

# Enlarging the window for radiative leptogenesis

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## Abstract

We investigate the scenario of resonant thermal leptogenesis, in which the leptonic asymmetries are generated through renormalization group corrections induced at the leptogenesis scale. In the framework of the standard model extended by three heavy Majorana neutrinos with masses  $M_1 = M_2 \ll M_3$  at some high scale, we show that the mass splitting and  $CP$ -violating effects induced by renormalization group corrections can lead to values of the  $CP$  asymmetries large enough for a successful leptogenesis. In this scenario, the low-energy neutrino oscillation data can also be easily accommodated. The possibility of having an underlying symmetry behind the degeneracy in the right-handed neutrino mass spectrum is also discussed.

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## 1. Introduction

Among the viable mechanisms to explain the matter–antimatter asymmetry observed in the universe, leptogenesis [1] has undoubtedly become one of the most compelling ones [2,3]. Indeed, its simplicity and close connection with low-energy neutrino physics render leptogenesis an attractive and eventually testable scenario. Unfortunately, even in its simplest realization through the well-known seesaw mechanism [4], the theory is plagued with too many parameters. To appreciate this point, let us recall that in the framework of the standard model (SM) extended with three heavy Majorana neutrinos  $N_i$  ( $i = 1, 2, 3$ ), the high-energy neutrino sector, characterized by the Dirac neutrino ( $m_D$ ) and the heavy Majorana neutrino ( $M_R$ ) mass matrices, has eighteen parameters. Of them, only nine combinations enter into the seesaw effective neutrino mass matrix  $m_D M_R^{-1} m_D^T$ , thus making difficult to establish a direct link between leptogenesis and low-energy phenomenology [5]. Furthermore, there are six  $CP$ -violating phases which are physically relevant at high energies, while only three combinations of them are potentially observable at low energies. Therefore,

no direct link between the sign of the baryon asymmetry and low-energy leptonic  $CP$  violation can be established, unless extra assumptions are introduced.

In a more economical framework [6], where only two heavy Majorana neutrinos are present, one is left with eleven high-energy parameters, three of which are physical phases. But since in this case one light neutrino is predicted to be massless, there remain only seven independent low-energy neutrino parameters, two of which are  $CP$ -violating phases. Thus, additional assumptions are usually required to completely determine the high-energy neutrino sector from low-energy observables. Typical examples are the introduction of texture zeros in the Yukawa matrices or the imposition of symmetries to constrain their structure [7]. In this respect, the heavy Majorana neutrino masses are rather unconstrained: they can range from the TeV region to the GUT scale, and the spectrum can be hierarchical, quasi-degenerate or even exactly degenerate [8]. Despite this arbitrariness, the heavy Majorana neutrino mass scale (and, consequently, the seesaw scale) turns out to be crucial for a successful implementation of the leptogenesis mechanism. In particular, the standard thermal leptogenesis scenario with hierarchical heavy Majorana neutrino masses ( $M_1 \ll M_2 < M_3$ ) requires  $M_1 \gtrsim 4 \times 10^8$  GeV [9], if  $N_1$  is in thermal equilibrium before it decays, or the more restrictive lower bound

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$M_1 \gtrsim 2 \times 10^9$  GeV [10] for a zero initial  $N_1$  abundance. Since this bound also determines the lowest reheating temperature allowed after inflation, it could be problematic in supersymmetric theories due to the overproduction of light particles like the gravitino.

Nevertheless, it is important to note that the above bounds are not model independent in the sense that they can be avoided, if the heavy Majorana neutrino spectrum is no longer hierarchical. Indeed, if at least two of the  $N_i$  are quasi-degenerate in mass, i.e.,  $M_1 \simeq M_2$ , then the leptonic  $CP$  asymmetry relevant for leptogenesis exhibits the resonant behaviour  $\varepsilon_1 \sim M_1/(M_2 - M_1)$  [11,12]. In this case, it is possible to show that the upper bound on the  $CP$  asymmetry is independent of the light neutrino masses and successful leptogenesis simply requires  $M_{1,2}$  to be above the electroweak scale for the sphaleron interactions to be effective. Of course, having such a degeneracy in the neutrino masses requires a compelling justification. Not being accidental, the quasi-degeneracy may arise, for instance, from some flavour symmetry softly broken at a high scale [13].

Another possibility which has been recently explored [14,15] relies on the fact that radiative effects, induced by the renormalization group (RG) running from high to low energies, can naturally lead to a sufficiently small neutrino mass splitting at the leptogenesis scale. In the latter case, non-vanishing and sufficiently large  $CP$  asymmetries, which are proportional to the charged-lepton  $\tau$  Yukawa coupling and weakly dependent on the heavy Majorana neutrino mass scale, are generated. Although in the SM framework the resulting baryon asymmetry turns out to be (by a factor of two) below the observed value [14], this mechanism can be successfully implemented in its minimal supersymmetric extension (MSSM) [15]. However, it is worth emphasizing that the above results have been obtained in a minimal seesaw scenario with only two heavy neutrinos. In such a case, low seesaw scales require extremely small Dirac neutrino Yukawa couplings for both  $N_1$  and  $N_2$ , in order to avoid too large low-energy neutrino masses. One may therefore ask whether the above problems can be overcome in a more realistic scenario where the effects of a third heavy Majorana neutrino  $N_3$  are also taken into account.

The purpose of this Letter is to further investigate the scenario of radiative leptogenesis proposed in Ref. [14], in which the leptonic  $CP$  asymmetries are generated through RG corrections induced at the leptogenesis scale. In the framework of the standard model extended by the addition of three heavy Majorana neutrinos with masses  $M_1 = M_2 \ll M_3$  at some high scale, we show that the mass splitting induced by the running of the heavy neutrino masses can lead to values of the  $CP$  asymmetries large enough for a successful leptogenesis. In this scenario, the observed baryon asymmetry and low-energy neutrino oscillation data can be easily reconciled. Moreover, since the results depend very weakly on the gap between the degeneracy and leptogenesis scales, low right-handed neutrino masses and reheating temperatures are acceptable, thus avoiding the well-known problem of overproduction of relic abundances in early universe. Finally, we shall also comment on possible symmetries which could explain the degeneracy of right-handed neutrino masses at high energies.

## 2. Radiative leptogenesis

In the SM extended by the addition of three right-handed neutrinos, the relevant Yukawa and heavy Majorana neutrino mass terms in the Lagrangian are

$$\mathcal{L} \propto \bar{\ell}_L Y_\ell \ell_R \phi^0 + \bar{\nu}_L Y_\nu N \phi^0 - \frac{1}{2} N^T C M_R N + \text{H.c.}, \quad (1)$$

where  $\ell$  and  $\nu$  refer to the charged-lepton and neutrino fields, respectively (family indices are omitted);  $Y_\ell$  and  $Y_\nu$  are the charged-lepton and Dirac neutrino Yukawa coupling matrices and  $\phi^0$  denotes the neutral component of the SM Higgs doublet. After integrating out the heavy Majorana neutrinos  $N$ , the light neutrino mass matrix, resulting from the seesaw mechanism, is given by

$$\mathcal{M} = -v^2 Y_\nu M_R^{-1} Y_\nu^T, \quad v \equiv \langle \phi^0 \rangle. \quad (2)$$

In the basis where  $Y_\ell$  and  $M_R$  are diagonal, all the parameter space can be conveniently spanned through the parametrisation [16],

$$Y_\nu = \frac{1}{v} U d^{1/2} R D^{1/2}, \quad (3)$$

where  $d = \text{diag}(m_1, m_2 e^{i\alpha}, m_3 e^{i\beta})$ ,  $D = \text{diag}(M_1, M_2, M_3)$ ;  $m_i$  are the light neutrino masses and  $\alpha, \beta$  are Majorana phases. The matrix  $R$  is an arbitrary  $3 \times 3$  complex orthogonal matrix, which can be parametrised in terms of complex angles  $\theta_i$  as

$$R = \begin{pmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ -s_1 c_3 - c_1 s_2 s_3 & c_1 c_3 - s_1 s_2 s_3 & c_2 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -s_1 s_2 c_3 - c_1 s_3 & c_2 c_3 \end{pmatrix}, \quad (4)$$

where  $s_i \equiv \sin \theta_i$ ,  $c_i \equiv \cos \theta_i$ . Finally, the matrix  $U$  is the standard Pontecorvo–Maki–Nakagawa–Sakata (PMNS) leptonic mixing matrix, which contains the  $CP$ -violating Dirac phase  $\delta$ . It turns out that the parametrisation (3) is also particularly convenient to disentangle the  $CP$ -violating phases relevant for leptogenesis from the low-energy phases. Indeed, the combination

$$H \equiv Y_\nu^\dagger Y_\nu = \frac{1}{v^2} D^{1/2} R^\dagger |d| R D^{1/2}, \quad (5)$$

which appears in physical quantities associated with leptogenesis is only sensitive to the phases in  $R$  and not to the phases  $\alpha, \beta$  and  $\delta$ . In terms of the matrix elements of  $R$ , the matrix  $H$  reads as

$$H_{ij} = \frac{\sqrt{M_i M_j}}{v^2} \sum_{k=1}^3 m_k R_{ki}^* R_{kj} \quad (i, j = 1, 2, 3). \quad (6)$$

Let us now discuss how the resonant leptogenesis mechanism works in the present framework. We assume an exact degeneracy of two heavy Majorana neutrinos, so that  $M_1 = M_2 \equiv M \ll M_3$  at a scale  $\Lambda$ , which is higher than the decoupling scale of the heaviest neutrino  $N_3$ . The parameter

$$\delta_N \equiv \frac{M_2}{M_1} - 1, \quad (7)$$

quantifies the degree of degeneracy between  $M_1$  and  $M_2$  at lower scales. Assuming that the interactions involving  $N_{1,2}$

are in thermal equilibrium at the time the heaviest neutrino  $N_3$  decays, only the leptonic  $CP$  asymmetries generated in the out-of-equilibrium decays of  $N_1$  and  $N_2$  will be relevant for leptogenesis. These asymmetries are given by

$$\varepsilon_j \simeq \frac{\text{Im}[H_{21}^2]}{16\pi H_{jj}\delta_N} \left(1 + \frac{\Gamma_i^2}{4M_j^2\delta_N^2}\right)^{-1},$$

$$\Gamma_i = \frac{H_{ii}M_i}{8\pi}, \quad i, j = 1, 2 \ (i \neq j), \quad (8)$$

where  $\Gamma_i$  are the tree-level decay widths. Notice that Eqs. (8) exhibit the expected resonant enhancement due to the mixing of two nearly degenerate heavy Majorana neutrinos [11]. In the present framework, a sufficiently small heavy neutrino mass splitting will be generated through the RG running effects. The latter turn out to be crucial in this case [14,15,17].

The evolution of the right-handed neutrino masses and the Dirac neutrino Yukawa matrix is given at one-loop by [18]

$$\frac{dM_i}{dt} = 2M_i H_{ii},$$

$$\frac{dY_\nu}{dt} = \left[ T - \frac{3}{4}g_Y^2 - \frac{9}{4}g_2^2 - \frac{3}{2}(Y_\ell Y_\ell^\dagger - Y_\nu Y_\nu^\dagger) \right] Y_\nu + Y_\nu A, \quad (9)$$

where

$$t \equiv \frac{1}{16\pi^2} \ln(\mu/\Lambda),$$

$$T = 3 \text{Tr}(Y_u Y_u^\dagger) + 3 \text{Tr}(Y_d Y_d^\dagger) + \text{Tr}(Y_\ell Y_\ell^\dagger) + \text{Tr}(Y_\nu Y_\nu^\dagger),$$

$Y_{u,d}$  are the up-quark and down-quark Yukawa matrices and  $g_{Y,2}$  are the gauge couplings. The matrix  $A$  is anti-Hermitian with

$$A_{jj} = 0,$$

$$A_{jk} = \frac{M_j + M_k}{M_k - M_j} \text{Re}(H_{jk}) + i \frac{M_k - M_j}{M_j + M_k} \text{Im}(H_{jk})$$

$$= -A_{jk}^*. \quad (10)$$

As is clear from the above equation, to avoid the singularity of  $A_{12}$  at the degeneracy scale  $\Lambda$ , we must require  $\text{Re}(H_{12}) = 0$ . This condition can always be guaranteed without loss of generality. Indeed, when  $M_1 = M_2$  there is the freedom to rotate the right-handed neutrino fields  $N_{1,2}$  with a real orthogonal matrix that does not change  $M_R$ , but rotates  $Y_\nu$  to the appropriate basis [14,15]. In terms of the parametrisation (3), this is equivalent to a redefinition of the real part of the complex angle  $\theta_1$ . In this sense,  $\text{Re}\theta_1$  is no longer a free parameter, i.e., it is constrained by the condition  $\text{Re}(H_{12}) = 0$ .

At this point it is worthwhile to comment on the number of physical parameters in the high-energy neutrino sector. From the previous analysis it is clear that in the case of two degenerate heavy Majorana neutrinos, there remain 16 physical parameters out of 18. A similar conclusion can be easily drawn by parametrising the Dirac neutrino Yukawa coupling matrix in the general form  $Y_\nu = VY_\Delta$ . Here  $V$  is a unitary matrix containing 3  $CP$ -violating phases and  $Y_\Delta$  is a lower triangular matrix with real diagonal entries and having in general 3 phases in the off-diagonal [5]. It is then straightforward to show that the

requirement  $\text{Re}(H_{12}) = 0$  leads to a constraint on one of the physical phases of  $Y_\Delta$ . This should not come as a surprise. Indeed, the correct counting of independent  $CP$ -violating phases always requires that one chooses an appropriate basis.

We now proceed with the estimate of the radiatively induced  $CP$  asymmetries at the leptogenesis scale  $\mu \approx M$  [14]. At a given scale  $\mu$ , the degeneracy parameter  $\delta_N$  is approximately given by

$$\delta_N(t) \simeq 2(H_{22} - H_{11})t. \quad (11)$$

From this simple expression we see that the lifting of the  $N_1$ – $N_2$  degeneracy requires  $H_{22} \neq H_{11}$ . Moreover, even if at the degeneracy scale  $\Lambda$  one has  $\text{Re}(H_{12}) = 0$ , a non-vanishing real part will be generated by quantum corrections. From Eqs. (9) and (10) we find

$$\text{Re}[H_{21}(t)] \simeq -\frac{3}{2}y_\tau^2 \text{Re}[(Y_\nu)_{31}^*(Y_\nu)_{32}]t. \quad (12)$$

Thus, neglecting the RG running of  $\text{Im}(H_{21})$ , one has for the  $CP$ -violating part appearing in the leptonic asymmetries,

$$\text{Im}[H_{21}^2(t)] \simeq -3y_\tau^2 \text{Im}[H_{21}(0)] \text{Re}[(Y_\nu)_{31}^*(Y_\nu)_{32}]t, \quad (13)$$

where  $y_\tau$  is the  $\tau$  Yukawa coupling. In terms of the elements of  $R$ , the quantity  $\text{Re}[(Y_\nu)_{31}^*(Y_\nu)_{32}]$  reads as

$$\text{Re}[(Y_\nu)_{31}^*(Y_\nu)_{32}] = \frac{M}{v^2} \sum_{i,j=1}^3 \sqrt{m_i m_j} \text{Re}[R_{i1}^* R_{j2} U_{3i}^* U_{3j}]. \quad (14)$$

Here, the dependence of the radiatively induced  $\text{Re}(H_{12})$  on the low-energy parameters is evident through the presence of the light neutrino masses  $m_i$  and the elements of the third row of the mixing matrix  $U$ . We also notice that only a small dependence on the mixing parameter  $U_{e3}$  is expected.

Substituting Eqs. (11) and (13) into Eq. (8) we obtain the following expressions for the leptonic  $CP$  asymmetries:

$$\varepsilon_{1,2} \simeq \varepsilon_{1,2}^0 (1 + D_{2,1})^{-1}, \quad (15)$$

where  $\varepsilon_{1,2}^0$  are the uncorrected  $CP$  asymmetries and  $D_{1,2}$  are correction factors which include the effects of the heavy Majorana decay widths

$$\varepsilon_j^0 \simeq \frac{3y_\tau^2}{32\pi} \frac{\text{Im}(H_{21}) \text{Re}[(Y_\nu)_{31}^*(Y_\nu)_{32}]}{H_{jj}(H_{22} - H_{11})}, \quad (16)$$

$$D_j \simeq \frac{1}{(32\pi)^2} \frac{H_{jj}^2}{(H_{22} - H_{11})^2 t^2}. \quad (17)$$

We note that the leptonic  $CP$  asymmetries do not depend explicitly on the heaviest mass  $M_3$ . Moreover, if the corrections due to the inclusion of the decay widths in the propagators are negligible, i.e.,  $D_j \ll 1$ , the asymmetries are also independent of the mass  $M$  of the two lightest right-handed Majorana neutrinos.

The expressions for the leptonic  $CP$  asymmetries in terms of the parameters  $\theta_i$  are quite long. Therefore, we will consider some interesting limiting cases for which simple analytical expressions can be obtained and the viability of the present mechanism is readily demonstrated. The simplest cases are clearly

those with a single non-vanishing parameter  $\theta_i$ . Since  $\theta_1 = \theta_2 = 0$  and  $\theta_1 = \theta_3 = 0$  imply  $H_{12} = 0$ , these would lead to vanishing asymmetries. Thus, we are left with the case  $\theta_2 = \theta_3 = 0$ , which we consider next.

### 2.1. The case $\theta_2 = \theta_3 = 0$

Since in this case the condition  $\text{Re}(H_{12}) = 0$  implies that the complex angle  $\theta_1$  is purely imaginary, i.e.,  $\theta_1 = i\omega_1$ , the elements of the matrix  $H$  relevant for leptogenesis are simply given by

$$\begin{aligned} H_{11} &= \frac{M}{v^2} (m_1 \cosh^2 \omega_1 + m_2 \sinh^2 \omega_1), \\ H_{22} &= \frac{M}{v^2} (m_1 \sinh^2 \omega_1 + m_2 \cosh^2 \omega_1), \\ H_{12} &= i \frac{M}{2v^2} \sinh(2\omega_1)(m_1 + m_2), \end{aligned} \quad (18)$$

so that

$$H_{11} - H_{22} = \frac{M}{v^2} (m_1 - m_2). \quad (19)$$

It is interesting to note that there is a direct connection between the induced heavy Majorana mass splitting parameter  $\delta_N$  (cf. Eq. (11)) and the low-energy neutrino mass spectrum.

Below the degeneracy scale  $\Lambda$ ,  $H_{12}$  develops a real part proportional to

$$\text{Re}[(Y_\mu)_{31}^* (Y_\nu)_{32}] \simeq \frac{M}{v^2} \sqrt{m_1 m_2} \text{Re}(U_{32}^* U_{31}). \quad (20)$$

According to Eq. (16), the uncorrected leptonic  $CP$ -asymmetries  $\varepsilon_1^0$  and  $\varepsilon_2^0$  are then given by

$$\begin{aligned} \varepsilon_1^0 &\simeq \frac{3y_\tau^2}{64\pi} \frac{(m_1 + m_2) \sqrt{m_1 m_2} \sinh(2\omega_1) \text{Re}(U_{32}^* U_{31})}{(m_1 - m_2)(m_1 \cosh^2 \omega_1 + m_2 \sinh^2 \omega_1)}, \\ \varepsilon_2^0 &\simeq \frac{3y_\tau^2}{64\pi} \frac{(m_1 + m_2) \sqrt{m_1 m_2} \sinh(2\omega_1) \text{Re}(U_{32}^* U_{31})}{(m_1 - m_2)(m_1 \sinh^2 \omega_1 + m_2 \cosh^2 \omega_1)}. \end{aligned} \quad (21)$$

The corrections due to the inclusion of the heavy Majorana neutrino decay widths are obtained from Eq. (17),

$$\begin{aligned} D_1 &\simeq \left[ \frac{\pi}{2} \frac{m_1 \cosh^2 \omega_1 + m_2 \sinh^2 \omega_1}{(m_2 - m_1) \ln(\Lambda/M)} \right]^2, \\ D_2 &\simeq \left[ \frac{\pi}{2} \frac{m_1 \sinh^2 \omega_1 + m_2 \cosh^2 \omega_1}{(m_2 - m_1) \ln(\Lambda/M)} \right]^2. \end{aligned} \quad (22)$$

It is interesting to analyse how the  $CP$ -asymmetries  $\varepsilon_{1,2}$  behave in some limiting cases. First, we notice that for  $m_1 = 0$ , the quantities  $\varepsilon_{1,2}^0$  vanish and no lepton asymmetry is generated. Consequently, in this case a lower bound on  $m_1$  is expected in order to reproduce the observed baryon asymmetry.<sup>1</sup> On the other hand, being  $\varepsilon_{1,2}^0$  proportional to  $(m_2 - m_1)^{-1}$ , one could expect an enhancement in the limit  $m_1 \simeq m_2$ . However, in this case the corrections due to the heavy Majorana decay widths become important, since  $D_{1,2} \propto (m_2 - m_1)^{-2}$ . From Eqs. (15),

(21) and (22) one gets that  $\varepsilon_{1,2} \propto m_2 - m_1$  which explains the suppression of the baryon asymmetry for  $m_1 \simeq m_2$ . This is the case of quasi-degenerate or inverted-hierarchical light neutrinos. In conclusion, when  $\theta_2 = \theta_3 = 0$  and the light neutrinos are hierarchical, one can expect an interval of intermediate values of the lightest mass  $m_1$  for which the radiative leptogenesis mechanism could lead to a sufficient baryon asymmetry.

In contrast to what happens in the standard thermal leptogenesis scenario, the  $CP$ -asymmetries in the present framework depend explicitly on the PMNS mixing matrix  $U$ . In the simple case under analysis, this dependence appears through the combination  $\text{Re}(U_{32}^* U_{31})$  as shown in Eqs. (21). Thus, the final value of the baryon asymmetry will depend on the particular values of  $U_{e3}$  and the low-energy  $CP$ -violating phases  $\alpha$ ,  $\beta$  and  $\delta$  (as well as on the neutrino mass-squared differences and the solar and atmospheric mixing angles, which, however, we assume already fixed by the data).

### 2.2. The case $m_1 = 0$

In spite of all the major experimental advances in the measurement of the neutrino mixing parameters, no information about leptonic  $CP$  violation is available yet. While the Dirac phase  $\delta$  can be potentially measured in future neutrino oscillation experiments, the only hope for probing the Majorana phases  $\alpha$  and  $\beta$  seems to reside in neutrinoless double  $\beta$  decay processes, which if observed, could provide only a single constraint on these phases [19]. In practical terms, this means that one cannot perform a perfect experiment to completely determine the effective neutrino mass matrix  $\mathcal{M}$  from input data. Nevertheless, if this matrix appears to be constrained so that the number of independent parameters is reduced, then it is reasonable to require this constraint to be weak-basis independent. One example of such a constraint is the condition  $\det \mathcal{M} = 0$  [20]. In this case, a massless neutrino is predicted and the spectrum is fully hierarchical. As already mentioned, a similar situation is verified in a minimal seesaw framework with only two right-handed heavy Majorana neutrinos, which automatically leads to  $m_1 = 0$ . It is therefore of interest to investigate whether the present mechanism is compatible with the above light neutrino mass spectrum.

First we notice that in the case that  $m_1 = 0$ , the so-called minimal seesaw scenario, which corresponds to the two heavy Majorana neutrino limit, can be obtained by setting  $\theta_1 = i\omega_1$ ,  $\theta_2 = \pi/2$  and  $\theta_3 = 0$ . Therefore, to present our analytical results we consider the simplest generalisation of the latter case by letting  $\theta_2 \equiv \omega_2$  to be an arbitrary real parameter. We then find

$$\begin{aligned} H_{11} &= \frac{M}{v^2} (m_2 \sinh^2 \omega_1 + m_3 \cosh^2 \omega_1 \sin^2 \omega_2), \\ H_{22} &= \frac{M}{v^2} (m_2 \cosh^2 \omega_1 + m_3 \sinh^2 \omega_1 \sin^2 \omega_2), \\ H_{12} &= i \frac{M}{2v^2} \sinh(2\omega_1) (m_2 + m_3 \sin^2 \omega_2). \end{aligned} \quad (23)$$

<sup>1</sup> This is no longer true in the most general case  $\theta_{2,3} \neq 0$ , as confirmed by the results of the next section.



Moreover,

$$H_{11} - H_{22} = \frac{M}{v^2} (-m_2 + m_3 \sin^2 \omega_2) \quad (24)$$

and

$$\text{Re}[(Y_\mu)_{31}^* (Y_\nu)_{32}] \simeq -\frac{M}{v^2} \sqrt{m_2 m_3} \sin \omega_2 \text{Re}(U_{32}^* U_{33}). \quad (25)$$

From the above equations it is clear that, contrarily to what happened in the previous case where the radiatively generated  $\delta_N$  and  $\text{Re}(H_{12})$  depended exclusively on low-energy parameters, these two quantities depend now also on the structure of the orthogonal matrix  $R$  through the parameter  $\omega_2$ .

The leptonic  $CP$ -asymmetries  $\varepsilon_i^0$  are given in this case by

$$\begin{aligned} \varepsilon_1^0 &\simeq \frac{3y_\tau^2}{64\pi} \frac{\sqrt{m_2 m_3} \sin \omega_2 \sinh(2\omega_1)}{(-m_2 + m_3 \sin^2 \omega_2)} \\ &\quad \times \frac{(m_2 + m_3 \sin^2 \omega_2) \text{Re}(U_{32}^* U_{33})}{(m_2 \sinh^2 \omega_1 + m_3 \cosh^2 \omega_1 \sin^2 \omega_2)}, \\ \varepsilon_2^0 &\simeq \frac{3y_\tau^2}{64\pi} \frac{\sqrt{m_2 m_3} \sin \omega_2 \sinh(2\omega_1)}{(-m_2 + m_3 \sin^2 \omega_2)} \\ &\quad \times \frac{(m_2 + m_3 \sin^2 \omega_2) \text{Re}(U_{32}^* U_{33})}{(m_2 \cosh^2 \omega_1 + m_3 \sinh^2 \omega_1 \sin^2 \omega_2)}, \end{aligned} \quad (26)$$

and the factors  $D_i$  read

$$\begin{aligned} D_1 &\simeq \left[ \frac{\pi}{2} \frac{m_2 \sinh^2 \omega_1 + m_3 \cosh^2 \omega_1 \sin^2 \omega_2}{(-m_2 + m_3 \sin^2 \omega_2) \ln(\Lambda/M)} \right]^2, \\ D_2 &\simeq \left[ \frac{\pi}{2} \frac{m_2 \cosh^2 \omega_1 + m_3 \sinh^2 \omega_1 \sin^2 \omega_2}{(-m_2 + m_3 \sin^2 \omega_2) \ln(\Lambda/M)} \right]^2. \end{aligned} \quad (27)$$

As expected, when  $\omega_2 = \pi/2$  the results of the minimal see-saw scenario considered in [14,15] are recovered. Thus, the new contributions coming from the mixing with the heaviest Majorana neutrino  $N_3$  turn out to be crucial in this case for the mechanism to be viable.

From Eqs. (26) and (27) it is clear that the leptonic  $CP$ -asymmetries  $\varepsilon_i$  vanish when

$$\sin^2 \omega_2 = \frac{m_2}{m_3} = \left( \frac{\Delta m_\odot^2}{\Delta m_a^2} \right)^{1/2}, \quad (28)$$

where  $\Delta m_\odot^2$  and  $\Delta m_a^2$  are the mass-squared differences measured in solar and atmospheric neutrino oscillation experiments, respectively. For values of  $\omega_2$  close to the above value, the contribution of the coefficients  $D_i$  becomes relevant. We also note that for small values of  $\omega_2$  one has  $\varepsilon_1/\varepsilon_2 \simeq \coth^2 \omega_1 > 1$ .

### 3. Numerical results and discussion

The most recent WMAP results and BBN analysis of the primordial deuterium abundance imply [21]

$$\eta_B = \frac{n_B}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}, \quad (29)$$

for the baryon-to-photon ratio of number densities. In the leptogenesis framework, once a lepton asymmetry has been generated by the out-of-equilibrium decays of the heavy Majorana

neutrinos, it will be converted into a baryon asymmetry by non-perturbative sphaleron interactions. The efficiency in producing the asymmetry is controlled by the parameters

$$K_i = \frac{\tilde{m}_i}{m_*}, \quad \tilde{m}_i = \frac{v^2 H_{ii}}{M_i}, \quad (30)$$

where  $m_* \simeq 10^{-3}$  eV is the so-called equilibrium neutrino mass. The resulting baryon asymmetry can be estimated as

$$\eta_B \simeq -10^{-2} (\kappa_1 \varepsilon_1 + \kappa_2 \varepsilon_2), \quad (31)$$

where  $\kappa_i < 1$  are the efficiency factors, which account for the washout effects. An accurate computation of these factors requires the solution of the relevant Boltzmann equations. In our numerical calculations we make use of the Boltzmann equations derived in Ref. [12], which are appropriate for resonant leptogenesis and, therefore, suitable to the cases considered here. We also remark that leptogenesis in the present framework always occurs in a strong washout regime. Indeed, from Eqs. (18), (23) and (30) it follows that

$$K_1 + K_2 > K_\odot \equiv \frac{(\Delta m_\odot^2)^{1/2}}{m_*} \simeq 9. \quad (32)$$

In this situation, the simple decay-plus-inverse-decay picture is applicable and the final baryon asymmetry is essentially independent of the initial conditions [22].

Our numerical computations proceed as follows. We start at  $\mu = M_Z$  with the best-fit values for the solar and atmospheric neutrino oscillation parameters [23]:

$$\begin{aligned} \tan^2 \theta_{12} &= 0.45, & \Delta m_\odot^2 &= 8.0 \times 10^{-5} \text{ eV}^2, \\ \tan^2 \theta_{23} &= 1.0, & \Delta m_a^2 &= 2.5 \times 10^{-3} \text{ eV}^2. \end{aligned} \quad (33)$$

For a given set of  $U_{e3}$ ,  $m_1$  and  $CP$ -violating phases, the low-energy effective neutrino mass matrix  $\mathcal{M} = U^\dagger \text{diag}(m_1, m_2, m_3)U^*$  is constructed. For the hierarchical (inverted-hierarchical) neutrino mass spectrum the lightest neutrino mass  $m_1$  ( $m_3$ ) is an input parameter. The two remaining masses are

$$m_2^2 = m_1^2 + \Delta m_\odot^2, \quad m_3^2 = m_1^2 + \Delta m_\odot^2 + \Delta m_a^2$$

for a hierarchical spectrum, and

$$m_1^2 = m_3^2 + \Delta m_a^2 - \Delta m_\odot^2, \quad m_2^2 = m_3^2 + \Delta m_a^2$$

for an inverted hierarchy. For a particular choice of the input parameters, the RG equations for the neutrino masses and mixing angles are solved up to the degeneracy scale, which we consider to be  $\Lambda \simeq 10^{16}$  GeV. At this stage, we do not consider the running effects due to the Dirac neutrino Yukawa couplings above the mass of the lightest heavy Majorana neutrino. We then define  $Y_\nu$  at the scale  $\Lambda$  as in Eq. (3), using a specific pattern for  $R$  and fixing the values of  $M_1 = M_2 = M$ . The value of  $M_3 > M$  is fixed by requiring the largest Dirac neutrino Yukawa coupling to be equal to the top-quark Yukawa coupling  $y_t$ . For the simplest viable scenario where  $R$  is parametrised by  $\theta_1 = i\omega_1$  and  $\theta_2 = \theta_3 = 0$ , this is equivalent to  $M_3 = y_t^2 v^2 / m_3$ .

All the couplings and masses are subsequently evolved down to the scale  $\mu = M$ , considering also the decoupling of  $N_3$ . At this scale the baryon asymmetry is computed as described at

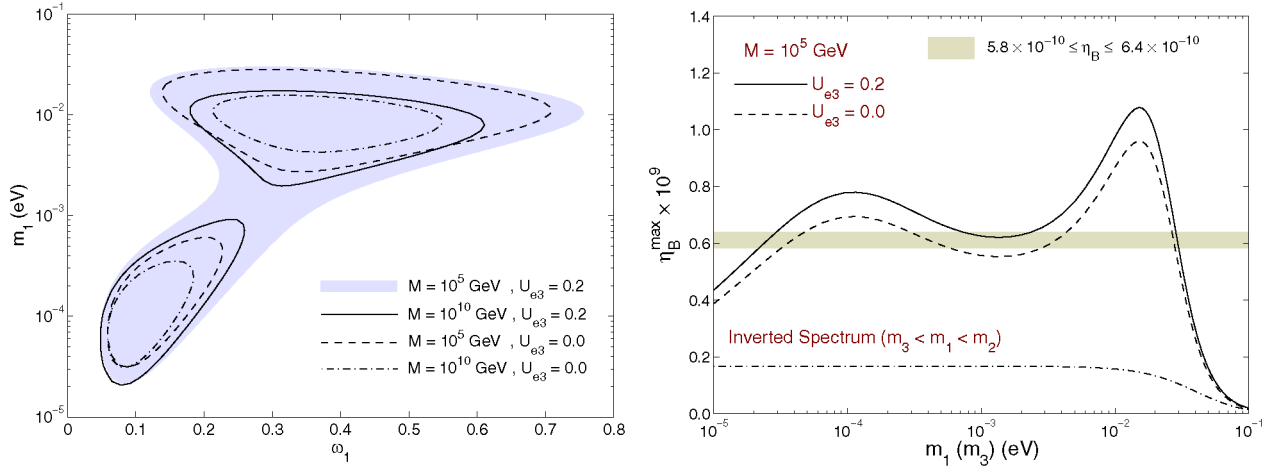


Fig. 1. The baryon asymmetry as a function of  $\omega_1$  in the case  $\theta_2 = \theta_3 = 0$  for  $U_{e3} = 0, 0.2$ . On the left, the regions in the  $(\omega_1, m_1)$ -plane where the value of  $\eta_B \geq 5.8 \times 10^{-10}$ . The results are presented for  $M = 10^5, 10^{10}$  GeV. The maximal value of  $\eta_B$  as a function of the lightest neutrino mass  $m_1$  is shown in the right plot for the same values of  $U_{e3}$  and  $M = 10^5$  GeV. The dash-dotted line corresponds to  $\eta_B^{\max}$  as a function of  $m_3$  for an inverted neutrino mass spectrum  $m_3 < m_1 < m_2$ .

the beginning of this section. Obviously, the two heavy Majorana neutrinos  $N_1$  and  $N_2$  are no longer degenerate at  $\mu = M$  due to radiative effects. Moreover, a non-trivial  $CP$ -violating part is generated due to the running of  $Y_\nu$ . We also evolve the effective neutrino mass operator from  $M$  down to  $M_Z$  in order to check whether the inclusion of  $Y_\nu$  and threshold corrections in the top-down running affects the values of the neutrino parameters initially considered. If so, the parameters at the degeneracy scale  $\Lambda$  are accordingly changed to achieve convergence.

In Fig. 1 we present the results of our numerical analysis for the case  $\theta_2 = \theta_3 = 0$ . The plot on the left shows the allowed region in the  $(\omega_1, m_1)$ -plane where  $\eta_B$  can be larger than the lower bound given in Eq. (29), i.e.,  $\eta_B \geq 5.8 \times 10^{-10}$ . The contours are given for  $U_{e3} = 0$  and  $U_{e3} = 0.2$ . The filled region was obtained following the full numerical procedure and considering  $M = 10^5$  GeV. Changing the scale  $M$  to  $10^{10}$  GeV does not alter the results significantly, as can be seen from the figure. This interesting feature of our scenario can be understood by noting that the uncorrected  $CP$ -asymmetries given in Eqs. (21) are independent of  $M$  and  $\Lambda$ . From the analysis of the plot we conclude that in this simple scenario radiative leptogenesis is compatible with the observed baryon asymmetry, provided that the lightest neutrino mass is in the range  $2 \times 10^{-5}$  eV  $\lesssim m_1 \lesssim 3 \times 10^{-2}$  eV.

The maximal value of  $\eta_B$  as a function of  $m_1$  is shown in the right plot of Fig. 1 for the same values of  $U_{e3}$  and  $M = 10^5$  GeV. It is interesting to note that for  $m_1 \lesssim 10^{-3}$  eV, the contributions coming from the decay widths are negligible,  $D_{1,2} \ll 1$ . This explains the small dependence of the results on the mass scale  $M$ . Moreover, in this region  $\varepsilon_1 \gg \varepsilon_2$  and  $K_2 \sim K_\odot \gg K_1$ , so that the washout is dominated by the inverse decays of  $N_2$ . On the other hand, for  $m_1 \simeq 10^{-2}$  eV one has  $D_1 \simeq D_2 \approx 1$  and the decay width corrections start to become relevant.

Finally, when  $m_1 \simeq m_2$ , which corresponds to quasi-degenerate or inverted-hierarchical light neutrinos, it follows directly

from Eqs. (21) and (22) that  $\varepsilon_1 \simeq \varepsilon_2$  and  $D_1 \simeq D_2 \gg 1$ . As a consequence, the generated leptonic  $CP$  asymmetries are suppressed by  $\Delta m_\odot^2$  and grow with  $\ln^2(\Lambda/M)$ ,

$$\varepsilon_1 \simeq \varepsilon_2 \simeq \frac{3y_\tau^2}{16\pi^3} \frac{\Delta m_\odot^2 \sinh(2\omega_1) \text{Re}(U_{32}^* U_{31})}{m_1^2 (2\cosh^2 \omega_1 - 1)^3} \ln^2\left(\frac{\Lambda}{M}\right). \quad (34)$$

In this situation, the expression for the baryon asymmetry is approximately given by

$$\eta_B \simeq 4 \times 10^{-12} \left(\frac{\Delta m_\odot^2}{8 \times 10^{-5} \text{ eV}^2}\right) \left(\frac{0.05 \text{ eV}}{m_1}\right)^3 \times \frac{\sinh(2\omega_1) \text{Re}(U_{32}^* U_{31})}{(2\cosh^2 \omega_1 - 1)^4} \ln^2\left(\frac{\Lambda}{M}\right). \quad (35)$$

In particular, for an inverted hierarchy with  $m_1 \approx \sqrt{\Delta m_a^2} \simeq 0.05$  eV, one can show that  $\eta_B$  is maximal when  $m_3 = 0$ ,  $\omega_1 \simeq 0.3$  and  $\text{Re}(U_{32}^* U_{31}) \simeq 1/4$ . Thus,  $\eta_B$  is bounded by

$$\eta_B \lesssim 3 \times 10^{-13} \ln^2\left(\frac{\Lambda}{M}\right). \quad (36)$$

When  $M = 10^5$  GeV and  $\Lambda = 10^{16}$  GeV, this upper bound corresponds to the plateau shown in the right plot of Fig. 1, obtained numerically for an inverted neutrino mass spectrum.

Similar plots are shown in Fig. 2 for the case  $m_1 = 0$ ,  $\theta_1 = i\omega_1$ ,  $\theta_2 = \omega_2$  and  $\theta_3 = 0$ . On the left, we present the allowed regions in the  $(\omega_1, \omega_2)$ -plane where the lower bound  $\eta_B = 5.8 \times 10^{-10}$  can be attained. As in Fig. 1, we take  $U_{e3} = 0, 0.2$ ,  $M = 10^5, 10^{10}$  GeV and  $M_3 = 10^{14}$  GeV. As expected, there are two distinct allowed regions separated by the line corresponding to the value of  $\omega_2$  given in Eq. (28),  $\omega_2 \simeq 0.44$ , where the leptonic asymmetries vanish. As can be seen from the figure, for values of  $\omega_2$  close to the above value there is a clear dependence on the mass parameter  $M$ . This has to do with the fact that in that region the corrections due to the  $N_{1,2}$  decay widths are significant,  $D_{1,2} \gtrsim 1$ . The maximal value of  $\eta_B$  as a function of  $\omega_2$  is shown in the right plot for the same values of  $U_{e3}$  and  $M = 10^5$  GeV.

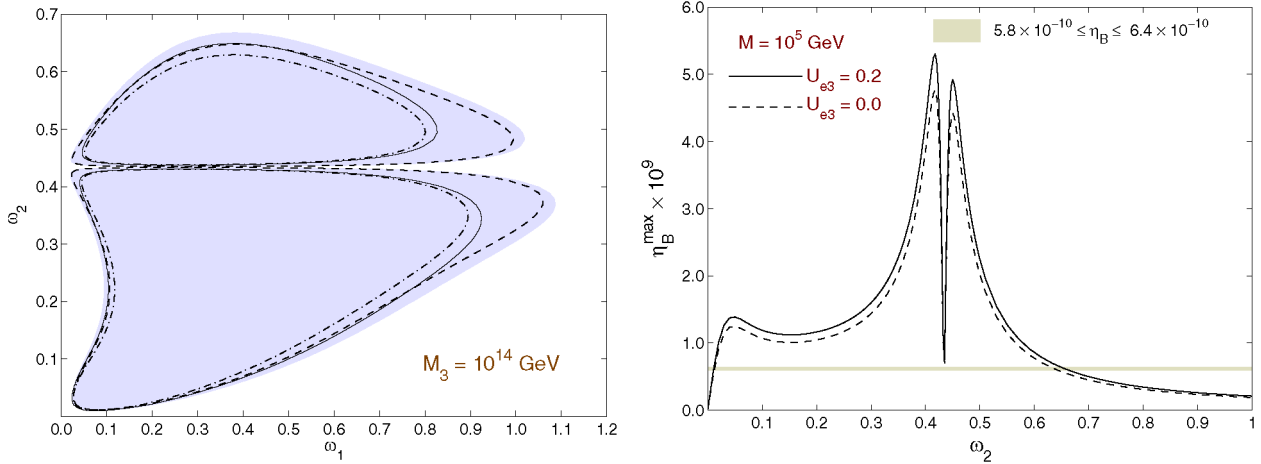


Fig. 2. The baryon asymmetry as a function of  $\omega_1$  and  $\omega_2$  in the case  $m_1 = 0, \theta_3 = 0$  for  $U_{e3} = 0, 0.2$ ;  $M = 10^5, 10^{10}$  GeV and  $M_3 = 10^{14}$  GeV. The line and colour schemes are the same as used in Fig. 1. On the left plot, the region in the  $(\omega_1, \omega_2)$ -plane where the value of  $\eta_B \geq 5.8 \times 10^{-10}$  is shown. The maximal value of  $\eta_B$  as a function of  $\omega_2$  is shown in the right plot for the same values of  $U_{e3}$  and  $M = 10^5$  GeV.

From the previous analysis we conclude that radiative leptogenesis in the framework of the SM extended with 3 heavy Majorana neutrinos is compatible with a hierarchical light neutrino mass spectrum. As anticipated by the analytical calculations, the small sensitivity of  $\eta_B$  to the value of  $U_{e3}$  is confirmed by our numerical results. This can be readily seen by comparing the curves for  $U_{e3} = 0$  and  $0.2$  in both Figs. 1 and 2.

#### 4. On a symmetry behind the heavy Majorana neutrino degeneracy

An essential aspect of the resonant leptogenesis framework is the requirement of having two quasi-degenerate heavy neutrino masses. In the radiative leptogenesis scenario, this quasi-degeneracy is generated by RG corrections, starting from a situation where there is an exact degeneracy between the heavy Majorana neutrino masses. This is, in our opinion, one of the most natural frameworks for the implementation of resonant leptogenesis.<sup>2</sup> It is therefore important to investigate whether this can be achieved by an underlying symmetry principle.

First, we should remark that, in order for our mechanism to work, any symmetry leading to exact heavy Majorana neutrino degeneracy between  $N_1$  and  $N_2$  has to be such that  $H_{11} - H_{22} \neq 0$ . This is crucial in order to radiatively generate the mass splitting  $\delta_N$ , as can be seen from Eq. (11). Moreover, Eqs. (12) and (13) require  $\text{Re}[(Y_\nu)_{31}^*(Y_\nu)_{32}] \neq 0$  and  $\text{Im}[H_{12}] \neq 0$ , otherwise the  $CP$ -violating effects needed for leptogenesis will be highly suppressed.

We now address the question of whether the degeneracy in the heavy Majorana neutrino mass spectrum could reflect the presence of an underlying symmetry at a high-energy scale. We shall briefly comment on two possible scenarios, based on simple discrete or Abelian symmetries. The more ambitious program of extending these symmetries to the quark and lep-

ton sectors of the full theory is beyond the scope of this Letter and will be presented elsewhere.

Let us assume that there is a  $Z_3 \times S_3$  symmetry, under which the right-handed neutrino fields  $N_i$  transform in the following way:

$$\begin{aligned} Z_3 : N_i &\rightarrow P_{ij} N_j, \\ S_3 : N_1 &\rightarrow N_2, N_2 \rightarrow N_3, N_3 \rightarrow N_1, \end{aligned} \quad (37)$$

where the matrix  $P$ , which defines the transformation properties of the  $N_i$  under the  $Z_3$  symmetry, is given by<sup>3</sup>

$$P = i\omega^* W, \quad W = \frac{1}{3} \begin{pmatrix} \omega & 1 & 1 \\ 1 & \omega & 1 \\ 1 & 1 & \omega \end{pmatrix}, \quad \omega = e^{i2\pi/3}. \quad (38)$$

It can be readily verified that the most general Majorana mass term which is consistent with the above symmetry is  $M_R = M_0 \Delta$ , where  $M_0$  is a high mass scale and  $\Delta$  is the  $3 \times 3$  democratic matrix,  $\Delta_{ij} = 1$ . We now assume that the  $Z_3 \times S_3$  symmetry is softly broken into  $S_3$ , so that the right-handed Majorana mass matrix has the form

$$M_R = M_0(\Delta + \epsilon \mathbb{1}), \quad (39)$$

where  $|\epsilon| \ll 1$ . One can easily verify that the eigenvalues of this matrix lead to required spectrum:  $M_1 = M_2 = |\epsilon| M_0$  and  $M_3 \simeq 3M_0$ . The parameter  $\epsilon$  reflects here the hierarchy between the scales  $M_3$  and  $M_{1,2}$ . Note that assuming  $|\epsilon| \ll 1$  is natural in the 't Hooft sense [27], since in the limit  $\epsilon \rightarrow 0$  the matrix  $M_R$  acquires a larger symmetry, namely,  $Z_3 \times S_3$ .

Next, we consider another possible explanation for the degeneracy in  $M_i$ , based on Abelian symmetries. In fact, one of the most popular schemes considered to explain the fermion mass and mixing patterns is the Froggatt–Nielsen mechanism [28] with spontaneously broken Abelian flavour symmetries [29]. Such flavour symmetries are assumed to be broken

<sup>2</sup> See Refs. [11,24] for some works where the question of heavy Majorana neutrino degeneracy has also been addressed.

<sup>3</sup> It is interesting to note that this  $Z_3$  symmetry is the minimal discrete symmetry which leads to a scenario with extended flavour democracy [25,26].

by  $\langle X \rangle / M_* = \epsilon \ll 1$ , where  $X$  is a scalar field and  $M_*$  is the fundamental mass scale of the theory. In order to try to explain the required heavy Majorana neutrino mass spectrum, we consider in the context of a supersymmetric theory a model with two Abelian flavour symmetries  $U(1)_X \times U(1)_{X'}$ . The two scalar fields  $X$  and  $X'$  are assumed to have charges  $Q(X) = (-1, -1)$  and  $Q(X') = (0, 1)$  under  $U(1)_X \times U(1)_{X'}$ . In this case, the effective superpotential contains the following non-renormalizable terms for the heavy Majorana neutrino masses

$$W_N = c_{ij} M_{B-L} \left( \frac{X}{M_*} \right)^{x_{ij}} \left( \frac{X'}{M_*} \right)^{x_{ij} - x'_{ij}} N_i N_j, \quad (40)$$

$$x_{ij}^{(0)} = n_i^{(0)} + n_j^{(0)},$$

where  $n_i^{(0)}$  is the charge of  $N_i$  under the  $U(1)_{X(X')}$  symmetry. The  $c_{ij}$  are order one coefficients not determined by the flavour symmetry and  $M_{B-L}$  is the typical  $B-L$  breaking scale. After spontaneous breaking of the  $U(1)$ , the heavy Majorana neutrino mass matrix is given by

$$(M_R)_{ij} = c_{ij} M_{B-L} \epsilon_1^{x_{ij}} \epsilon_2^{x_{ij} - x'_{ij}}, \quad (41)$$

$$\epsilon_1 = \frac{\langle X \rangle}{M_*}, \quad \epsilon_2 = \frac{\langle X' \rangle}{M_*},$$

with  $\epsilon_{1,2} \ll 1$ . Since the appearance of non-renormalizable terms with negative powers of the superfields  $X$  and  $X'$  is forbidden by the holomorphicity of the superpotential, the  $U(1)$  charges  $n_i^{(0)}$  have to be such that  $x_{ij} \geq 0$  and  $x_{ij} - x'_{ij} \geq 0$ . Otherwise,  $c_{ij}$  must be set to zero (holomorphic zeros) [30]. This property can be used to justify a heavy Majorana mass spectrum of the type:  $M_1 = M_2 \ll M_3 \simeq M_{B-L}$ . Indeed, it is easy to see that imposing the conditions

$$x_{ij} \geq 0 \wedge x_{ij} - x'_{ij} \geq 0, \quad (i, j) = (1, 2), (3, 3),$$

$$x_{ij} < 0 \vee x_{ij} - x'_{ij} < 0, \quad (i, j) \neq (1, 2), (3, 3), \quad (42)$$

one obtains the following structure:

$$(M_R)_{12} = c_{12} \epsilon_1^{x_{12}} \epsilon_2^{x_{12} - x'_{12}} M_{B-L},$$

$$(M_R)_{33} = c_{33} \epsilon_1^{x_{33}} \epsilon_2^{x_{33} - x'_{33}} M_{B-L},$$

$$(M_R)_{ij} = 0, \quad (i, j) \neq (1, 2), (3, 3), \quad (43)$$

which leads to  $M_1 = M_2 = |(M_R)_{12}|$  and  $M_3 = |(M_R)_{33}|$ . There is, however, a caveat on this approach. It is well known that in these schemes, besides the usual canonical terms  $N_i^\dagger N_i$ , the Kähler potential receives non-renormalizable contributions involving powers of  $X/M_*$  and  $X'/M_*$ . As emphasized in Ref. [31], these extra terms may fill the supersymmetric zeros corresponding to negative powers of the scalar fields in the superpotential. This is a consequence of the superfield redefinitions which bring back the Kähler potential to the canonical form. As a result, the superpotential couplings get modified [32]. In the present case, one can show that, after the  $U(1)$  spontaneous symmetry breaking, the Kähler potential reads

$$K = N_i^\dagger C_{ij} N_j, \quad C_{ij} = (\delta_{ij} + k_{ij} \epsilon_1^{|a_{ij}|} \epsilon_2^{|a'_{ij}|}), \quad (44)$$

where  $k_{ij}$  are coefficients,  $a_{ij} = n_j - n_i$  and  $a'_{ij} = n_j - n_i + n'_i - n'_j$ .

Obviously, the transformation which redefines the superfields  $N_i$  to the canonical basis will depend on the choice of the charges  $n_i$  and  $n'_i$ . For illustration, let us take the following set of  $n_i$  and  $n'_i$  charges

$$n_i = (2, -1, 0), \quad n'_i = (3, -4, 0), \quad (45)$$

which obey the conditions given in Eq. (42), and let us assume  $\epsilon_1 \simeq \epsilon_2 \equiv \epsilon$ . From the above charge configuration it follows that the uncorrected  $M_R$  leads to  $M_1 = M_2 \simeq \epsilon^3 M_{B-L}$  and  $M_3 \simeq M_{B-L}$ . One can show that the redefinition of the heavy neutrino superfields performed to recover the canonical form of the Kähler potential lifts the degeneracy between  $N_1$  and  $N_2$  with a corresponding  $\delta_N = M_2/M_1 - 1 \sim \epsilon^3$ . Therefore, in this case the radiative leptogenesis framework would make sense only if the RG corrections to  $\delta_N$  (cf. Eq. (11)) are larger than  $\epsilon^3$ . Clearly, this will depend on the size of the Dirac neutrino Yukawa couplings. To conclude, it is worth emphasizing that if one invokes this kind of  $U(1)$  flavour symmetries to explain degenerate or quasi-degenerate spectra, as it is in the case of resonant leptogenesis, these effects should be properly taken into account.

## 5. Conclusions

We have presented an appealing and economical scenario of resonant leptogenesis, based on the radiative generation of the leptonic  $CP$  asymmetries. In particular, we have studied the mechanism of radiative leptogenesis [14] in the more general  $3 \times 3$  SM seesaw framework with a heavy Majorana neutrino mass spectrum  $M_1 \simeq M_2 \ll M_3$ . We have shown that even for simple flavour structures of the Dirac neutrino Yukawa coupling matrix, one can successfully generate the cosmological baryon asymmetry and, simultaneously, accommodate the low-energy neutrino data. The key ingredients for the viability of the mechanism are the heavy Majorana mass splitting and the  $CP$ -violating effects induced at the leptogenesis scale by renormalization group corrections. As far as leptogenesis is concerned, our conclusions are quite independent of the specific values of the heavy Majorana mass scales  $M$  and  $M_3$ , as well as of the degeneracy scale  $\Lambda$ . We have also seen that the mechanism works in a wide region of the low-energy neutrino parameter space. In contrast with the minimal seesaw scenario with only two heavy Majorana neutrinos, we have concluded that the present framework is compatible with a fully hierarchical light neutrino mass spectrum. Furthermore, from the simple limiting cases considered, an upper bound on the lightest neutrino mass  $m_1 \lesssim 0.03$  eV was obtained. Obviously, this bound is expected to get modified if one considers non-minimal structures for the neutrino Yukawa coupling matrix.

We have also presented a brief discussion on possible symmetries which could lead to an exact mass degeneracy between  $N_1$  and  $N_2$  at a high-energy scale. For instance, the soft breaking of a specific  $Z_3 \times S_3$  symmetry to  $S_3$  by a small parameter  $\epsilon$  naturally leads to a heavy Majorana mass spectrum of the type  $M_{1,2} = \epsilon M_3$ . Alternatively, flavour structures based on  $U(1)$



Abelian symmetries can also explain such a degeneracy. However, as it was stressed, the application of such symmetries to explain exact or quasi-degenerate mass spectra should be done with care. Indeed, one should properly take into account the corrections which appear when the Kähler potential is brought to its canonical form by a redefinition of the heavy Majorana neutrino fields.

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