Natural braneworld inflation and baryogenesis

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Abstract

In natural inflation models, the inflaton is a pseudo Nambu–Goldstone boson and the flatness of the potential is protected by shift symmetries. In this framework, a successful inflation requires the global symmetry to be spontaneously broken at a scale close to the Planck mass. Such a high value of the spontaneous breaking scale may not be legitimate in an effective field theory. On the other hand, if natural inflation occurs during the nonconventional high-energy era in braneworld cosmology, the conditions for the inflaton slow rolling can be eased and the spontaneous breaking scale can be lowered to values below the Planck scale. We examine the observational constraints on this scenario and study the possibility that the baryon asymmetry of the universe is generated by the decay of the pseudo Nambu–Goldstone boson during the reheating era.
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1. Introduction

Today there is a compelling evidence that the early universe underwent a period of cosmological inflation [1,2]. Inflation not only elegantly solves several cosmological issues, including the horizon and flatness problems, but it can also provide the initial conditions required for the structure formation in the universe. In particular, the observations by the Cosmic Background Explorer (COBE) and, more recently, the Wilkinson Microwave Anisotropy Probe (WMAP) satellites [3]

confirm the predictions of a flat universe with a nearly scale-invariant spectrum of adiabatic perturbations.

Despite the existence of several cosmologically viable inflaton potentials, the construction of a "naturally" flat potential is a difficult task from the particle physics viewpoint. In the simplest chaotic inflationary scenarios, inflaton field values above the four-dimensional Planck mass, $\phi \gtrsim M_P$, are typically required to allow for a sufficiently long period of inflation. Thus, one expects nonrenormalizable quantum corrections of the order of $\mathcal{O}[(\phi/M_P)^n]$ to destroy the flatness of the potential. Moreover, the inflaton must couple to matter fields to efficiently reheat the universe after the inflationary era. Such couplings could also

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destabilize the potential. Among the few known candidates which preserve the scalar potential nearly flat and protected against radiative corrections, supersymmetry has undoubtedly received most of the attention so far. However, it has recently been argued [4] that this symmetry alone cannot naturally provide a satisfactory model of inflation, if supergravity effects are taken into account. Another natural candidate for the inflaton is provided by nonlinearly realized symmetries, such as those which involve a pseudo Nambu–Goldstone boson (PNGB) [5] or extra components of gauge fields propagating in extra dimensions [4,6,7].

In the simplest variant of natural inflation [5], the role of the inflaton is played by a PNGB, and the flatness of the potential is protected by a shift symmetry under $\phi \to \phi + \text{const}$, which remains after the global symmetry is spontaneously broken. An explicit breaking of the shift symmetry leads typically to a potential of the form

$$V(\phi) = \Lambda^4 \big[1 - \cos(\phi/f) \big], \tag{1}$$

where ϕ is the canonically normalized field, f is the spontaneous breaking scale and Λ is the scale at which the soft explicit breaking takes place.[1] For large values of f, the potential can be flat. However, in this framework the slow-roll requirements set the bound $f \gtrsim M_P$. Such high values could be problematic and difficult to justify in an effective field theory [4]. Moreover, one can expect the quantum gravity effects to explicitly break the global symmetry. A possible solution to the above problems is to consider higher-dimensional cosmological models, where our four-dimensional world is viewed as a three-brane embedded in a higher-dimensional bulk.[2] For instance, in the 5D version of natural inflation considered in Ref. [6], the inflaton is the extra component of a gauge field, which propagates in the bulk, and the flatness of its potential, coming only from nonlocal effects, is not spoiled by higher-dimensional operators or quantum gravity effects.

In this Letter we shall consider another possible scenario for extra-natural inflation. We shall assume

that the inflaton is described by a standard four-dimensional PNGB, with a potential given by Eq. (1), but inflation occurs during the nonconventional high-energy regime of the theory [10]. In braneworld cosmology, modifications to the Friedmann equation [11] (due to the linear dependence of the expansion rate H on the energy density ρ at early times) can ease the slow-roll conditions and enable inflation to take place at field values far below M_P [12,13], thus avoiding the well-known difficulties with higher-order nonrenormalizable terms. Since in this framework the observational data does not put direct limits on the symmetry breaking scales, but rather on the ratios f/M_5 and Λ/M_5, it is possible to obtain lower values of the natural inflation scale for low values of the five-dimensional fundamental Planck mass M_5. The latter is only constrained by demanding the brane terms in the Friedmann equation to play a negligible role at the big bang nucleosynthesis scale $\sim \mathcal{O}(\text{MeV})$.

After the inflationary epoch, the cold inflaton-dominated universe undergoes a reheating phase, during which the inflaton oscillates about the minimum of its potential, giving rise to particle and entropy production. In addition to entropy creation, the right abundance of baryons must also be created. It is then interesting to ask whether the inflaton field could also solve another outstanding cosmological puzzle, namely, the explanation of the baryon asymmetry of the universe. A particularly attractive and viable mechanism is the so-called spontaneous baryogenesis [14], which can naturally occur in the early universe, if the CPT symmetry is violated. In this framework, the PNGB field associated with the broken baryon number plays a crucial role. If the PNGB is derivatively coupled to a baryon current J_B^μ with an effective Lagrangian of the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{f} \partial_\mu \phi \, J_B^\mu, \tag{2}$$

a net baryon asymmetry can be produced either in a Hubble-damped regime or in a regime characterized by the decays of the inflaton as it oscillates about its minimum [14]. In the latter case, and assuming that the PNGB field couples to fermions which carry baryon number, the resulting asymmetry turns out to be proportional to $\Gamma \phi_i^3 / f$, where ϕ_i is the inflaton field value at the onset of reheating and Γ is the inflaton decay width [15,16].

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[1] Note that the flatness of the PNGB potential is natural in the 't Hooft sense [8]: in the limit $\Lambda \to 0$ the shift symmetry is restored.

[2] For an alternative solution, based on the presence of two axions with a potential provided by two anomalous gauge groups, see Ref. [9].

By combining the constraints coming from natural inflation with the ones inferred for the present baryon-to-entropy ratio, it is then possible to put limits on the reheating temperature and the relevant scales required for natural inflation and baryogenesis to be successfully realized in the above braneworld scenario.

2. Natural braneworld inflation

In a cosmological braneworld scenario, where the space–time on the brane is described by a flat Friedmann–Robertson–Walker metric, the Friedmann equation receives an additional term quadratic in the energy density[3] [11],

$$H^2 = \frac{8\pi}{3M_P^2}\rho\left(1 + \frac{\rho}{2\lambda}\right), \tag{3}$$

where λ is the brane tension, which is related to the fundamental five-dimensional Planck scale M_5 through the equation

$$\lambda = \frac{3}{4\pi}\frac{M_5^6}{M_P^2}. \tag{4}$$

At sufficiently low energies, $\rho \ll \lambda$, the Friedmann equation of standard cosmology is recovered. For nucleosynthesis to take place successfully, the change in the expansion rate due to the ρ^2-term in the Friedmann equation (3) must be sufficiently small at scales $\sim \mathcal{O}(\text{MeV})$. This leads to the lower bound $\lambda \gtrsim (1\,\text{MeV})^4$. A more stringent bound, $\lambda \gtrsim (1\,\text{TeV})^4$, can be obtained by requiring the theory to reduce to Newtonian gravity on scales larger than 1 mm.

In the slow-roll approximation and at high energies ($V \gg \lambda$), the total number of e-folds during inflation is given by [12]

$$N \simeq -\frac{4\pi}{M_P^2\lambda}\int_{\phi_I}^{\phi_F}\frac{V^2}{V'}\,d\phi, \tag{5}$$

where ϕ_I and ϕ_F are the values of the scalar field at the beginning and at the end of the expansion, respectively. The value ϕ_F can be computed from the condition $\max\{\epsilon(\phi_F), |\eta(\phi_F)|\} = 1$, where ϵ and η are

the slow-roll parameters, given in the high-energy approximation by

$$\epsilon \simeq \frac{M_P^2\lambda}{4\pi}\frac{V'^2}{V^3}, \qquad \eta \simeq \frac{M_P^2\lambda}{4\pi}\frac{V''}{V^2}. \tag{6}$$

The spectra of scalar [12] and tensor [17,18] perturbations at the Hubble radius crossing are also key parameters during inflation. Their expressions at high energies read as

$$A_s^2 = \frac{64\pi}{75M_P^6\lambda^3}\frac{V^6}{V'^2}, \qquad A_t^2 = \frac{8V^3}{25M_P^4\lambda^2}. \tag{7}$$

The scale dependence of the scalar perturbations is described by the spectral tilt

$$n_s - 1 = \frac{d\ln A_s^2}{d\ln k} \simeq -6\epsilon + 2\eta, \tag{8}$$

and its running is given by

$$\alpha_s = \frac{dn_s}{d\ln k} \simeq 16\epsilon\eta - 18\epsilon^2 - 2\xi, \tag{9}$$

where

$$\xi \simeq \frac{M_P^4\lambda^2}{(4\pi)^2}\frac{V'V'''}{V^4}. \tag{10}$$

Finally, the tensor power spectrum amplitude can be parameterized by the tensor-to-scalar ratio

$$r_s = 16\frac{A_t^2}{A_s^2} = 24\epsilon. \tag{11}$$

We also remark that the tensor spectral index n_t is not an independent parameter since it is related to r_s by the inflationary consistency condition $n_t = -r_s/8$, which holds independently of the brane tension λ [18].

In natural inflation, with the potential given as in Eq. (1), the following inflationary constraints are obtained from Eqs. (5)–(11):

$$N_* = \frac{1}{x}\left[\left(\sin^2\chi_F - \sin^2\chi_*\right) + \ln\left(\frac{\cos^2\chi_F}{\cos^2\chi_*}\right)\right],$$

$$A_s = \left(\frac{2}{75\pi^2 x^3}\right)^{1/2}\left(\frac{\Lambda}{f}\right)^2\frac{\sin^5\chi_*}{\cos\chi_*},$$

$$n_s = 1 - \frac{x}{\sin^4\chi_*}\left(1 + 4\cos^2\chi_*\right),$$

$$\alpha_s = -\frac{2x^2\cos^2\chi_*}{\sin^8\chi_*}\left(3 + 2\cos^2\chi_*\right),$$

$$r_s = 24x \frac{\cos^2 \chi_*}{\sin^4 \chi_*}, \qquad (12)$$

where N_* is the number of e-folds before the end of inflation, at which observable perturbations are generated, and

$$\chi = \frac{\phi}{2f}, \qquad x = \frac{\lambda}{8\pi}\left(\frac{M_P}{f\Lambda^2}\right)^2. \qquad (13)$$

For a given value of the parameter x, the value of the field at the end of inflation can be determined from the condition $\epsilon(\chi_F) = 1$, leading to the equation

$$2\sin^2 \chi_F = -x + \sqrt{x^2 + 4x}. \qquad (14)$$

For $x \ll 1$, we obtain $\sin \chi_F \simeq x^{1/4}(1 - \sqrt{x}/4)$.

In our numerical estimates we shall use the COBE normalization $A_s \simeq 2 \times 10^{-5}$ and the following WMAP bounds on the inflationary parameters [19]

$$0.94 \leqslant n_s \leqslant 1.00, \qquad |\alpha_s| \leqslant 0.02,$$
$$r_s \leqslant 0.14, \qquad (15)$$

which apply to models of inflation with a negative curvature ($\eta < 0$). We also recall that in standard cosmology a reasonable fiducial value for the number of e-folds is $N_* = 55$. This number is expected to be higher in the brane scenario [20,21]. In what follows we assume $N_* = 60$. It is worth noticing that for the potential (1) the spectral index n_s is very weakly dependent on N_* and its running α_s is negligible, $\alpha_s \approx -5 \times 10^{-4}$. Therefore, if small-scale microwave background observations indicate a significant negative running of the spectral index, natural braneworld inflation in its simplest version of Eq. (1) will be excluded.

The numerical solution of Eq. (12) is presented in Fig. 1. We see that natural inflation in the brane constrains the parameter x to the range $5.2 \times 10^{-3} \lesssim x \lesssim 5.5 \times 10^{-2}$. The lower bound comes from the WMAP upper bound on r_s, while the upper bound comes from the minimum allowed value for n_s. This leads to the following constraints on the spontaneous and explicit symmetry breaking scales,
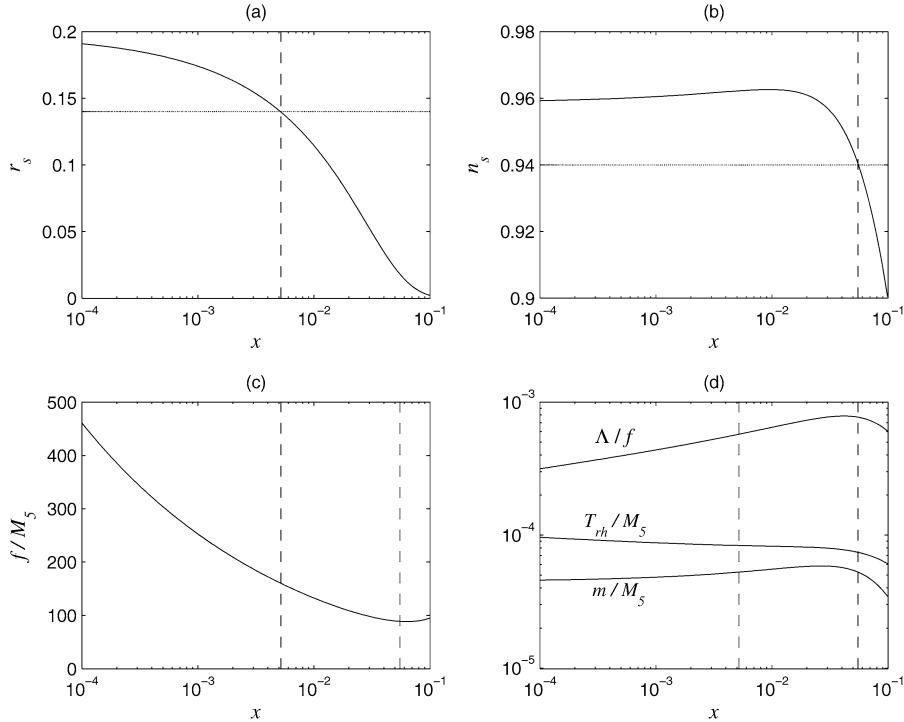

Fig. 1. Inflationary parameters r_s, n_s ((a) and (b)), the symmetry breaking scales f, Λ ((c) and (d)) and the inflaton mass $m = \Lambda^2/f$ (d) in the natural braneworld inflationary scenario. The plots are given as functions of the dimensionless parameter x defined in Eq. (13). The region delimited by the vertical dashed lines corresponds to the parameter space allowed by the WMAP bounds on n_s and r_s.

$$90 \lesssim \frac{f}{M_5} \lesssim 160,$$

$$7 \times 10^{-2} \lesssim \frac{\Lambda}{M_5} \lesssim 9 \times 10^{-2}, \tag{16}$$

which imply the hierarchy of scales $6 \times 10^{-4} \lesssim \Lambda/f \lesssim 8 \times 10^{-4}$ and an inflaton mass $5 \times 10^{-5} M_5 \lesssim m = \Lambda^2/f \lesssim 6 \times 10^{-5} M_5$. It is also worth noticing that in the present framework the density fluctuation spectrum is red tilted ($0.94 \lesssim n_s \lesssim 0.96$) and a significant amount of gravitational waves is predicted ($0.02 \lesssim r_s \lesssim 0.14$), which could be detectable at forthcoming experiments such as the PLANCK satellite.

We remark that the above analysis has been done assuming that the high-energy approximation is valid, i.e., $V/\lambda \gg 1$, so that inflation occurs in the non-conventional braneworld era. This requires $M_5 \ll 10^{17}$ GeV. On the other hand, in the range of x allowed by inflation, one obtains $\chi \sim \mathcal{O}(1)$. Therefore, from Eq. (16) it follows that the inflaton field $\phi \sim f \sim \mathcal{O}(10^2) \times M_5 \ll M_P$ in the high-energy regime, i.e., natural inflation indeed takes place at field values below the Planck scale.

3. Reheating and baryogenesis

After inflation, the cold inflaton-dominated universe undergoes a phase of reheating, during which the inflaton decays into ordinary particles and the universe becomes radiation dominated. Assuming an instantaneous conversion of the inflaton energy into radiation, one can identify $\rho = \rho_{\text{rad}} = (\pi^2/30) g_* T^4$, where g_* is the effective number of relativistic degrees of freedom; $g_* \sim 100$ in the standard model for temperatures above the electroweak scale. The reheating temperature T_{rh} is then obtained from the requirement that the expansion rate of the universe equals the inflaton decay width, i.e., $H(T_{\text{rh}}) = \Gamma$. In the braneworld scenario, this leads to the relation

$$T_{\text{rh}}^4 = \frac{T_t^4}{2} \left[-1 + \sqrt{1 + \frac{45}{\pi^3 g_*} \left(\frac{\Gamma M_P}{T_t^2} \right)^2} \right], \tag{17}$$

where T_t is the transition temperature from brane cosmology to standard cosmology, defined through the relation $\rho(T_t) = 2\lambda$ so that

$$T_t^2 = \frac{2}{\pi} \sqrt{\frac{15\lambda}{g_*}} = \frac{3}{\pi} \sqrt{\frac{5}{\pi g_*}} \frac{M_5^3}{M_P}. \tag{18}$$

During the oscillating phase, the inflaton field evolves according to the well-known equation of a damped harmonic oscillator, $\ddot{\phi} + \Gamma \dot{\phi} + m^2 \phi = 0$. If the inflaton field couples derivatively to the baryon current through the interaction Lagrangian of Eq. (2), the net baryon number density produced during reheating can be estimated as [16]

$$n_B \equiv n_b - n_{\bar{b}} = \frac{\Gamma \phi_i^3}{2f}, \tag{19}$$

where ϕ_i is the value of the field at the onset of the reheating epoch.

When the expansion is negligible ($H < \Gamma$), the baryon number can be obtained from Eq. (19) by replacing ϕ_i by the field value at T_{rh},

$$\phi_{\text{rh}} = \pi \sqrt{\frac{g_*}{15}} \frac{T_{\text{rh}}^2}{m}. \tag{20}$$

Since the entropy density after thermalization is given by $s = 2\pi^2 g_* T_{\text{rh}}^3/45$, the baryon-to-entropy ratio reads then as

$$\frac{n_B}{s} = \frac{\pi}{4} \sqrt{\frac{3g_*}{5}} \frac{\Gamma}{f} \left(\frac{T_{\text{rh}}}{m} \right)^3. \tag{21}$$

On the other hand, the process of particle production starts at earlier times, when $H \simeq m > \Gamma$. Since the generation of the asymmetry is more efficient at that time, the final baryon-to-entropy ratio is larger than that of Eq. (21) approximately by a factor

$$\kappa = \mathcal{D} \left(\frac{\phi_i}{\phi_{\text{rh}}} \right)^3, \tag{22}$$

where $\phi_i = \phi(H \simeq m)$ and

$$\mathcal{D} = \left(\frac{\Gamma}{m} \right)^2 \frac{1 + m M_P/\sqrt{3\pi\lambda}}{1 + \Gamma M_P/\sqrt{3\pi\lambda}} \tag{23}$$

is a dilution factor due to the expansion of the universe from the time $t \sim 1/m$ to $t \sim 1/\Gamma$. The final baryon-to-entropy ratio is therefore given by

$$\frac{n_B}{s} = \frac{\pi \kappa}{4} \sqrt{\frac{3g_*}{5}} \frac{\Gamma}{f} \left(\frac{T_{\text{rh}}}{m} \right)^3. \tag{24}$$

Let us note that $\kappa \simeq m/\Gamma$ in standard cosmology [16], while in the high-energy regime of brane cosmology we obtain $\kappa \simeq (m/\Gamma)^{1/2}$.

Using Eqs. (17) and (18), the decay width can be written in the form

$$\Gamma = \frac{2\pi}{3} \sqrt{\frac{\pi g_*}{5}} \frac{T_{\mathrm{rh}}^2}{M_P} \left[1 + \left(\frac{T_{\mathrm{rh}}}{T_t} \right)^4 \right]^{1/2}. \tag{25}$$

Moreover, the field ratio $\phi_i/\phi_{\mathrm{rh}}$ will be given by

$$\frac{\phi_i}{\phi_{\mathrm{rh}}} = \frac{1}{\sqrt{2}} \left(\frac{T_t}{T_{\mathrm{rh}}} \right)^2 \left[-1 + \sqrt{1 + \frac{45}{\pi^3 g_*} \left(\frac{m M_P}{T_t^2} \right)^2} \right]^{1/2}. \tag{26}$$

Thus, in the high-energy regime of brane cosmology, i.e., when $\rho \gg 2\lambda$, one obtains from Eq. (24),

$$\frac{n_B}{s} = \frac{\pi^2 g_*}{10} \sqrt{\frac{\pi}{6}} \frac{T_{\mathrm{rh}}^5}{f m^{5/2} M_5^{3/2}}, \tag{27}$$

while in standard cosmology,

$$\frac{n_B}{s} = \frac{\pi}{4} \sqrt{\frac{3g_*}{5}} \frac{T_{\mathrm{rh}}^3}{f m^2}. \tag{28}$$

The recent observations by WMAP [3] imply that $n_B/s \simeq 9 \times 10^{-11}$, a value which is in remarkable agreement with the independent determination of this quantity from big bang nucleosynthesis. Using the above value, we can put bounds on the reheating temperature required for baryogenesis to take place during the radiation era. The results are presented in Fig. 2, where T_{rh} is plotted as a function of the fundamental 5D Planck mass M_5. We notice that T_{rh} is quite insensitive to the variations of the dimensionless parameter x defined in Eq. (13), in the range allowed by natural inflation (cf. Fig. 1). In Fig. 2 we present the results for $x = 0.03$, assuming $g_* = 100$. From the figure we see that baryogenesis through PNGB decays can occur in the high-energy regime of brane cosmology for $M_5 \lesssim 6 \times 10^{11}$ GeV and $T_{\mathrm{rh}} \lesssim 4 \times 10^7$ GeV, at much lower reheating temperatures than in standard cosmology.

One may wonder whether the baryon asymmetry produced by the mechanism described above will survive without being diluted by baryon number violating processes in thermal equilibrium. In fact, if nonzero baryon and lepton numbers are generated such that $B - L \neq 0$, the asymmetry will not be washed out by the anomalous electroweak sphaleron processes. On the other hand, if an equal amount of baryon and lepton asymmetries is generated such that $B - L$ vanishes, the final asymmetry will be diluted
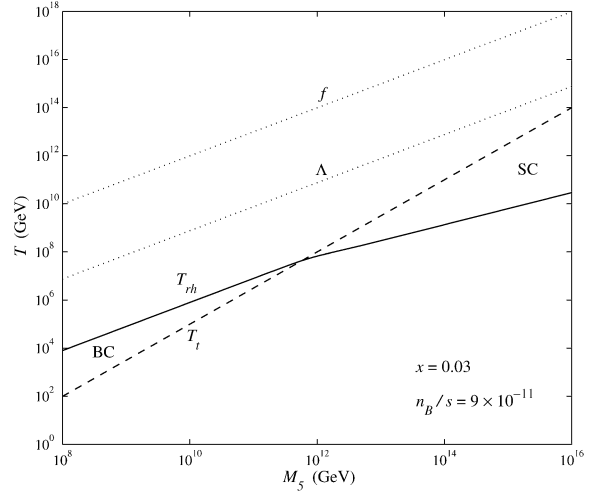

Fig. 2. The reheating temperature T_{rh} (as a function of the 5D Planck mass M_5) required for a successful baryogenesis in the natural braneworld inflationary scenario. The dashed line corresponds to the transition temperature from brane cosmology (BC) to standard cosmology (SC). For comparison, the symmetry breaking scales f and Λ required for inflation are also shown (dotted lines).

by such processes, unless the reheating temperature is below the critical temperature of the electroweak phase transition $T_{\mathrm{ew}} \sim 100$ GeV. At higher temperatures, i.e., before the electroweak symmetry breaking, the sphaleron rate is roughly $\Gamma_{\mathrm{ws}} \simeq 25\alpha_{\mathrm{w}}^5 T$ [22], where $\alpha_{\mathrm{w}} \simeq 1/30$ is the weak coupling constant. Thus, electroweak sphalerons in standard cosmology are in thermal equilibrium ($\Gamma_{\mathrm{ws}} \gtrsim H$) at temperatures $T_{\mathrm{ew}} \lesssim T \lesssim 10^{12}$ GeV. In brane cosmology, this bound depends on the fundamental scale M_5 and we find $T_{\mathrm{ew}} \lesssim T \lesssim 10^{-2} M_5$, in the high-energy braneworld era.

4. Conclusion

A natural way to obtain a flat inflaton potential is to invoke some approximate shift symmetry, which involves a pseudo Nambu–Goldstone boson or extra components of gauge fields living in extra dimensions. On the other hand, modifications to the expansion rate of the universe at earlier times turn out to be crucial for processes that took place in early universe. In this Letter we have examined the observational constraints on the scenario of natural braneworld inflation, assuming that inflation occurs during the high-energy regime of the theory. In this framework, the scale at which the

global symmetry is spontaneously broken can be easily lowered to values far below the Planck mass M_P, thus protecting the theory from dangerous nonrenormalizable operators which could spoil the flatness of the potential.

The explanation of the baryon asymmetry of the universe is another unresolved issue. Here we have considered the possibility that this asymmetry is generated by the decay of the inflaton during the reheating era. If the inflaton couples derivatively to a baryon current, a net asymmetry can be generated as the field coherently oscillates about its minimum and decays into ordinary matter. In this case, the spontaneous symmetry breaking scale f also describes the physics of baryon number violation. In this regard, there may be additional constraints on the implementation of the mechanism, such as those arising from proton stability. Finally, if the baryon creation occurs in the braneworld era, then sufficiently low reheating temperatures and low values of the fundamental five-dimensional Planck mass M_5 can yield the right magnitude of the observed baryon-to-entropy ratio.

Acknowledgements

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