Supernovae constraints on dark energy and modified gravity models

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Abstract. We use the Type Ia Supernova *gold* sample to constrain the parameters of dark energy models namely the Cardassian, Dvali-Turner (DT) and generalized Chaplygin gas (GCG) models. In our best fit analysis for these dark energy proposals we consider flat and the non-flat priors. For all models, we find that relaxing the flatness condition implies that data favors a positive curvature; moreover, the GCG model is nearly flat, as required by Cosmic Microwave Background (CMB) observations.

1. Introduction

Various proposals have been put forward to explain recent observations indicating that the universe is accelerating. A possible explanation is that the universe is filled with dark energy in the form of an exotic component, the generalized Chaplygin gas, with negative equation of state [1, 2, 3]. The striking feature of this model is that it allows for an unification of dark energy and dark matter [3].

Another possible explanation for the accelerated expansion of the Universe could be the infrared modification of gravity one should expect from extra dimensional physics, which would lead to a modification of the effective Friedmann equation at late times. A concrete model has been suggested by Dvali, Gabadadze and Porrati [5] and later generalized by Dvali and Turner [4]. Another possibility is the modification of the Friedmann equation by the introduction of an additional nonlinear term proportional to ρ^n, the so-called Cardassian model [6].

Currently type Ia supernovae (SNe Ia) observations provide the most direct way to probe the dark energy component at low to medium redshifts. Recently, supernovae data has been analysed by various groups and it was shown that it yields relevant constraints on some cosmological parameters. In particular, it is possible to conclude that, when one considers the full supernova data set, the decelerating model is ruled out with a significant confidence level [7]. It is also shown that one can measure the current value of the dark energy equation of state with higher accuracy and the data prefers the phantom kind of equation of state, $w_X < -1$. Furthermore, the most significant result of that analysis is that, without a flat prior, supernovae data does not favour a flat ΛCDM model at least up to 68% confidence level, which is consistent with other

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Table 1. Best fit parameters for the Cardassian, DT and GCG models, considering flat and non-flat priors.

Cardassian model	Ω_m	n	Ω_k	χ^2
Flat Prior	0.49	-1.4	$-$	173.7
Non-Flat Prior	0.21	-3.1	0.47	173.2
DT model	Ω_m	β	Ω_k	χ^2
Flat Prior	0.51	-19.2	$-$	174.7
Non-Flat Prior	0.24	-60.0	0.43	174.0
GCG model	A_s	α	Ω_k	χ^2
Flat Prior	0.93	2.8	$-$	174.2
Non-Flat Prior	0.97	4.0	0.02	174.5

cosmological observations. In what concerns the equation of state of the dark energy component, it has been shown in Ref. [8], using the same set of supernovae data, that the best fit equation of state of dark energy evolves rapidly from $w_X \simeq 0$ in the past to $w_X \sim -1$ at present, which suggests that a time varying dark energy fits the data better than the ΛCDM model.

In this paper, we analyze the Cardassian, the DT and the GCG models using so called *gold* sample SNe Ia compilation of data by Riess *et al.* [9]; we consider both flat and non-flat priors. For more details see Ref. [10].

2. Observational constraints from supernovae data
The observations of supernovae measure essentially the apparent magnitude m, which is related to the (dimensionless) luminosity distance D_L by

$$m(z) = \mathcal{M} + 5\log_{10} D_L(z) \ , \tag{1}$$

where

$$D_L(z) \equiv H_0(1+z) \int_0^z \frac{1}{H(z')} dz' \ . \tag{2}$$

Also,

$$\mathcal{M} = M + 5\log_{10}\left(\frac{c/H_0}{1 \text{ Mpc}}\right) + 25 \ , \tag{3}$$

where M is the absolute magnitude which is believed to be constant for all supernovae of type Ia. For our analysis, we consider the set of supernovae data recently compiled by Riess *et al.* [9] known as the *gold* sample. The data points in this sample are given in terms of the distance modulus $\mu_{\text{obs}}(z) \equiv m(z) - M_{\text{obs}}(z)$ and the χ^2 is calculated from

$$\chi^2 = \sum_{i=1}^n \left[\frac{\mu_{\text{obs}}(z_i) - \mathcal{M}' - 5\log_{10} D_{L\text{th}}(z_i; c_\alpha)}{\sigma_{\mu_{\text{obs}}}(z_i)}\right]^2 \ , \tag{4}$$

where $\mathcal{M}' = \mathcal{M} - M_{\text{obs}}$ is a free parameter and $D_{L\text{th}}(z; c_\alpha)$ is the theoretical prediction for the dimensionless luminosity distance of a supernova at a particular distance, for a given model with parameters c_α. The errors $\sigma_{\mu_{\text{obs}}}(z)$ take into account the effects of peculiar motions. Minimization of the χ^2, with respect to $\mathcal{M}'$, Ω_m, Ω_k and the respective model parameter(s) leads to the results summarized in Table 1. The best fit value for $\mathcal{M}'$ is 43.3 for all models.

3. Cardassian model

We first consider the Cardassian model [6]; in this model, the universe is composed only of radiation and matter and the increasing expansion rate is given by

$$H^2 = \frac{8\pi}{3M_{\mathrm{Pl}}^2}\left(\rho + b\rho^n\right) - \frac{k}{a^2} \tag{5}$$

where $M_{\mathrm{Pl}} = 1.22 \times 10^{19}\mathrm{GeV}$ is the 4-dimensional Planck mass, b and n are constants, and we have added a curvature term to the original Cardassian model. The new term dominates only recently, hence in order to get the recent acceleration in the expansion rate, $n < 2/3$ is required.
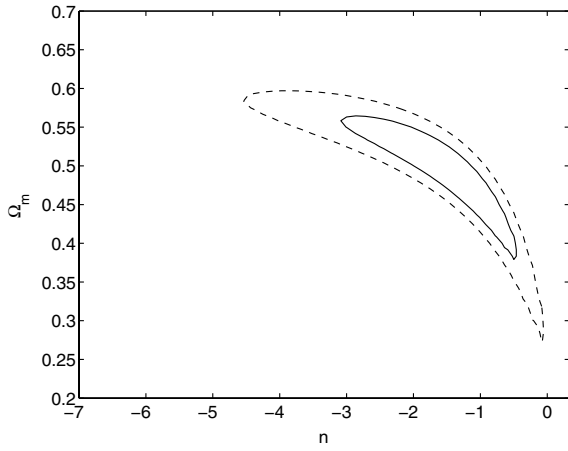

Figure 1. Confidence contours for the flat Cardassian model. The solid and dashed lines represent the 68% and 95% confidence regions, respectively.

At present, the universe is matter dominated and Eq. (5) can be rewritten as

$$\left(\frac{H}{H_0}\right)^2 = \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + (1 - \Omega_m - \Omega_k)(1+z)^{3n} \ , \tag{6}$$

where H_0 is the present value of the Hubble constant and $\Omega_k = -\frac{k}{a_0^2 H_0^2}$ is the present curvature parameter. Notice that the case $n = 0$ corresponds to the ΛCDM model.
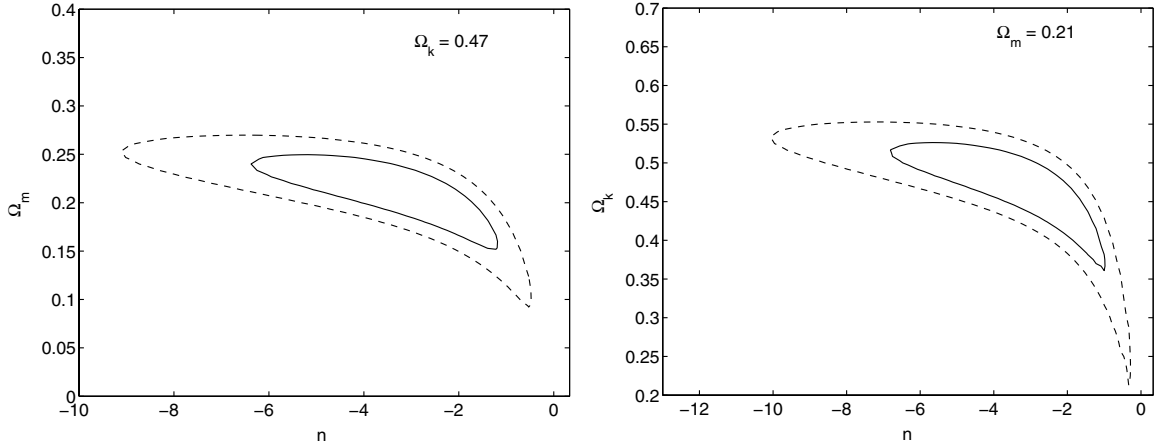

Figure 2. As for Figure 1, for the non-flat Cardassian model.

For the flat case, we have only two parameters and their best fit values are $\{\Omega_m, n\} = \{0.49, -1.4\}$. In Fig. 1 we show the 68% and 95% confidence contour plots, where it is clear that the ΛCDM model is excluded at a 95% confidence level.

If we relax the flat prior, we find that the best fit values are $\Omega_k = 0.47$, $\Omega_m = 0.21$ and $n = -3.1$. Moreover (see Fig. 2) the case $n = 0$ is again excluded at 95% confidence level.
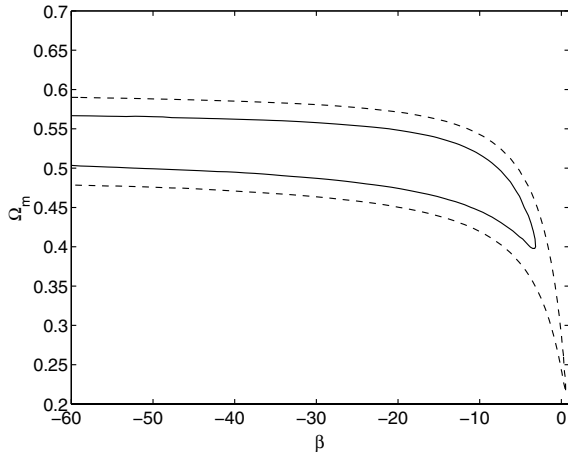

Figure 3. Confidence contours for the flat DT model. The solid and dashed lines represent the 68% and 95% confidence regions, respectively.

4. Dvali-Turner model

The second model we consider is the one proposed by Dvali & Turner [4], where the Friedmann equation is modified by the addition of a new term which may arise in theories with extra dimensions [5]

$$H^2 - \frac{H^\beta}{r_c^{2-\beta}} = \frac{8\pi}{3M_{\mathrm{Pl}}^2}\rho - \frac{k}{a^2} \ , \tag{7}$$

where r_c is a crossover scale which sets the scale beyond which the laws of the 4-dimensional gravity breakdown and become 5-dimensional and we have introduced a curvature term.
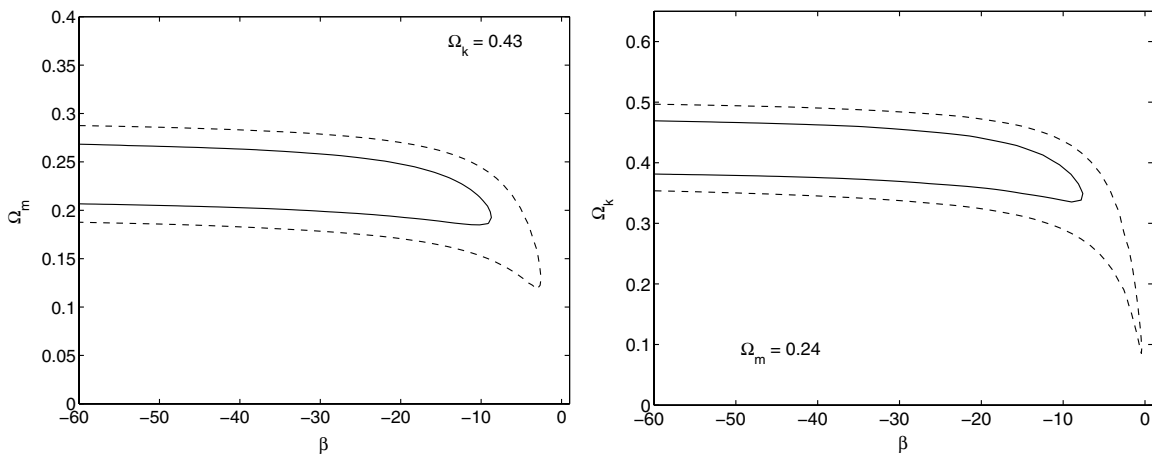

Figure 4. Confidence contours for the non-flat DT model.

If we assume that the universe is matter dominated the Friedmann expansion law can be

written as

$$\left(\frac{H}{H_0}\right)^2 = \Omega_m(1+z)^3 + (1 - \Omega_m - \Omega_k)\left(\frac{H}{H_0}\right)^{\beta} + \Omega_k(1+z)^2 . \tag{8}$$

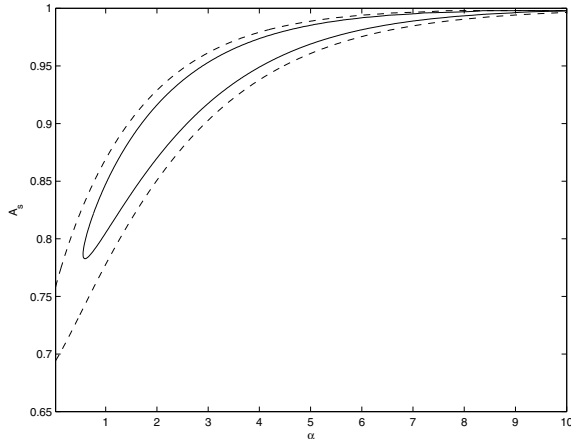Notice that β is the only parameter of the model: for $\beta = 0$ the new term behaves like a

Figure 5. Confidence contours for the flat GCG model.

cosmological constant and the case $\beta = 1$ corresponds to the Dvali-Gabadadze-Porrati (DGP) model [5]. The requirement that the new term does not interfere with the formation of large-scale structure leads to the bound $\beta \leq 1$.
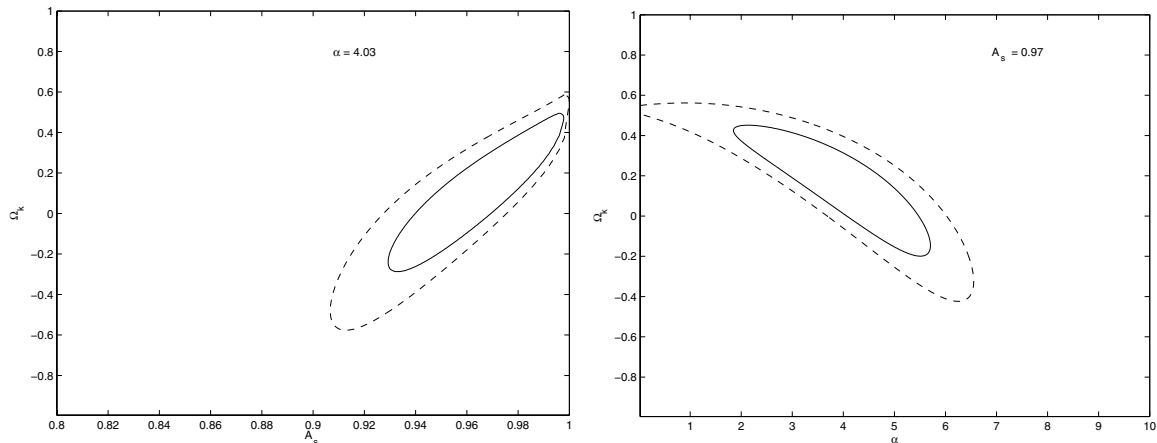
Figure 6. Confidence contours for the non-flat GCG model.

From Fig. 3, it is clear that β is very weakly constrained and can become arbitrarily large and negative. Moreover, both the ΛCDM model ($\beta = 0$) and the DGP model ($\beta = 1$), are strongly disfavoured. Fixing $\beta = 1$ (DGP model), we get $\Omega_m = 0.17$.

If we relax the flat prior, we find that the best fit results are for $\{\Omega_m, \Omega_k, \beta\} = \{0.24, 0.43, -60, 0\}$. As we can see from the contour plots shown in Fig. 4, β can become arbitrarily large and negative, if we allow Ω_m to be large. Moreover, both ΛCDM and DGP models are disfavoured at 95% C.L.

5. Generalized Chaplygin gas model

Finally, we consider the generalized Chaplygin gas model, which is characterized by the equation of state

$$p_{ch} = -\frac{A}{\rho_{ch}^{\alpha}} \ , \tag{9}$$

where A and α are positive constants. For $\alpha = 1$, the equation of state is reduced to the Chaplygin gas scenario [1]. Integrating the energy conservation equation with the equation of state (9), one gets [3]

$$\rho_{ch} = \rho_{ch0} \left[A_s + \frac{(1 - A_s)}{a^{3(1+\alpha)}} \right]^{1/(1+\alpha)} \ , \tag{10}$$

where ρ_{ch0} is the present energy density of GCG and $A_s \equiv A/\rho_{ch0}^{(1+\alpha)}$. Hence, ρ_{ch}, interpolates between a dust dominated phase, $\rho_{ch} \propto a^{-3}$, in the past and a de-Sitter phase, $\rho_{ch} = -p_{ch}$, at late times. This property makes the GCG model an interesting candidate for the unification of dark matter and dark energy. Notice that $\alpha = 0$ corresponds to the ΛCDM model.

The Friedmann equation for a non-flat unified GCG model is given by

$$\left(\frac{H}{H_0} \right)^2 = (1 - \Omega_k) \left[A_s + (1 - A_s)(1+z)^{3(1+\alpha)} \right]^{1/(1+\alpha)} + \Omega_k(1+z)^2 \ . \tag{11}$$

This model has been thoroughly scrutinized from the observational point of view; for a CMB power spectrum compatibility analysis see e.g. Ref. [11] and for a previous supernovae data analysis see Ref. [12] and references therein. The 68% and 95% confidence level contours for the flat case are shown in Fig. 5. Notice that the ΛCDM model is consistent at 95% C.L although it is ruled out at 68% C.L.

When we relax the condition of flat prior, the best fit model becomes very close to flat ($\Omega_k = 0.02$) which is a a new result. The confidence contours for this case are shown in Fig. 6.

6. Conclusions

We performed a likelihood analysis of the latest type Ia supernovae data for three models: the modified gravity Cardassian and Dvali-Turner models and the generalized Chaplygin gas model of unification of dark energy and dark matter. We find that SNe Ia most recent data allows, in all cases, for non-trivial constraints on model parameters as summarized in Table 1. We find that, for all models, relaxing the flatness condition implies that data favors a positive curvature and the GCG model is nearly flat in this case. The fact that the *gold* sample of supernovae data prefers a flat GCG model, which is consistent with CMB observations, leads us to conclude that the GCG is a better choice among the three alternative models that we have considered.

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