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Leptonic CP violation and neutrino mass models

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Abstract. We discuss leptonic mixing and CP violation at low and high energies, emphasizing possible connections between leptogenesis and CP violation at low energies, in the context of lepton flavour models. Furthermore, we analyse weak-basis invariants relevant for leptogenesis and for CP violation at low energies. These invariants have the advantage of providing a simple test of the CP properties of any lepton flavour model.

Contents

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1. Introduction

The experimental data on atmospheric and solar neutrinos have provided evidence for non-vanishing neutrino masses and for non-trivial leptonic mixing [1]. These important discoveries rendered even more pressing the fundamental question of understanding the spectrum of fermion masses and the pattern of their mixing. In the Standard Model (SM) neutrinos are strictly massless. No Dirac mass terms can arise in the SM due to the absence of right-handed (rh) neutrinos and no left-handed (lh) Majorana masses can be generated at tree level due to the simple Higgs structure of the SM. Furthermore, no Majorana masses can be generated in higher orders due to the exact B–L conservation. Therefore, the discovery of neutrino masses and leptonic mixing provides clear evidence for physics beyond the SM.

It is remarkable that a simple extension of the SM, through the introduction of rh neutrinos, leads to non-vanishing but naturally small neutrino masses. With the addition of rh neutrinos to the SM, the most general Lagrangian consistent with renormalizability and gauge invariance leads to both Dirac and rh Majorana neutrino mass terms. The natural scale for the Dirac neutrino masses is v, the scale of electroweak symmetry breaking. On the other hand, since the rh neutrinos transform trivially under $SU(2) \times U(1)$, the rh Majorana mass term is gauge invariant and as a result its scale V can be much larger, being identified with the scale of lepton number violation. In the context of grand unified theories (GUT) this scale can be naturally taken as the GUT scale. The presence of both Majorana and Dirac masses of the above indicated order of magnitude, automatically leads to light neutrinos with masses of order v^2/V, through the see-saw mechanism [2]. Strictly speaking, in order to have naturally small neutrino masses it is not necessary to introduce rh neutrinos, one may have only lh neutrinos, provided lepton number violation occurs at a high-energy scale. The introduction of rh neutrinos is well motivated in the framework of some GUT theories like S0(10) and it has the special appeal of establishing a possible connection between neutrinos and the generation of the baryon asymmetry of the universe (BAU). In fact, one of the most attractive mechanisms to generate BAU is baryogenesis through leptogenesis [3], a scenario where the out-of-equilibrium decays of heavy rh neutrinos create a lepton asymmetry which is later converted into a baryon asymmetry by B + L violating (but B–L conserving) sphaleron interactions [4].

It is well known [5] that pure gauge theories do not violate CP. In fact, the fermionic sector (kinetic energy terms and fermion interactions with vector bosons) as well as the vector boson sector of gauge theories are always CP symmetric. The same is true for the couplings of scalars with gauge fields. In the SM, CP violation in the quark sector arises from the simultaneous presence of charged current gauge interactions and complex Yukawa couplings [6]. In general, for three or more generations there is no CP transformation which leaves invariant both the Yukawa couplings and the charged current gauge interactions. This leads to the well-known Kobayashi–Maskawa mechanism [7] of CP violation operating in the quark sector. In the leptonic sector and in the context of the SM, there is no CP violation since for massless neutrinos leptonic mixing in the charged currents can always be rotated away through a redefinition of neutrino fields. In any extension of the SM with non-vanishing neutrino masses and mixing, there is in general leptonic CP violation. In the case of an extension of the SM consisting of the addition of three rh neutrinos, one has in general both leptonic CP violation at low energies, visible for example through neutrino oscillations and CP violation at high energies relevant for the generation of baryogenesis through leptogenesis.

In this paper, we review leptonic mixing and CP violation at low and high energies, with emphasis on the possible connection between leptogenesis and low-energy data as well as on the analysis of weak-basis (WB) invariants relevant for CP violation. In fact, by writing the most general CP transformation for the fermion fields in a WB one can derive simple conditions for CP conservation which can be applied without going to the physical basis. This strategy was followed for the first time in the context of the SM in [8]. These invariants provide a simple way of testing whether a specific lepton flavour model [9] leads to CP violation either at low or high energies. The crucial advantage of these invariants stems from the fact that for any lepton flavour model, they can be calculated in any WB, without requiring cumbersome changes of basis. The paper is organized as follows. In section 2, we establish our notation introducing the various leptonic mass terms, derive necessary conditions for CP invariance and identify the independent CP violating phases, both in a WB and in the mass eigenstate basis. In section 3, we derive WB invariants which are relevant for CP violation at low energies, as well as WB invariants sensitive to CP violation at high energies relevant for leptogenesis. In section 4, we analyse the special limit of exactly degenerate neutrino masses. The relationship between low-energy CP violation and CP violation at high energies is discussed in section 5. Finally, in section 6, we present our summary and conclusions.

2. Neutrino mass terms

We consider a simple extension of the SM where three rh neutrinos (one per generation) are introduced. In this case, the most general form for the leptonic mass terms after spontaneous symmetry breaking is

$$\begin{aligned}\mathcal{L}_m &= -\,[\tfrac{1}{2}\nu_L^{0T} C m_L \nu_L^0 + \overline{\nu_L^0} m_D \nu_R^0 + \tfrac{1}{2}\nu_R^{0T} C M_R \nu_R^0 + \overline{l_L^0} m_l l_R^0] + \text{h.c.}\\ &= -\,[\tfrac{1}{2} n_L^T C \mathcal{M}^* n_L + \overline{l_L^0} m_l l_R^0] + \text{h.c.},\end{aligned} \tag{1}$$

where m_L, M_R, denote the lh and rh neutrino Majorana mass matrices, while m_D, m_l stand for the neutrino Dirac mass matrix and the charged lepton mass matrix, respectively. The generation at tree level of a mass term of the form $\nu_L^{0T} C m_L \nu_L^0$ also requires the extension of the Higgs sector (e.g., a Higgs triplet). The introduction of the column vector $n_L = (\nu_L^0, (\nu_R^0)^c)$ allows one to write $\mathcal{L}_m$ in a more compact form, with the 6×6 matrix $\mathcal{M}$ given by

$$\mathcal{M} = \begin{pmatrix} m_L^* & m_D \\ m_D^T & M_R \end{pmatrix}. \tag{2}$$

The mass terms in $\mathcal{L}_m$ contain all the information on CP violation arising from the charged gauge interactions, irrespective of the mechanism which generates the lepton mass terms and will be analysed in the next subsection. An enlarged Higgs sector will in general provide new sources of CP violation which we do not discuss in this work. In fact most of our analysis will be done in the framework of the minimal Higgs structure (no Higgs triplets), thus implying that the term m_L in equation (1) is absent. The corresponding matrix $\mathcal{M}$ has then a zero block entry in its upper left block.

For simplicity, in most of our discussions in this paper, we will consider that the number of rh neutrinos equals the number of lh neutrinos. It should be pointed out that this is not required for the generation of appropriate neutrino masses.

2.1. The general case

In this subsection, we study leptonic CP violation in the case corresponding to the most general mass terms given by equation (1). There are two aspects in which leptonic CP non-conservation differs from CP violation in the quark sector. One aspect has to do with the fact that being neutral, neutrinos can have both Majorana and Dirac mass terms. The other one results from the fact that the full leptonic mixing matrix appearing in the charged currents is a 3×6 matrix, consisting of the first three lines of a 6×6 unitary matrix. Of course, in the low-energy limit, where only the light neutrinos are active, the leptonic mixing is described by a 3×3 unitary matrix. For the analysis of leptonic mixing and CP violation mediated through the charged gauge bosons the relevant part of the Lagrangian is $\mathcal{L}_m$ given by equation (1) together with the charged gauge interaction

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+ \overline{l_L^0} \gamma^\mu \nu_L^0 + \text{h.c.} \tag{3}$$

The simplest way of determining the number of independent CP violating phases [10] is by working in a conveniently chosen WB and analysing the restrictions on the Lagrangian implied by CP invariance. We follow this approach, but also identify the CP violating phases appearing in the charged weak interactions, written in the mass eigenstate basis.

The most general CP transformation which leaves the gauge interaction invariant is

$$\text{CP} l_L^0 (\text{CP})^\dagger = U' \gamma^0 C \overline{l_L^0}^T, \qquad \text{CP} l_R^0 (\text{CP})^\dagger = V' \gamma^0 C \overline{l_R^0}^T, \qquad \text{CP} \nu_L^0 (\text{CP})^\dagger = U' \gamma^0 C \overline{\nu_L^0}^T,$$
$$\text{CP} \nu_R^0 (\text{CP})^\dagger = W' \gamma^0 C \overline{\nu_R^0}^T, \qquad \text{CP} W_\mu^+ (\text{CP})^\dagger = -(-1)^{\delta_{0\mu}} W_\mu^-, \tag{4}$$

where U', V' and W' are unitary matrices acting in the flavour space. This transformation combines the CP transformation of a single fermion field with a WB transformation [11]. Invariance of the mass terms under the above CP transformation, requires that the following relations have to be satisfied:

$$U'^{\text{T}} m_L U' = -m_L^*, \tag{5}$$

$$W'^{\text{T}} M_R W' = -M_R^*, \tag{6}$$

$$U'^{\dagger} m_D W' = m_D{}^*, \tag{7}$$

$$U'^{\dagger} m_l V' = m_l{}^*. \tag{8}$$

It can be easily seen that if there are unitary matrices U', V', W' satisfying equations (5)–(8) in one particular WB, then a solution exists for any other WB. In order to analyse the implications of the above conditions, it is convenient to choose the WB, where both m_L and M_R are real diagonal. In this WB and assuming the eigenvalues of m_L and M_R to be all non-zero and non-degenerate, equations (5) and (6) constrain U' and W' to be of the form

$$U' = \text{diag}\,(\exp(\text{i}\alpha_1), \exp(\text{i}\alpha_2), \ldots \exp(\text{i}\alpha_n)), \tag{9}$$

$$W' = \text{diag}\,(\exp(\text{i}\beta_1), \exp(\text{i}\beta_2), \ldots \exp(\text{i}\beta_n)), \tag{10}$$

where n denotes the number of generations. Here, we are assuming, for simplicity that there is an equal number of fields ν_L^0 and ν_R^0. The phases α_i and β_i have to satisfy

$$\alpha_i = (2p_i + 1)\frac{\pi}{2}, \qquad \beta_i = (2q_i + 1)\frac{\pi}{2} \tag{11}$$

with p_i, q_i integer numbers. Then equations (7) and (8) constrain m_D and $m_l m_l^\dagger \equiv h_l$ in the following way:

$$\text{phase}(m_D)_{ij} = (p_i - q_j)\frac{\pi}{2}, \tag{12}$$

$$\text{phase}(h_l)_{ij} = (p_i - q_j)\frac{\pi}{2}. \tag{13}$$

As a result, CP invariance restricts all the phases of m_D and h_l to be either zero or $\pm\pi/2$. Since in general m_D is an arbitrary $n \times n$ complex matrix whilst h_l is an arbitrary $n \times n$ Hermitian matrix the number of independent CP restrictions is

$$N_g = n^2 + \frac{n(n-1)}{2}. \tag{14}$$

For three generations $N_g = 12$. It is clear that if the number of rh fields were n' rather than n, the matrix m_D would have dimension $n \times n'$ and N_g would be given by

$$N_g' = nn' + \frac{n(n-1)}{2}. \tag{15}$$

It can be checked that this number of CP restrictions coincides with the number of CP violating phases which arise in the leptonic mixing matrix of the charged weak current after all leptonic masses have been diagonalized. Let us now choose the WB such that m_l is already diagonal, real and positive. The diagonalization of the $2n \times 2n$ matrix $\mathcal{M}$, which in general is given by equation (2), is performed via the unitary transformation

$$V^{\mathrm{T}}\mathcal{M}^* V = \mathcal{D}, \tag{16}$$

for $\mathcal{D} = \text{diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_{\nu_1}, M_{\nu_2}, M_{\nu_3})$, with m_{ν_i} and M_{ν_i} denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write V and $\mathcal{D}$ in the following block form:

$$V = \begin{pmatrix} K & R \\ S & T \end{pmatrix}, \tag{17}$$

$$\mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix}. \tag{18}$$

The neutrino weak-eigenstates are related to the mass eigenstates by

$$\nu_{i\,L}^0 = V_{i\alpha}\nu_{\alpha L} = (K, R)\begin{pmatrix} \nu_{iL} \\ N_{iL} \end{pmatrix} \quad \begin{pmatrix} i = 1, 2, 3 \\ \alpha = 1, 2, \ldots 6 \end{pmatrix} \tag{19}$$

and thus the leptonic charged current interactions are given by

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}(\overline{l_{iL}}\gamma_\mu K_{ij}\nu_{jL} + \overline{l_{iL}}\gamma_\mu R_{ij}N_{jL})W^\mu + \text{h.c.} \tag{20}$$

with K and R being the charged current couplings of charged leptons to the light neutrinos ν_j and to the heavy neutrinos N_j, respectively. From equation (17), we see that K and R correspond to the first n rows of the $2n \times 2n$ unitary matrix V which diagonalizes the full neutrino mass matrix $\mathcal{M}^*$. The most general $n \times 2n$ leptonic mixing matrix can then be exactly parametrized by the first n rows of a $2n \times 2n$ unitary matrix provided that it is chosen in such a way that a minimal number of phases appears in these first n rows. This is the case of the parametrization proposed in [12]. Its particularization for a 6×6 matrix is given by

$$V = \hat{V}P, \tag{21}$$

where $P = \text{diag}\,(1, \exp(\mathrm{i}\sigma_1), \exp(\mathrm{i}\sigma_2), \ldots, \exp(\mathrm{i}\sigma_5))$ and $\hat{V}$ is given by

$$\begin{aligned}\hat{V} = {} & O_{56}I_6(\delta_{10})O_{45}O_{46}I_5(\delta_9)I_6(\delta_8)\left(\prod_{j=4}^{6} O_{3j}\right)I_4(\delta_7)I_5(\delta_6)I_6(\delta_5) \\ & \times \left(\prod_{j=3}^{6} O_{2j}\right)I_3(\delta_4)I_4(\delta_3)I_5(\delta_2)I_6(\delta_1)\left(\prod_{j=2}^{6} O_{1j}\right),\end{aligned} \tag{22}$$

where O_{ij} are orthogonal matrices mixing the ith and jth generation and $I_j(\delta_k)$ are unitary diagonal matrices of the form

$$I_j(\delta_k) = \begin{pmatrix} 1 & & & & & & \\ & \cdot & & & & & \\ & & 1 & & & & \\ & & & e^{i\delta_k} & & & \\ & & & & 1 & & \\ & & & & & \cdot & \\ & & & & & & 1 \end{pmatrix} \leftarrow j. \tag{23}$$

This parametrization is particularly useful, for instance, in models with vectorial quarks [13]. It can be readily verified that the first three rows of $\hat{V}$, contain seven phases. The Majorana character of the physical neutrinos does not allow for the five phases in P to be rotated away and we are finally left with 12 phases in the mixing matrix (K, R). The generalization to $n + n'$ dimensional unitary matrices leads to $\frac{1}{2}(n-1)(n-2+2n')$ phases in the first n rows of $\hat{V}$ [13] which, together with the $(n+n'-1)$ phases that cannot be rotated away, adds up to $nn' + \frac{n(n-1)}{2}$ thus coinciding with the general result obtained in equation (15).

2.2. The case of minimal see-saw

The minimal see-saw case corresponds to $\mathcal{L}_m$ with no lh Majorana mass terms included, together with the assumption that the bare rh Majorana mass terms are much larger than the weak scale.

From equations (2), (16), (17) and (18), with $m_L = 0$, one obtains

$$S^\dagger m_D^T K^* + K^\dagger m_D S^* + S^\dagger M_R S^* = d, \tag{24}$$

$$S^\dagger m_D^T R^* + K^\dagger m_D T^* + S^\dagger M_R T^* = 0, \tag{25}$$

$$T^\dagger m_D^T R^* + R^\dagger m_D T^* + T^\dagger M_R T^* = D. \tag{26}$$

We assume, as before, that we are already in a WB where m_l is real and diagonal. These equations allow us to derive the following relations which hold to an excellent approximation:

$$S^\dagger = -K^\dagger m_D M_R^{-1}, \tag{27}$$

$$-K^\dagger m_D \frac{1}{M_R} m_D^T K^* = d. \tag{28}$$

It is clear from equation (27) that S is of order m_D/M_R and therefore is very suppressed. Equation (28) is the usual see-saw formula with the matrix K frequently denoted by V_{PMNS}, the Pontecorvo, Maki, Nakagawa and Sakata matrix [14]. Although the block K in equation (17) is not a unitary matrix its deviations from unitarity are of the order m_D^2/M_R^2. It is from equation (28) that the low-energy physics of the leptonic sector is derived. The decoupling limit corresponds to an effective theory with only lh neutrinos and a Majorana mass matrix, m_{eff} defined as

$$m_{eff} = -m_D \frac{1}{M_R} m_D^T, \tag{29}$$

showing that for m_D of the order of the electroweak scale and M_R of the scale of grand unification, the smallness of light neutrino masses is a natural consequence of the see-saw mechanism [2]. From the relation $\mathcal{M}^* V = V^* \mathcal{D}$ and taking into account the zero entry in $\mathcal{M}$, one derives the following exact relation:

$$R = m_D T^* D^{-1}. \tag{30}$$

This equation plays an important role in the connection between low- and high-energy physics in the leptonic sector. If we choose to work in a WB where both m_l and M_R are diagonal, equation (26) shows that $T = 1$ up to corrections of order m_D^2/M_R^2, leading to an excellent approximation to

$$R = m_D D^{-1}. \tag{31}$$

The matrices K and R are again the charged current couplings. The counting of the number of physical CP violating phases can be done in various ways [15]–[17]. The simplest approach [17] is by choosing a WB where M_R and m_l are simultaneously real and diagonal. From the CP transformations given by equation (4), we now obtain conditions of equations (6), (7) and (8). Once again, equation (6) constrains the matrix W' to be of the form of equation (10) with β_i given by equation (11). Multiplying equation (8) by its Hermitian conjugate, with m_l real and diagonal, one concludes that U' has to be of the form of equation (9), where in this case the α_i are arbitrary phases. From equations (7), (10) and (9) it follows then that CP invariance constrains the matrix m_D to satisfy

$$\arg(m_D)_{ij} = \tfrac{1}{2}(\alpha_i - \beta_j). \tag{32}$$

Note that the β_i are fixed up to discrete ambiguities whilst the α_i are free. Therefore, CP invariance constrains the matrix m_D to have only n free phases α_i. Since m_D is an arbitrary matrix, with n^2 independent phases, it is clear that the number of independent CP restrictions is given by:

$$N_m = n^2 - n. \tag{33}$$

In the minimal see-saw model, for three generations, there are six CP violating phases instead of the 12 of the general case. The decrease in the number of independent phases is to be expected since in this case m_L, which in general is a complex symmetric matrix and would have six phases for three generations, is not present in the theory. We may still use the explicit parametrization given before by equations (21) and (22). Yet, now the angles and phases introduced are no longer independent parameters, there will be special constraints among them. The number of mixing angles [16] is also $(n^2 - n)$, i.e., six mixing angles for three generations. The exact form of these constraints can be derived from $\mathcal{M}^* = V^* \mathcal{D} V^\dagger$ taking into account that $\mathcal{M}$ has a zero entry in the upper left block, which implies

$$K^* d K^\dagger + R^* D R^\dagger = 0. \tag{34}$$

An important physical question is how to distinguish experimentally minimal see-saw from the general case. This is obviously a very difficult (if not impossible) task, since it would require the knowledge of the heavy neutrino masses as well as a detailed knowledge of the matrix R. So far, we have not made any assumption on the type of hierarchy in the light neutrino masses (i.e. normal hierarchy, inverted hierarchy or almost degeneracy). Recently it was argued that in grand unified models with minimal see-saw inverted hierarchy for light neutino masses is theoretically disfavoured [18].

At this stage, it is useful to compare the number of physical parameters—three light and three heavy neutrino masses, three charged lepton masses, six mixing angles and six CP violating phases, giving a total of 21 parameters—to the number of parameters present in the WB where M_R and m_l are simultaneously real and diagonal. In this case these two matrices contain six real parameters. Since m_D is a 3×3 general matrix, it contains nine real parameters and six phases due to the possibility of rotating away three phases on its left-hand side. Thus, there are also 21 parameters in this WB. Obviously, not all WB have the property of containing the minimum number of parameters. It is useful to parametrize m_D as a product of a unitary matrix U times a Hermitian matrix H, which can be done without loss of generality

$$m_D = UH = P_\xi \hat{U}_\rho P_\alpha \hat{H}_\sigma P_\beta. \tag{35}$$

In the second equality, a maximum number of phases were factored out of U and H leaving them with one phase each—ρ and σ respectively, and $P_\xi = \mathrm{diag}\,(\exp(\mathrm{i}\xi_1), \exp(\mathrm{i}\xi_2), \exp(\mathrm{i}\xi_3))$, $P_\alpha = \mathrm{diag}\ (1, \exp(\mathrm{i}\alpha_1), \exp(\mathrm{i}\alpha_2))$ and $P_\beta = \mathrm{diag}\,(1, \exp(\mathrm{i}\beta_1), \exp(\mathrm{i}\beta_2))$. The phases in P_ξ can be eliminated by rotating simultaneously ν_L^0 and l_L^0. Alternatively, one may write m_D, without loss of generality, as the product of a unitary times a lower triangular matrix [19]. This choice may be particularly useful in specific scenarios and it is easy to show how the six independent phases may be chosen [17].

3. WB invariants and CP violation

In this section, we derive simple conditions for CP conservation in the form of WB invariants which have to vanish in order for CP invariance to hold. These conditions are very useful, since they allow us to determine whether or not a given Lagrangian violates CP without the need to go to any special WB or to the physical basis. This is specially relevant in the analysis of lepton flavour models, where the various matrices of Yukawa couplings may have special textures in flavour space reflecting, for example, the existence of a lepton flavour symmetry. In the presence of texture zeros, WB invariants provide the simplest method to investigate whether a specific lepton flavour model leads to leptonic CP violation at low energies or whether the model allows for CP violation at high energies, necessary to generate BAU through leptogenesis.

The method to build WB invariants relevant for CP violation was first proposed in [8] to the quark sector and was soon afterwards extended to the low-energy physics of the leptonic sector [10]; the WB invariant relevant for CP violation with three degenerate light neutrinos was obtained later in [20]. In [17], similar conditions relevant for leptogenesis in the minimal see-saw model with three generations were derived. This approach has been widely applied in the literature [21] to the study of CP violation in many different scenarios.

It was shown in the previous section that CP invariance of the charged gauge currents requires the existence of unitary matrices U', V', W' satisfying equations (5)–(8) or just (6)–(8) depending on whether m_L is introduced. These matrices have different forms in different WB. On the other hand, physically meaningful quantities must be invariant under WB transformations. In order to derive conditions for CP invariance expressed in terms of WB invariants, we combine these equations in a non-trivial way and eliminate the dependence on the above unitary matrices by using the fact that traces and determinants are invariant under similarity transformations. In the next subsections, we present and discuss conditions relevant for different physical situations.

3.1. WB invariants relevant for CP violation at low energies

The different terms of $\mathcal{L}_m$ lead to conditions (5)–(8) for CP invariance. The strategy outlined above can be applied directly to this Lagrangian [10] leading among other interesting possibilities, to the following WB invariant CP conserving condition:

$$\mathrm{tr}[(m_L^* m_L)^a, {h_l}^b]^q = 0, \tag{36}$$

with $h_l = m_l m_l^\dagger$, a, b and q are integers and q is odd. An analogous condition with m_L and h_l replaced by M_R and $h_D = m_D^\dagger m_D$ also holds. In the framework of minimal see-saw, m_L is not present at tree level. However, the low-energy limit of the minimal see-saw corresponds to an effective theory with only lh neutrinos, with an effective Majorana mass matrix m_{eff} given by equation (29) in terms of m_D and M_R. Invariance under CP of the effective Lagrangian implies the following condition for m_{eff}:

$$U'^\dagger m_{eff} U'^* = -m_{eff}^*, \tag{37}$$

which is analogous to equation (5) with m_L replaced by m_{eff}^*. This implies that the conditions relevant to discuss the CP properties of the leptonic sector at low-energies are similar to those involving m_L and h_l in [10] and can be translated into, for instance,

$$\mathrm{tr}[(m_{eff}\ m_{eff}^*)^a,\ {h_l}^b]^q = 0, \tag{38}$$

$$\mathrm{Im}\,\mathrm{tr}[(h_l)^c(m_{eff}\ m^*_{eff})^d(m_{eff}\ h^*_l\ m^*_{eff})^e(m_{eff}\ m^*_{eff})^f] = 0, \tag{39}$$

$$\mathrm{Im}\,\det[(m^*_{eff}\ h_l\ m_{eff}) + r(h^*_l\ m^*_{eff}\ m_{eff})] = 0, \tag{40}$$

where $a, b, \ldots, f$ are integers, q is odd and r is an arbitrary real number. These relations are necessary conditions for CP invariance. The non-vanishing of any of these WB invariants implies CP violation. However, these relations may not be sufficient to guarantee CP invariance. In fact, there are cases where some of them vanish automatically and yet CP may be violated.

It is well known that the minimal structure that can lead to CP violation in the leptonic sector is two generations of lh Majorana neutrinos requiring that their masses be non degenerate and that none of them vanishes. In this case, it was proved [10] that the condition

$$\mathrm{Im}\,\mathrm{tr}\ Q = 0 \tag{41}$$

with $Q = h_l m_{eff} m^*_{eff} m_{eff} h^*_l m^*_{eff}$ is a necessary and sufficient condition for CP invariance.

In the realistic case of three generations of light neutrinos there are three independent CP violating phases relevant at low-energies. In the physical basis they appear in the V_{PMNS} matrix—one of them is a Dirac-type phase analogous to the one appearing in the Cabibbo, Kobayashi and Maskawa matrix, V_{CKM}, of the quark sector and the two additional ones can be factored out of V_{PMNS} but cannot be rephased away due to the Majorana character of the neutrinos. Selecting from the necessary conditions a subset of restrictions which are also sufficient for CP invariance is in general not trivial. For three generations it was shown that the following four conditions are sufficient [10] to guarantee CP invariance

$$\mathrm{Im}\,\mathrm{tr}[h_l\ (m_{eff}\ m^*_{eff})(m_{eff}\ h^*_l\ m^*_{eff})] = 0, \tag{42}$$

$$\mathrm{Im}\,\mathrm{tr}[h_l\ (m_{eff}\ m^*_{eff})^2(m_{eff}\ h^*_l\ m^*_{eff})] = 0, \tag{43}$$

$$\mathrm{Imtr}[h_l(m_{eff}\ m^*_{eff})^2(m_{eff}\ h^*_l\ m^*_{eff})\ (m_{eff}\ m^*_{eff})] = 0, \tag{44}$$

$$\mathrm{Im}\,\det[(m^*_{eff}\ h_l\ m_{eff}) + (h^*_l\ m^*_{eff}\ m_{eff})] = 0 \tag{45}$$

provided that neutrino masses are non-zero and non-degenerate. It can be easily seen that these conditions are trivially satisfied in the case of complete degeneracy ($m_1 = m_2 = m_3$). Yet there may still be CP violation in this case, as will be discussed in section 4.

Leptonic CP violation at low-energies can be detected through neutrino oscillations which are sensitive to the Dirac-type phase, but insensitive to the Majorana-type phases in V_{PMNS}. In any given model, the strength of Dirac-type CP violation can be obtained from the following low-energy WB invariant:

$$Tr[h_{eff}, h_l]^3 = 6i\Delta_{21}\Delta_{32}\Delta_{31}\mathrm{Im}\{(h_{eff})_{12}(h_{eff})_{23}(h_{eff})_{31}\}, \tag{46}$$

where $h_{eff} = m_{eff}m_{eff}{}^\dagger$ and $\Delta_{21} = (m_\mu{}^2 - m_e{}^2)$ with analogous expressions for Δ_{31} and Δ_{32}. This invariant is, of course a special case of equation (38). For three lh neutrinos there is a Dirac-type CP violation if and only if this invariant does not vanish. This quantity can be computed in any WB and can also be fully expressed in terms of physical observables since

$$\mathrm{Im}\{(h_{eff})_{12}(h_{eff})_{23}(h_{eff})_{31}\} = -\Delta m^2_{21}\Delta m^2_{31}\Delta m^2_{32}\mathcal{J}_{CP}, \tag{47}$$

where the Δm^2_{ij}'s are the usual light neutrino mass squared differences and $\mathcal{J}_{CP}$ is the imaginary part of an invariant quartet of the leptonic mixing matrix U_ν, appearing in the difference of the CP-conjugated neutrino oscillation probabilities, such as $P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_\mu)$. It is given by

$$\mathcal{J}_{CP} \equiv \mathrm{Im}[(U_\nu)_{11}(U_\nu)_{22}(U_\nu)^*_{12}(U_\nu)^*_{21}] = \tfrac{1}{8}\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\cos(\theta_{13})\sin\delta, \tag{48}$$

where θ_{ij} and δ are the mixing angles and the Dirac-type phase appearing in the standard parametrization adopted in [22]. The most salient feature of leptonic mixing is the fact that two of the mixing angles $(\theta_{12}, \theta_{23})$ are large, with only θ_{13} being small. This opens the possibility of detecting leptonic CP violation through neutrino oscillations, which requires $\mathcal{J}_{CP}$ to be of order 10^{-2}, a value that can be achieved, provided θ_{13} is not extremely small (at present one only has an experimental bound $\theta_{13} < 0.26$). A similar invariant condition is useful in the quark sector [8], where the corresponding $\mathcal{J}_{CP}$ is of the order 10^{-5}. The search for CP violation in the leptonic sector at low-energies is at present one of the major experimental challenges in neutrino physics. Experiments with superbeams and neutrino beams from muon storage rings (neutrino factories) have the potential [23] to measure directly the Dirac phase δ through CP and T asymmetries or indirectly through oscillation probabilities which are themselves CP conserving but also depend on δ. An alternative method [24] is to measure the area of unitarity triangles defined for the leptonic sector [25].

3.2. *WB invariants relevant for leptogenesis*

One of the most plausible scenarios for the generation of the BAU is the leptogenesis mechanism [3] where a CP asymmetry generated through the out-of-equilibrium L-violating decays of the heavy Majorana neutrinos leads to a lepton asymmetry which is subsequently transformed into a baryon asymmetry by (B + L)-violating sphaleron processes [4].

In this section, we consider thermal leptogenesis in the minimal see-saw scenario. In what follows the notation will be simplified into m and M for m_D and M_R. The lepton number asymmetry, ε_{N_j}, arising from the decay of the jth heavy Majorana neutrino is defined in terms of the family number asymmetry $\Delta A^j{}_i = N^j{}_i - \overline{N}^j{}_i$ by

$$\varepsilon_{N_j} = \frac{\sum_i \Delta A^j{}_i}{\sum_i (N^j{}_i + \overline{N^j}_i)}, \tag{49}$$

the sum in i runs over the three flavours $i = \mathrm{e}\mu\tau$. The evaluation of ε_{N_j}, involves the computation of the interference between the tree-level diagram and one-loop diagrams for the decay of the heavy Majorana neutrino N^j into charged leptons $l_i^{\pm}(i = \mathrm{e}, \mu, \tau)$ which leads to [26]

$$\begin{aligned}\varepsilon_{N_j} &= \frac{g^2}{{M_W}^2}\sum_{k\neq j}\left[\mathrm{Im}((m^\dagger m)_{jk}(m^\dagger m)_{jk})\frac{1}{16\pi}\left(I(x_k) + \frac{\sqrt{x_k}}{1-x_k}\right)\right]\frac{1}{(m^\dagger m)_{jj}} \\ &= \frac{g^2}{{M_W}^2}\sum_{k\neq j}\left[(M_k)^2\mathrm{Im}((R^\dagger R)_{jk}(R^\dagger R)_{jk})\frac{1}{16\pi}\left(I(x_k) + \frac{\sqrt{x_k}}{1-x_k}\right)\right]\frac{1}{(R^\dagger R)_{jj}},\end{aligned} \tag{50}$$

where M_k denote the heavy neutrino masses, the variable x_k is defined as $x_k = \frac{{M_k}^2}{{M_j}^2}$ and $I(x_k) = \sqrt{x_k}(1 + (1 + x_k)\log(\frac{x_k}{1+x_k}))$. From equation (50) it can be seen that the lepton-number asymmetry is only sensitive to the CP-violating phases appearing in $m^\dagger m$ in the WB, where $M_R \equiv M$ is diagonal (notice that this combination is insensitive to rotations of the lh neutrinos). Making use of the parametrization given by equation (35) for $m_D \equiv m$ it becomes clear that leptogenesis is only sensitive to the phases β_1, β_2 and σ. The second equality of equation (50) is established with the help of equation (31).

WB invariant conditions relevant for leptogenesis must be sensitive to these three phases, clearly meaning that they must be expressed in terms of $h = m^\dagger m$. From condition equation (7), we obtain

$$W'^\dagger h W' = h^*. \tag{51}$$

Only the matrix M is also sensitive to the W' rotation. From condition equation (6), we derive

$$W'^\dagger H W' = H^*, \tag{52}$$

where $H = M^\dagger M$. From these two new conditions, together with equation (6) it can be readily derived that CP invariance requires [17]:

$$I_1 \equiv \mathrm{Im}\,\mathrm{Tr}[hHM^*h^*M] = 0, \tag{53}$$

$$I_2 \equiv \mathrm{Im}\,\mathrm{Tr}[hH^2M^*h^*M] = 0, \tag{54}$$

$$I_3 \equiv \mathrm{Im}\,\mathrm{Tr}[hH^2M^*h^*MH] = 0 \tag{55}$$

as well as many other expressions of the same type. These conditions can be computed in any WB and are necessary and sufficient to guarantee that CP is conserved at high energies. This was shown by going to the WB where M is real and diagonal. In this basis the I_i's are then of the form:

$$\begin{aligned} I_1 = {} & M_1M_2({M_2}^2 - {M_1}^2)\mathrm{Im}({h_{12}}^2) + M_1M_3({M_3}^2 - {M_1}^2)\mathrm{Im}({h_{13}}^2) \\ & + M_2M_3({M_3}^2 - {M_2}^2)\mathrm{Im}({h_{23}}^2), \end{aligned} \tag{56}$$

$$\begin{aligned} I_2 = {} & M_1M_2({M_2}^4 - {M_1}^4)\mathrm{Im}({h_{12}}^2) + M_1M_3({M_3}^4 - {M_1}^4)\mathrm{Im}({h_{13}}^2) \\ & + M_2M_3({M_3}^4 - {M_2}^4)\mathrm{Im}({h_{23}}^2), \end{aligned} \tag{57}$$

$$\begin{aligned} I_3 = {} & {M_1}^3{M_2}^3({M_2}^2 - {M_1}^2)\mathrm{Im}({h_{12}}^2) + {M_1}^3{M_3}^3({M_3}^2 - {M_1}^2)\mathrm{Im}({h_{13}}^2) \\ & + {M_2}^3{M_3}^3({M_3}^2 - {M_2}^2)\mathrm{Im}({h_{23}}^2) = 0. \end{aligned} \tag{58}$$

These are a set of linear equations in terms of the variables $\mathrm{Im}({h_{ij}}^2) = \mathrm{Im}((m^\dagger m)_{ij}(m^\dagger m)_{ij})$ appearing in equation (50). The determinant of the coefficients of this set of equations is

$$\mathrm{Det} = {M_1}^2{M_2}^2{M_3}^2\Delta^2{}_{21}\Delta^2{}_{31}\Delta^2{}_{32}, \tag{59}$$

where $\Delta_{ij} = ({M_i}^2 - {M_j}^2)$. Non-vanishing of the determinant implies that all imaginary parts of $(h_{ij})^2$ should vanish, in order for equations (53)–(55) to hold. Conversely, the non-vanishing of any of the I_i implies CP violation at high energies, relevant for leptogenesis.

4. The case of degenerate neutrinos

The present experimental data on light neutrino masses rule out the exact degeneracy. However, since neutrino oscillations measure neutrino mass differences and not the absolute mass scale, both hierarchical neutrino masses and quasi-degenerate neutrino masses are allowed, by present experimental data. If neutrinos turn out to be almost degenerate the limit of exact degeneracy may be relevant in explaining some of the features of the leptonic mixing matrix, for instance, the observed smallness of one of the mixing angles. An open question is whether there is a model in which this limit corresponds to a symmetry of the Lagrangian. In this case, the quasi-degeneracy of neutrino masses would be natural, since it would correspond to small deviations of an exact symmetry. In the case of Dirac neutrinos, the limit of exact mass degeneracy is trivial, since there is no mixing or CP violation in that limit. The situation is entirely different for Majorana neutrinos, since in that case one can have both mixing and CP violation even in the limit of exact degeneracy. The proof is simple [10] and follows from equation (37) together with

$$U'^{\dagger} h_l U' = h_l^*, \tag{60}$$

which is readily obtained from equation (8). Let us consider the low-energy limit, where only lh neutrinos are relevant and assume that there are three lh Majorana neutrinos with exact degenerate masses. Without loss of generality, one can choose to work in a WB where the effective lh neutrino mass matrix is diagonal, real. Since, we are assuming the exact degeneracy limit, the mass matrix is just proportional to the unit matrix. We have seen that invariance under CP requires equation (37) to be satisfied by some unitary matrix U'. In the case of degeneracy and in the WB we have chosen, equation (37) is satisfied provided that $U' = iO$ (with O an orthogonal matrix). In addition, we still have the freedom to make a change of WB such that m_{eff} is unchanged and $\mathrm{Re}h_l$ becomes diagonal. In this basis equation (60) can be split into

$$O^T(\mathrm{Re}h_l)O = \mathrm{Re}h_l, \tag{61}$$

$$O^T(\mathrm{Im}h_l)O = -\mathrm{Im}h_l. \tag{62}$$

From equation (61) and assuming $\mathrm{Re}h_l$ to be non-degenerate the matrix O is constrained to be of the form $O = \mathrm{diag}(\epsilon_1, \ldots, \epsilon_n)$ with $\epsilon_i = \pm 1$. This in turn implies from equation (62) that, in the general case of non vanishing $(\mathrm{Im}h_l)_{ij}$, the ϵ_i have to obey the conditions

$$\epsilon_i \cdot \epsilon_j = -1, \quad i \neq j. \tag{63}$$

Clearly these conditions cannot be simultaneously satisfied for more than two generations.

In the general case of three light neutrinos V_{PMNS} can be parametrized by three angles and three phases. In the limit of exact degeneracy, in general mixing cannot be rotated away and V_{PMNS} is parametrized by two angles and one CP violating phase. We shall denote the corresponding leptonic mixing matrix by U_0. It has been shown [20] that in general this matrix cannot be rotated away. Only in the case where the theory is CP invariant and the three degenerate neutrinos have the same CP parity can U_0 be rotated away.

In the WB, where the charged lepton mass matrix is diagonal, real and positive the neutrino mass matrix is diagonalized by the transformation

$$U_0^{\dagger} \cdot m_{eff} \cdot U_0^* = \mu \cdot \mathbb{1}, \tag{64}$$

where μ is the common neutrino mass. Let us define the dimensionless matrix $Z_0 = m_{eff}/\mu$. From equation (64), we obtain

$$Z_0 = U_0 \cdot U_0^T, \tag{65}$$

which is unitary and symmetric. The matrix Z_0 can be written without loss of generality as

$$Z_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} c_\theta & s_\theta & 0 \\ s_\theta & z_{22} & z_{23} \\ 0 & z_{23} & z_{33} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix}. \tag{66}$$

Unitarity of Z_0 implies that either s_θ or z_{23} must vanish. The case $s_\theta = 0$ automatically leads to CP invariance. Assuming $s_\theta \neq 0$, the most general form for the symmetric unitary matrix Z_0 is then given by

$$Z_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} c_\theta & s_\theta & 0 \\ s_\theta & -c_\theta & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix}. \tag{67}$$

This choice of Z_0 does not include the trivial case where CP is a good symmetry and all neutrinos have the same CP parity. In fact, in the CP conserving case where $\mathrm{e}^{\mathrm{i}\alpha} = \pm 1$ one has $\mathrm{Tr}(Z_0) = -\det(Z_0) = \pm 1$ corresponding to the eigenvalues $(1, -1, 1)$ and $(1, -1, -1)$ and permutations. It is well known [27] that different relative signs correspond to different CP parities. From equations (65) and (67), we conclude that the mixing matrix U_0 must be of the form

$$U_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) & 0 \\ \sin(\dfrac{\theta}{2}) & -\cos(\frac{\theta}{2}) & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\alpha/2} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{i} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{68}$$

up to an arbitrary orthogonal transformation $U_0 \to U_0 \cdot O$. Notice that U_0 cannot be rotated away due to the fact that it is not an orthogonal matrix, even in the CP conserving case. The matrix U_0 is parametrized by two angles θ, ϕ and one-phase α. In the limit of exact degeneracy a necessary and sufficient condition [20] for CP invariance is

$$G \equiv \mathrm{Tr}[(m_{eff}^* \cdot h_l \cdot m_{eff}), h_l^*]^3 = 0. \tag{69}$$

In the WB where h_l is diagonal, i.e., $h_l = \mathrm{diag}\,(m_e^2,\ m_\mu^2,\ m_\tau^2)$, it can be written as

$$G = 6\mathrm{i}\ \Delta_m \mathrm{Im}[(Z_0)_{11}^*(Z_0)_{22}^*(Z_0)_{12}(Z_0)_{21}] = -\frac{3\mathrm{i}}{2}\Delta_m \cos(\theta)\sin^2(\theta)\sin^2(2\phi)\sin(\alpha), \tag{70}$$

where $\Delta_m = \mu^6(m_\tau^2 - m_\mu^2)^2(m_\tau^2 - m_e^2)^2(m_\mu^2 - m_e^2)^2$ is a multiplicative factor which contains the different masses of the charged leptons and the common neutrino mass μ. One may wonder whether equation (68) would be a realistic mixing matrix for the case of three non-degenerate neutrinos. It has been shown [20] that this is indeed the case. In fact this matrix corresponds to $\sin\theta_{13} = 0$ (of the standard parametrization), solar neutrino data only constrain the angle θ, whilst atmospheric neutrino data only constrain ϕ. Neutrinoless double beta decay depends on θ and light neutrino masses. The angle α can be factored out in U_0 and is thus a Majorana-type phase.

Heavy Majorana neutrinos may play a crucial role in the generation of the BAU. If these particles are indeed responsible for BAU they must obey certain constraints (such as a lower limit in their mass). It is common to assume heavy neutrino masses to be hierarchical in the study of thermal leptogenesis since this corresponds to the simplest scenario, which is sometimes called minimal leptogenesis. Currently, there are no direct experimental constraints on heavy neutrino masses and the possibility of quasi-degenerate heavy Majorana neutrinos remains open.

5. On the relation between low-energy CP violation and CP violation required for leptogenesis

5.1. A brief summary of low-energy data

There has been great experimental progress in the determination of leptonic masses and mixing in the last few years. The evidence for solar and atmospheric neutrino oscillations is now solid. The pattern of leptonic mixing (V_{PMNS}) is very different from that of the quark sector (V_{CKM}), since only one of the leptonic mixing angles, θ_{13}, is small. The latest great progress reported is in the measurement of the square mass difference relevant for solar oscillations, Δm^2_{21}, and is due to recent KamLAND results [28]. KamLAND is a terrestial long baseline experiment which has great sensitivity to Δm^2_{21}, but it does not constrain θ_{12} much better than the current set of solar experiments. The combined result including those of SNO [29] and previous solar experiments [30] is for the 1σ range [1]

$$\Delta m^2_{21} = 8.2^{+0.3}_{-0.3} \times 10^{-5}\mathrm{eV}^2, \tag{71}$$

$$\tan^2\theta_{12} = 0.39^{+0.05}_{-0.04} \tag{72}$$

and corresponds to the large mixing angle solution (LMA) of the Mikheev, Smirnov and Wolfenstein effect [31] with the upper island excluded. On the other hand, atmospheric neutrino results from Super-Kamiokande [32] and recent important progress by K2K [33], which is also a terrestrial long baseline experiment, are consistent with, for the 1σ range [1]:

$$\Delta m^2_{32} = 2.2^{+0.6}_{-0.4} \times 10^{-3}\mathrm{eV}^2, \tag{73}$$

$$\tan^2\theta_{23} = 1.0^{+0.35}_{-0.26}. \tag{74}$$

The present bounds for $\sin^2\theta_{13}$ from the CHOOZ experiment [34] have been somewhat relaxed since they depend on Δm^2_{31} and this value went down. Assuming the range for Δm^2_{32} from Super-Kamiokande and K2K, the 3σ bound [1] lies in $\sin^2\theta_{13} < 0.05$–0.07. A higher value for the angle θ_{13} is good news for the prospectives of detection of low-energy leptonic CP violation, mediated through a Dirac-type phase, whose strength is given by $\mathcal{J}_{CP}$ defined in section 3. Direct kinematic limits on neutrino masses [35] from Mainz and Troitsk and neutrinoless double beta decay experiments [36] when combined with the given square mass differences exclude light neutrino masses higher than order 1 eV. Non-vanishing light neutrino masses also have an important impact in cosmology. Recent data from the Wilkinson microwave anisotropy probe, WMAP [37, 38], together with other data, put an upper bound on the sum of light neutrino masses of 0.7 eV.

In the context of the see-saw mechanism, the smallness of light neutrino masses is related to the existence of heavy neutrinos. These heavy neutrinos may in turn play an important cosmological role via the generation of BAU through leptogenesis. Since leptogenesis requires CP violation at high energies, one may ask whether there is a connection between CP violation at low energies and CP violation at high energies. This question will be addressed in the next subsection.

5.2. *On the need for a lepton flavour symmetry*

The expression for the lepton-number asymmetry resulting from the decay of heavy Majorana neutrinos is given by equation (50). Yet leptogenesis is a complicated thermodynamical non-equilibrium process and depends on additional parameters. The simplest scenario corresponds to heavy hierarchical neutrinos, where M_1 is much smaller than M_2 and M_3. The case of almost degeneracy of heavy neutrinos has been considered by several authors [39] and corresponds to a resonant enhancement of ε_{N_j}. In the hierarchical case, the baryon asymmetry depends only on four parameters [40]: the mass M_1 of the lightest heavy neutrino, together with the corresponding CP asymmetry ε_{N_1} in their decays, as well as the effective neutrino mass $\tilde{m}_1$ defined as [41]

$$\tilde{m}_1 = (m^\dagger m)_{11}/M_1, \tag{75}$$

in the WB where M is diagonal, real and positive and, finally, the sum of all light neutrino masses squared, $\bar{m}^2 = m_1^2 + m_2^2 + m_3^2$. It has been shown that this sum controls an important class of washout processes. Successful leptogenesis would require ε_{N_1} of order 10^{-8}, if washout processes could be neglected, in order to reproduce the observed ratio of baryons to photons [37]

$$\frac{n_B}{n_\gamma} = (6.1^{+0.3}_{-0.2}) \times 10^{-10}. \tag{76}$$

Leptogenesis is a non-equilibrium process that takes place at temperatures $T \sim M_1$. This imposes an upper bound on the effective neutrino mass $\tilde{m}_1$ given by the 'equilibrium neutrino mass' [42]

$$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v^2}{M_{Pl}} \simeq 10^{-3}\mathrm{eV}, \tag{77}$$

where M_{Pl} is the Planck mass ($M_{Pl} = 1.2 \times 10^{19}$ GeV), $v = \langle \phi^0 \rangle / \sqrt{2} \simeq 174$ GeV is the weak scale and g_* is the effective number of relativistic degrees of freedom in the plasma and equals 106.75 in the SM case. Yet, it has been shown [43] that successful leptogenesis is also possible for $\tilde{m}_1 > m_*$, in the range from $\sqrt{\Delta m_{12}^2}$ to $\sqrt{\Delta m_{23}^2}$. In this case the full out-of-equilibrium condition is not verified and the temperature where the inverse decays freeze out is smaller than M_1. This range is particularly interesting since it corresponds to the strong washout regime where theoretical uncertainties are small. Furthermore, the value of the baryonic asymmetry of the universe can still be large enough and is entirely determined by neutrino properties. The square root of the sum of all neutrino masses squared $\bar{m}$ is constrained, in the case of normal hierarchy, to be below 0.20 eV [43], which corresponds to an upper bound on light neutrino masses very close to 0.10 eV. This result is sensitive to radiative corrections which depend on top and Higgs masses as well as on the treatment of thermal corrections. In [44], a slightly higher value of 0.15 eV is found. This bound can be relaxed for instance in various scenarios including models with quasi-degenerate heavy neutrinos [39], non-thermal leptogenesis scenarios [45] or also theories with Higgs triplets [46] leading to non-minimal see-saw mechanism. In the limit $M_1 \ll M_2, M_3,\ \varepsilon_{N_1}$ can be simplified into

$$\varepsilon_{N_1} \simeq -\frac{3}{16\pi v^2}\left(I_{12}\frac{M_1}{M_2} + I_{13}\frac{M_1}{M_3}\right), \tag{78}$$

where

$$I_{1i} \equiv \frac{\mathrm{Im}[(m^\dagger m)^2_{1i}]}{(m^\dagger m)_{11}} \tag{79}$$

and a lower bound on the lightest heavy neutrino mass M_1 is derived. Depending on the cosmological scenario, the range for minimal M_1 varies from order 10^7 to 10^9 Gev [40, 44].

Viability of leptogenesis is thus closely related to low-energy parameters, in particular the light neutrino masses. This raises the question of whether the same is true for CP violation at both low and high energies. Part of the answer to this question [47] is given here in section 3.2 where it was shown that leptogenesis depends only on three of the phases of the parametrization introduced by equation (35), to $\beta_1,\ \beta_2$ and σ, whilst the phases in V_{PMNS} depend on all six phases [17]. The question remains whether a CP conserving low-energy theory (no Dirac-type and no Majorana-type phases) would still allow for high-energy CP violation. The answer is yes [48], since the matrix m can be parametrized in such a way that V_{PMNS} cancels out in the product $m^\dagger m$ and all the additional phases remaining in this product cancel out in m_{eff}. As a result, any connection between CP violation at low and at high energies is model dependent. More specifically, in order to establish the above connection, one has to restrict the number of free parameters in the lepton flavour sector. An elegant way of obtaining such restrictions is through the introduction of a lepton-flavour symmetry. There is another motivation for restricting the number of free parameters in the lepton flavour sector. This has to do with the fact that, contrary to what happens in the quark sector, without lepton flavour restrictions, it is not possible to fully reconstruct the low energy neutrino mass matrix from low-energy data obtainable through feasible experiments [49].

Several authors have studied the connection between CP violation at low and at high energies in various interesting scenarios [50]. An important motivation for such studies is the attempt to show whether or not the BAU was generated through leptogenesis.

5.3. Towards a minimal scenario

A particular minimal scenario allowing to establish a link between BAU generated through leptogenesis and CP violation at low energies was considered in [51]. The starting point was to write m, the Dirac-type neutrino mass matrix, as the product of a unitary times a lower triangular matrix in the WB where M and m_l are diagonal and real. As pointed out before there is no lack of generality in choosing this parametrization. The strategy was then to simplify this matrix m in order to obtain physical constraints. Starting from

$$m_D = UY_\triangle, \tag{80}$$

with $Y_\triangle$ of the form

$$Y_\triangle = \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21}e^{i\phi_{21}} & y_{22} & 0 \\ y_{31}e^{i\phi_{31}} & y_{32}e^{i\phi_{32}} & y_{33} \end{pmatrix}, \tag{81}$$

where y_{ij} are real positive numbers, it follows that U does not play any role for leptogenesis since it cancels out in the product $m^\dagger m$. It is clear that a necessary condition for a direct link between leptogenesis and low-energy CP violation to exist is the requirement that the matrix U contains no CP violating phases. The simplest possible choice, corresponding to $U = \mathbb{1}$, was made. Next, further simplifying restrictions were imposed on $Y_\triangle$ in order to obtain minimal scenarios based on the triangular decomposition. These correspond to special zero textures together with assumptions on the hierarchy of the different entries. Only two patterns with one additional zero in $Y_\triangle$ were found to be consistent with low-energy physics (either with hierarchical heavy neutrinos or two-fold quasi degeneracy)

$$\begin{pmatrix} y_{11} & 0 & 0 \\ y_{21}e^{i\phi_{21}} & y_{22} & 0 \\ 0 & y_{32}e^{i\phi_{32}} & y_{33} \end{pmatrix}, \qquad \begin{pmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & 0 \\ y_{31}e^{i\phi_{31}} & y_{32}e^{i\phi_{32}} & y_{33} \end{pmatrix}. \tag{82}$$

In both cases, there are two independent phases. A further simplification is to assume one of these phases to vanish. Special examples were built and it was shown that it is possible to obtain viable leptogenesis in this class of models and at the same time obtain specific predictions for low-energy physics once the known experimental constraints are imposed. In particular all the textures considered predicted the existence of low-energy CP violating effects in the range of sensitivity of future long baseline experiments. It should be noted that strong hierarchies in the entries of masses matrices could in principle be generated by the Froggatt–Nielsen mechanism [52].

The question of whether the sign of the BAU can be related to CP violation in neutrino oscillation experiments was addressed by considering models with only two heavy neutrinos [53]. In this case, the Dirac mass matrix has dimension 3×2. The interesting examples correspond to textures of the form given above in equation (82) with the third column eliminated and corresponds to the most economical extension of the SM leading to leptogenesis. With the elimination of the third column one more phase in the third row can be rotated away, hence only one physical phase remains. In fact, there are fewer parameters in this case and these are strongly constrained by low-energy physics thus leading to a definite relative sign between $\mathrm{Im}\,(m^\dagger m)^2_{12}$ and $\sin 2\delta$ (with δ the Dirac-type phase of V_{PMNS}).

6. Summary and conclusions

We have reviewed leptonic CP violation and neutrino mass models, with emphasis on the use of WB invariants to study CP violation at low and high energies, as well as on the possible connection between leptonic CP violation at low energies and CP violation required for the generation of the BAU through leptogenesis. We have identified the WB invariant which measures the strength of Dirac-type CP violation at low energies for three generations of light neutrinos and have presented the simplest WB invariants which are sensitive to CP violation required by leptogenesis. These WB invariants are specially relevant for the study of any given lepton-flavour model, where Yukawa couplings are constrained by lepton-flavour symmetries leading, for example, to texture zeros in the leptonic mass matrices. The usefulness of the invariants stems from the fact that they can be applied in any WB, without having to perform any cumbersome change of basis.

Most of our analysis was done in the framework of the minimal see-saw mechanism, where there is a closer connection between low-energy data and leptogenesis. We have also considered some special cases such as the limit of exact degeneracy, illustrating the fact that for three Majorana neutrinos, both leptonic mixing and CP violation can exist even in the limit where neutrinos are exactly degenerate.

In conclusion, neutrino physics provides an invaluable tool to the study of the question of leptonic flavour and CP violation at low energies, while at the same time having profound implications to the physics of the early universe, in particular to the generation of the BAU.

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