S_i^0 S_j^0\partial S_k^0},$$

$$g_{ijk}^{S'^0S'^0S'^0}=\frac{\partial^3\mathcal{L}}{\partial S_i'^0\partial S_j'^0\partial S_k'^0}. \tag{B17}$$

a. Neutral-scalar–neutral-scalar–neutral-scalar

$$g_{ijk}^{S'^0S'^0S'^0}=-\frac{1}{4}(g^2+g'^2)u_m(\hat{\delta}_{mi}\hat{\delta}_{jk}+\hat{\delta}_{mj}\hat{\delta}_{ik}+\hat{\delta}_{mk}\hat{\delta}_{ij}), \tag{B18}$$

where we have defined

$$u_m\equiv(v_d,v_u,v_1,v_2,v_3), \quad \hat{\delta}_{ij}\equiv\mathrm{diag}(+,-,+,+,+). \tag{B19}$$

For future reference we also define

$$v_m\equiv(v_1,v_2,v_3) \tag{B20}$$

while δ_{ij} without the hat is the usual Kronecker delta.

b. Scalar–pseudoscalar–pseudoscalar

$$g_{ijk}^{S'^0P'^0P'^0}=-\frac{1}{4}(g^2+g'^2)u_m\hat{\delta}_{mi}\hat{\delta}_{jk}. \tag{B21}$$

c. Scalar–charged-scalar–charged-scalar

We define

$$g_{ijk}^{S'^0S'^+S'^-}=\begin{pmatrix} g_{iHH}^{S'^0S'^+S'^-} & g_{iHL}^{S'^0S'^+S'^-} & g_{iHR}^{S'^0S'^+S'^-} \\ (g_{iHL}^{S'^0S'^+S'^-})^\dagger & g_{iLL}^{S'^0S'^+S'^-} & g_{iLR}^{S'^0S'^+S'^-} \\ (g_{iHR}^{S'^0S'^+S'^-})^\dagger & (g_{iLR}^{S'^0S'^+S'^-})^\dagger & g_{iRR}^{S'^0S'^+S'^-}\end{pmatrix}, \tag{B22}$$

where

$$\begin{aligned}(g_{iHH}^{S'^0S'^+S'^-})_{jk}=&\frac{1}{4}g^2[-v_u(\delta_{i1}\delta_{j1}\delta_{k2}+\delta_{i2}\delta_{j1}\delta_{k1}\\&+\delta_{i1}\delta_{j2}\delta_{k1}+\delta_{i2}\delta_{j2}\delta_{k2})-v_d(\delta_{i1}\delta_{j1}\delta_{k1}\\&+\delta_{i1}\delta_{j2}\delta_{k2}+\delta_{i2}\delta_{j1}\delta_{k2}+\delta_{i2}\delta_{j2}\delta_{k1})\\&+v_m\delta_{i-2,m}(\delta_{j1}\delta_{k1}-\delta_{j2}\delta_{k2})]\\&-\frac{1}{4}g'^2u_m\hat{\delta}_{im}\hat{\delta}_{jk}-\frac{1}{2}v_m\delta_{j1}\delta_{k1}\\&\times(h_Eh_E^\dagger+h_E^*h_E^T)_{m,i-2},\end{aligned} \tag{B23}$$

$$\begin{aligned}(g_{iHL}^{S'^0S'^+S'^-})_{jk}=&-\frac{1}{4}g^2[\delta_{i-2,k}(v_d\delta_{j1}+v_u\delta_{j2})+v_m\delta_{ij}\delta_{mk}]\\&+\frac{1}{2}v_m(h_E^*h_E^T)_{mk}\delta_{i1}\delta_{j1}\\&+\frac{1}{2}v_d\delta_{j1}(h_E^*h_E^T)_{i-2,k},\end{aligned} \tag{B24}$$

$$\begin{aligned}(g_{iHR}^{S'^0S'^+S'^-})_{jk}=&\frac{1}{\sqrt{2}}\epsilon_m(h_E^*)_{mk}(\delta_{i1}\delta_{j2}+\delta_{i2}\delta_{j1})\\&+\frac{1}{\sqrt{2}}(A_E^*)_{i-2,k}\delta_{j1}+\frac{1}{\sqrt{2}}\mu(h_E^*)_{i-2,k}\delta_{j2},\end{aligned} \tag{B25}$$

$$\begin{aligned}(g_{iLL}^{S'^0S'^+S'^-})_{jk}=&\frac{1}{4}(g^2-g'^2)u_m\hat{\delta}_{im}\delta_{jk}-\frac{1}{4}g^2v_m\\&\times(\delta_{i-2,j}\delta_{mk}+\delta_{i-2,k}\delta_{mj})-(h_E^*h_E^T)_{jk}v_d\delta_{i1},\end{aligned} \tag{B26}$$

$$(g_{iLR}^{S'^0S'^+S'^-})_{jk}=-\frac{1}{\sqrt{2}}\delta_{i1}(A_E^*)_{jk}+\frac{1}{\sqrt{2}}\mu\delta_{i2}(h_E^*)_{jk}, \tag{B27}$$

$$\begin{aligned}(g_{iRR}^{S'^0S'^+S'^-})_{jk}=&\frac{1}{2}g'^2u_m\hat{\delta}_{im}\delta_{jk}-v_d\delta_{i1}(h_E^Th_E^*)_{jk}\\&-\frac{1}{2}v_m[(h_E^*)_{i-2,k}(h_E)_{mj}\\&+(h_E)_{i-2,j}(h_E^*)_{mk}].\end{aligned} \tag{B28}$$

d. Scalar–up-squarks–up-squarks

With the definition

$$\mathcal{L}=g_{ijk}^{S'^0\tilde{u}'\tilde{u}'*}S_i'^0\tilde{u}_j'\tilde{u}_k'^*+\cdots \tag{B29}$$

we get

$$g_{ijk}^{S'^0\tilde{u}'\tilde{u}'*}=\begin{pmatrix} g_{iLL}^{S'^0\tilde{u}'\tilde{u}'*} & g_{iLR}^{S'^0\tilde{u}'\tilde{u}'*} \\ g_{iRL}^{S'^0\tilde{u}'\tilde{u}'*} & g_{iRR}^{S'^0\tilde{u}'\tilde{u}'*}\end{pmatrix}, \tag{B30}$$

where

$$g_{iLL}^{S'^0\tilde{u}'\tilde{u}'*}=u_m\hat{\delta}_{im}\left(-\frac{1}{4}g^2+\frac{1}{12}g'^2\right)\mathcal{I}-v_u(h_Uh_U^\dagger)\delta_{i2},$$

$$g_{iLR}^{S'^0\tilde{u}'\tilde{u}'*}=-\frac{1}{\sqrt{2}}\delta_{i2}A_U+\frac{1}{\sqrt{2}}\mu h_U\delta_{i1}-\frac{1}{\sqrt{2}}h_U\epsilon_m\delta_{i-2,m},$$

$$g_{iRL}^{S'^0\tilde{u}'\tilde{u}'*}=g_{iLR}^{S'^0\tilde{u}'\tilde{u}'*},$$

$$g_{iRR}^{S'^0\tilde{u}'\tilde{u}'*}=-\frac{1}{3}u_m\hat{\delta}_{im}g'^2\mathcal{I}-v_u(h_U^T h_U^*)\delta_{i2}, \tag{B31}$$

where $\mathcal{I}$ is the unit 3×3 matrix.

e. Scalar–down-squarks–down-squarks

With the definition

$$\mathcal{L}=g_{ijk}^{S'^0\tilde{d}'\tilde{d}'*}S_i'^0\tilde{d}_j'\tilde{d}_k'^*+\cdots \tag{B32}$$

we get

$$g_{ijk}^{S'^0\tilde{d}'\tilde{d}'*}=\begin{pmatrix} g_{iLL}^{S'^0\tilde{d}'\tilde{d}'*} & g_{iLR}^{S'^0\tilde{d}'\tilde{d}'*} \\ g_{iRL}^{S'^0\tilde{d}'\tilde{d}'*} & g_{iRR}^{S'^0\tilde{d}'\tilde{d}'*}\end{pmatrix}, \tag{B33}$$

where

$$g_{iLL}^{S'^0\tilde{d}'\tilde{d}'*}=u_m\hat{\delta}_{im}\left(\frac{1}{4}g^2+\frac{1}{12}g'^2\right)\mathcal{I}-v_d(h_D h_D^\dagger)\delta_{i1},$$

$$g_{iLR}^{S'^0\tilde{d}'\tilde{d}'*}=-\frac{1}{\sqrt{2}}\delta_{i1}A_D+\frac{1}{\sqrt{2}}\mu h_D\delta_{i2},$$

$$g_{iRL}^{S'^0\tilde{d}'\tilde{d}'*}=(g_{iLR}^{S'^0\tilde{d}'\tilde{d}'*})^*,$$

$$g_{iRR}^{S'^0\tilde{d}'\tilde{d}'*}=\frac{1}{6}u_m\hat{\delta}_{im}g'^2\mathcal{I}-v_d(h_D^T h_D^*)\delta_{i1}. \tag{B34}$$

f. Scalar-W^+-W^-

With the definition

$$\mathcal{L}=g_i^{S'^0W^+W^-}S_i'^0W^+W^-+\cdots \tag{B35}$$

we get

$$g_i^{S'^0W^+W^-}=g\frac{m_W}{v}(v_d\delta_{i1}+v_u\delta_{i2}+v_m\delta_{i-2,m}), \tag{B36}$$

where

$$v=\sqrt{v_d^2+v_u^2+v_1^2+v_2^2+v_3^2}. \tag{B37}$$

g. Scalar-Z^0-Z^0

With the definition

$$\mathcal{L}=\frac{1}{2}g_i^{S'^0Z^0Z^0}S_i'^0Z^0Z^0+\cdots \tag{B38}$$

we get

$$g_i^{S'^0Z^0Z^0}=\frac{g}{\cos\theta_W}\frac{m_Z}{v}(v_d\delta_{i1}+v_u\delta_{i2}+v_m\delta_{i-2,m}). \tag{B39}$$

h. Scalar-quark-quark

With the definition

$$\mathcal{L}=g_{ijk}^{S'^0\overline{u}u}S_i'^0\overline{u_j}u_k+g_{ijk}^{S'^0\overline{d}d}S_i'^0\overline{d_j}d_k+\cdots \tag{B40}$$

we get

$$g_{ijk}^{S'^0\overline{u}u}=-\frac{1}{\sqrt{2}}(h_U)_{jk}\delta_{i2} \tag{B41}$$

and

$$g_{ijk}^{S'^0\overline{d}d}=-\frac{1}{\sqrt{2}}(h_D)_{jk}\delta_{i1}. \tag{B42}$$

i. Scalar-chargino-chargino and scalar-neutralino-neutralino

With the definition

$$\mathcal{L}=\overline{\chi_i^-}(O_{Lijk}^{\mathrm{cch}'}P_L+O_{Rijk}^{\mathrm{cch}'}P_R)\chi_j^-S_i'^0 + \frac{1}{2}\overline{\chi_i^0}(O_{Lijk}^{\mathrm{nnh}'}P_L+O_{Rijk}^{\mathrm{nnh}'}P_R)\chi_j^0S_i'^0 \tag{B43}$$

we have

$$\begin{aligned}O_{Lijk}^{\mathrm{cch}'}=-\frac{\epsilon_j}{\sqrt{2}}[&g(\mathbf{V}_{i1}^*\mathbf{U}_{j2}^*\delta_{k1}+\mathbf{V}_{i2}^*\mathbf{U}_{j1}^*\delta_{k2}+\mathbf{V}_{i1}^*\mathbf{U}_{j3}^*\delta_{k3}\\ &+\mathbf{V}_{i1}^*\mathbf{U}_{j4}^*\delta_{k4}+\mathbf{V}_{i1}^*\mathbf{U}_{j5}^*\delta_{k5})+(h_{E11}\mathbf{U}_{j3}^*\mathbf{V}_{i3}^*\\ &+h_{E22}\mathbf{U}_{j4}^*\mathbf{V}_{i4}^*+h_{E33}\mathbf{U}_{j5}^*\mathbf{V}_{i5}^*)\delta_{k1}-(h_{E11}\mathbf{U}_{j2}^*\mathbf{V}_{i3}^*\delta_{k3}\\ &+h_{E22}\mathbf{U}_{j2}^*\mathbf{V}_{i4}^*\delta_{k4}+h_{E33}\mathbf{U}_{j2}^*\mathbf{V}_{i5}^*\delta_{k5})],\end{aligned} \tag{B44}$$

$$O_{Rijk}^{\mathrm{cch}'}=(O_{Ljik}^{\mathrm{cch}'})^*$$

and

$$\begin{aligned}O_{Lijk}^{\mathrm{nnh}'}=\eta_j\frac{1}{2}&(-g\mathbf{N}_{i2}^*\mathbf{N}_{j3}^*+g'\mathbf{N}_{i1}^*\mathbf{N}_{j3}^*-g\mathbf{N}_{j2}^*\mathbf{N}_{i3}^*+g'\mathbf{N}_{j1}^*\mathbf{N}_{i3}^*)\\ &\times(\delta_{k1}-\delta_{k2}+\delta_{k3}+\delta_{k4}+\delta_{k5}),\\ O_{Rijk}^{\mathrm{nnh}'}=(O_{Ljik}^{\mathrm{nnh}'})^*.&\end{aligned} \tag{B45}$$

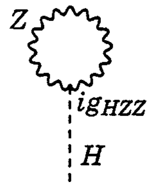
APPENDIX C: TADPOLES

1. Gauge boson and ghost tadpoles

We will consider the gauge boson and ghost tadpoles in an arbitrary R_ξ gauge to show that the dependence on ξ cancels out. We will do it for any model.

a. General Z^0 boson tadpole

We write down the tadpole contribution from the Z^0 for a general theory with the coupling g_{HZZ} to the Higgs boson:



$$iT_Z = \frac{1}{2} i\, g_{HZZ} \int \frac{d^d p}{(2\pi)^d}\, G_\mu{}^\mu(p) \tag{C1}$$

where

$$G^\mu_\mu = (-i)\left[\frac{4}{p^2 - M_Z^2} - (1-\xi)\frac{p^2}{(p^2-M_Z^2)(p^2-\xi M_Z^2)}\right] \tag{C2}$$

and the factor $\frac{1}{2}$ is a symmetry factor. Now we do some transformations in the second term of the Z^0 propagator $G(\xi)$,

$$\begin{aligned} G(\xi) &= (1-\xi)\frac{p^2}{p^2-M_Z^2}\frac{1}{p^2-\xi M_Z^2} \\ &= \frac{1}{p^2-M_Z^2} - \xi\frac{1}{p^2-\xi M_Z^2} \end{aligned} \tag{C3}$$

and therefore we can write

$$G^\mu_\mu = (-i)\left[\frac{3}{p^2-M_Z^2} + \xi\frac{1}{p^2-\xi M_Z^2}\right]. \tag{C4}$$

Then

$$\begin{aligned} iT_Z &= \frac{1}{2} i g_{HZZ}(-i)\frac{i}{16\pi^2}[3A_0(M_Z^2) + \xi A_0(\xi M_Z^2)] \\ &= \frac{i}{16\pi^2}\frac{1}{2} g_{HZZ}[3A_0(M_Z^2) + \xi A_0(\xi M_Z^2)], \end{aligned} \tag{C5}$$

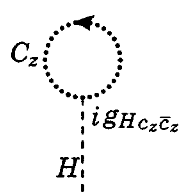
where we have used the definition

$$\frac{i}{16\pi^2} A_0(m^2) \equiv \int \frac{d^d p}{(2\pi)^d}\frac{1}{p^2 - m^2}. \tag{C6}$$

As $A_0(\xi m^2)$ grows for large ξ as ξm^2 we conclude that T_Z grows as ξ^2. This dependence has to cancel against other diagrams. It is easy to realize that the Goldstone of the Z^0 will not do it because, although its mass depend on ξ, its contribution to the tadpole will only grow as ξ because its coupling to H does not depend on ξ. But the ghost coupling to H does depend ξ as we will see.

b. General Z^0 ghost tadpole

Let us then calculate the tadpole of the ghost of the Z^0. We have



$$iT_{c_z} = (-1)\, i\, g_{Hc_z\bar{c}_z} \int \frac{d^d p}{(2\pi)^d}\frac{i}{p^2 - \xi M_Z^2} \tag{C7}$$

where the factor (-1) is because of the anticommutative properties of the ghosts. Using the definition of A_0 we get

$$iT_{c_z} = \frac{i}{16\pi^2} g_{Hc_z\bar{c}_z} A_0(\xi M_Z^2). \tag{C8}$$

Adding the two contributions together we obtain

$$\begin{aligned} iT_Z + iT_{c_z} = \frac{i}{16\pi^2}\Bigg[&\frac{3}{2} g_{HZZ} A_0(M_Z^2) \\ &+ \left(\frac{1}{2} g_{HZZ}\xi + g_{Hc_z\bar{c}_z}\right) A_0(\xi M_Z^2)\Bigg]. \end{aligned} \tag{C9}$$

We see that for the ξ dependence to cancel one must have

$$\frac{1}{2} g_{HZZ}\xi + g_{Hc_z\bar{c}_z} = 0. \tag{C10}$$

As we will show below this is true for the SM, MSSM, and also for the bilinear R-parity model. Then the contribution from the Z^0 and neutral ghost tadpoles is, for any model, gauge independent and given by

$$iT_Z + iT_{c_z} = \frac{i}{16\pi^2}\frac{3}{2} g_{HZZ} A_0(M_Z^2). \tag{C11}$$

c. The $W^{\pm}$ boson and $c^{\pm}$ ghost tadpoles

The calculation for the $W^{\pm}$ boson and charged ghosts is very similar. The main differences are that the W tadpole does not have a factor $\frac{1}{2}$ and that there are *two* ghosts for the $W^{\pm}$. Therefore we have

$$iT_W+iT_{c_W^+}+iT_{c_W^-}=\frac{i}{16\pi^2}[3g_{HWW}A_0(M_W^2)+(g_{HWW}\xi+g_{Hc_W^+\overline{c_W^+}}+g_{Hc_W^-\overline{c_W^-}})A_0(\xi M_W^2)]. \tag{C12}$$

We see that the ξ dependence will cancel out if

$$g_{HWW}\xi+g_{Hc_W^+\overline{c_W^+}}+g_{Hc_W^-\overline{c_W^-}}=0. \tag{C13}$$

We will show below that this is true in general. Then the contribution from the $W^{\pm}$ and charged ghost tadpoles is, for any model, gauge independent and given by

$$iT_W+iT_{c_W^+}+iT_{c_W^-}=\frac{i}{16\pi^2}3g_{HWW}A_0(M_W^2). \tag{C14}$$

d. The standard model

Now let us see how the cancellation occurs in the standard model. The relevant couplings for the Z^0 are

$$g_{HZZ}=\frac{g}{\cos\theta_W}M_Z,$$

$$g_{Hc_z\overline{c_z}}=-\frac{g}{2\cos\theta_W}\xi M_Z \tag{C15}$$

and we immediately see that Eq. (C10) is verified. For the $W^{\pm}$ we have

$$g_{HWW}=gM_W,$$

$$g_{Hc_W^+\overline{c_W^+}}=-\frac{g}{2}\xi M_W,$$

$$g_{Hc_W^-\overline{c_W^-}}=-\frac{g}{2}\xi M_W \tag{C16}$$

satisfying Eq. (C13).

e. Bilinear R-parity model

In the bilinear R-parity model the relevant couplings are

$$g_i^{S'^0c_W^+\overline{c^+W}}=-\frac{g}{2}\xi\frac{m_W}{v}(v_d\delta_{i1}+v_u\delta_{i2}+v_m\delta_{i-2,m}),$$

$$g_i^{S'^0c_W^-\overline{c^-W}}=-\frac{g}{2}\xi\frac{m_W}{v}(v_d\delta_{i1}+v_u\delta_{i2}+v_m\delta_{i-2,m}), \tag{C17}$$

and

$$g_i^{S'^0c_z\overline{c_z}}=-\frac{g}{2\cos\theta_W}\xi\frac{m_Z}{v}(v_d\delta_{i1}+v_u\delta_{i2}+v_m\delta_{i-2,m}). \tag{C18}$$

Then using Eqs. (B36), (B39), (C17), (C18) in Eqs. (C13), (C10) we see that the same cancellation occurs.

2. General tadpole expressions

After showing the gauge invariance of the gauge boson tadpoles together with their ghosts we give now the general tadpole in a compact form. We will write them for the unrotated neutral Higgs H'^0 because that is what is needed for substitution into Eq. (9). The general form can be written as $(X=W^{\pm},Z^0,S^{\pm},H^0,A^0,\tilde{u},\tilde{d},u,d)$,

$$T^X_{H_i'^0}=\frac{1}{16\pi^2}P_i^X, \tag{C19}$$

where

$$P_i^W=3g_i^{S'^0W^+W^-}A_0(M_W^2),$$

$$P_i^Z=\frac{3}{2}g_i^{S'^0Z^0Z^0}A_0(M_Z^2),$$

$$P_i^{S^{\pm}}=-\sum_{k=1}^{8}{}'\; g_{ikk}^{S'^0S^+S^-}A_0(m_k^2),$$

$$P_i^{S^0}=-\sum_{k=1}^{5}\frac{1}{2}g_{ikk}^{S'^0S^0S^0}A_0(m_k^2),$$

$$P_i^{P^0}=-\sum_{k=1}^{5}{}'\frac{1}{2}g_{ikk}^{S'^0P^0P^0}A_0(m_k^2),$$

$$P_i^{\tilde{u}}=-\sum_{k=1}^{6}3g_{ikk}^{S'^0\tilde{u}\tilde{u}^*}A_0(m_k^2),$$

$$P_i^{\tilde{d}}=-\sum_{k=1}^{6}3g_{ikk}^{S'^0\tilde{d}\tilde{d}^*}A_0(m_k^2)$$

$$P_i^{\chi^{\pm}}=-\sum_{k=1}^{5}(-1)\frac{O_{Lkki}^{\text{cch}'}+O_{Rkki}^{\text{cch}'}}{2}4m_kA_0(m_k^2),$$

$$P_i^{\chi^0}=-\sum_{k=1}^{7}\left(-\frac{1}{2}\right)\frac{O_{Lkki}^{\text{nnh}'}+O_{Rkki}^{\text{nnh}'}}{2}4m_kA_0(m_k^2),$$

$$P_i^u=-\sum_{k=1}^{3}(-3)g_{ikk}^{H'^0\overline{u}u}4m_kA_0(m_k^2),$$

$$P_i^d = -\sum_{k=1}^{3} (-3) g_{ikk}^{H'^0 \overline{d}d} 4 m_k A_0(m_k^2), \tag{C20}$$

where Σ' means that we sum over all fields *except* for the Goldstone boson. As explained in Sec. VI B the contribution of the goldstones is added to the self-energies to achieve gauge invariance.

APPENDIX D: ONE-LOOP SELF-ENERGIES

In this section we write down the contribution of the several self-energy diagrams in the $\xi = 1$ gauge.

1. The *W* and *Z* loops

The contribution of the W and Z loops to the functions Σ^V and Π^V can be written in the form $(X = W, Z)$,

$$\Sigma_{ij}^V = -\frac{1}{16\pi^2} \sum_k F_{ijk}^X B_1(p^2, m_k^2, m_X^2),$$

$$\Pi_{ij}^V = -\frac{1}{16\pi^2} \sum_k G_{ijk}^X m_k B_0(p^2, m_k^2, m_X^2) \tag{D1}$$

with

$$F_{ijk}^W = 2(O_{Ljk}^{\mathrm{ncw}} O_{Lki}^{\mathrm{cnw}} + O_{Rjk}^{\mathrm{ncw}} O_{Rki}^{\mathrm{cnw}}),$$

$$G_{ijk}^W = -4(O_{Ljk}^{\mathrm{ncw}} O_{Rki}^{\mathrm{cnw}} + O_{Rjk}^{\mathrm{ncw}} O_{Lki}^{\mathrm{cnw}}), \tag{D2}$$

and

$$F_{ijk}^Z = (O_{Ljk}^{\mathrm{nnz}} O_{Lki}^{\mathrm{nnz}} + O_{Rjk}^{\mathrm{nnz}} O_{Rki}^{\mathrm{nnz}}),$$

$$G_{ijk}^Z = -2(O_{Ljk}^{\mathrm{nnz}} O_{Rki}^{\mathrm{nnz}} + O_{Rjk}^{\mathrm{nnz}} O_{Lki}^{\mathrm{nnz}}). \tag{D3}$$

2. The scalar loops

All the scalar contributions can be written in the form $(X = S^{\pm}, S^0, P^0, \tilde{u}, \tilde{d})$,

$$\Sigma_{ij}^V = -\frac{1}{16\pi^2} \sum_r \sum_k F_{ijkr}^X B_1(p^2, m_k^2, m_r^2),$$

$$\Pi_{ij}^V = -\frac{1}{16\pi^2} \sum_r \sum_k G_{ijkr}^X m_k B_0(p^2, m_k^2, m_r^2) \tag{D4}$$

with

$$F_{ijkr}^{S^{\pm}} = (O_{Rjkr}^{\mathrm{ncs}} O_{Lkir}^{\mathrm{cns}} + O_{Ljkr}^{\mathrm{ncs}} O_{Rkir}^{\mathrm{cns}}),$$

$$G_{ijkr}^{S^{\pm}} = (O_{Ljkr}^{\mathrm{ncs}} O_{Lkir}^{\mathrm{cns}} + O_{Rjkr}^{\mathrm{ncs}} O_{Rkir}^{\mathrm{cns}}), \tag{D5}$$

$$F_{ijkr}^{S^0} = \frac{1}{2}(O_{Ljkr}^{\mathrm{nnh}} O_{Lkir}^{\mathrm{nnh}} + O_{Rjkr}^{\mathrm{nnh}} O_{Rkir}^{\mathrm{nnh}}),$$

$$G_{ijkr}^{S^0} = \frac{1}{2}(O_{Ljkr}^{\mathrm{nnh}} O_{Rkir}^{\mathrm{nnh}} + O_{Rjkr}^{\mathrm{nnh}} O_{Lkir}^{\mathrm{nnh}}), \tag{D6}$$

$$F_{ijkr}^{P^0} = -\frac{1}{2}(O_{Rjkr}^{\mathrm{nna}} O_{Lkir}^{\mathrm{nna}} + O_{Ljkr}^{\mathrm{nna}} O_{Rkir}^{\mathrm{nna}}),$$

$$G_{ijkr}^{P^0} = -\frac{1}{2}(O_{Ljkr}^{\mathrm{nna}} O_{Lkir}^{\mathrm{nna}} + O_{Rjkr}^{\mathrm{nna}} O_{Rkir}^{\mathrm{nna}}), \tag{D7}$$

$$F_{ijkr}^{\tilde{u}} = (O_{Rjkr}^{\mathrm{nus}} O_{Lkir}^{\mathrm{uns}} + O_{Ljkr}^{\mathrm{nus}} O_{Rkir}^{\mathrm{uns}}),$$

$$G_{ijkr}^{\tilde{u}} = (O_{Ljkr}^{\mathrm{nus}} O_{Lkir}^{\mathrm{uns}} + O_{Rjkr}^{\mathrm{nus}} O_{Rkir}^{\mathrm{uns}}), \tag{D8}$$

$$F_{ijkr}^{\tilde{d}} = (O_{Rjkr}^{\mathrm{nds}} O_{Lkir}^{\mathrm{dns}} + O_{Ljkr}^{\mathrm{nds}} O_{Rkir}^{\mathrm{dns}}),$$

$$G_{ijkr}^{\tilde{d}} = (O_{Ljkr}^{\mathrm{nds}} O_{Lkir}^{\mathrm{dns}} + O_{Rjkr}^{\mathrm{nds}} O_{Rkir}^{\mathrm{dns}}). \tag{D9}$$

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