

Unification versus proton decay in $SU(5)$

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Abstract

We investigate unification constraints in the simplest renormalizable non-supersymmetric $SU(5)$ framework. We show that in the scenario where the Higgs sector is composed of the **5**-, **24**-, and **45**-dimensional representations the proton could be practically stable. We accordingly demonstrate that of all the $SU(5)$ scenarios only the non-renormalizable one with the **5**-, **24**-, and **15**-dimensional Higgs multiplets can be verified if low-energy supersymmetry is not realized in nature.

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1. Introduction

Grand unified theories (GUTs) are considered to be among the most appealing scenarios for physics beyond the Standard Model. Qualitatively they always predict (i) unification of gauge couplings of the Standard Model and (ii) proton decay. The first feature cannot be directly probed since unification takes place at a very high energy scale—the so-called GUT scale. However the second feature *can* be probed and it offers the only realistic way of testing grand unification. It is thus important to single out and investigate viable models of grand unification; where proton decay is not only well predicted, but also experimentally accessible in both current and future proton decay experiments.

Out of all grand unified theories the scenarios based on $SU(5)$ gauge symmetry are arguably the most predictive ones for proton decay. Recall, of all simple gauge groups that allow SM embedding only $SU(5)$ has unique single-step symmetry breaking pattern. This allows rather accurate determination

of high energy scales relevant for proton decay. And, in its non-supersymmetric version, $SU(5)$ GUT avoids uncertainties pertaining to the exact nature as well as the relevant scale of supersymmetry breaking; both of those features are yet to be established experimentally. Moreover, it simplifies the discussion on the dominant source(s) of proton decay. All these appealing properties single out non-supersymmetric $SU(5)$ as the theory for proton decay. We accordingly focus our attention on its simplest realistic realizations.

Our starting point is the $SU(5)$ model proposed long ago by Georgi and Glashow [1]. Their model offers partial matter unification by accommodating i th generation of matter fields in the $\bar{\mathbf{5}}_i$ - and $\mathbf{10}_i$ -dimensional representations. The scalar sector is composed of a **24**-dimensional Higgs representation and a **5**-dimensional Higgs multiplet. The SM singlet in **24** breaks $SU(5)$ symmetry down to the Standard Model, while the SM $SU(2)$ doublet in **5** accomplishes electroweak symmetry breaking. The model, however, is not realistic; the gauge couplings do not unify, neutrinos are massless and $m_{\mu(e)} = m_{s(d)}$ at the GUT scale.

There are two possible model building approaches that lead to simple yet realistic extensions of the Georgi–Glashow (GG) model in view of generation of realistic fermion masses. One approach is to allow for higher-dimensional operators which modify bad mass relations $m_{\mu(e)} = m_{s(d)}$ [2]. That

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approach requires no additional Higgs fields to be introduced to fix those relations. And, the strength of required corrections in the charged sector might allow one to place an upper bound on the scale where the UV completion of the unified theory takes place. Of course, in order to improve unification more split representations need to be present. The other approach is to stick with renormalizable operators. The latter approach requires addition of a **45**-dimensional Higgs multiplet [3]. (Recall that the tensor product $\mathbf{10} \otimes \bar{\mathbf{5}} = \mathbf{5} \oplus \mathbf{45}$.) Within that approach, as far as the neutrino masses are concerned, one can either introduce some fermion singlets—the right-handed neutrinos—or a **15**-dimensional Higgs representations (or both).

The simplest non-renormalizable model based on $SU(5)$ has been subject of a recent investigation [4]. It has been shown that the model, with the Higgs sector composed of **5**, **24** and **15**, can be tested at future proton decay experiments and at future collider experiments, for example at LHC. The first possibility is due to existence of an upper bound on the proton decay lifetime. Namely, $\tau_p \leq 1.4 \times 10^{36} (0.015 \text{ GeV}^3/\alpha)^2$ years, where α is the nucleon matrix element. The second one is based on potential production of light leptoquarks [4]. See Ref. [5] for the study of several phenomenological and cosmological issues in this context.

In this work we want to investigate the minimal extension of the Georgi–Glashow model within the renormalizable framework. Namely, we want to study predictions of a model with the Higgs sector made out of **5**, **45** and **24** representations. We initially assume that the matter sector contains right-handed neutrinos to generate neutrino masses through the type-I see-saw mechanism [7]. However, we also discuss the case when there is an extra **15**-dimensional representation that generates neutrino mass via type-II see-saw [8].

The Letter is organized as follows: In Section 1 we describe the minimal renormalizable $SU(5)$ and consequently study the unification constraints. In Section 2 we compare different scenarios based on $SU(5)$ and discuss possibility to test them. In the last section we summaries our results.

2. Minimal renormalizable $SU(5)$

The Higgs sector of the minimal renormalizable $SU(5)$ is composed of the **5**-, **24**-, and **45**-dimensional representations. Their SM $SU(3) \times SU(2) \times U(1)$ decomposition is given by:

$$\mathbf{5} = H_1 + T = (\mathbf{1}, \mathbf{2}, 1/2) + (\mathbf{3}, \mathbf{1}, -1/3),$$

$$\begin{aligned} \mathbf{24} &= \Sigma_8 + \Sigma_3 + \Sigma_{(3,2)} + \Sigma_{(\bar{3},2)} + \Sigma_{24} \\ &= (\mathbf{8}, \mathbf{1}, 0) + (\mathbf{1}, \mathbf{3}, 0) + (\mathbf{3}, \mathbf{2}, -5/6) \\ &\quad + (\bar{\mathbf{3}}, \mathbf{2}, 5/6) + (\mathbf{1}, \mathbf{1}, 0), \end{aligned}$$

$$\begin{aligned} \mathbf{45} &= \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 + H_2 \\ &= (\mathbf{8}, \mathbf{2}, 1/2) + (\bar{\mathbf{6}}, \mathbf{1}, -1/3) \\ &\quad + (\mathbf{3}, \mathbf{3}, -1/3) + (\bar{\mathbf{3}}, \mathbf{2}, -7/6) \\ &\quad + (\mathbf{3}, \mathbf{1}, -1/3) + (\bar{\mathbf{3}}, \mathbf{1}, 4/3) + (\mathbf{1}, \mathbf{2}, 1/2), \end{aligned}$$

where we also set our notation. The Yukawa potential for charged fermions read as:

$$\begin{aligned} V_{\text{Yukawa}} &= (Y_1)_{ij} \mathbf{10}_i^{\alpha\beta} \bar{\mathbf{5}}_j^\alpha (\mathbf{5}_H^*)^\beta + (Y_2)_{ij} \mathbf{10}_i^{\alpha\beta} \bar{\mathbf{5}}_j^\delta (\mathbf{45}_H^*)_\delta^{\alpha\beta} \\ &\quad + \epsilon_{\alpha\beta\gamma\delta r} ((Y_3)_{ij} \mathbf{10}_i^{\alpha\beta} \mathbf{10}_j^{\gamma\delta} \mathbf{5}_H^r \\ &\quad + (Y_4)_{ij} \mathbf{10}_i^{\alpha\beta} \mathbf{10}_j^{m\gamma} (\mathbf{45}_H)_m^{\delta r}), \quad i = 1, \dots, 3, \end{aligned} \quad (1)$$

where the field **45** satisfies the following conditions:

$$(\mathbf{45})_\delta^{\alpha\beta} = -(\mathbf{45})_\delta^{\beta\alpha}, \quad \sum_{\alpha=1}^5 (\mathbf{45})_\alpha^{\alpha\beta} = 0, \quad (2)$$

$$\sum_{i=1}^3 \langle \mathbf{45} \rangle_i^{i5} = -\langle \mathbf{45} \rangle_4^{45} \quad (v_{45} = \langle \mathbf{45} \rangle_1^{15} = \langle \mathbf{45} \rangle_2^{25} = \langle \mathbf{45} \rangle_3^{35}). \quad (3)$$

In this model the masses for charged fermions are given by:

$$M_D = Y_1 v_5^* + Y_2 v_{45}^*, \quad (4)$$

$$M_E = Y_1 v_5^* - 3Y_2 v_{45}^*, \quad (5)$$

$$M_U = Y_3 v_5 + Y_4 v_{45}, \quad (6)$$

where $Y_3 = Y_3^T$ and $\langle \mathbf{5} \rangle = v_5$. Y_1 , Y_2 , and Y_4 are arbitrary 3×3 matrices. (Note the Georgi–Jarlskog [6] factor in Eq. (5).) Clearly, there are enough parameters in the Yukawa sector to fit all charged fermions masses. For previous studies in this context see [12]. Now, let us understand the unification constraints within this model.

2.1. Unification of gauge interactions

Necessary conditions for the successful gauge coupling unification can be expressed via two equalities. (See Ref. [9] for details.) These are

$$\begin{aligned} \frac{B_{23}}{B_{12}} &= \frac{5 \sin^2 \theta_w - \alpha_{\text{em}}/\alpha_s}{8 \cdot 3/8 - \sin^2 \theta_w}, \\ \ln \frac{M_{\text{GUT}}}{M_Z} &= \frac{16\pi}{5\alpha_{\text{em}}} \frac{3/8 - \sin^2 \theta_w}{B_{12}}, \end{aligned} \quad (7)$$

where all experimentally measured quantities on the right-hand sides are to be taken at M_Z energy scale. The first one is the so-called “B-test” and the second one is the “GUT scale relation”. In what follows we use [10] $\sin^2 \theta_w = 0.23120 \pm 0.00015$, $\alpha_{\text{em}}^{-1} = 127.906 \pm 0.019$ and $\alpha_s = 0.1187 \pm 0.002$ to obtain:

$$\frac{B_{23}}{B_{12}} = 0.719 \pm 0.005, \quad \ln \frac{M_{\text{GUT}}}{M_Z} = \frac{184.9 \pm 0.2}{B_{12}}. \quad (8)$$

The left-hand sides, on the other hand, depend on particular particle content of the theory at hand and corresponding mass spectrum. More precisely, $B_{ij} = B_i - B_j$, where B_i coefficients are given by:

$$B_i = b_i + \sum_I b_{iI} r_I, \quad r_I = \frac{\ln M_{\text{GUT}}/M_I}{\ln M_{\text{GUT}}/M_Z}. \quad (9)$$

b_i are the SM coefficients while b_{iI} are the one-loop coefficients of any additional particle I of mass M_I ($M_Z \leq M_I \leq M_{\text{GUT}}$). (Recall, for the case of n light Higgs doublet fields

Table 1
Contributions to the B_{ij} coefficients. The masses of the Higgs doublets are taken to be at M_Z

	2HSM	T	V	Σ_8	Σ_3	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6
B_{23}	4	$-\frac{1}{6}r_T$	$-\frac{7}{2}r_V$	$-\frac{1}{2}r_{\Sigma_8}$	$\frac{1}{3}r_{\Sigma_3}$	$-\frac{2}{3}r_{\Phi_1}$	$-\frac{5}{6}r_{\Phi_2}$	$\frac{3}{2}r_{\Phi_3}$	$\frac{1}{6}r_{\Phi_4}$	$-\frac{1}{6}r_{\Phi_5}$	$-\frac{1}{6}r_{\Phi_6}$
B_{12}	$\frac{36}{5}$	$\frac{1}{15}r_T$	$-7r_V$	0	$-\frac{1}{3}r_{\Sigma_3}$	$-\frac{8}{15}r_{\Phi_1}$	$\frac{2}{15}r_{\Phi_2}$	$-\frac{9}{3}r_{\Phi_3}$	$\frac{17}{15}r_{\Phi_4}$	$\frac{1}{15}r_{\Phi_5}$	$\frac{16}{15}r_{\Phi_6}$

$b_1 = 40/10 + n/10$, $b_2 = -20/6 + n/6$ and $b_3 = -7$.) Relevant B_{ij} -coefficient contributions in our scenario are listed in Table 1.

There are five SM multiplets that mediate proton decay in this model. These are the superheavy gauge bosons $V(= (\mathbf{3}, \mathbf{2}, -5/6) + (\bar{\mathbf{3}}, \mathbf{2}, 5/6))$, the $SU(3)$ triplet T , Φ_3 , Φ_5 and Φ_6 . The least model dependent and usually the most dominant proton decay contribution comes from gauge boson mediation. Its strength is set by M_V and α_{GUT} —the value of gauge coupling at M_{GUT} . In what follows, we identify M_V with the GUT scale, i.e., we set $M_V \equiv M_{GUT}$. Clearly, we are interested in the regime where $M_V (= M_{GUT})$ is above experimentally established bounds. Now, how large M_{GUT} is primarily depends on masses of Σ_3 , Φ_1 , and Φ_3 through their negative contribution to B_{12} . If they are light enough they render gauge contributions to proton decay innocuous. However, Φ_3 field cannot be very light due to proton decay constraints. At the same time, it cannot be at the GUT scale since B-test cannot be satisfied using solely Σ_3 and/or Φ_1 . Clearly, proton decay constraints will thus create tension between successful unification and possible values for M_{Φ_3} and M_{GUT} .

We again note that contributions from fields in Σ_3 and Φ_1 cannot sufficiently modify B-test. This is because the SM fails rather badly, i.e., $B_{23}^{SM}/B_{12}^{SM} \simeq 0.51$, so that large corrections to B_{23} and B_{12} are needed. Thus, we always need to use contribution coming from the field Φ_3 to some extent. This contradicts previous studies [11] where successful unification was claimed with Φ_3 field kept at the GUT scale. Unification constraints in the context of the model with the same Higgs content as ours have also been studied before in [9]. However, authors did not notice that Φ_3 in general mediates nucleon decay. Moreover, even if the model violates baryon number single 45-dimensional representation is sufficient for successful unification contrary to the remarks in Ref. [9].

The Φ_3 contributions to proton decay are coming from interactions $Y_4 Q^T \sigma_2 \Phi_3 Q$ and $Y_2 Q^T \sigma_2 \Phi_3^* L$ (for a review on proton decay see [14]). Our calculation shows that Φ_3 should be heavier than 10^{10} GeV in order not to conflict experimental data. (Of course, this rather naive estimate holds if one assumes most natural values for Yukawa couplings.) If for some reason one of the two couplings is absent or suppressed the bound on Φ_3 would cease to exist. For example, if we choose Y_4 to be symmetric matrix, the coupling $Y_4 Q^T \sigma_2 \Phi_3 Q$ vanishes. Therefore, Φ_3 could be very light.

There are four critical mass parameters (M_{Φ_1} , M_{Φ_3} , M_{Σ_3} and M_{GUT}) and two equations that govern unification. So, we show in Fig. 1 a contour plot of M_{Φ_1} (solid line) and M_{Φ_3} (dash-dot line) in the M_{GUT} - M_{Σ_3} plane in order to present the full parameter space for successful gauge coupling unification.

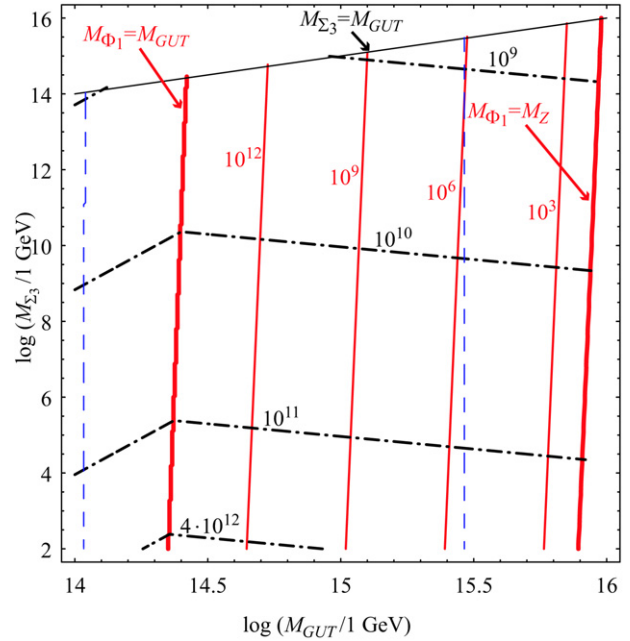


Fig. 1. Plot of lines of constant value of M_{Φ_1} (solid line) and M_{Φ_3} (dash-dot line) in the $\log(M_{GUT}/1 \text{ GeV})$ - $\log(M_{\Sigma_3}/1 \text{ GeV})$ plane. To generate the plot we require exact one-loop unification with central values for the gauge couplings as given in the text. All the masses are in the GeV units. The viable gauge coupling unification region is bounded from the right (below) by the requirement that $M_{\Phi_1}(M_{\Sigma_3}) \geq M_Z$. Two dashed lines represent lower bounds on the GUT scale due to the proton decay experimental limits. The left (right) one is generated under the assumption of suppression (enhancement) of the flavor dependent part of proton decay amplitudes. Both lines correspond to $\alpha = 0.015 \text{ GeV}^3$.

Fig. 1 shows that Σ_3 alone cannot generate unification. So, the proton decay mediating field Φ_3 needs to be below the GUT scale. Its mass varies between 10^9 GeV and 10^{12} GeV in the shown region. Recall, M_{Φ_3} must be above 10^{10} GeV unless some additional symmetry or cancellation is assumed. And, the lighter the Φ_3 field is the higher the GUT scale gets. Basically, change in the Φ_3 mass by a factor of 10^3 corresponds to change in the GUT scale by a factor of 10. This can be traced back to its rather significant impact on B_{12} .

The viable gauge coupling unification region in Fig. 1 is bounded from the right (below) by requirement that $M_{\Phi_1}(M_{\Sigma_3}) \geq M_Z$. And, the plot is valid only in the region where $M_{\Sigma_3} \leq M_{GUT}$. Clearly, there are two qualitatively distinct regions separated by the $M_{\Phi_1} = M_{GUT}$ curve. To the left of the $M_{\Phi_1} = M_{GUT}$ curve only Σ_3 and Φ_3 play the role in unification and hence the change in slope of M_{Φ_3} as one crosses it.

To help the reader we also plot current bounds (dashed lines) on the GUT scale that stem from experimental bounds on pro-

ton decay lifetime and the $M_V = M_{\text{GUT}}$ relation. There are two of them. The one on the left corresponds to lower bound on the GUT scale in the case of suppression of the flavor dependent part of the total proton decay amplitude [4]. The right one corresponds to maximally enhanced partial amplitude for $p \rightarrow \pi^0 e^+$. (In both cases we use experimental limit $\tau_p \geq 5.0 \times 10^{33}$ years [10].) More precisely, by using the flavor freedom of the $d = 6$ gauge mediated proton decay amplitudes, one can specify lower bounds for suppressed (enhanced) scenario on the GUT scale to be $M_{\text{GUT}} \geq 3.0 \times 10^{14} \sqrt{\alpha_{\text{GUT}}} \sqrt{\alpha/0.003 \text{ GeV}^3}$ ($M_{\text{GUT}} \geq 8.0 \times 10^{15} \sqrt{\alpha_{\text{GUT}}} \sqrt{\alpha/0.003 \text{ GeV}^3}$). The lines shown are generated for $\alpha = 0.015 \text{ GeV}^3$ [15], where α is the nucleon matrix element. Note that any “intermediate” scenario for fermion masses falls in between in terms of the M_{GUT} bounds. (See [13] for more details.) Clearly, realistic scenario where only Φ_3 corrects the SM running with all other fields at M_{GUT} is possible.

Fig. 1 was generated under simplifying assumption that only Σ_3 , Φ_1 , and Φ_3 are allowed to be below M_{GUT} . But, in general, other fields could venture below the GUT scale too. If we allow for such a scenario and place a lower limit on Φ_3 mass to be 10^{10} GeV in order to avoid rapid proton decay the maximal value of the GUT scale comes out to be $3 \times 10^{16} \text{ GeV}$. At the same time $M_{\Sigma_3} = M_{\Phi_1} = M_Z$ and $M_{\Sigma_8} = 4 \times 10^5 \text{ GeV}$. All other fields play no significant role and are at the GUT scale.

In the previous discussion we have assumed that superheavy right-handed neutrinos generate observed neutrino masses. If we do not want singlets in the theory, we have to introduce the **15** of Higgs to generate neutrino masses through the type II see-saw mechanism. There is a difference between the two scenarios from the point of view of proton decay. Namely, the singlets do not significantly affect the running while split multiplets in **15** could do that. Moreover, **15** contains scalar leptoquarks that, through the mixing with the **5** of Higgs could also mediate proton decay. (Recall, $\mathbf{15} = \Phi = (\Phi_a, \Phi_b, \Phi_c) = (\mathbf{1}, \mathbf{3}, \mathbf{1}) + (\mathbf{3}, \mathbf{2}, 1/6) + (\mathbf{6}, \mathbf{1}, -2/3)$.) In the case that **15** is included in the model additional contributions to the B_{ij} are shown in Table 2.

There are two fields in **15** that can improve unification; these are Φ_a and Φ_b . Φ_a has very small contribution to B_{12} and very large contribution to B_{23} . This means that its impact on the GUT scale is not significant. Φ_b , on the other hand, has large impact on the GUT scale relation but, in general, it mediates proton decay and it is probably better to keep it heavy.

In any case, the Φ_3 contribution to the running of the gauge couplings is crucial to achieve high scale unification in agreement with experimental data. If its contribution to the decay of the proton is set to zero by additional symmetry the unification

scale could be very large. Therefore, since in that case the most important contributions to the decay of the proton are the gauge $d = 6$ ones, we can conclude that proton could be *stable* for all practical purposes in the minimal renormalizable $SU(5)$.

2.2. Testing minimal realistic $SU(5)$ models

As we discussed in previous sections there are two simple candidates for unification based on $SU(5)$. In the first scenario the Higgs sector is composed of **5**, **24**, and **15**, and higher-dimensional operators are used to modify the relation $m_{\mu(e)} = m_{s(d)}$ [4]. Let us call this model GUT-I. The second scenario is the one discussed in this work—the renormalizable model with a Higgs sector composed of **5**, **24**, and **45**. Let us call this GUT-II. There are two possibilities to test the GUT-I [4]. One is through proton decay in the current and next generation of experiments and the other is through the production of light leptoquarks in future colliders. In this work we have concluded that GUT-II model cannot be tested through proton decay since the lifetime of the nucleon could be very large. Therefore, we can say that the only GUT candidate based on $SU(5)$ which can be falsified in the near future is the model presented in Ref. [4].

3. Summary

We have studied the possibility to achieve unification without supersymmetry in a minimal realistic grand unified theory based on $SU(5)$, where the Higgs sector is composed of the **5**-, **24**-, and **45**-dimensional representations. We have pointed out that the proton could be practically stable in this scenario. We have accordingly concluded that the best candidate to be tested is the GUT model with **5**-, **24**-, and **15**-dimensional representations in the Higgs sector.

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Table 2
Contributions of an extra **15** to B_{ij} coefficients

	Φ_a	Φ_b	Φ_c
B_{23}	$\frac{2}{3} r \Phi_a$	$\frac{1}{6} r \Phi_b$	$-\frac{5}{6} r \Phi_c$
B_{12}	$-\frac{1}{15} r \Phi_a$	$-\frac{7}{15} r \Phi_b$	$\frac{8}{15} r \Phi_c$

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