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Zeros of the inverted neutrino mass matrix

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Abstract

I investigate viable textures with two texture zeros for the inverted neutrino mass matrix, and present the predictions of those textures for the neutrino masses and for lepton mixing. By using an Abelian symmetry and one or two heavy scalar singlets, I construct realizations of those textures in the context of seesaw models.

Particle physics was highlighted in the last decade, among other achievements, by the discovery of neutrino oscillations and, hence, of the massiveness of neutrinos. If one assumes the existence of only three light neutrinos, then lepton mixing should be parametrized by a 3×3 unitary matrix

$$U = \mathrm{diag}\left(1, e^{i\rho_1}, e^{i\rho_2}\right)\bar{U}\,\mathrm{diag}\left(e^{i\sigma_1}, e^{i\sigma_2}, e^{i\sigma_3}\right), \tag{1}$$

$$\bar{U} = \begin{pmatrix} -c_2c_3 & c_2s_3 & s_2e^{-i\delta} \\ c_1s_3 + s_1s_2c_3e^{i\delta} & c_1c_3 - s_1s_2s_3e^{i\delta} & s_1c_2 \\ s_1s_3 - c_1s_2c_3e^{i\delta} & s_1c_3 + c_1s_2s_3e^{i\delta} & -c_1c_2 \end{pmatrix}, \tag{2}$$

where $s_j = \sin\theta_j$ and $c_j = \cos\theta_j$ for $j = 1, 2, 3$, the θ_j being angles of the first quadrant. The matrix U connects, in the charged weak current $\bar{\ell}U\gamma^\mu[(1-\gamma_5)/2]\nu$, the charged-lepton fields $\bar{\ell} = (\bar{e}, \bar{\mu}, \bar{\tau})$ to the physical (mass-eigenstate) neutrino fields $\nu = (\nu_1, \nu_2, \nu_3)^T$. In (1), the phases ρ_1 and ρ_2 are unobservable, since they can be eliminated through rephasings of the μ and τ fields; observable phases are only the 'Dirac phase' δ and—if the ν_j are self-conjugate fields, as I shall assume—the 'Majorana phases' $2(\sigma_1 - \sigma_3)$ and $2(\sigma_2 - \sigma_3)$. If one denotes the mass of ν_j by m_j, then we know [1] that (i) $\Delta m^2_\odot \equiv m_2^2 - m_1^2 \simeq 8.1 \times 10^{-5}$ eV^2; (ii) $\Delta m^2_{\mathrm{atm}} \equiv |m_3^2 - m_1^2| \simeq 2.2 \times 10^{-3}$ eV^2; (iii) the solar mixing angle θ_3 is large, $s_3^2 \simeq 0.30$, but far from the 'maximal' value $\pi/4$; (iv) the

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atmospheric mixing angle θ_1 is most likely maximal, with $0.34 < s_1^2 < 0.68$ at 3σ level; (v) θ_2 may well vanish, with $s_2^2 < 0.047$ at 3σ level.

In the weak basis where the charged-lepton mass matrix is diagonal, the neutrino Majorana mass matrix $\mathcal{M}_\nu$, which is symmetric, is diagonalized by U as

$$U^T \mathcal{M}_\nu U = \mathrm{diag}(m_1, m_2, m_3). \tag{3}$$

In 2002 Frampton, Glashow, and Marfatia (FGM) [2] speculated that $\mathcal{M}_\nu$ may display a 'texture' such that two of its matrix elements are zero.[1] This assumption encompasses several viable possibilities; FGM listed them, together with the corresponding predictions for the neutrino masses and for the parameters of the mixing matrix. Models which embody FGMs hypothesis have been constructed under the paradigm of $\mathcal{M}_\nu$ generated by the vacuum expectation values of scalar triplets added to the Standard Model for that purpose [3], and under the paradigm of $\mathcal{M}_\nu$ generated through the seesaw mechanism [4,5]. In this mechanism, which is much favoured on theoretical grounds, $\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D$, where M_D is the Dirac neutrino mass matrix, which connects the flavour-eigenstate neutrinos to some right-handed neutrinos, and M_R is the Majorana mass matrix of those (super-heavy) right-handed neutrinos. In the context of the seesaw mechanism, it seems natural to assume *a texture for* $\mathcal{M}_\nu^{-1}$;[2] indeed, zeros of $\mathcal{M}_\nu^{-1}$ are identical with zeros of M_R when M_D is a square diagonal matrix, and this situation is easy to enforce in a seesaw model with an Abelian symmetry and a relatively small number of singlet Higgs fields [5]. It is the purpose of this Letter to study two-zero textures for $\mathcal{M}_\nu^{-1}$.

Some textures of $\mathcal{M}_\nu$ with two zeros are equivalent to two-zero textures of $\mathcal{M}_\nu^{-1}$ [4]. This happens, in particular, with four textures shown to be viable by FGM:

$$\begin{aligned}
&\text{case } A_1\text{:} \quad \mathcal{M}_\nu \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix} \quad \Leftrightarrow \quad \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & 0 \\ \times & 0 & 0 \end{pmatrix},\\
&\text{case } A_2\text{:} \quad \mathcal{M}_\nu \sim \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix} \quad \Leftrightarrow \quad \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & \times & \times \\ \times & 0 & 0 \\ \times & 0 & \times \end{pmatrix},\\
&\text{case } B_3\text{:} \quad \mathcal{M}_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix} \quad \Leftrightarrow \quad \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix},\\
&\text{case } B_4\text{:} \quad \mathcal{M}_\nu \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix} \quad \Leftrightarrow \quad \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix},
\end{aligned}$$

where the symbol $\times$ denotes non-zero matrix elements, and the nomenclature in the first column is the one of FGM. I remind that, besides these four viable cases, FGM found three other ones:

$$\begin{aligned}
&\text{case } B_1\text{:} \quad \mathcal{M}_\nu \sim \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix},\\
&\text{case } B_2\text{:} \quad \mathcal{M}_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix},\\
&\text{case } C\text{:} \quad \mathcal{M}_\nu \sim \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}.
\end{aligned}$$

[1] Since $\mathcal{M}_\nu$ is necessarily symmetric, $(\mathcal{M}_\nu)_{\alpha\beta} = (\mathcal{M}_\nu)_{\beta\alpha} = 0$ is counted as *only one* zero matrix element whenever $\alpha \neq \beta$.

[2] I assume $\mathcal{M}_\nu$ to be non-singular.

It turns out that, besides the cases $A_{1,2}$ and $B_{3,4}$ studied by FGM, there are three extra realistic two-zero textures for $\mathcal{M}_\nu^{-1}$:

$$\text{case } B_5\text{:}\quad \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix},$$

$$\text{case } B_6\text{:}\quad \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix},$$

$$\text{case } D\text{:}\quad \mathcal{M}_\nu^{-1} \sim \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix},$$

where the nomenclature in the first column is new.

It follows from (3) that, after discarding the unphysical phases $\rho_{1,2}$,

$$\mathcal{M}_\nu = \bar{U}^* \operatorname{diag}(\bar{m}_1, \bar{m}_2, \bar{m}_3) \bar{U}^\dagger, \tag{4}$$

where $\bar{m}_j \equiv m_j e^{-2i\sigma_j}$. Clearly then,

$$\mathcal{M}_\nu^{-1} = \bar{U} \operatorname{diag}\left(\frac{1}{\bar{m}_1}, \frac{1}{\bar{m}_2}, \frac{1}{\bar{m}_3}\right) \bar{U}^T. \tag{5}$$

Therefore, each two-zero texture for $\mathcal{M}_\nu$ or $\mathcal{M}_\nu^{-1}$ is equivalent to a set of two equations

$$\bar{m}_1 = k_1 \bar{m}_3, \qquad \bar{m}_2 = k_2 \bar{m}_3, \tag{6}$$

where k_1 and k_2 are functions of the parameters of $\bar{U}$. It follows from (6) that

$$R_\nu \equiv \frac{\Delta m_\odot^2}{\Delta m_{\text{atm}}^2} = \frac{|k_2|^2 - |k_1|^2}{|1 - |k_1|^2|}. \tag{7}$$

This quantity is experimentally known to be small, $R_\nu \simeq 0.037$.

In order to obtain simple expressions for k_1 and k_2 it is convenient to define $\epsilon \equiv s_2 e^{i\delta}$ and to make series expansions in $|\epsilon|$, since this parameter is experimentally known to be small. Using the notation $t_j = \tan\theta_j$, one then uses, to first order in $|\epsilon|$,

$$\bar{U} \approx \begin{pmatrix} -c_3 & s_3 & \epsilon^* \\ c_1 s_3\left(1 + \frac{\epsilon t_1}{t_3}\right) & c_1 c_3 (1 - \epsilon t_1 t_3) & s_1 \\ s_1 s_3\left(1 - \frac{\epsilon}{t_1 t_3}\right) & s_1 c_3\left(1 + \frac{\epsilon t_3}{t_1}\right) & -c_1 \end{pmatrix}. \tag{8}$$

One finds that all B cases yield, to first order in $|\epsilon|$,

$$k_1 \approx k\left(1 + \frac{x}{t_3}\right), \qquad k_2 \approx k(1 - t_3 x), \tag{9}$$

where $k = -t_1^2$ for cases B_1, B_3, and B_5, while

$$x = \frac{\epsilon}{t_1^3} + \frac{\epsilon^*}{t_1}, \quad \text{for case } B_1, \tag{10}$$

$$x = -\frac{\epsilon}{t_1} - \epsilon^* t_1, \quad \text{for case } B_3, \tag{11}$$

$$x = \epsilon t_1 + \epsilon^* t_1^3, \quad \text{for case } B_5. \tag{12}$$

The results for cases B_2, B_4, and B_6 are identical with those for cases B_1, B_3, and B_5, respectively, with t_1 substituted by $-1/t_1$ in both k and x [2]. All B cases yield

$$R_\nu \approx 2|k|^2 \left(t_3 + \frac{1}{t_3} \right) \left| \frac{\operatorname{Re} x}{1 - |k|^2} \right|, \tag{13}$$

and this shows that the atmospheric mixing angle cannot be maximal in any of the B cases, since R_ν becomes too large when t_1 (and hence $|k|$) is too close to 1. On the other hand, a small $|\epsilon| = s_2$ has the power to suppress R_ν in all B cases.

Let us now analyze case D. The relevant equations are in this case

$$\frac{\bar{U}_{21}^2}{\bar{m}_1} + \frac{\bar{U}_{22}^2}{\bar{m}_2} + \frac{\bar{U}_{23}^2}{\bar{m}_3} = 0, \qquad \frac{\bar{U}_{31}^2}{\bar{m}_1} + \frac{\bar{U}_{32}^2}{\bar{m}_2} + \frac{\bar{U}_{33}^2}{\bar{m}_3} = 0. \tag{14}$$

If

$$s_2 = 0, \qquad s_1 = c_1, \tag{15}$$

then Eqs. (14) are linearly dependent and read simply

$$\frac{s_3^2}{\bar{m}_1} + \frac{c_3^2}{\bar{m}_2} + \frac{1}{\bar{m}_3} = 0. \tag{16}$$

This condition has been studied in [6]. It leads to a mass spectrum $m_1 < m_2 < m_3$. The mass m_1 may either be of order $\sqrt{\Delta m_\odot^2}$ or be larger than a value of order $\sqrt{\Delta m_{\text{atm}}^2}$; in particular, an almost-degenerate mass spectrum is allowed. If, for definiteness, one uses the central values $s_3^2 = 0.3$, $m_2^2 - m_1^2 = 8.1 \times 10^{-5}\ \text{eV}^2$, and $m_3^2 - m_1^2 = 2.2 \times 10^{-3}\ \text{eV}^2$, then one obtains that

$$\begin{aligned} &\text{either} \quad 3.17 \times 10^{-3}\ \text{eV} < m_1 < 8.28 \times 10^{-3}\ \text{eV}, \\ &\text{or} \qquad\ \ m_1 > 1.44 \times 10^{-2}\ \text{eV}. \end{aligned} \tag{17}$$

Next looking for a solution of (14) which does not satisfy the assumptions (15), one obtains

$$\bar{m}_1 = u \frac{t_3 z}{t_3 z - 1} \bar{m}_3, \qquad \bar{m}_2 = u \frac{z}{z + t_3} \bar{m}_3, \tag{18}$$

where

$$u = \frac{-1 + 2\epsilon \cot 2\theta_1 \cot 2\theta_3 - \epsilon^2}{c_2^2}, \tag{19}$$

$$z = \epsilon \tan 2\theta_1. \tag{20}$$

Eqs. (18)–(20) are *exact*. Note that $|z|$ is not necessarily small, since $|\tan 2\theta_1|$ is experimentally known to be large. On the other hand, and for the same reason, u is certainly very close to -1. It follows from (18) that

$$\frac{s_3^2}{\bar{m}_1} + \frac{c_3^2}{\bar{m}_2} - \frac{1}{u \bar{m}_3} = 0, \tag{21}$$

an equation which is almost identical to (16) since $u \approx -1$. Thus, the approximate range (17) still holds. The mixing-matrix parameter z is given by

$$z = \frac{t_3 \bar{m}_2}{u \bar{m}_3 - \bar{m}_2}. \tag{22}$$

With the $\bar{m}_j$ satisfying (21), one obtains

$$|z| = \frac{m_1 m_2}{\sqrt{s_3^2 |u|^2 m_2^2 m_3^2 + c_3^2 |u|^2 m_1^2 m_3^2 - m_1^2 m_2^2}}, \tag{23}$$

$$\mathrm{Re}\, z = \frac{1}{2c_3s_3} \frac{-s_3^4|u|^2m_2^2m_3^2 + c_3^4|u|^2m_1^2m_3^2 + (s_3^2 - c_3^2)m_1^2m_2^2}{s_3^2|u|^2m_2^2m_3^2 + c_3^2|u|^2m_1^2m_3^2 - m_1^2m_2^2}. \tag{24}$$

The effective mass relevant for neutrinoless $\beta\beta$ decay is in case D [6]

$$\langle m\rangle = \left|\bar{m}_1^*\bar{U}_{e1}^2 + \bar{m}_2^*\bar{U}_{e2}^2 + \bar{m}_3^*\bar{U}_{e3}^2\right| = \left|\frac{c_2^2\bar{m}_1\bar{m}_2}{u\bar{m}_3} + \bar{m}_3\epsilon^2\right| \approx \frac{m_1m_2}{m_3}. \tag{25}$$

All the textures for $\mathcal{M}_\nu^{-1}$ in this Letter can be obtained in a simple way in models based on the seesaw mechanism; one just needs to follow the methods of [5]. Let there be three right-handed neutrinos ν_{Rj}, which add to the standard model's right-handed charged leptons ℓ_{Rj} and lepton doublets $D_{Lj} = (\nu_{Lj}, \ell_{Lj})^T$. Suppose for definiteness that one wanted to produce a model with $\mathcal{M}_\nu^{-1}$ as in case B_5. One possibility (among others [5]) would consist in introducing an Abelian symmetry group $\mathbb{Z}_4$ under which the leptons of family 1, i.e., those with $j=1$, remained invariant, the leptons of family 2 changed sign, and the leptons of family 3 were multiplied by i. Assuming the existence of only one (the standard model's) Higgs doublet, and assuming that Higgs doublet to be invariant under $\mathbb{Z}_4$, it follows immediately from this arrangement that both the charged-lepton mass matrix and the neutrino Dirac mass matrix M_D are diagonal. Then, since $\mathcal{M}_\nu^{-1} = -M_D^{-1}M_RM_D^{T\,-1}$ in the seesaw mechanism, zeros in $\mathcal{M}_\nu^{-1}$ are equivalent to zeros in the right-handed neutrino Majorana mass matrix M_R. The bilinears $\nu_{Rj}\nu_{Rj'}$ transform under $\mathbb{Z}_4$ as

$$\begin{pmatrix} 1 & -1 & i \\ -1 & 1 & -i \\ i & -i & -1 \end{pmatrix}. \tag{26}$$

For $j=j'=1$ and $j=j'=2$ the bilinear $\nu_{Rj}\nu_{Rj'}$ is $\mathbb{Z}_4$-invariant, hence the corresponding mass terms are directly present in the Lagrangian. Further introducing in the theory one complex scalar singlet S transforming under $\mathbb{Z}_4$ as $S \to iS$, we see that S has an Yukawa coupling to $\nu_{R2}\nu_{R3}$ while S^* couples to $\nu_{R1}\nu_{R3}$. Hence, the vacuum expectation value (VEV) of S produces the $(2,3)$ and $(1,3)$ matrix elements of M_R. The matrix elements $(M_R)_{33}$ and $(M_R)_{12}$ remain zero, as required by the texture B_5, since they violate $\mathbb{Z}_4$ and since there is no scalar singlet with Yukawa couplings appropriate to generate them.

Case B_6 can be produced in an analogous way to case B_5. For case D, one needs once again a symmetry $\mathbb{Z}_4$ and a complex singlet S transforming into iS under $\mathbb{Z}_4$. The first-generation leptons are $\mathbb{Z}_4$-invariant, the second-generation ones transform with i, and the third-generation ones with $-i$. The bilinears $\nu_{Rj}\nu_{Rj'}$ now transform as

$$\begin{pmatrix} 1 & i & -i \\ i & -1 & 1 \\ -i & 1 & -1 \end{pmatrix}, \tag{27}$$

and therefore the $(2,2)$ and $(3,3)$ matrix elements of M_R are zero.

Cases $A_{1,2}$ and $B_{3,4}$ are more demanding; I dwell on case A_1 for definiteness. I introduce a symmetry $\mathbb{Z}_8$ under which the leptons of the first family remain invariant, those of the second family change sign, and those of the third family are multiplied by $\omega = \exp(i\pi/4)$. The Higgs doublet is, as before, unique and invariant under this $\mathbb{Z}_8$ symmetry. The $\nu_{Rj}\nu_{Rj'}$ transform as

$$\begin{pmatrix} 1 & \omega^4 & \omega \\ \omega^4 & 1 & \omega^5 \\ \omega & \omega^5 & \omega^2 \end{pmatrix}. \tag{28}$$

The $(1,1)$ and $(2,2)$ matrix elements of M_R are $\mathbb{Z}_8$-invariant. The $(1,2)$ matrix element requires the presence of a real scalar singlet which changes sign under $\mathbb{Z}_8$. The $(1,3)$ matrix element is generated by the Yukawa coupling to a complex scalar singlet which gets multiplied by ω^7 under $\mathbb{Z}_8$. The other entries of M_R remain zero in the absence of any further scalar singlets.

Thus, all the textures for $\mathcal{M}_\nu^{-1}$ advocated in this Letter can be easily justified by seesaw models supplemented with an Abelian symmetry and one or two scalar singlets with VEVs at the seesaw scale. This implementation of the textures operates both at the seesaw scale and at any other scale. Indeed, in the standard model *with only one Higgs doublet*, the matrices $\mathcal{M}_\nu$ at any two energy scales μ_1 and μ_2 are related by [7]

$$\mathcal{M}_\nu(\mu_1) = I\mathcal{M}_\nu(\mu_2)I, \tag{29}$$

where the matrix I (which depends on μ_1 and μ_2) is *diagonal*, positive, and non-singular. It follows from (29) that any zero matrix element of $\mathcal{M}_\nu$, or of $\mathcal{M}_\nu^{-1}$, at a given energy scale, remains zero at any other energy scale.

To summarize, I have shown in this Letter that two-zero textures for the inverted neutrino mass matrix are quite easy to obtain in the context of seesaw models. There are seven such textures which are not in disagreement with the available data on neutrino masses and lepton mixing; four of those seven viable textures coincide with textures for $\mathcal{M}_\nu$ previously studied by FGM. The other three textures are new, and one of them in particular—called 'case D' in this Letter—produces the interesting prediction (16) for the neutrino masses and Majorana phases.

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References

[1] M. Maltoni, T. Schwetz, M. Tórtola, J.W.F. Valle, in: F. Halzen, M. Lindner, A. Suzuki (Eds.), New. J. Phys. 6 (2004) 122, Focus Issue on Neutrino Physics, hep-ph/0405172.
[2] P.H. Frampton, S.L. Glashow, D. Marfatia, Phys. Lett. B 536 (2002) 79, hep-ph/0201008;
See also: Z.-Z. Xing, Phys. Lett. B 530 (2002) 159, hep-ph/0201151.
[3] M. Frigerio, S. Kaneko, E. Ma, M. Tanimoto, Phys. Rev. D 71 (2005) 011901, hep-ph/0409187.
[4] A. Kageyama, S. Kaneko, N. Shimoyama, M. Tanimoto, Phys. Lett. B 538 (2002) 96, hep-ph/0204291.
[5] W. Grimus, A.S. Joshipura, L. Lavoura, M. Tanimoto, Eur. J. Phys. C 36 (2004) 227, hep-ph/0405016.
[6] W. Grimus, L. Lavoura, Phys. Lett. B 572 (2003) 189, hep-ph/0305046.
[7] P.H. Chankowski, Z. Płuciennik, Phys. Lett. B 316 (1993) 312, hep-ph/9306333;
K.S. Babu, C.N. Leung, J. Pantaleone, Phys. Lett. B 319 (1993) 191, hep-ph/9309223;
S. Antusch, M. Drees, J. Kersten, M. Lindner, M. Ratz, Phys. Lett. B 519 (2001) 238, hep-ph/0108005;
P.H. Chankowski, S. Pokorski, Int. J. Mod. Phys. A 17 (2002) 575, hep-ph/0110249.