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General formulae for $f_1 \to f_2\gamma$

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Abstract. At one-loop level the decay $f_1 \to f_2\gamma$, where f_1 and f_2 are two spin-1/2 particles with the same electric charge, is mediated by a boson B and a spin-1/2 fermion F. The boson B may have either spin 0 – interacting with the fermions through the Dirac matrices 1 and γ_5 – or spin 1 – with $V+A$ and $V-A$ couplings to the fermions. I give general formulae for the one-loop electroweak amplitude of $f_1 \to f_2\gamma$ in all these cases.

1 Introduction

Radiative decays like $\mu \to e\gamma$ and $b \to s\gamma$ provide an important testing ground for many models in particle physics. In particular, the experimental bounds on flavor-changing leptonic radiative decays [1] are planned to improve, in some cases by a few orders of magnitude [2], and the relevance of those decays in tests of new physics will certainly increase.

It is important for model builders to be able to compute expeditiously the predictions of their models for radiative decays. Unfortunately, QCD effects are important and blur the picture in hadronic decays like $b \to s\gamma$. On the other hand, in flavor-changing leptonic decays only the electroweak theory is relevant, and simple, closed formulae may be produced.

The amplitude for $\mu \to e\gamma$ in the standard electroweak theory with lepton mixing (either light or heavy neutrinos) has been given by Cheng and Li [3]. However, that amplitude has been computed for gauge bosons with exclusively left-handed interactions. Recently, the same authors together with He [4] have computed the amplitude for $\mu \to e\gamma$ following from a general Yukawa interaction, confirming earlier results by Hisano et al. [5].

In this paper I give simple formulae for the amplitude of $f_1 \to f_2\gamma$ following from either a general (axial-)vector interaction or a general Yukawa interaction. My formulae are more general than the ones given in the references above, since

(1) I allow for arbitrary electric charges of the fermions f_1 and f_2, and of the internal fields – a fermion F and a boson B – in the one-loop diagram responsible for the decay;

(2) I do not neglect the masses of f_1 and f_2 in the loop integrals;

(3) I allow for a general gauge interaction, with both $V-A$ and $V+A$ components. The last point is important since gauge bosons displaying $V+A$ interactions are present in many theories. In particular,

(1) In the left–right-symmetric model [6] there is a charged gauge boson $W_{\mathrm{R}}^{\pm}$ coupling to the fermions like $V+A$ and, as a matter of fact, the observed $W^{\pm}$ is supposed to have a small $W_{\mathrm{R}}^{\pm}$ component;

(2) In models with vector-like fermions [7] – like for instance the E_6 grand unified theory, which has both vector-like charge-$-1/3$ quarks and vector-like charge--1 leptons – the neutral gauge bosons couple to flavor-changing currents while retaining both $V-A$ and $V+A$ couplings;

(3) In the 3-3-1 model [8], based on the electroweak gauge group $SU(3) \times U(1)$, both singly and doubly charged vector bosons exist, and they have both $V-A$ and $V+A$ couplings to the fermions.

The one-loop computation of $f_1 \to f_2\gamma$ is non-trivial since there are both vertex-type diagrams – in which the photon attaches to either the internal boson B or the internal fermion F – and self-energy-type diagrams – in which the photon attaches to either f_1 or f_2. One must write the (divergent) two-point integrals in terms of three-point integrals in order to be able to add the diagrams of both types. When one does that one finds that the full vertex is both gauge-invariant and finite, as it ought to be.

The plan of this paper is as follows. In Sect. 2 I give the notation for the gauge-invariant amplitude. In Sect. 3 I define the relevant three-point finite loop integrals in terms of which the amplitude will be written. In Sect. 4 I give the amplitude resulting from the Yukawa couplings to a spin-0 boson. In Sect. 5 I give the amplitude following from the couplings of the fermions to an intermediate vector boson. The results of this work are summarized in Sect. 6.

2 Notation for the vertex

I want to compute the process $f_1(p_1) \to f_2(p_2)\gamma(q)$, where $q = p_1 - p_2$. The fermion f_1 has mass m_1 while f_2 has mass m_2. The fermions are on mass shell: $p_1^2 = m_1^2$ and $p_2^2 = m_2^2$. The fermions f_1 and f_2 are represented by spinors u_1 and $\bar{u}_2$, respectively, which satisfy $\not{p}_1 u_1 = m_1 u_1$ and $\bar{u}_2 \not{p}_2 = m_2 \bar{u}_2$.

The amplitude for the decay is $e\epsilon_\mu^*(q) M^\mu$, where $\epsilon_\mu^*(q)$ is the polarization vector of the outgoing photon and e is the electric charge of the positron. Gauge invariance implies that $q_\mu M^\mu$ must be zero; therefore M^μ must be of the form

$$M^\mu = \bar{u}_2 \left(\sigma_\mathrm{L} \Sigma_\mathrm{L}^\mu + \sigma_\mathrm{R} \Sigma_\mathrm{R}^\mu + \delta_\mathrm{L} \Delta_\mathrm{L}^\mu + \delta_\mathrm{R} \Delta_\mathrm{R}^\mu\right) u_1 , \tag{1}$$

where σ_L, σ_R, δ_L, and δ_R are numerical coefficients with the dimension of inverse mass, and

$$\Sigma_\mathrm{L}^\mu = \left(p_1^\mu + p_2^\mu\right)\gamma_\mathrm{L} - \gamma^\mu\left(m_2\gamma_\mathrm{L} + m_1\gamma_\mathrm{R}\right), \tag{2}$$
$$\Sigma_\mathrm{R}^\mu = \left(p_1^\mu + p_2^\mu\right)\gamma_\mathrm{R} - \gamma^\mu\left(m_2\gamma_\mathrm{R} + m_1\gamma_\mathrm{L}\right), \tag{3}$$
$$\Delta_\mathrm{L}^\mu = q^\mu\gamma_\mathrm{L} + \frac{q^2}{m_2^2 - m_1^2}\gamma^\mu\left(m_2\gamma_\mathrm{L} + m_1\gamma_\mathrm{R}\right), \tag{4}$$
$$\Delta_\mathrm{R}^\mu = q^\mu\gamma_\mathrm{R} + \frac{q^2}{m_2^2 - m_1^2}\gamma^\mu\left(m_2\gamma_\mathrm{R} + m_1\gamma_\mathrm{L}\right). \tag{5}$$

The matrices $\gamma_\mathrm{L} = (1-\gamma_5)/2$ and $\gamma_\mathrm{R} = (1+\gamma_5)/2$ are the projectors of chirality. If we define $\sigma^{\mu\nu} = (\mathrm{i}/2)\left[\gamma^\mu, \gamma^\nu\right]$, then M^μ may alternatively be written as

$$M^\mu = \bar{u}_2 \Big[\mathrm{i}\sigma^{\mu\nu} q_\nu \left(\sigma_\mathrm{L}\gamma_\mathrm{L} + \sigma_\mathrm{R}\gamma_\mathrm{R}\right) + \delta_\mathrm{L}\Delta_\mathrm{L}^\mu + \delta_\mathrm{R}\Delta_\mathrm{R}^\mu\Big] u_1 . \tag{6}$$

Only the coefficients σ_L and σ_R are relevant to the physical decay $f_1 \to f_2\gamma$, because $\epsilon_\mu^*(q)\, q^\mu = 0$ and $q^2 = 0$ for an on-shell photon. The coefficients δ_L and δ_R are important when $f_1(p_1) \to f_2(p_2)\gamma(q)$ is just a sub-process of a more complex decay, like for instance $f_1(p_1) \to f_2(p_2)\, e^+e^-$. In this paper I shall only give σ_L and σ_R [1]. The partial width for $f_1 \to f_2\gamma$ is

$$\Gamma = \frac{\left(m_1^2 - m_2^2\right)^3 \left(\left|\sigma_\mathrm{L}\right|^2 + \left|\sigma_\mathrm{R}\right|^2\right)}{16\pi m_1^3} . \tag{7}$$

3 The basic integrals

The expressions for the coefficients σ_L and σ_R will be given in terms of a few loop integrals. Denote

$$D_B = k^2 - m_B^2 , \tag{8}$$
$$D_{1F} = \left(k + p_1\right)^2 - m_F^2 , \tag{9}$$
$$D_{2F} = \left(k + p_2\right)^2 - m_F^2 . \tag{10}$$

Then, I define

$$a = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{D_B D_{1F} D_{2F}} , \tag{11}$$
$$c_1 p_1^\theta + c_2 p_2^\theta = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^\theta}{D_B D_{1F} D_{2F}} , \tag{12}$$
$$d_1 p_1^\theta p_1^\psi + d_2 p_2^\theta p_2^\psi + f\left(p_1^\theta p_2^\psi + p_2^\theta p_1^\psi\right) + x g^{\theta\psi} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^\theta k^\psi}{D_B D_{1F} D_{2F}} . \tag{13}$$

In the formulae for $\sigma_\mathrm{L,R}$ only the finite coefficients a, c_1, c_2, d_1, d_2, and f occur; the divergent x cancels out with the two-point integrals.

Conversely, let

$$D_{1B} = \left(k - p_1\right)^2 - m_B^2 , \tag{14}$$
$$D_{2B} = \left(k - p_2\right)^2 - m_B^2 , \tag{15}$$
$$D_F = k^2 - m_F^2 . \tag{16}$$

Then,

$$\bar{a} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{D_{1B} D_{2B} D_F} , \tag{17}$$
$$\bar{c}_1 p_1^\theta + \bar{c}_2 p_2^\theta = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^\theta}{D_{1B} D_{2B} D_F} , \tag{18}$$
$$\bar{d}_1 p_1^\theta p_1^\psi + \bar{d}_2 p_2^\theta p_2^\psi + \bar{f}\left(p_1^\theta p_2^\psi + p_2^\theta p_1^\psi\right) + \bar{x} g^{\theta\psi} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^\theta k^\psi}{D_{1B} D_{2B} D_F} . \tag{19}$$

The functions a, c_1, c_2, and so on are just a variant of the well-known Passarino–Veltman [9] decomposition of tensor integrals. In the standard notation of those authors, one has, in particular,

$$a = \frac{\mathrm{i}}{16\pi^2} C_0\left(m_1^2, q^2, m_2^2, m_B^2, m_F^2, m_F^2\right), \tag{20}$$
$$c_1 = \frac{\mathrm{i}}{16\pi^2} C_1\left(m_1^2, q^2, m_2^2, m_B^2, m_F^2, m_F^2\right), \tag{21}$$
$$c_2 = \frac{\mathrm{i}}{16\pi^2} C_2\left(m_1^2, q^2, m_2^2, m_B^2, m_F^2, m_F^2\right), \tag{22}$$
$$d_1 = \frac{\mathrm{i}}{16\pi^2} C_{11}\left(m_1^2, q^2, m_2^2, m_B^2, m_F^2, m_F^2\right), \tag{23}$$
$$d_2 = \frac{\mathrm{i}}{16\pi^2} C_{22}\left(m_1^2, q^2, m_2^2, m_B^2, m_F^2, m_F^2\right), \tag{24}$$
$$f = \frac{\mathrm{i}}{16\pi^2} C_{12}\left(m_1^2, q^2, m_2^2, m_B^2, m_F^2, m_F^2\right). \tag{25}$$

These functions may be numerically computed, for (almost) all values of their arguments, using packages [10] which have been developed following work by van Oldenborgh [11].

When one uses the approximation $m_1^2 = m_2^2 = 0$ together with $q^2 = 0$ the integrals may be computed easily. Defining $t = m_F^2/m_B^2$, one obtains

[1] Hisano et al. give partial results for δ_L and δ_R in (15) and (18) of their paper [5]

$$a = \frac{\mathrm{i}}{16\pi^2 m_B^2} \left[\frac{-1}{t-1} + \frac{\ln t}{(t-1)^2} \right], \tag{26}$$

$$\begin{aligned} c_1 = c_2 &\equiv c \\ &= \frac{\mathrm{i}}{16\pi^2 m_B^2} \left[\frac{t-3}{4\,(t-1)^2} + \frac{\ln t}{2\,(t-1)^3} \right], \end{aligned} \tag{27}$$

$$\begin{aligned} d_1 = d_2 = 2f &\equiv d \\ &= \frac{\mathrm{i}}{16\pi^2 m_B^2} \left[\frac{-2t^2+7t-11}{18\,(t-1)^3} + \frac{\ln t}{3\,(t-1)^4} \right], \end{aligned} \tag{28}$$

$$\bar{a} = \frac{\mathrm{i}}{16\pi^2 m_B^2} \left[\frac{1}{t-1} - \frac{t \ln t}{(t-1)^2} \right], \tag{29}$$

$$\begin{aligned} \bar{c}_1 = \bar{c}_2 &\equiv \bar{c} \\ &= \frac{\mathrm{i}}{16\pi^2 m_B^2} \left[\frac{3t-1}{4\,(t-1)^2} - \frac{t^2 \ln t}{2\,(t-1)^3} \right], \end{aligned} \tag{30}$$

$$\begin{aligned} \bar{d}_1 = \bar{d}_2 = 2\bar{f} &\equiv \bar{d} \\ &= \frac{\mathrm{i}}{16\pi^2 m_B^2} \left[\frac{11t^2-7t+2}{18\,(t-1)^3} - \frac{t^3 \ln t}{3\,(t-1)^4} \right]. \end{aligned} \tag{31}$$

4 Results for a Yukawa interaction

The fermions f_1 and f_2 may have an Yukawa interaction with a spin-0 boson B and with another spin-1/2 fermion F, assumed to be distinct from both f_1 and f_2. Let us write that interaction as

$$\begin{aligned} &\mathcal{L}_{\mathrm{Yukawa}} \\ &= \sum_{i=1}^{2} \left[B \bar{F} \left(L_i \gamma_{\mathrm{L}} + R_i \gamma_{\mathrm{R}} \right) f_i + B^* \bar{f}_i \left(L_i^* \gamma_{\mathrm{R}} + R_i^* \gamma_{\mathrm{L}} \right) F \right], \end{aligned} \tag{32}$$

with arbitrary dimensionless numerical coefficients L_1, L_2, R_1, and R_2. I denote

$$\lambda = L_2^* L_1 \,, \tag{33}$$
$$\rho = R_2^* R_1 \,, \tag{34}$$
$$\zeta = L_2^* R_1 \,, \tag{35}$$
$$\upsilon = R_2^* L_1 \,. \tag{36}$$

The electric charges of f_1 and f_2, in units of e, are Q_f; the electric charge of F is Q_F and the electric charge of B is Q_B. Obviously, from (32),

$$Q_f = Q_F - Q_B \,. \tag{37}$$

Otherwise I allow for arbitrary Q_f, Q_F, and Q_B.

Let us consider the consequences of the Yukawa interaction in (32) for the vertex $f_1 (p_1) \to f_2 (p_2) \gamma (q)$. There will in general be four diagrams for that vertex: two self-energy diagrams in which the photon attaches either to f_1 or to f_2; one diagram in which the photon attaches to F; and another diagram in which the photon attaches to B. The self-energy diagrams are proportional to Q_f, and the other two diagrams are proportional to Q_F and Q_B, respectively. One uses (37) to write the vertex as the sum of two terms, one of them proportional to Q_F and the other one proportional to Q_B.

The mass of the scalar boson B is denoted m_B and the mass of the fermion F is denoted m_F. With the loop integrals defined in the previous section I construct

$$k_1 = m_1 \left(c_1 + d_1 + f \right), \tag{38}$$
$$k_2 = m_2 \left(c_2 + d_2 + f \right), \tag{39}$$
$$k_3 = m_F \left(c_1 + c_2 \right), \tag{40}$$

and

$$\bar{k}_1 = m_1 \left(-\bar{c}_1 + \bar{d}_1 + \bar{f} \right), \tag{41}$$
$$\bar{k}_2 = m_2 \left(-\bar{c}_2 + \bar{d}_2 + \bar{f} \right), \tag{42}$$
$$\bar{k}_3 = m_F \left(-\bar{a} + \bar{c}_1 + \bar{c}_2 \right). \tag{43}$$

The results for σ_{L} and σ_{R} are written in terms of these functions:

$$\begin{aligned} \sigma_{\mathrm{L}} = &\, Q_F \left(\rho k_1 + \lambda k_2 + \upsilon k_3 \right) \\ &+ Q_B \left(\rho \bar{k}_1 + \lambda \bar{k}_2 + \upsilon \bar{k}_3 \right), \end{aligned} \tag{44}$$

$$\begin{aligned} \sigma_{\mathrm{R}} = &\, Q_F \left(\lambda k_1 + \rho k_2 + \zeta k_3 \right) \\ &+ Q_B \left(\lambda \bar{k}_1 + \rho \bar{k}_2 + \zeta \bar{k}_3 \right). \end{aligned} \tag{45}$$

The results in (38)–(45) do not involve any approximations and they are fully general – they hold even when the photon is off-shell, $q^2 \neq 0$. One may want to keep the mass prefactors in the $k_1, k_2, \ldots, \bar{k}_3$ of (38)–(43), while computing $c_1 + d_1 + f, c_2 + d_2 + f, \ldots, -\bar{a} + \bar{c}_1 + \bar{c}_2$ in the approximation $m_1^2 = m_2^2 = 0$ (and $q^2 = 0$). One uses (26)–(31) and obtains

$$\begin{aligned} (-\mathrm{i})\, 16\pi^2 m_B^2 \left(c + \frac{3}{2}\, d \right) = &\, \frac{t^2 - 5t - 2}{12\,(t-1)^3} \\ &+ \frac{t \ln t}{2\,(t-1)^4}, \end{aligned} \tag{46}$$

$$\begin{aligned} (-\mathrm{i})\, 16\pi^2 m_B^2 \left(-\bar{c} + \frac{3}{2}\, \bar{d} \right) = &\, \frac{2t^2 + 5t - 1}{12\,(t-1)^3} \\ &- \frac{t^2 \ln t}{2\,(t-1)^4}, \end{aligned} \tag{47}$$

$$(-\mathrm{i})\, 16\pi^2 m_B^2 \left(2c \right) = \frac{t-3}{2\,(t-1)^2} + \frac{\ln t}{(t-1)^3}, \tag{48}$$

$$(-\mathrm{i})\, 16\pi^2 m_B^2 \left(-\bar{a} + 2\bar{c} \right) = \frac{t+1}{2\,(t-1)^2} - \frac{t \ln t}{(t-1)^3}. \tag{49}$$

The functions in the right-hand sides of (46)–(49) have been given in (16) and (19) of [5], and then again in [4], where they were called $H(r)$, $G(r)$, $K(r)$, and $I(r)$, respectively (with $r = t - 1$ and apart from a common factor 2). They are all positive definite, decreasing functions, which start at $t = 0$ with a value smaller than 1 and tend to 0 as t^{-1} when $t \to \infty$. The exception is the function in the right-hand side of (48), which tends to infinity as $-3/2 - \ln t$ in the limit $t \to 0$.

5 Results for a gauge interaction

Now suppose that the fermions f_1 and f_2 interact with a (neutral or charged) vector boson B_α and with another fermion F [2], assumed to be distinct from both f_1 and f_2, the interaction Lagrangian being

$$\mathcal{L}_{\text{gauge}} = \sum_{i=1}^{2} \Big[B_\alpha \bar{F} \gamma^\alpha \left(L_i' \gamma_{\text{L}} + R_i' \gamma_{\text{R}} \right) f_i + B_\alpha^* \bar{f}_i \gamma^\alpha \left(L_i'^* \gamma_{\text{L}} + R_i'^* \gamma_{\text{R}} \right) F \Big] , \quad (50)$$

with arbitrary dimensionless numerical coefficients L_1', L_2', R_1', and R_2'. I use the notation

$$\lambda' = L_2'^* L_1' , \quad (51)$$
$$\rho' = R_2'^* R_1' , \quad (52)$$
$$\zeta' = L_2'^* R_1' , \quad (53)$$
$$\upsilon' = R_2'^* L_1' . \quad (54)$$

The electric charge of F is Q_F, in units of e, and the electric charge of B_α is Q_B. Again, (37) holds. The mass of B_α is m_B and the mass of F is m_F.

The massive gauge field B_α has associated with it a scalar "would-be Goldstone boson" φ, while φ^* is associated with B_α^*. The Yukawa interaction of f_1 and f_2 with F and with the "would-be Goldstone bosons" φ and φ^* is given by[3]

$$\begin{aligned}\mathcal{L}_\varphi = {} & \varphi \frac{\mathrm{i}}{m_B} \sum_{i=1}^{2} \bar{F} \Big[\left(R_i' m_i - L_i' m_F \right) \gamma_{\text{L}} + \left(L_i' m_i - R_i' m_F \right) \gamma_{\text{R}} \Big] f_i \\ & + \varphi^* \frac{\mathrm{i}}{m_B} \sum_{i=1}^{2} \bar{f}_i \Big[\left(L_i'^* m_F - R_i'^* m_i \right) \gamma_{\text{R}} + \left(R_i'^* m_F - L_i'^* m_i \right) \gamma_{\text{L}} \Big] F . \end{aligned} \quad (55)$$

I assume that, just as in the standard model (SM), the three-gauge-boson vertex of a photon A_μ with outgoing momentum q, an incoming B_α with incoming momentum p, and an incoming B_β^* with incoming momentum $\bar{p}$ (obviously $p + \bar{p} = q$) has the following Feynman rule:

$$\mathrm{i} e Q_B \Big[g^{\alpha\beta} \left(p - \bar{p} \right)^\mu - g^{\mu\alpha} \left(q + p \right)^\beta + g^{\mu\beta} \left(q + \bar{p} \right)^\alpha \Big] . \quad (56)$$

Furthermore, I assume that the vertex of A_μ with (incoming) B_α^* and φ has Feynman rule $e Q_B m_B g^{\mu\alpha}$, while the vertex of A_μ with B_α and φ^* is $-e Q_B m_B g^{\mu\alpha}$. This is, once again, analogous to what happens in the SM.

One adds the contributions from diagrams with B_α with those from diagrams with φ and with those from diagrams with both B_α and φ. All diagrams must be computed in the same gauge – I have used the Feynman–'t Hooft gauge, in which the propagators of both B_α and φ have poles exclusively at the physical mass m_B. One obtains

$$\begin{aligned}\sigma_{\text{L}} = {} & Q_F \left(\rho' y_1 + \lambda' y_2 + \upsilon' y_3 + \zeta' y_4 \right) \\ & + Q_B \left(\rho' \bar{y}_1 + \lambda' \bar{y}_2 + \upsilon' \bar{y}_3 + \zeta' \bar{y}_4 \right) , \end{aligned} \quad (57)$$

$$\begin{aligned}\sigma_{\text{R}} = {} & Q_F \left(\lambda' y_1 + \rho' y_2 + \zeta' y_3 + \upsilon' y_4 \right) \\ & + Q_B \left(\lambda' \bar{y}_1 + \rho' \bar{y}_2 + \zeta' \bar{y}_3 + \upsilon' \bar{y}_4 \right) , \end{aligned} \quad (58)$$

with

$$y_1 = m_1 \Big[2a + 4c_1 + 2c_2 + 2d_1 + 2f + \frac{m_F^2}{m_B^2} \left(-c_2 + d_1 + f \right) + \frac{m_2^2}{m_B^2} \left(c_2 + d_2 + f \right) \Big] , \quad (59)$$

$$y_2 = m_2 \Big[2a + 2c_1 + 4c_2 + 2d_2 + 2f + \frac{m_F^2}{m_B^2} \left(-c_1 + d_2 + f \right) + \frac{m_1^2}{m_B^2} \left(c_1 + d_1 + f \right) \Big] , \quad (60)$$

$$y_3 = m_F \Big[-4a - 4c_1 - 4c_2 + \frac{m_F^2}{m_B^2} \left(c_1 + c_2 \right) - \frac{m_1^2}{m_B^2} \left(c_1 + d_1 + f \right) - \frac{m_2^2}{m_B^2} \left(c_2 + d_2 + f \right) \Big] , \quad (61)$$

$$y_4 = -\frac{m_1 m_2 m_F}{m_B^2} \left(d_1 + d_2 + 2f \right) , \quad (62)$$

and

$$\bar{y}_1 = m_1 \Big[2\bar{c}_2 + 2\bar{d}_1 + 2\bar{f} + \frac{m_F^2}{m_B^2} \left(\bar{a} - 2\bar{c}_1 - \bar{c}_2 + \bar{d}_1 + \bar{f} \right) + \frac{m_2^2}{m_B^2} \left(-\bar{c}_2 + \bar{d}_2 + \bar{f} \right) \Big] , \quad (63)$$

$$\bar{y}_2 = m_2 \Big[2\bar{c}_1 + 2\bar{d}_2 + 2\bar{f} + \frac{m_F^2}{m_B^2} \left(\bar{a} - \bar{c}_1 - 2\bar{c}_2 + \bar{d}_2 + \bar{f} \right) + \frac{m_1^2}{m_B^2} \left(-\bar{c}_1 + \bar{d}_1 + \bar{f} \right) \Big] , \quad (64)$$

$$\bar{y}_3 = m_F \Big[-4\bar{c}_1 - 4\bar{c}_2 + \frac{m_F^2}{m_B^2} \left(-\bar{a} + \bar{c}_1 + \bar{c}_2 \right) + \frac{m_1^2}{m_B^2} \left(\bar{c}_1 - \bar{d}_1 - \bar{f} \right) + \frac{m_2^2}{m_B^2} \left(\bar{c}_2 - \bar{d}_2 - \bar{f} \right) \Big] , \quad (65)$$

$$\bar{y}_4 = \frac{m_1 m_2 m_F}{m_B^2} \left(-\bar{a} + 2\bar{c}_1 + 2\bar{c}_2 - \bar{d}_1 - \bar{d}_2 - 2\bar{f} \right) . \quad (66)$$

Just as in the previous section, the results in (57)–(66) are completely general – they hold even when $q^2 \neq 0$. One may want to keep the mass prefactors in y_1–y_3 and in $\bar{y}_1$–$\bar{y}_3$ while computing the functions inside the square

[2] I use the same notation F as in the previous section for the fermion with which f_1 and f_2 interact, although the fermion F will not in general be the same in the Yukawa interaction and in the gauge interaction. In the same vein, I use identical notations $m_{B,F}$ and $Q_{B,F}$ for the masses and electric charges, respectively, of the intermediate boson and fermion

[3] The phases of φ and φ^* are implicitly defined through (55) in a convenient way

brackets in the limit $m_1^2 = m_2^2 = 0$ (and $q^2 = 0$). With $t = m_F^2/m_B^2$ as before, one obtains

$$(-\mathrm{i})\, 16\pi^2 m_B^2 \left[2a + 6c + 3d + t\left(-c + \frac{3}{2}\, d\right)\right] = \frac{-5t^3 + 9t^2 - 30t + 8}{12\,(t-1)^3} + \frac{3t^2 \ln t}{2\,(t-1)^4}\,, \tag{67}$$

$$(-\mathrm{i})\, 16\pi^2 m_B^2 \left[2\bar{c} + 3\bar{d} + t\left(\bar{a} - 3\bar{c} + \frac{3}{2}\, \bar{d}\right)\right] = \frac{-4t^3 + 45t^2 - 33t + 10}{12\,(t-1)^3} - \frac{3t^3 \ln t}{2\,(t-1)^4}\,, \tag{68}$$

$$(-\mathrm{i})\, 16\pi^2 m_B^2 \left(-4a - 8c + 2tc\right) = \frac{t^2 + t + 4}{2\,(t-1)^2} - \frac{3t \ln t}{(t-1)^3}\,, \tag{69}$$

$$(-\mathrm{i})\, 16\pi^2 m_B^2 \left[-8\bar{c} + t\left(-\bar{a} + 2\bar{c}\right)\right] = \frac{t^2 - 11t + 4}{2\,(t-1)^2} + \frac{3t^2 \ln t}{(t-1)^3}\,. \tag{70}$$

The function in the right-hand side of (68) has been given in [3]; the functions in the right-hand sides of (67), (69), and (70) are new. The functions in (69) and (70) are positive definite and decrease continuously from 2 at $t = 0$ to $1/2$ at $t \to \infty$; the functions in (67) and (68) are negative definite and increase from a value larger than -1 at $t = 0$ to a value smaller than 0 at $t \to \infty$.

6 Conclusions

The amplitude for the decay $f_1 \to f_2\gamma$ involves two relevant operators, $\Sigma_{\mathrm{L}}^{\mu}$ and $\Sigma_{\mathrm{R}}^{\mu}$ given in (2) and (3). When f_1 and f_2 interact with a scalar boson B and with a fermion F ($F \neq f_1$ and $F \neq f_2$) as in (32), the coefficients of those operators in the amplitude, σ_{L} and σ_{R}, receive contributions as in (38)–(45). When f_1 and f_2 interact with a vector boson B_α and with a fermion F through (50), σ_{L} and σ_{R} receive the contributions in (57)–(66). All these results are completely general – they do not depend on the values of the kinematical variables $p_1^2 = m_1^2$, $p_2^2 = m_2^2$, and q^2. The finite loop integrals a, c_1, c_2, d_1 and so on in the general expressions for σ_{L} and σ_{R} are defined through (8)–(19).

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