S-matrix approach for pion re-scattering in π^0 production

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Abstract. The pion re-scattering operator for pion production, obtained in time-ordered perturbation theory, is used as a reference result to study retardation effects in the exchanged pion propagator and off-shell dependencies intrinsic to production models. We found that the S-matrix construction of the re-scattering operator describes these effects well.

In a previous paper [1] we made the connection to the usual non-relativistic representation of the pion re-scattering operator by following the approach of Refs. [2]. We started from the covariant two-nucleon Feynman amplitudes including final and initial state interaction (FSI and ISI, respectively). Neglecting the negative energy contributions in the nucleon propagators and integrating over the energy of the exchanged pion, the resulting amplitudes were transformed into those following from time-ordered perturbation theory. The irreducible "stretched box diagrams" (with more than one meson in flight in the intermediate states), giving a very small contribution, can also be neglected [1]. The full amplitude for FSI in the lowest order Born approximation is

$$\mathscr{M}_{FSI} = \int \frac{d^3q'}{(2\pi)^3} T_{NN}^{FSI} \frac{1}{F_1+F_2-\omega_1-\omega_2+i\varepsilon} \hat{O}_{rs}, \tag{1}$$

$$\hat{O}_{rs} = -\frac{1}{2\omega_\pi}\left[\frac{V(\omega_\pi)}{E_2-\omega_2-\omega_\pi}+\frac{V(-\omega_\pi)}{E_1-\omega_1-E_\pi-\omega_\pi}\right] = \frac{1}{2}\tilde{V}\,G_\pi, \tag{2}$$

$$\tilde{V} = \frac{1}{\omega_\pi}\left[(E_1-\omega_1-E_\pi-\omega_\pi)V(\omega_\pi)+(E_2-\omega_2-\omega_\pi)V(-\omega_\pi)\right], \tag{3}$$

$$G_\pi = -\frac{1}{(E_1-\omega_1-E_\pi-\omega_\pi)(E_2-\omega_2-\omega_\pi)}, \tag{4}$$

where the effective pion re-scattering operator was factorized[1] into an effective pion re-scattering vertex $\tilde{V}$ and an effective pion propagator G_π. Here, $V(\omega_\pi)$ is the product of the πN amplitude with the πNN vertex. As in Ref. [1] the standard χPT re-scattering vertex is employed. We note that for ISI, the effective pion propagator has logarithmic singularities. They were included in the calculation. In order to get the amplitude closer to the considered Feynman diagram, we tested also the replacement of the T-matrix in Eq. (1) by a simple scalar σ meson exchange.

The S-matrix technique [3] is commonly used in quantum mechanical reductions of covariant diagrams, in particular to obtain the important short-range Z-diagrams [4]. In

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that approach, the energy of the exchanged pion is obtained by energy conservation on the vertex assuming that particles in the intermediate state are on shell. It gives for G_π:

$$G_\pi^{on} = \frac{1}{\left(\Delta_1 + \Delta_2\right)^2 - \omega_\pi^2} \tag{5}$$

with $2\Delta_1 = \omega_1 + E_\pi - E_1$ and $2\Delta_2 = E_2 - \omega_2$ for FSI. Various approximations to the result in Eq. (2) were suggested earlier in Refs. [1, 2, 4, 5] and are also investigated here, namely, the "$E - E'$", the fixed kinematics and the static approximations. They consider respectively the energy of the exchanged pion to be $E_2 - \omega_2$, $m_\pi/2$ and zero.

RESULTS AND CONCLUSIONS

Our numerical results refer to the $NN \to (NN)\pi$ transition in the partial waves ${}^3P_0 \to ({}^1S_0)s_0$. On the upper panels, Fig. 1 shows the results obtained for the study of retardation, i. e., the energy considered in the exchanged pion propagator. One sees that all considered approximations (dotted, dashed and cross-dotted lines) taken only for the effective pion propagator do not differ much from each other and are close to the reference result (solid line), as already found on Ref. [1].

The lower panels of Fig. 1 study the approximations on the full operator $\hat{O}_{rs}$ in (1). The amplitudes with the S-matrix operator O^S (dotted line with crosses) are the closest to the reference result (full line). The same approach both for the operator and for V_σ (dash-dotted line on the left panel) increases slightly the gap from the reference result. The fixed-kinematics version of $\hat{O}_{rs}$, $\hat{O}_{fk}$, works well for small values of the excess energy Q, but starts to deviate rapidly with increasing Q (dotted line). The static approximation $\hat{O}_{st}$ for the re-scattering operator (dashed line) overestimates largely the amplitude (1).

All these findings for the amplitudes manifest themselves also in the results for the cross section. As found in Ref. [1], the S-matrix prescription for the pion energy, when applied only to the re-scattering vertex $V(\omega_\pi)$, but not in its multiplicative kinematic factors, overestimates the amplitude (1) by almost a factor of 5. Nevertheless, and this is the key point of this paper, this deviation is dramatically reduced if the S-matrix approach is used consistently in the whole effective operator. This procedure amounts to extend the on-shell approximation used in Ref. [1] to the full operator $\tilde{V}$, which includes kinematical factors which differently weigh the two dominant time-ordered diagrams.

The re-scattering operator considered in this paper for the neutral pion production in the isoscalar πN channel thus indeed seems to be relatively unimportant: its enhancement reported in previous papers followed from a non-appropriate (static or fixed-kinematics) treatment of the energy dependence of the effective operator[5]. Nevertheless, the re-scattering mechanism is filtered differently by other spin/isospin channels in pion production reactions. For charged pion production reactions the general irreducible re-scattering operator comprises also the dominant isovector Weinberg-Tomozawa term of the πN amplitude, and its importance is therefore enhanced. Our results indicate that also in those channels its contribution can be estimated using the simple S-matrix effective operator.

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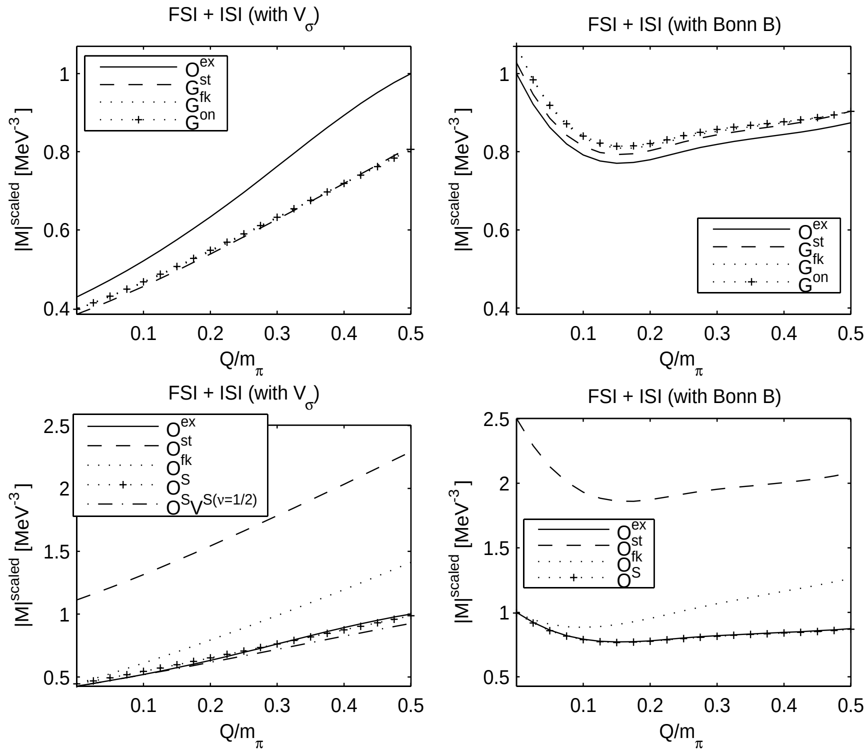

FIGURE 1. Absolute values of the FSI + ISI amplitude as a function of the excess energy $Q = 2E - 2M - E_\pi$ (in units of m_π). The right(left) panels correspond to the amplitudes with σ exchange (Bonn B) for the NN interaction. The amplitudes are taken at the maximum pion momentum q_π^{max}, determined by Q. The solid line is the reference calculation. The dashed, dotted and cross-dotted lines correspond to the static, fixed-kinematics and on-shell approximations, respectively. The corresponding operators are G^{st}, G^{fk}, G^{on} and O^{st}, O^{fk}, O^{S}. All amplitudes were normalized by a factor defined by the maximum value of the exact result. The upper panels correspond to approximations for the pion propagator G_π only; the lower panels to approximations for the whole operator $\hat{O}_{rs}$.

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