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Non-vanishing U_{e3} and $\cos 2\theta_{23}$ from a broken Z_2 symmetry

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Abstract

It is shown that the neutrino mass matrices in the flavour basis yielding a vanishing U_{e3} are characterized by invariance under a class of Z_2 symmetries. A specific Z_2 in this class also leads to a maximal atmospheric mixing angle θ_{23}. The breaking of that Z_2 can be parameterized by two dimensionless quantities, ϵ and ϵ'; the effects of $\epsilon, \epsilon' \neq 0$ are studied perturbatively and numerically. The induced value of $|U_{e3}|$ strongly depends on the neutrino mass hierarchy. We find that $|U_{e3}|$ is less than 0.07 for a normal mass hierarchy, even when $\epsilon, \epsilon' \sim 30\%$. For an inverted mass hierarchy $|U_{e3}|$ tends to be around 0.1 but can be as large as 0.17. In the case of quasi-degenerate neutrinos, $|U_{e3}|$ could be close to its experimental upper bound 0.2. In contrast, $|\cos 2\theta_{23}|$ can always reach its experimental upper bound 0.28. We propose a specific model, based on electroweak radiative corrections in the MSSM, for ϵ and ϵ'. In that model, both $|U_{e3}|$ and $|\cos 2\theta_{23}|$, could be close to their respective experimental upper bounds if neutrinos are quasi-degenerate.

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1. Introduction

In recent years, the observation of solar [1,2] and atmospheric [3] neutrino oscillations has dramatically improved our knowledge of neutrino masses and lepton mixing. The neutrino mass-squared differences Δ_{sun} and Δ_{atm}, and the mixing angles $\tan^2\theta_{\text{sun}}$ and $\sin^2 2\theta_{\text{atm}}$, are now quite well determined. The third mixing angle, represented by the matrix element U_{e3} of the lepton mixing matrix U (MNS matrix [4]), is constrained to be small by the non-observation of neutrino oscillations at the CHOOZ experiment [5].

In spite of all this progress, the available information on neutrino masses and lepton mixing is not sufficient to uncover the mechanism of neutrino mass generation. In particular, we do not yet know whether the observed features of lepton mixing are due to some underlying flavour symmetry, or they are mere mathematical coincidences [6] of the seesaw mechanism. Two features of lepton mixing which would suggest a definite symmetry are the small magnitudes of U_{e3} and $\cos 2\theta_{23}$, where θ_{23} is one of the angles in the standard parameterization of the MNS matrix and coincides with the atmospheric mixing angle θ_{atm} when $U_{e3}=0$. The best-fit value for θ_{23} in a two-generation analysis [3] of the atmospheric data is $\theta_{23}=\pi/4$, corresponding to $\cos 2\theta_{23}=0$. Likewise, $|U_{e3}|$ is required to be small: $|U_{e3}|\leqslant 0.26$ at 3σ from a combined analysis of the atmospheric and CHOOZ data [7]. This smallness strongly hints at some flavour symmetry.

There are many examples of symmetries which can force U_{e3} and/or $\cos 2\theta_{23}$ to vanish. Both quantities vanish in the extensively studied bi-maximal mixing Ansatz [8–11], which can be realized through a symmetry [12]. One can also make both U_{e3} and $\cos 2\theta_{23}$ zero while leaving the solar mixing angle arbitrary [13,14]. Alternatively, it is possible to force only U_{e3} to be zero, by imposing a discrete Abelian [15] or non-Abelian [16] symmetry; conversely, one can obtain maximal atmospheric mixing but a free U_{e3} by means of a non-Abelian symmetry or a non-standard CP symmetry [17].

The symmetries mentioned above need not be exact. It is important to consider perturbations of those symmetries from the phenomenological point of view and to study quantitatively [18] the magnitudes of U_{e3} and $\cos 2\theta_{23}$ possibly generated by such perturbations.

This paper is a study of a special class of symmetries and of the consequences of their perturbative violation. We show in Section 2 that U_{e3} vanishes if the neutrino mass matrix in the flavour basis is invariant under a class of Z_2 symmetries. The solar and atmospheric mixing angles, as well as the neutrino masses, remain unconstrained by these Z_2 symmetries. Those Z_2 symmetries thus constitute a general class of symmetries leading only to a vanishing U_{e3}. We point out that there is a special Z_2 in this class which leads, furthermore, to maximal atmospheric mixing. We consider more closely that specific Z_2 in Section 3, wherein we study departures from the symmetric limit. We parameterize perturbations of the Z_2-invariant mass matrix in terms of two complex parameters, and derive general expressions for U_{e3} and $\cos 2\theta_{23}$ in terms of those parameters; we also present detailed numerical estimates of U_{e3} and $\cos 2\theta_{23}$. Section 4 is devoted to the study of the specific perturbation which is induced by the electroweak radiative corrections to a Z_2-invariant neutrino mass matrix defined at a high scale. We discuss a specific model for this scenario. In the concluding Section 5 we make a comparison of the predictions for $|U_{e3}|$ and $\cos 2\theta_{23}$ obtained within various frameworks.

2. Vanishing U_{e3} from a class of Z_2 symmetries

The neutrino masses and lepton mixing are completely determined by the neutrino mass matrix in the flavour basis—the basis where the charged-lepton mass matrix is diagonal—which we denote as $\mathcal{M}_{\nu f}$. In this section we look for effective symmetries of $\mathcal{M}_{\nu f}$ which may lead to a vanishing U_{e3}.

One knows [19] that the lepton-number symmetry $L_e - L_\mu - L_\tau$ implies (i) a vanishing solar mass-squared difference Δ_{sun}, (ii) a maximal solar mixing angle θ_{23}, and (iii) a vanishing U_{e3}, while it keeps the atmospheric mixing angle unconstrained; one must introduce [20] a significant breaking of $L_e - L_\mu - L_\tau$ in order to correct the predictions (i) and (ii). A better symmetry seems to be the μ–τ interchange symmetry [13], which implies vanishing U_{e3} and maximal θ_{23}, but leaves both the neutrino masses and the solar mixing angle unconstrained; this is consistent with the present experimental results. The μ–τ interchange symmetry can be physically realized in a model based on the discrete non-Abelian group D_4 [14]; a variation of this model [16] keeps the prediction $U_{e3} = 0$ but leaves the atmospheric mixing angle arbitrary. Recently, Low [15] has considered models wherein $\mathcal{M}_{\nu f}$ has, due to a discrete Abelian symmetry, a structure leading to $U_{e3} = 0$.

We now show that there exists a class $Z_2(\gamma, \alpha)$ of discrete symmetries of the Z_2 type which encompasses all the models discussed above and enforces a form of $\mathcal{M}_{\nu f}$ leading to $U_{e3} = 0$. This class is parametrized by an angle γ ($0 < \gamma < 2\pi$) and a phase α ($0 \leqslant \alpha < 2\pi$). The symmetry $Z_2(\gamma, \alpha)$ is defined by the 3×3 matrix

$$S(\gamma, \alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & e^{-i\alpha}\sin\gamma \\ 0 & e^{i\alpha}\sin\gamma & -\cos\gamma \end{pmatrix}. \tag{1}$$

This matrix is unitary; indeed, it satisfies

$$\left[S(\gamma, \alpha)\right]^2 = 1_{3\times 3}, \tag{2}$$

$$\left[S(\gamma, \alpha)\right]^T = \left[S(\gamma, \alpha)\right]^*. \tag{3}$$

Eq. (2) means that $S(\gamma, \alpha)$ is a realization of the group Z_2. We define the $Z_2(\gamma, \alpha)$ invariance of $\mathcal{M}_{\nu f}$ by

$$\left[S(\gamma, \alpha)\right]^T \mathcal{M}_{\nu f} S(\gamma, \alpha) = \mathcal{M}_{\nu f}. \tag{4}$$

If one writes

$$\mathcal{M}_{\nu f} = \begin{pmatrix} \tilde{X} & \tilde{A} & \tilde{B} \\ \tilde{A} & \tilde{C} & \tilde{D} \\ \tilde{B} & \tilde{D} & \tilde{E} \end{pmatrix}, \tag{5}$$

where all the matrix elements are complex in general, then Eq. (4) is equivalent to

$$\frac{\tilde{B}}{\tilde{A}} - e^{-i\alpha} \tan\frac{\gamma}{2} = 0,$$
$$\left(e^{i\alpha}\tilde{E} - e^{-i\alpha}\tilde{C}\right)\sin\gamma + 2\tilde{D}\cos\gamma = 0. \tag{6}$$

Let us first prove that the $Z_2(\gamma,\alpha)$ invariance of $\mathcal{M}_{\nu f}$ implies $U_{e3} = 0$. The matrix $S(\gamma,\alpha)$ has a unique eigenvalue -1 corresponding to the eigenvector

$$v = \begin{pmatrix} 0 \\ \exp(-i\alpha/2)\sin(\gamma/2) \\ -\exp(i\alpha/2)\cos(\gamma/2) \end{pmatrix}. \tag{7}$$

Eq. (4), together with $S(\gamma,\alpha)v = -v$, imply that $[S(\gamma,\alpha)]^T(\mathcal{M}_{\nu f}v) = -(\mathcal{M}_{\nu f}v)$. Then, Eq. (3), together with the fact that the eigenvalue -1 of $S(\gamma,\alpha)$ is unique, implies that $\mathcal{M}_{\nu f}v \propto v^*$. Now, $\mathcal{M}_{\nu f}$ determines the lepton mixing matrix—MNS matrix—U according to

$$\mathcal{M}_{\nu f} = U^* \operatorname{diag}(m_1, m_2, m_3) U^\dagger, \tag{8}$$

where m_1, m_2, and m_3 are the (real and non-negative) neutrino masses. Thus, if we write $U = (u_1, u_2, u_3)$, then the column vectors u_j satisfy $\mathcal{M}_{\nu f}u_j = m_j u_j^*$ for $j = 1, 2, 3$. The fact that $\mathcal{M}_{\nu f}v \propto v^*$ therefore means that, apart from a phase factor, v is one of the columns of the MNS matrix, hence $U_{e3} = 0$.

Let us next prove the converse of the above, i.e., that $U_{e3} = 0$ implies that there is some angle γ and phase α such that $\mathcal{M}_{\nu f}$ is $Z_2(\gamma,\alpha)$-invariant. If $U_{e3} = 0$ then U may be parametrized by two angles $\vartheta_{1,2}$ and five phases $\chi_{1,2,3,4,5}$ as

$$U = \begin{pmatrix} e^{i\chi_1}\cos\vartheta_1 & e^{i\chi_2}\sin\vartheta_1 & 0 \\ -e^{i\chi_3}\sin\vartheta_1\cos\vartheta_2 & e^{i(\chi_2+\chi_3-\chi_1)}\cos\vartheta_1\cos\vartheta_2 & e^{i\chi_4}\sin\vartheta_2 \\ e^{i\chi_5}\sin\vartheta_1\sin\vartheta_2 & -e^{i(\chi_2+\chi_5-\chi_1)}\cos\vartheta_1\sin\vartheta_2 & e^{i(\chi_4+\chi_5-\chi_3)}\cos\vartheta_2 \end{pmatrix}. \tag{9}$$

When one computes $\mathcal{M}_{\nu f}$ through Eq. (8) one then finds that it satisfies Eq. (6) with $\gamma/2 = \vartheta_2$ and $\alpha = \chi_5 - \chi_3 + \pi$.

One has thus proved the *equivalence* of $U_{e3} = 0$ with the existence of some angle γ and phase α such that $\mathcal{M}_{\nu f}$ is $Z_2(\gamma,\alpha)$-invariant.

It should be stressed that $Z_2(\gamma,\alpha)$ will not usually be a symmetry of the full model, nor is it necessarily the remaining symmetry of some larger symmetry operating at a high scale. Some examples may help making this clear:

- The μ–τ interchange symmetry [13], which corresponds to $\cos\gamma = 0$, $e^{i\alpha}\sin\gamma = 1$, cannot be a symmetry of the full theory, since the masses of the μ and τ charged leptons are certainly different; thus, that symmetry must be broken in the charged-lepton mass matrix, but that breaking must occur in such a way that it remains unseen—at least at tree level—in the form of $\mathcal{M}_{\nu f}$. Moreover, the μ–τ interchange symmetry predicts $\cos 2\theta_{23} = 0$ together with $U_{e3} = 0$.

- Many models based on $\bar{L} = L_e - L_\mu - L_\tau$ lead to [19]

$$\mathcal{M}_{\nu f} = \begin{pmatrix} x & y & ry \\ y & z & rz \\ ry & rz & r^2 z \end{pmatrix}. \tag{10}$$

  In this case $\cos\gamma = (1-|r|^2)/(1+|r|^2)$ and $e^{i\alpha}\sin\gamma = 2r^*/(1+|r|^2)$. The symmetry $Z_2(\gamma,\alpha)$ is not a subgroup of the original $\bar{L}$ symmetry, rather it occurs accidentally as a consequence of the specific particle content of the models and of the particular way in which $\bar{L}$ is softly broken. The mass matrix in Eq. (10) predicts $m_3 = 0$ together with $U_{e3} = 0$.
- The softly-broken D_4 model [16] has

$$\mathcal{M}_{\nu f}^{-1} = \begin{pmatrix} x & y & t \\ y & z & 0 \\ t & 0 & z \end{pmatrix}, \tag{11}$$

  together with the condition $\arg y^2 = \arg t^2$. In this case $\cos\gamma = (y^2 - t^2)/(y^2 + t^2)$ and $e^{i\alpha}\sin\gamma = 2yt/(y^2+t^2)$. The fact that the (μ,τ) matrix element of $\mathcal{M}_{\nu f}^{-1}$ is zero, and the fact that its (μ,μ) and (τ,τ) matrix elements remain equal, are just reflections of the limited particle content used to break the original D_4 symmetry softly.

Thus, the symmetry $Z_2(\gamma,\alpha)$ may be fundamental, effective, or accidental, depending on the specific model at hand.

Considering Eq. (9) more carefully one notices that the phase $\alpha = \chi_5 - \chi_3 + \pi$ is physically meaningless, since it can be removed through a rephasing of the charged-lepton fields. Let us then set $\alpha = 0$. In that case, the $\mathcal{M}_{\nu f}$ satisfying Eq. (6) can be written in the form

$$\mathcal{M}_{\nu f} = \begin{pmatrix} X & \sqrt{2}A\cos(\gamma/2) & \sqrt{2}A\sin(\gamma/2) \\ \sqrt{2}A\cos(\gamma/2) & B + C\cos\gamma & C\sin\gamma \\ \sqrt{2}A\sin(\gamma/2) & C\sin\gamma & B - C\cos\gamma \end{pmatrix}. \tag{12}$$

The eigenvalue corresponding to the eigenvector in Eq. (7) is $B - C$.

Specific choices of the parameters in Eq. (12) give different models. The model with $B = C = X = 0$ corresponds to $L_e - L_\mu - L_\tau$ symmetry [19]. The model with $\gamma = \pi/2$ corresponds to μ–τ interchange symmetry [13]. The D_4 model in [16] has $\tilde{X} = \tilde{A}^2/\tilde{D}$. Likewise, various models in [15] can be shown to have a $\mathcal{M}_{\nu f}$ which is formally identical to the matrix in Eq. (12).

In this paper we modify the standard parametrization for U by multiplying its third row by -1, i.e., we use

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ -s_{23}s_{12} + c_{23}s_{13}c_{12}e^{i\delta} & s_{23}c_{12} + c_{23}s_{13}s_{12}e^{i\delta} & -c_{23}c_{13} \end{pmatrix} \times \operatorname{diag}\left(e^{i\rho}, e^{i\sigma}, 1\right). \tag{13}$$

Then, if we let $U_{e3} = s_{13}e^{-i\delta} = 0$, Eq. (8) reduces to Eq. (12) with $\gamma/2 = \theta_{23}$ and

$$\begin{aligned}
X &= c_{12}^2 m_1 e^{-2i\rho} + s_{12}^2 m_2 e^{-2i\sigma}, \\
A &= -\frac{c_{12}s_{12}}{\sqrt{2}}\left(m_1 e^{-2i\rho} - m_2 e^{-2i\sigma}\right), \\
B &= \frac{1}{2}\left(s_{12}^2 m_1 e^{-2i\rho} + c_{12}^2 m_2 e^{-2i\sigma} + m_3\right), \\
C &= \frac{1}{2}\left(s_{12}^2 m_1 e^{-2i\rho} + c_{12}^2 m_2 e^{-2i\sigma} - m_3\right).
\end{aligned} \tag{14}$$

3. Non-zero U_{e3}, $\cos 2\theta_{23}$ from Z_2 breaking

Models with $U_{e3} = 0$ can be divided in two different categories:

- Those in which the solar scale also vanishes, along with U_{e3}. These are obtained by setting $m_1 = m_2$ in Eq. (14). In these models, the perturbation which generates the solar scale can be expected to also generate U_{e3}, and one may find [18,21] correlations between them.
- Models in which the solar scale is present already at the zeroth order. These are represented by Eq. (12) without additional restrictions on its parameters, except possibly $\gamma = \pi/4$.

We consider here the more general second category, but fix $\gamma = \pi/4$, i.e., we consider models with vanishing U_{e3} and $\cos 2\theta_{23}$. $\mathcal{M}_{\nu f}$ can be explicitly written in this case as

$$\mathcal{M}_{\nu f} = U_0^* \operatorname{diag}(m_1, m_2, m_3) U_0^\dagger, \tag{15}$$

where U_0 is obtained from Eq. (13) by setting $s_{13} = 0$ and $\theta_{23} = \pi/4$. One then has

$$\mathcal{M}_{\nu f} = \begin{pmatrix} X & A & A \\ A & B & C \\ A & C & B \end{pmatrix}. \tag{16}$$

Consider a general perturbation $\delta\mathcal{M}_{\nu f}$ to Eq. (16). The matrix $\delta\mathcal{M}_{\nu f}$ is a general complex symmetric matrix, but part of it can be absorbed through a redefinition of the parameters in Eq. (16). The remaining part can be written, without loss of generality, as

$$\delta\mathcal{M}_{\nu f} = \begin{pmatrix} 0 & \epsilon_1 & -\epsilon_1 \\ \epsilon_1 & \epsilon_2 & 0 \\ -\epsilon_1 & 0 & -\epsilon_2 \end{pmatrix}. \tag{17}$$

The perturbation is controlled by two parameters, ϵ_1 and ϵ_2, which are complex and model-dependent. We want to study their effects perturbatively, i.e., we want to assume ϵ_1 and ϵ_2 to be small. This smallness can be quantified by saying either that they are smaller than the largest element in $\mathcal{M}_{\nu f}$, or that the perturbation to a given matrix element of $\mathcal{M}_{\nu f}$ is smaller than the element itself. We adopt the latter alternative and define two dimensionless parameters:

$$\epsilon_1 \equiv \epsilon A, \qquad \epsilon_2 \equiv \epsilon' B. \tag{18}$$

Thus, we have the neutrino mass matrix with Z_2 breaking as follows:

$$\mathcal{M}_{\nu f} = \begin{pmatrix} X & A(1+\epsilon) & A(1-\epsilon) \\ A(1+\epsilon) & B(1+\epsilon') & C \\ A(1-\epsilon) & C & B(1-\epsilon') \end{pmatrix}, \tag{19}$$

where we shall assume ϵ and ϵ' to be small, $|\epsilon|, |\epsilon'| \ll 1$.

One finds that, to first order in ϵ and ϵ', the only effect of the $\delta\mathcal{M}_{\nu f}$ in Eq. (17) is to generate non-zero U_{e3} and $\cos 2\theta_{23}$. The neutrino masses, as well as the solar angle, do not receive any corrections. U_{e3} and $\cos 2\theta_{23}$ are of the same order as ϵ and ϵ'. Define

$$\hat{m}_1 \equiv m_1 e^{-2i\rho}, \tag{20}$$

$$\hat{m}_2 \equiv m_2 e^{-2i\sigma}, \tag{21}$$

and

$$\bar{\epsilon} \equiv (\hat{m}_1 - \hat{m}_2)\epsilon, \tag{22}$$

$$\bar{\epsilon}' \equiv \frac{\hat{m}_1 s_{12}^2 + \hat{m}_2 c_{12}^2 + m_3}{2}\epsilon'. \tag{23}$$

Then, we get

$$\begin{aligned} U_{e3} = {} & \frac{s_{12}c_{12}}{m_3^2 - m_2^2}\left(\bar{\epsilon} s_{12}^2 \hat{m}_2^* + \bar{\epsilon}^* s_{12}^2 m_3 - \bar{\epsilon}' \hat{m}_2^* - \bar{\epsilon}'^* m_3\right) \\ & + \frac{s_{12}c_{12}}{m_3^2 - m_1^2}\left(\bar{\epsilon} c_{12}^2 \hat{m}_1^* + \bar{\epsilon}^* c_{12}^2 m_3 + \bar{\epsilon}' \hat{m}_1^* + \bar{\epsilon}'^* m_3\right), \end{aligned} \tag{24}$$

$$\begin{aligned} \cos 2\theta_{23} = {} & \mathrm{Re}\left\{ \frac{2c_{12}^2}{m_3^2 - m_2^2}\left(\bar{\epsilon} s_{12}^2 - \bar{\epsilon}'\right)(\hat{m}_2 + m_3)^* \right. \\ & \left. - \frac{2s_{12}^2}{m_3^2 - m_1^2}\left(\bar{\epsilon} c_{12}^2 + \bar{\epsilon}'\right)(\hat{m}_1 + m_3)^* \right\}. \end{aligned} \tag{25}$$

The meaningful phases in $\mathcal{M}_{\nu f}$ are the ones of rephasing-invariant quartets. Since $\mathcal{M}_{\nu f}$ is symmetric, there are three such phases which are linearly independent. (Correspondingly, there are three physical phases in the MNS matrix: δ, 2ρ, and 2σ.) One easily sees that, in the first-order approximation in ϵ and ϵ', the imaginary parts of those two small parameters are meaningless when taken separately; only $\mathrm{Im}(2\epsilon - \epsilon')$ is physically meaningful to this order. Indeed, one can manipulate Eqs. (24) and (25) to obtain

$$\begin{aligned} \cos 2\theta_{23} = {} & \left\{ \frac{c_{12}^2}{m_2^2 - m_3^2}\left[m_3^2 + c_{12}^2 m_2^2 + s_{12}^2\,\mathrm{Re}(\hat{m}_1 \hat{m}_2^* + \hat{m}_1 m_3) \right.\right. \\ & \left. + \left(1 + c_{12}^2\right)\mathrm{Re}(\hat{m}_2 m_3)\right] \\ & + \frac{s_{12}^2}{m_1^2 - m_3^2}\left[m_3^2 + s_{12}^2 m_1^2 + c_{12}^2\,\mathrm{Re}(\hat{m}_2 \hat{m}_1^* + \hat{m}_2 m_3) \right. \end{aligned}$$

$$
\begin{aligned}
&+\left(1+s_{12}^2\right)\mathrm{Re}(\hat m_1 m_3)\Big]\bigg\}\,\mathrm{Re}\,\epsilon' \\
&+2c_{12}^2 s_{12}^2\left[\frac{m_2^2-\mathrm{Re}(\hat m_1\hat m_2^*+\hat m_1 m_3-\hat m_2 m_3)}{m_2^2-m_3^2}\right. \\
&\left.+\frac{m_1^2-\mathrm{Re}(\hat m_1\hat m_2^*-\hat m_1 m_3+\hat m_2 m_3)}{m_1^2-m_3^2}\right]\mathrm{Re}\,\epsilon \\
&+\frac{c_{12}^2 s_{12}^2(m_1^2-m_2^2)\,\mathrm{Im}(\hat m_1\hat m_2^*+\hat m_1 m_3+\hat m_2^* m_3)}{(m_3^2-m_1^2)(m_3^2-m_2^2)}\,\mathrm{Im}(2\epsilon-\epsilon'),
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\frac{U_{e3}}{c_{12}s_{12}} = \frac{1}{2}\bigg\{&\frac{1}{m_2^2-m_3^2}\left[s_{12}^2\hat m_1\hat m_2^*+s_{12}^2\hat m_1^* m_3+\left(1+c_{12}^2\right)\hat m_2^* m_3+m_3^2+c_{12}^2 m_2^2\right] \\
&+\frac{1}{m_3^2-m_1^2}\left[c_{12}^2\hat m_1^*\hat m_2+c_{12}^2\hat m_2^* m_3+\left(1+s_{12}^2\right)\hat m_1^* m_3+m_3^2+s_{12}^2 m_1^2\right]\bigg\}\,\mathrm{Re}\,\epsilon' \\
&+\left[\frac{s_{12}^2}{m_2^2-m_3^2}\left(m_2^2-\hat m_1\hat m_2^*-\hat m_1^* m_3+\hat m_2^* m_3\right)\right. \\
&\left.+\frac{c_{12}^2}{m_3^2-m_1^2}\left(m_1^2-\hat m_1^*\hat m_2+\hat m_1^* m_3-\hat m_2^* m_3\right)\right]\mathrm{Re}\,\epsilon \\
&+\frac{i}{2}\left[\frac{s_{12}^2}{m_2^2-m_3^2}\left(m_2^2-\hat m_1\hat m_2^*+\hat m_1^* m_3-\hat m_2^* m_3\right)\right. \\
&\left.+\frac{c_{12}^2}{m_3^2-m_1^2}\left(m_1^2-\hat m_1^*\hat m_2-\hat m_1^* m_3+\hat m_2^* m_3\right)\right]\mathrm{Im}(2\epsilon-\epsilon').
\end{aligned}
\tag{27}
$$

The induced values of $|U_{e3}|$ and $|\cos 2\theta_{23}|$ are strongly correlated to neutrino mass hierarchies. This makes it possible to draw some general conclusions even if we do not know the magnitudes of ϵ, ϵ'. In Table 1 we give expressions and values for $|U_{e3}|$ and $|\cos 2\theta_{23}|$ in case of the hierarchical ($m_1 < m_2 < m_3$), inverted ($m_1 \approx m_2 \sim \sqrt{\Delta_{\mathrm{atm}}} \gg m_3$) and quasi-degenerate neutrino spectrum. CP conservation is assumed but we distinguish two different cases (a) the Dirac solar pair corresponding to $\sigma = \rho = 0$ and the pseudo-Dirac solar pair with[1] $\rho = \pi/2$, $\sigma = 0$. We have also given approximate values in some cases assuming the common degenerate mass $m \sim 0.3$ eV.

It follows from the Table 1 and Eqs. (26), (27) that:

- The first-order contribution to U_{e3} given in Eq. (27) vanish identically if $\hat m_1 = \hat m_2$. As a consequence of this, U_{e3} gets suppressed by a factor $\mathcal{O}(\frac{\Delta_{\mathrm{sun}}}{\Delta_{\mathrm{atm}}})$ for the inverted or quasi-degenerate spectrum with $\rho = \sigma = 0$. Similar suppression also occurs in case of the normal neutrino mass hierarchy even when $\rho \neq \sigma$. U_{e3} need not be suppressed in other cases and can be large.

[1] The physically different case with $\rho = 0$, $\sigma = \pi/2$ has similar results.

Table 1
Leading order predictions for $|U_{e3}|$, $|\cos 2\theta_{23}|$ in case of different neutrino mass hierarchies with CP conservation. The numerical estimates are based on the best fit values of neutrino parameters and the quasi-degenerate mass $m = 0.3$ eV

Normal hierarchy	$\lvert U_{e3}\rvert \approx c_{12}s_{12}\sqrt{\frac{\Delta_{\rm sun}}{\Delta_{\rm atm}}}(\epsilon + \frac{\epsilon'}{2}) \approx 0.09(\epsilon + \frac{\epsilon'}{2})$
$m_1 \ll m_2$; $m_2^2 \approx \Delta_{\rm sun}$; $m_3^2 \approx \Delta_{\rm atm}$	$\lvert\cos 2\theta_{23}\rvert \approx \epsilon'$
Inverted hierarchy	$\lvert U_{e3}\rvert \approx \frac{\Delta_{\rm sun}}{2\Delta_{\rm atm}} s_{12}c_{12}(\epsilon - \frac{\epsilon'}{2}) \approx 0.009(\epsilon - \frac{\epsilon'}{2})$
$\sigma = 0$; $\rho = 0$	$\lvert\cos 2\theta_{23}\rvert \approx \epsilon'$
$\sigma = 0$; $\rho = \pi/2$	$\lvert U_{e3}\rvert \approx \frac{1}{2}\sin 4\theta_{12}(\epsilon - \frac{\epsilon'}{2}) \approx 0.4(\epsilon - \frac{\epsilon'}{2})$
	$\lvert\cos 2\theta_{23}\rvert \approx 2(\epsilon \sin^2 2\theta_{12} + \frac{\epsilon'}{2}\cos^2 2\theta_{12})$
Quasi-degenerate	$\lvert U_{e3}\rvert \approx 2\epsilon' c_{12}s_{12}\frac{m^2}{\Delta_{\rm atm}}\frac{\Delta_{\rm sun}}{\Delta_{\rm atm}} \approx 1.6\epsilon$
$\sigma = 0$; $\rho = 0$	$\lvert\cos 2\theta_{23}\rvert \approx 4\frac{m^2}{\Delta_{\rm atm}}\epsilon' \approx 180\epsilon'$
$\sigma = 0$; $\rho = \pi/2$	$\lvert U_{e3}\rvert \approx 4\frac{m^2}{\Delta_{\rm atm}}c_{12}s_{12}(\epsilon s_{12}^2 + \frac{\epsilon'}{2}c_{12}^2) \approx 81(\epsilon s_{12}^2 + \frac{\epsilon'}{2}c_{12}^2)$
	$\lvert\cos 2\theta_{23}\rvert \approx 8\frac{m^2}{\Delta_{\rm atm}}c_{12}^2(\epsilon s_{12}^2 + \frac{\epsilon'}{2}c_{12}^2) \approx 259(\epsilon s_{12}^2 + \frac{\epsilon'}{2}c_{12}^2)$

- In contrast to U_{e3}, $\cos 2\theta_{23}$ is almost as large as ϵ, ϵ' if neutrino mass spectrum is normal or inverted. It gets enhanced compared to these parameters if the spectrum is quasi-degenerate.
- In case of the quasi-degenerate spectrum, both $|\cos 2\theta_{23}|$ and $|U_{e3}|$ can become quite large and reach the present experimental limits. Especially, the enhancement factors are large in case of the pseudo-Dirac solar pair ($\rho = \pi/2$, $\sigma = 0$). U_{e3} and $\cos 2\theta_{23}$ are in fact proportional to each other in this particular case. The parameters ϵ, ϵ' are constrained to be lower than 10^{-2} for the quasi-degenerate spectrum.

The perturbative expressions given above may not be reliable for some values of ϵ, ϵ' due to large enhancement factor of $\mathcal{O}(\frac{m^2}{\Delta_{\rm atm}})$ and one should do a numerical analysis. We now discuss results of such analysis in various circumstances. Scattered plots of the predicted values for $|\cos 2\theta_{23}|$ and $|U_{e3}|$ are given in Fig. 1 in the case of normal neutrino mass hierarchy. CP conservation ($\rho = \sigma = 0$, real ϵ, ϵ') is assumed. Neutrino masses and θ_{12} do not receive any corrections at $O(\epsilon, \epsilon')$ and hence do not appreciably change by perturbations. We therefore randomly varied these input parameters in the experimentally allowed regions. m_1 was varied up to m_2. On the other hand, ϵ, ϵ' are unknown unless the symmetry breaking is specified, so these are varied randomly in the range −0.3–0.3 with the condition that the output parameters should lie in the 90% C.L. limit [2,7]:

$$\begin{aligned} &0.33 \leqslant \tan^2\theta_{\rm sun} \leqslant 0.49, \qquad 7.7\times 10^{-5} \leqslant \Delta_{\rm sun} \leqslant 8.8\times 10^{-5}\ {\rm eV}^2, \quad 90\%\ {\rm C.L.},\\ &0.92 \leqslant \sin^2 2\theta_{\rm atm}, \qquad 1.5\times 10^{-3} \leqslant \Delta_{\rm atm} \leqslant 3.4\times 10^{-3}\ {\rm eV}^2, \quad 90\%\ {\rm C.L.} \end{aligned} \tag{28}$$

The $|U_{e3}|$ is forced to be small less than 0.025, in Fig. 1 as would be expected from the foregoing discussion. The value ~ 0.025 at the upper end arises from the (assumed) bound $|\epsilon|, |\epsilon'| \leqslant 0.3$. Since $|U_{e3}|$ is proportional to ϵ, ϵ', it increases if the bound on ϵ, ϵ'

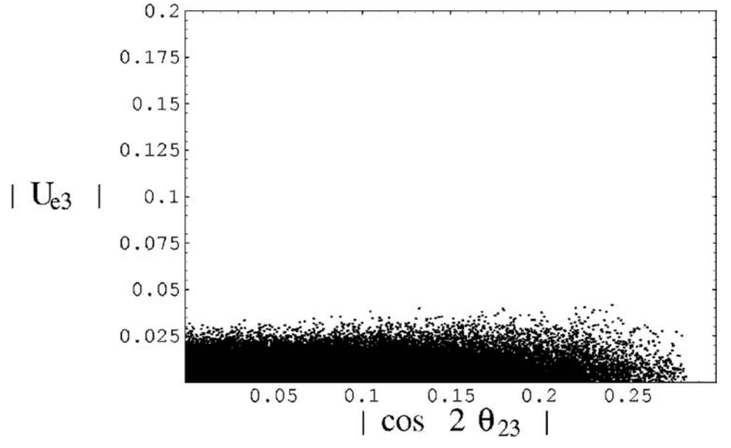

Fig. 1. The scattered plots showing the allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ in case of the normal neutrino mass hierarchy. ϵ, ϵ' are randomly varied in the range -0.3–0.3. The Majorana phases are chosen as $\rho = 0$, $\sigma = 0$.

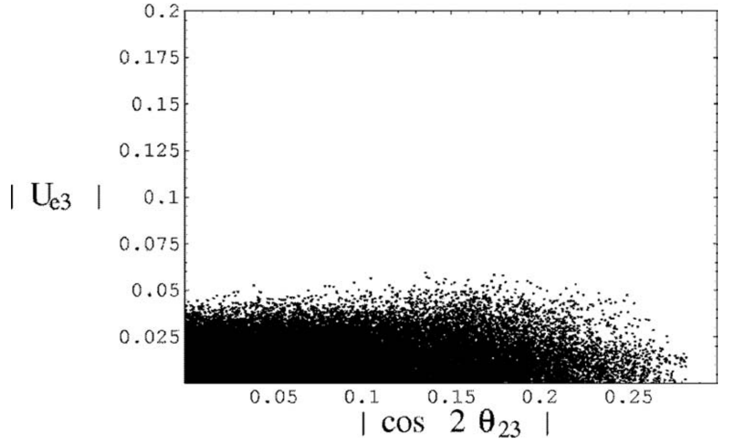

Fig. 2. The allowed values $|\cos 2\theta_{23}|$ and $|U_{e3}|$ for $\rho = \pi/4$, $\sigma = 0$ and the normal neutrino mass hierarchy. The other parameters are the same as in Fig. 1.

is loosened. However, $|\epsilon| \leqslant 0.3$ is a reasonable bound due to assume if Z_2 breaking is perturbative. On the other hand, $|\cos 2\theta_{23}|$ can assume large values as seen from Fig. 1. The present bound $\sin^2 2\theta_{23} > 0.92$ from the atmospheric experiments gets translated to $|\cos 2\theta_{23}| < 0.28$ which constrains $|\epsilon'| \leqslant 0.2$ in our analyses.

The non-maximal value for θ_{23} gives rise to interesting physical effects such as excess of the e-like events in the atmospheric neutrino data in the sub-GeV region [22], different matter dependent survival probabilities for the ν_μ and the $\bar{\nu}_\mu$ [23]. These can be searched for in the future atmospheric [24] and the long baseline experiments. The values $|\cos 2\theta_{23}| > 0.1$ are expected to be probed in these experiments [25]. These values occur quite naturally for a reasonably large range of parameters. In order to find the phase dependence of our

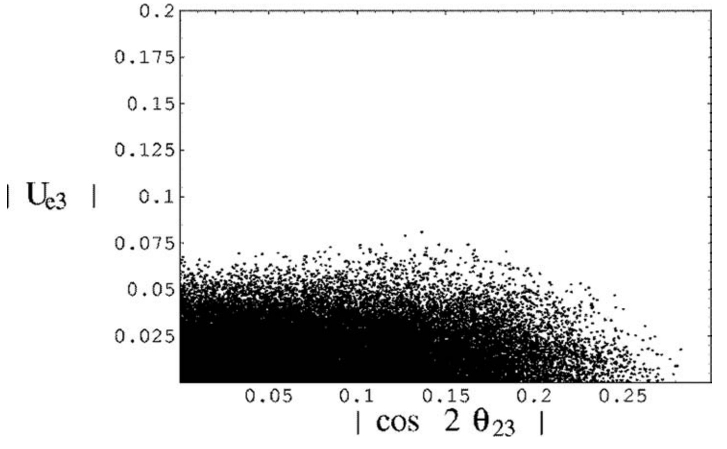

Fig. 3. The allowed values $|\cos 2\theta_{23}|$ and $|U_{e3}|$ for $\rho = \pi/2$, $\sigma = 0$ and the normal neutrino mass hierarchy. The other parameters are the same as in Fig. 1.

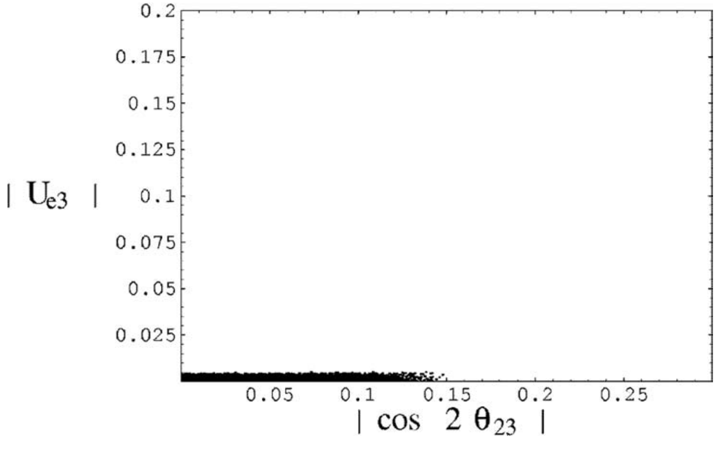

Fig. 4. The allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ for $\rho = 0$, $\sigma = 0$ in case of the inverted neutrino mass hierarchy. The ϵ, ϵ' are varied randomly in the range -0.3–0.3 while m_3 is varied up to 10^{-2} eV.

results, we show the results in the cases of ($\rho = \pi/4$, $\sigma = 0$) and ($\rho = \pi/2$, $\sigma = 0$). The phase dependence is found in the prediction of $|U_{e3}|$, which increases up to 0.075.

The region $|U_{e3}| > 0.07$ is expected to be probed in the long baseline experiments with the conventional or super beams [26] and in the reactor experiments [27]. The smaller values for $|U_{e3}| \sim 0.025$ can be reached only at the neutrino factory [28]. Most of the region displayed in Figs. 1–3 therefore seem inaccessible to the near future neutrino experiments aimed at searching for $|U_{e3}|$.

Scattered plots for the predicted values for $|U_{e3}|$ and $|\cos 2\theta_{23}|$ are given in Fig. 4 in case of the inverted hierarchy of the neutrino masses. The value of $|U_{e3}|$ is even more suppressed compared to the corresponding case displayed in Fig. 1. This suppression is due to the

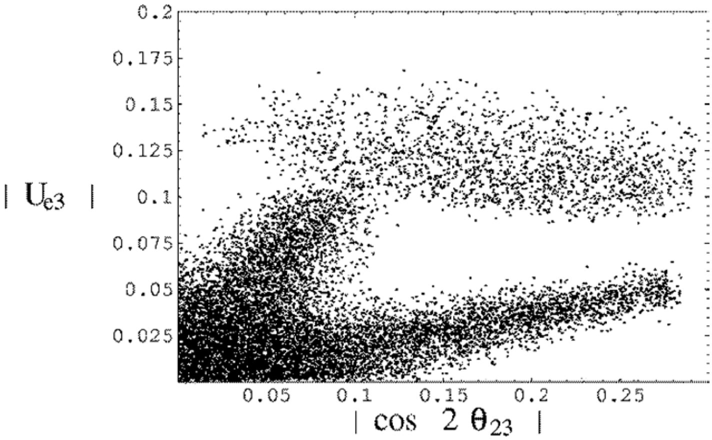

Fig. 5. The allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ for $\rho = \pi/4$, $\sigma = 0$ in case of the inverted neutrino mass hierarchy.

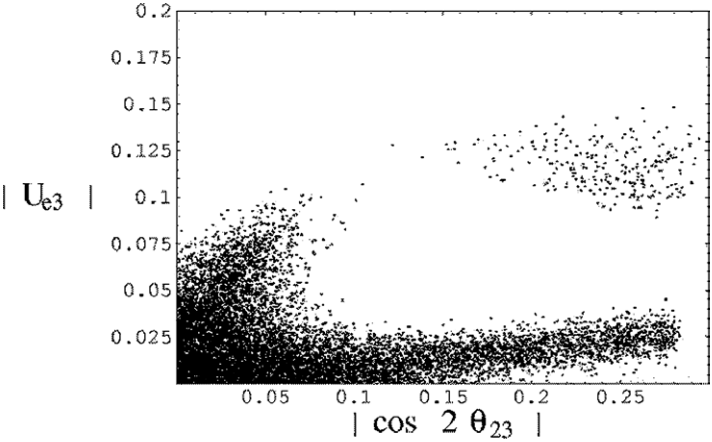

Fig. 6. The allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ for $\rho = \pi/2$, $\sigma = 0$ in case of the inverted neutrino mass hierarchy.

strong cancellation between m_1 and m_2, which is seen in Table 1. However, the Majorana phases spoil this cancellation, and so $|U_{e3}|$ could be larger as seen in Figs. 5 and 6, where the two cases ($\rho = \pi/4$, $\sigma = 0$) and ($\rho = \pi/2$, $\sigma = 0$) are displayed, respectively. Thus, the effect of the Majorana phases is very important in the inverted hierarchy. The isolated points in Fig. 6 follows from the tuning of the parameters ϵ and ϵ'. Apart from this tuning, the allowed values of $|U_{e3}|$ are moderate ~ 0.1 but will be explored in the future long baseline and reactor experiments.

The parameter ϵ' is constrained strongly $|\epsilon'| \leqslant 0.03$ in case of the quasi-degenerate neutrino masses due to an enhancement factor $\mathcal{O}(\frac{m^2}{\Delta_{\mathrm{atm}}})$ present in this case, as seen in Table 1. $|\epsilon|$ is however not constrained as strongly and we take $|\epsilon| \leqslant 0.3$. The scattered

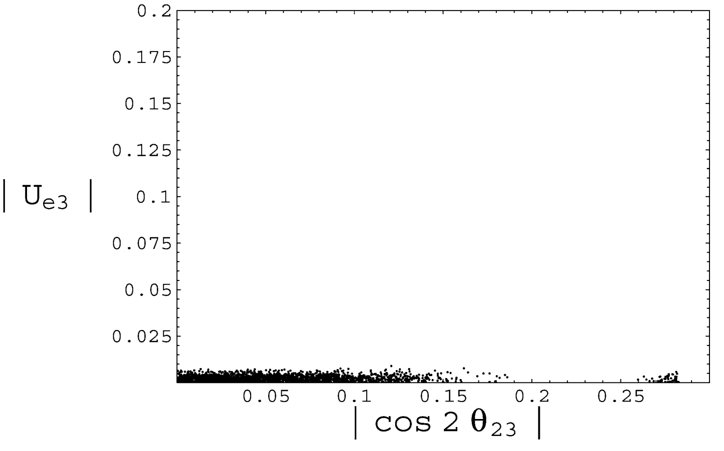

Fig. 7. The scattered plots of the allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ with $|\epsilon| \leqslant 0.3$ and $|\epsilon'| \leqslant 0.03$ and the quasi-degenerate neutrino masses. The Majorana phases are chosen as $\rho = 0$, $\sigma = 0$. The degenerate mass scale is fixed at $m = 0.3$ eV.

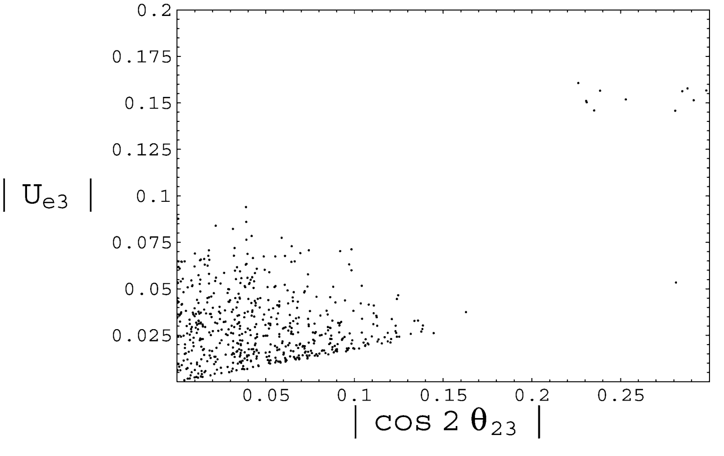

Fig. 8. The allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ in the quasi-degenerate neutrino masses. The Majorana phases are chosen as $\rho = \pi/4$, $\sigma = 0$. The degenerate mass scale is fixed at $m = 0.3$ eV.

plots for the predicted values for $|U_{e3}|$ and $|\cos 2\theta_{23}|$ are given in Fig. 7. The value of $|U_{e3}|$ is expected to be $O(0.01)$. There are partial cancellations among contributions from m_1, m_2, m_3 when $\rho = \sigma$. However, different choice for the Majorana phases spoil this cancellation and $|U_{e3}|$ could be large as seen in Figs. 8 and 9, which correspond to ($\rho = \pi/4$, $\sigma = 0$) and ($\rho = \pi/2$, $\sigma = 0$), respectively. It is found that $|U_{e3}|$ could increase to 0.1 in these cases.

In the above analyses, we fixed $\sigma = 0$ because only the relative phase $\rho - \sigma$ is essential in determining the masses and mixing angles in the case of the hierarchical and inverted hierarchical neutrino masses. However, σ dependence is non-trivial for the degenerate masses. We show the results for ($\rho = 0$, $\sigma = \pi/2$) and ($\rho = \pi/4$, $\sigma = \pi/2$) in

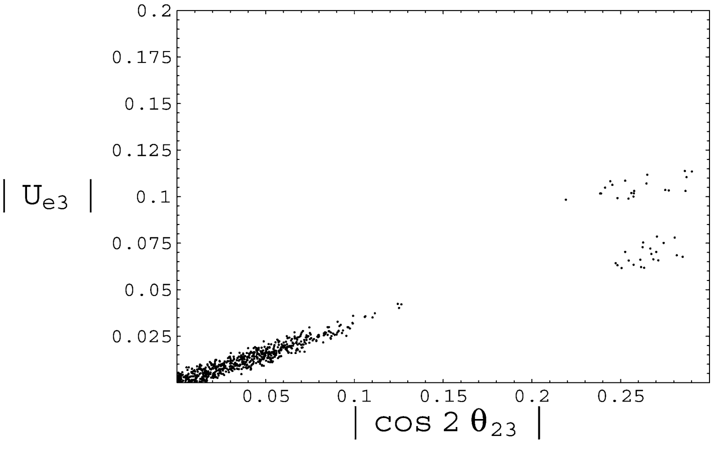

Fig. 9. The allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ in the quasi-degenerate neutrino masses. The Majorana phases are chosen as $\rho = \pi/2$, $\sigma = 0$. The degenerate mass scale is fixed at $m = 0.3$ eV.

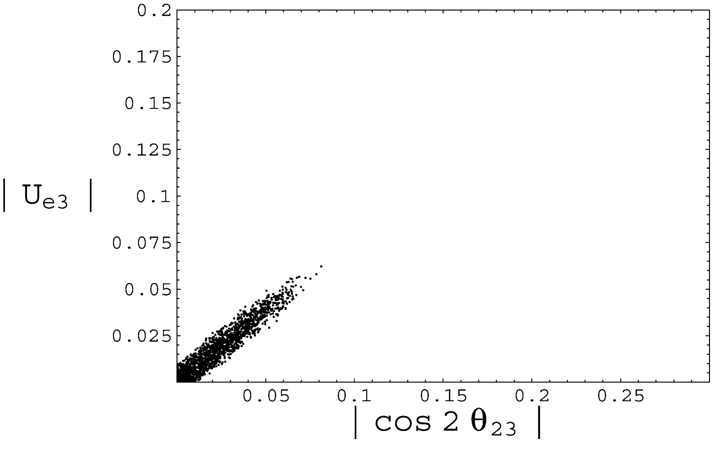

Fig. 10. The allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ in the quasi-degenerate neutrino masses. The Majorana phases are chosen as $\rho = 0$, $\sigma = \pi/2$. The degenerate mass scale is fixed at $m = 0.3$ eV.

Figs. 10 and 11 respectively. It is noted that $|U_{e3}|$ could be as large as 0.2 for the case $\rho = \pi/4$, $\sigma = \pi/2$ but values $\leqslant 0.1$ are more probable as seen from the density of points.

Before ending this section, we wish to point out an interesting aspect of this analysis. Since U_{e3} is zero in the absence of the perturbation, the CP-violating Dirac phase δ relevant for neutrino oscillations is undefined at this stage. CP violation is present through the Majorana phases ρ and σ. Turning on perturbation leads to non-zero U_{e3} and also to a non-zero Dirac phase even if perturbation is real. Moreover, δ generated this way can be large and independent of the strength of perturbation parameters. This phenomenon was noticed [29] in the specific case of the radiative generation of U_{e3}. This occurs here also for a more general perturbation.

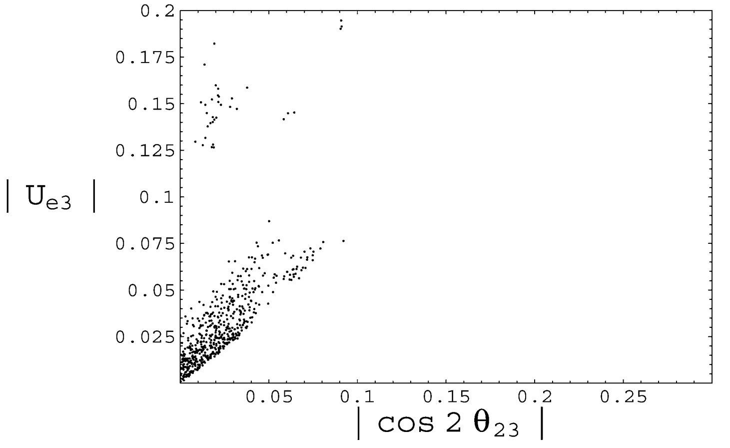

Fig. 11. The allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ in the quasi-degenerate neutrino masses. The Majorana phases are chosen as $\rho = \pi/4$, $\sigma = \pi/2$. The degenerate mass scale is fixed at $m = 0.3$ eV.

As an example, let us consider the limit $\epsilon' = 0$ and a real ϵ. Since $U_{\mu 3}$ is almost maximal and real, δ is approximately given by

$$\tan\delta \approx \frac{m_1 m_2 \sin 2(\rho-\sigma) - m_3 m_1 \sin 2\rho + m_2 m_3 \sin 2\sigma + \mathcal{O}(\frac{\Delta_{\rm sun}}{\Delta_{\rm atm}})\,\mathrm{Im}(\mathcal{Z})}{m_1^2 c_{12}^2 - m_2^2 s_{12}^2 - m_1 m_2 \cos 2\theta_{12} \cos 2(\rho-\sigma) + m_3 m_1 \cos 2\rho - m_2 m_3 \cos 2\sigma + \mathcal{O}(\frac{\Delta_{\rm sun}}{\Delta_{\rm atm}})\,\mathrm{Re}(\mathcal{Z})}, \tag{29}$$

where $\mathcal{Z} \equiv (\hat{m}_2^*(\hat{m}_1 - \hat{m}_2) + m_3(\hat{m}_1 - \hat{m}_2)^*) s_{12}^2$. It follows from above that irrespective of the specific mass hierarchy, the induced δ would be large if ρ and σ are large and not finetuned.

4. Radiatively generated U_{e3} and $\cos 2\theta_{23}$

The ϵ, ϵ' were treated as independent parameters so far. They can be related in specific models. We now consider one example which is based on the electroweak breaking of the Z_2 symmetry in the MSSM. We assume that neutrino masses are generated at some high scale M_X and the effective neutrino mass operator describing them is Z_2 symmetric with the result that $U_{e3} = \cos 2\theta_{23} = 0$ at M_X. This symmetry is assumed to be broken spontaneously in the Yukawa couplings of the charged leptons. This breaking would radiatively induce non-zero U_{e3} and $\cos 2\theta_{23}$ [30]. This can be calculated by using the renormalization group equations (RGEs) of the effective neutrino mass operator [31–33]. These equations depend upon the detailed structure of the model below M_X. We assume here that theory below M_X is the MSSM and use the RGEs derived in this case. Subsequently we will give an example which realizes our assumptions.

Integration of the RGEs allows us [31–33] to relate the neutrino mass matrix $\mathcal{M}_{\nu f}(M_X)$ to the corresponding matrix at the low scale which we identify here with the Z mass M_Z:

$$\mathcal{M}_{\nu f}(M_Z) \approx I_g I_t \big(I \mathcal{M}_{\nu f}(M_X) I\big), \tag{30}$$

where $I_{g,t}$ are calculable numbers depending on the gauge and top quark Yukawa couplings. I is a flavour dependent matrix given by

$$I \approx \mathrm{diag}(1+\delta_e, 1+\delta_\mu, 1+\delta_\tau) \tag{31}$$

with

$$\delta_\alpha \approx c\left(\frac{m_\alpha}{4\pi v}\right)^2 \ln\frac{M_X}{M_Z}, \tag{32}$$

where $c = \frac{3}{2}, -\frac{1}{\cos^2\beta}$ in case of the Standard Model (SM) and the Minimal Supersymmetric Standard Model (MSSM) respectively [31]. v refers to the vacuum expectation value for the SM Higgs doublet.

We have implicitly neglected possible threshold effects. Inclusion of these effects would not modify the analysis if threshold effects are flavour blind as would be approximately true [34] in case of the minimal supergravity scenario with universal boundary conditions.

$\mathcal{M}_{\nu f}(M_X)$ is given by Eq. (16). From this we can write $\mathcal{M}_{\nu f}(M_Z)$ as follows when the muon and the electron Yukawa couplings are neglected:

$$\mathcal{M}_{\nu f}(M_Z) = \begin{pmatrix} X & A' & A' \\ A' & B' & C' \\ A' & C' & B' \end{pmatrix} + \begin{pmatrix} 0 & A'\epsilon & -A'\epsilon \\ A'\epsilon & B'\epsilon' & 0 \\ -A'\epsilon & 0 & -B'\epsilon' \end{pmatrix} + O\left(\delta_\tau^2\right), \tag{33}$$

where

$$C' = C(1+\delta_\tau), \qquad A' = A\left(1+\frac{\delta_\tau}{2}\right), \qquad B' = B(1+\delta_\tau), \qquad \epsilon = \frac{\epsilon'}{2} = -\frac{\delta_\tau}{2} \tag{34}$$

and A, B, C are defined in Eq. (14). Note that m_1, m_2 and m_3 defined previously are no longer mass eigenvalues because of the changes $A \to A'$, $B \to B'$ and $C \to C'$. Using the above equations, we get from Eq. (24)

$$\begin{aligned} U_{e3} &\simeq -\frac{\delta_\tau s_{12} c_{12}}{2(m_3^2 - m_1^2)}\left[m_1^2 + 2m_3\hat{m}_1^* + m_3^2\right] + \frac{\delta_\tau s_{12} c_{12}}{2m_3^2 - m_2^2}\left[m_2^2 + 2\hat{m}_2^* m_3 + m_3^2\right], \\ \cos 2\theta_{23} &\simeq \frac{\delta_\tau s_{12}^2}{m_3^2 - m_1^2}\left[m_1^2 + 2m_3\hat{m}_1^* + m_3^2\right] + \frac{\delta_\tau c_{12}^2}{m_3^2 - m_2^2}\left[m_2^2 + 2\hat{m}_2^* m_3 + m_3^2\right]. \end{aligned} \tag{35}$$

It is easily seen that the effect of the radiative corrections is enhanced in the case of the quasi-degenerate neutrino masses with opposite phase $|\rho - \sigma| = \pi/2$ as previous works presented [32,33]. In the MSSM, the parameter δ_τ is negative and its absolute value can become quite large for large $\tan\beta$, e.g., for $\tan\beta \sim 50$, $|\delta_\tau| \sim 0.075$. However, large $\tan\beta$ is not favoured because the renormalization of parameters A, B, C as in Eq. (34) also shifts the value of the solar angle and solar mass compared to their values in the $\delta_\tau \to 0$ limit. One now gets

$$\Delta_{\mathrm{sun}} \cos 2\theta_{\mathrm{sun}} \approx \Delta_{21} \cos 2\theta_{12} + 2\delta_\tau \left|m_1 e^{-2i\rho} s_{12}^2 + m_2 e^{-2i\sigma} c_{12}^2\right|^2. \tag{36}$$

Here, $\Delta_{21} \equiv m_2^2 - m_1^2$ and θ_{12} correspond to the values of the solar scale and angle at M_X. The radiative corrections add a negative contribution to $\Delta_{\mathrm{sun}} \cos 2\theta_{\mathrm{sun}}$ in case of the MSSM

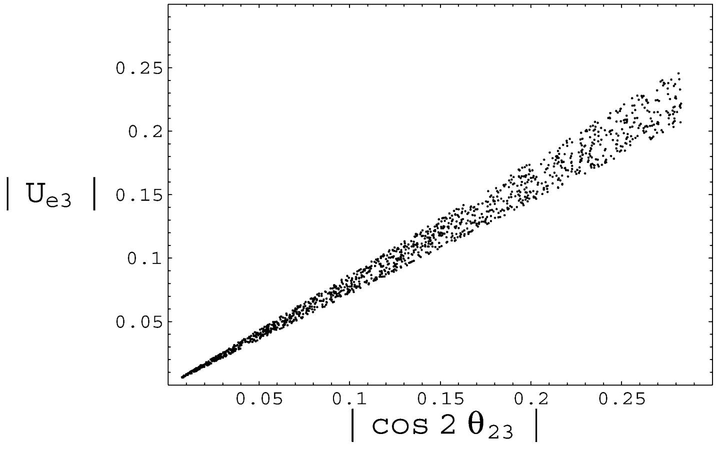

Fig. 12. The scattered plots of the allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ in case of the radiatively broken Z_2 and the quasi-degenerate neutrino masses $m = 0.3$ eV. The Majorana phases are chosen as $\rho = 0$, $\sigma = \pi/2$.

and can spoil the LMA solution (which need positive $\Delta_{\text{sun}} \cos 2\theta_{\text{sun}}$) if Δ_{21} is small or $|\delta_\tau|$ is large. This provides a constraint on possible values of δ_τ and consequently on $|U_{e3}|$, $|\cos 2\theta_{23}|$ that can be generated in the model. For example, requiring that the first term dominates over the second term in Eq. (36) implies

$$|\delta_\tau| \leqslant \left(\frac{\Delta_{21}}{2m^2 \cos 2\theta_{12}}\right) \approx 10^{-3}, \tag{37}$$

where we assumed CP conservation, the quasi-degenerate spectrum, $\sigma = \pi/2$; $\rho = 0$, $m \approx 0.3$ eV, and $\Delta_{21} \sim 8 \times 10^{-5}$ eV^2. The values for $|U_{e3}|$ and $|\cos 2\theta_{23}|$ implied by the above constraint are quite small. Notice however that one can loosen the bound on δ_τ by choosing significantly larger value Δ_{21} than 8×10^{-5} eV^2. The cancellations between two terms in Eq. (36) can still lead to physical solar scale.

Results of the numerical analysis are shown in Fig. 12 in case of the quasi-degenerate spectrum with $m = 0.3$ eV; $\sigma = \pi/2$, $\rho = 0$. The θ_{12}, Δ_{21} and $\tan\beta$ at high scale are varied randomly, then the allowed choices which reproduce the parameters as in Eq. (28) at the low scale are determined. Both $|U_{e3}|$ and $|\cos 2\theta_{23}|$ can reach their respective experimental bound. The near proportionality between the two can be understood from their expressions given in Table 1. We find numerically that $\tan\beta$ is constrained to be lower than 20 in this case. The forthcoming experiments will be able to test this relationship between $|U_{e3}|$ and $|\cos 2\theta_{23}|$. It may be useful to note our numerical results of $|U_{e3}|$ in the cases of the normal-hierarchy and inverted-one of the neutrino masses. In both cases, $|U_{e3}|$ reaches at most 0.025. These results are consistent with one in Ref. [30].

Let us now give an example which realizes our assumptions. One needs a Z_2-invariant neutrino mass matrix and a charged lepton mass matrix which break it at the high scale. This breaking is required to be spontaneous. This can be done without invoking additional Higgs doublets provided one introduces several singlet fields. The model below is based on the MSSM augmented with two pairs of the standard model singlet fields denoted by (η_1, η_2) and $(\bar{\eta}_1, \bar{\eta}_2)$. We impose a discrete $Z_4 \times Z_4$ symmetry under which various superfields transform as follows:

$$\begin{aligned}&\left(\mu^c, \tau^c, \bar{\eta}_1, \eta_1^*\right) \sim (i, 1), \qquad (D_e, \eta_2, \bar{\eta}_2^*) \sim (1, i),\\ &e^c \sim (1, -1), \qquad D_- \sim (-1, 1),\end{aligned} \tag{38}$$

where $D_\alpha(\alpha^c)$ denote the leptonic doublets (singlets) with flavour $\alpha = e, \mu, \tau$; $D_\pm \equiv \frac{D_\tau \pm D_\mu}{\sqrt{2}}$. The D_+ and the standard Higgs superfields $H_{u,d}$ transform as singlets. We assume that the $Z_4 \times Z_4$ symmetry is broken by the vacuum expectation values of the η-fields at a scale only slightly lower than the neutrino mass scale M_X. As a result, non-renormalizable terms involving these fields can give sizable contributions to Yukawa couplings as in the Froggatt–Nielsen mechanism [35].

The following dimension 5 terms in the superpotential contribute to the charged lepton masses:

$$W_Y = D_+\left(\Gamma_\mu \mu^c + \Gamma_\tau \tau^c\right)\frac{H_d \eta_1}{M_X} + D_-\left(\Gamma'_\mu \mu^c + \Gamma'_\tau \tau^c\right)\frac{H_d \bar{\eta}_1}{M_X} + \Gamma_e D_e e^c \frac{H_d \eta_2}{M_X}. \tag{39}$$

The neutrino masses follow from the following non-renormalizable operators invariant under the $Z_4 \times Z_4$ symmetry:

$$\begin{aligned}W_\nu = {}&\frac{\alpha}{M_X}(D_+ H_u)^T (D_+ H_u) + \frac{\beta}{M_X}(D_- H_u)^T (D_- H_u)\\ &+ \frac{\gamma}{M_X}(D_+ H_u)^T (D_e H_u)\frac{\bar{\eta}_2}{M_X},\end{aligned} \tag{40}$$

where we have suppressed the Lorentz and $SU(2)$ indices. Eq. (39) leads to the charged lepton mass matrix

$$\mathcal{M}_l = \begin{pmatrix} a_e & 0 & 0 \\ 0 & a_\mu - a'_\mu & a_\tau - a'_\tau \\ 0 & a_\mu + a'_\mu & a_\tau + a'_\tau \end{pmatrix}, \tag{41}$$

where

$$\begin{aligned}&a_e = \Gamma_e \frac{\langle H_d^0\rangle\langle\eta_2\rangle}{M_X}, \qquad a_\alpha = \frac{\Gamma_\alpha}{\sqrt{2}}\frac{\langle H_d^0\rangle\langle\eta_1\rangle}{M_X},\\ &a'_\alpha = \frac{\Gamma'_\alpha}{\sqrt{2}}\frac{\langle H_d^0\rangle\langle\bar{\eta}_1\rangle}{M_X} \quad (\alpha = \mu, \tau).\end{aligned} \tag{42}$$

The neutrino mass matrix has the Z_2 invariant form of Eq. (16) but with $X = 0$. This together with the charged lepton mass matrix in Eq. (41) imply that the $U_{e3} = 0$ at the tree level. In the limit $a_\alpha = a'_\alpha$, Eq. (41) leads to a massless muon and also corrections to θ_{23} from the charged leptons vanish. In this limit, the model is equivalent to the Z_2 model with $\gamma = \pi/4$. The imposition of equality $a_\alpha = a'_\alpha$ is technically natural in the context of supersymmetric theory. Small departure from it would lead to the muon mass and a contribution $\theta_{23l} \approx \mathcal{O}(\frac{m_\mu}{m_\tau})$ from the diagonalization of the charged lepton matrix to θ_{23}. In this case one gets the more general model represented by Eq. (12). U_{e3} still remains zero at M_X.

The model discussed above reduces to the MSSM below the $Z_4 \times Z_4$ breaking scale. $\mathcal{M}_{\nu f}$ in this case is invariant under a Z_2 symmetry which interchanges D_μ with D_τ. This Z_2 however is not a symmetry of the charged lepton Yukawa couplings, Eq. (39). Even in the neutrino sector, the Z_2 invariance is only approximate one and is broken by the terms of $\mathcal{O}(\langle\eta\rangle^2/M_X^2)$ where $\langle\eta\rangle$ generically denotes the vacuum expectation value for any of the singlet fields. The parameter $\lambda \sim \langle\eta\rangle/M_X$ determines the tau lepton mass in Eq. (39) and is required to be $\geqslant \mathcal{O}(10^{-2})$ if the Yukawa couplings Γ_α are to remain below 1. This means that the neglected non-leading terms in Eqs. (39), (40) are typically $\mathcal{O}(10^{-2})$ smaller than the leading ones.

The breakdown of the Z_2 symmetry and a non-zero U_{e3} arise in the model from the non-leading terms not displayed in Eqs. (39), (40). The charged lepton mass matrix gets additional contributions from the following $Z_4 \times Z_4$ invariant dimension six terms in the super potential:

$$D_e\left(\beta_{e\mu}\mu^c + \beta_{e\tau}\tau^c\right)H_d\frac{\eta_1\bar{\eta}_2}{M_X^2} + D_+ e^c H_d\frac{\beta_e\eta_2^2 + \bar{\beta}_e\bar{\eta}_2^2}{M_X^2}. \tag{43}$$

The corrected charged lepton mass matrix then has the following form

$$\mathcal{M}_l = \begin{pmatrix} a_e & \lambda_{e\mu} & \lambda_{e\tau} \\ \lambda_e & a_\mu - a'_\mu & a_\tau - a'_\tau \\ \lambda_e & a_\mu + a'_\mu & a_\tau + a'_\tau \end{pmatrix}. \tag{44}$$

Here, $\lambda_{e,e\mu,e\tau}$ can be read-off from Eq. (43). These are suppressed compared to the leading terms in Eq. (39) by $\lambda = \frac{\langle\eta\rangle}{M_X}$ where $\langle\eta\rangle$ refers to a typical vacuum expectation of any of the singlet fields. An estimate of λ can be obtained by noting that it determines the tau lepton mass in Eq. (39) and is required to be $\geqslant \mathcal{O}(10^{-2})$ if the Yukawa couplings Γ_α are to remain below 1. This means that the terms $\lambda_{e\alpha}$, λ_e in Eq. (44) can be $\mathcal{O}(m_\mu)$ if the relevant Yukawa couplings are $\mathcal{O}(1)$. They can therefore significantly affect the e–μ sector and would lead to a large electron mass and e–μ mixing. This requires assuming suppression in some of the Yukawa couplings. While different choices are possible, we give an example which is particularly interesting. This corresponds to choosing $a_e \ll m_e$; $a_\alpha = a'_\alpha \approx \mathcal{O}(m_\tau)$; $\lambda_{e\tau} \sim \lambda_{e\mu} \sim \mathcal{O}(m_e)$ and $\lambda_e \sim \mathcal{O}(m_\mu)$. The $\lambda_{e\alpha}$ contribute to the electron mass and the corresponding Yukawa couplings $\beta_{e\alpha}$ need to be suppressed $\beta_{e\alpha} \sim \mathcal{O}(\frac{m_e}{m_\mu})$. One gets correct pattern for the charged lepton masses and a contribution of $\mathcal{O}(\frac{m_e}{m_\tau})$ to U_{e3} from the charged lepton sector. The radiatively induced U_{e3} can be larger than this as seen from Fig. 12.

The non-leading terms break Z_2 in the neutrino sector also and lead to a direct contribution to U_{e3}. This comes from the terms of the type

$$\frac{(\eta_1^2, \bar{\eta}_1^2)}{M_X^3}(D_+H_u)^T D_- H_u, \qquad \frac{\bar{\eta}_2}{M_X^4}\left(\eta_1^2, \bar{\eta}_1^2\right)(D_-H_u)^T D_e H_u,$$
$$\frac{(\eta_2^2, \bar{\eta}_2^2)}{M_X^3}(D_eH_u)^T D_e H_u. \tag{45}$$

These terms are typically suppressed by $\mathcal{O}(10^{-2})$ compared to the corresponding leading terms displayed in Eq. (40).

5. Conclusions

The neutrino mixing matrix contains two small parameters $|U_{e3}|$ and $\cos 2\theta_{23}$ which would influence the outcome of the future neutrino experiments. This paper was devoted to study of these parameters within a specific theoretical framework. The vanishing of $|U_{e3}|$ was shown to follow from a class of Z_2 symmetries of $\mathcal{M}_{\nu f}$. This symmetry can be used to parameterize all models with zero U_{e3}. A specific Z_2 in this class also leads to the maximal atmospheric neutrino mixing angle. We showed that breaking of this can be characterized by two dimensionless parameters ϵ, ϵ' and we studied their effects perturbatively and numerically.

It was found that the magnitudes of $|U_{e3}|$ and $|\cos 2\theta_{23}|$ are strongly dependent upon the neutrino mass hierarchies and CP-violating phases. The $|U_{e3}|$ gets strongly suppressed in case of the inverted or quasi-degenerate neutrino spectrum if $\rho = \sigma$ while similar suppression occurs in the case of normal hierarchy independent of this phase choice. The choice $\rho \neq \sigma$ can lead to a larger values ~ 0.1 for $|U_{e3}|$ which could be close to the experimental value in some cases with inverted or quasi-degenerate spectrum. In contrast, the $|\cos 2\theta_{23}|$ could be large, near its present experimental limit in most cases studied. For the normal and inverted mass spectrum, the magnitude of $\cos 2\theta_{23}$ is similar to the magnitudes of the perturbations ϵ, ϵ' while it can get enhanced compared to them if the neutrino spectrum is quasi-degenerate.

The phenomenological implications of the present scheme are distinct from various other schemes discussed in the literature [8–11,18,21]. Ref. [21] considered various neutrino mass textures which lead to zero solar scale, $U_{e3} = 0$ and $\cos 2\theta_{23} = 0$, and applied random perturbations to them. In this approach, both $|U_{e3}|$ and $|\cos 2\theta_{23}|$ were found to be similar in contrast to the present case which predicts $|U_{e3}| \leqslant |\cos 2\theta_{23}|$. The approach of [21] predicts large $|\cos 2\theta_{23}|$ of $\mathcal{O}(\sqrt{\Delta_{\text{sun}}/\Delta_{\text{atm}}})$ for the normal neutrino mass hierarchy and small $\mathcal{O}(\Delta_{\text{sun}}/\Delta_{\text{atm}})$ in the other cases. This is quite different from our results as seen in Table 1.

An alternative proposal is to make assumptions on the leptonic mixing matrices $U_{\nu,l}$. The cases considered correspond to a bi-maximal form for U_ν with a small corrections from U_l [9] or its converse [10]. If U_ν is bi-maximal and U_l gives small corrections than one finds rather large $|U_{e3}|$ near the present limit and moderate $|\cos 2\theta_{23}|$, e.g., $|\cos 2\theta_{23}| \leqslant 0.12$ in the specific scheme considered in [11]. The converse case with the bi-maximal U_l and U_ν with a typical form of the CKM matrix is characterized by small $|U_{e3}| \sim 0.02$ and small $|\cos 2\theta_{23}| \leqslant 0.08$ [11].

One sees clear distinctions in the predictions of various models and it should be possible to rule out some of them once the challenging task of the experimental determination of $|U_{e3}|$ and $|\cos 2\theta_{23}|$ is accomplished.

Note. After this work was completed, we found a paper by Mohapatra with the similar discussion based on the μ–τ interchange symmetry [36].

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References

[1] Super-Kamiokande Collaboration, S. Fukuda, et al., Phys. Rev. Lett. 86 (2001) 5651;
Super-Kamiokande Collaboration, S. Fukuda, et al., Phys. Rev. Lett. 86 (2001) 5656;
SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 87 (2001) 071301;
SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011301;
SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011302;
SNO Collaboration, Q.R. Ahmad, et al., nucl-ex/0309004.

[2] KamLAND Collaboration, K. Eguchi, et al., Phys. Rev. Lett. 90 (2003) 0212021;
KamLAND Collaboration, T. Araki, et al., hep-ex/0406035.

[3] Super-Kamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 81 (1998) 1562;
Super-Kamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 82 (1999) 2644;
Super-Kamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 82 (1999) 5194.

[4] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

[5] CHOOZ Collaboration, M. Apollonio, et al., Phys. Lett. B 466 (1999) 415.

[6] A.Yu. Smirnov, Phys. Rev. D 48 (1993) 3264;
G. Dutta, A.S. Joshipura, Phys. Rev. D 51 (1995) 3838;
M. Honda, S. Kaneko, M. Tanimoto, Phys. Lett. B 593 (2004) 165;
I. Dorsner, A.Yu. Smirnov, Nucl. Phys. B 698 (2004) 386.

[7] G.L. Fogli, E. Lisi, M. Marrone, D. Montanino, A. Palazzo, A.M. Rotunno, Phys. Rev. D 67 (2003) 073002;
J.N. Bahcall, M.C. Gonzalez-Garcia, C. Peña-Garay, JHEP 0302 (2003) 009;
M. Maltoni, T. Schwetz, J.W.F. Valle, Phys. Rev. D 67 (2003) 093003;
P.C. Holanda, A.Yu. Smirnov, JHEP 0302 (2003) 001;
V. Barger, D. Marfatia, Phys. Lett. B 555 (2003) 144;
M. Maltoni, T. Schwetz, M. Tórtola, J.W.F. Valle, Phys. Rev. D 68 (2003) 113010.

[8] S.M. Barr, I. Dorsner, Nucl. Phys. B 585 (2000) 79.

[9] C. Guinti, M. Tanimoto, Phys. Rev. D 66 (2002) 113006;
C. Guinti, M. Tanimoto, Phys. Rev. D 66 (2002) 056013;
P.H. Frampton, S.T. Petcov, W. Rodejohann, Nucl. Phys. B 687 (2004) 31;
W. Rodejohann, Phys. Rev. D 70 (2000) 073010.

[10] G. Altarelli, F. Feruglio, I. Masina, Nucl. Phys. B 689 (2004) 157;
S. Antusch, S.F. King, Phys. Lett. B 591 (2004) 104;
A. Romanino, Phys. Rev. D 70 (2004) 013003;
M. Raidal, Phys. Rev. Lett. 93 (2004) 161801.

[11] H. Minakata, A.Yu. Smirnov, Phys. Rev. D 70 (2004) 073009.

[12] S. Nussinov, R.N. Mohapatra, Phys. Rev. D 60 (1999) 013002.

[13] W. Grimus, L. Lavoura, Acta Phys. Pol. B 34 (2003) 5393;
W. Grimus, L. Lavoura, JHEP 0107 (2001) 045;
E. Ma, Phys. Rev. D 66 (2002) 117301;
E. Ma, G. Rajasekaran, Phys. Rev. D 68 (2003) 071302.

[14] W. Grimus, L. Lavoura, Phys. Lett. B 572 (2003) 189.

[15] C.I. Low, Phys. Rev. D 70 (2004) 073013.

[16] W. Grimus, A.S. Joshipura, S. Kaneko, L. Lavoura, M. Tanimoto, JHEP 0407 (2004) 078.

[17] K.S. Babu, E. Ma, J.W.F. Valle, Phys. Lett. B 552 (2003) 207;
W. Grimus, L. Lavoura, Phys. Lett. B 579 (2004) 113.
[18] A.S. Joshipura, hep-ph/0411154.
[19] A.S. Joshipura, Phys. Rev. D 60 (1999) 053002;
A.S. Joshipura, S.D. Rindani, Phys. Lett. B 464 (1999) 239;
R.N. Mohapatra, et al., Phys. Lett. B 474 (2000) 355;
R.N. Mohapatra, et al., Eur. Phys. J. C 14 (2000) 85;
L. Lavoura, W. Grimus, JHEP 0009 (2000) 007;
L. Lavoura, Phys. Rev. D 62 (2000) 093011;
W. Grimus, L. Lavoura, Phys. Rev. D 62 (2000) 093062;
H. Nishiura, K. Matsuda, T. Fukuyama, Phys. Rev. D 60 (1999) 013006;
H. Nishiura, K. Matsuda, T. Fukuyama, Phys. Rev. D 61 (2000) 053001;
H.S. Goh, R.N. Mohapatra, S.P. Ng, Phys. Lett. B 570 (2003) 215;
M. Bando, S. Kaneko, M. Obara, M. Tanimoto, Phys. Lett. B 580 (2004) 229.
[20] M. Frigerio, A.Yu. Smirnov, Phys. Rev. D 67 (2003) 013007.
[21] A. de Gouvêa, Phys. Rev. D 69 (2004) 093007.
[22] O.L.G. Peres, A.Yu. Smirnov, Nucl. Phys. B 680 (2004) 479.
[23] S. Choubey, P. Roy, Phys. Rev. Lett. 93 (2004) 021803.
[24] M.C. Gonzalez-Garcia, M. Maltoni, A.Yu. Smirnov, Phys. Rev. D 70 (2004) 093005.
[25] H. Minakata, M. Sonoyama, H. Sugiyama, Phys. Rev. D 70 (2004) 113012;
H. Minakata, Talk given at the Fujihara Seminar: Neutrino Mass and Seesaw Mechanism, KEK, Japan, 23–25 February, 2004;
S. Antusch, P. Huber, J. Kersten, T. Schwetz, W. Winter, Phys. Rev. D 70 (2004) 097302.
[26] M. Komatsu, P. Migliozzi, F. Terranova, J. Phys. G 29 (2003) 443;
J.J. Gómez-Cadenas, et al., hep-ph/0105297;
D. Ayres, et al., hep-ex/0210005;
P. Huber, M. Lindner, M. Rolinec, T. Schwetz, W. Winter, Phys. Rev. D 70 (2004) 073014.
[27] K. Anderson, et al., hep-ex/0402041;
H. Minakata, H. Sugiyama, O. Yasuda, K. Inoue, F. Suekane, Phys. Rev. D 68 (2003) 033017;
Double-CHOOZ Collaboration, F. Aedellier, et al., hep-ex/0405032;
KASKA Collaboration, F. Suekane, et al., hep-ex/0407016.
[28] C. Albright, et al., hep-ex/0008064;
M. Apollonio, et al., hep-ph/0210192.
[29] A.S. Joshipura, N. Singh, S. Rindani, Nucl. Phys. B 660 (2003) 362;
T. Miura, T. Shindou, E. Takasugi, Phys. Rev. D 66 (2002) 093002.
[30] S. Antusch, J. Kersten, M. Lindner, M. Ratz, Nucl. Phys. B 674 (2003) 401.
[31] P.H. Chankowski, Z. Pluciennik, Phys. Lett. B 316 (1993) 312;
K.S. Babu, C.N. Leung, J. Pantaleone, Phys. Lett. B 319 (1993) 191;
P.H. Chankowski, W. Krolikowski, S. Pokorski, Phys. Lett. B 473 (2000) 109;
J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro, Nucl. Phys. B 573 (2000) 652.
[32] M. Tanimoto, Phys. Lett. B 360 (1995) 41;
J. Ellis, G.K. Leontaris, S. Lola, D.V. Nanopoulos, Eur. Phys. J. C 9 (1999) 389;
J. Ellis, S. Lola, Phys. Lett. B 458 (1999) 310;
J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro, Nucl. Phys. B 556 (1999) 3;
J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro, JHEP 9909 (1999) 015;
J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro, Nucl. Phys. B 569 (2000) 82;
M. Carena, J. Ellis, S. Lola, C.E.M. Wagner, Eur. Phys. J. C 12 (2000) 507.
[33] N. Haba, Y. Matsui, N. Okamura, M. Sugiura, Eur. Phys. J. C 10 (1999) 677;
N. Haba, Y. Matsui, N. Okamura, M. Sugiura, Prog. Theor. Phys. 103 (2000) 145;
N. Haba, N. Okamura, Eur. Phys. J. C 14 (2000) 347;
N. Haba, N. Okamura, M. Sugiura, Prog. Theor. Phys. 103 (2000) 367.
[34] B. Brahmachari, E.J. Chun, Phys. Lett. B 596 (2004) 184.
[35] C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B 147 (1979) 277.
[36] R.N. Mohapatra, JHEP 0410 (2004) 027.