Confining quark-model suggestion against $D_s^*(2317)$ and $D_s^*(2460)$ as chiral partners of standard D_s

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This paper presents the first study of mesons with a quark and an antiquark with different and finite masses in a simple confining and chiral invariant quark-antiquark interaction, leading to spontaneous chiral symmetry breaking and to constituent quarks. In the false chiral invariant vacuum, the chiral partners are degenerate, and tachyons occur in the light-light spectrum. In the true vacuum, most of the standard nonrelativistic quark-model spectra should be recovered except for the pion and other particular constraints. The calibration problem of chiral quark models is also addressed here. The detailed inspection of the different contributions to the D and D_s masses suggests that the challenging recently observed $D_s^*(2317)$ and $D_s^*(2460)$ mesons might not fit as global chirally rotated quark-antiquark D_s mesons.

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I. INTRODUCTION

Recently, the discovery of the new $D_s^*(2317)$ and $D_s^*(2460)$ [1–3] revived the interest in chiral partners. Ten years before the discovery, Nowak, Rho, Zahed, Bardeen, and Hill [4,5] already predicted that the standard pseudoscalar meson $D_s(1968)$ and the standard vector meson $D_s^*(2112)$ would have two chiral partners, respectively, a scalar and an axial vector. The chiral partnership conjecture was revived [6,7] with the claim that the chiral partners are, respectively, the $D_s^*(2317)$ and $D_s^*(2460)$. Indeed, mesons can be arranged in parity multiplets. In the false chiral invariant vacuum, the scalars and pseudoscalars, or the vectors and axial vectors, are degenerate. In the true, chiral symmetry breaking vacuum, their masses split and the important question is, does this splitting explain the new D_s resonances?

Importantly, neither the quark model nor quenched lattice QCD are able to describe the $D_s^*(2317)$ and $D_s^*(2460)$ as standard quark-antiquark mesons. The new D_s^* do not fit in the spectrum of standard quark-antiquark mesons, which is governed by the quark constituent masses and by a confining potential, together with well-known hyperfine, spin-orbit and tensor potentials [8]; see Table I. Quenched lattice QCD, which only accesses the quark-antiquark spectrum, confirms that these D_s^* masses are too light for standard $q\bar{q}$ mesons [9,10].

However, here I show that the confining quark model is compatible with spontaneous chiral symmetry breaking. This leads to a logical incompatibility. Either it is false that the quark model cannot account for the new D_s^* narrow resonances, or it is false that the chiral models can. Thus these two different claims should be scrutinized in detail. Because this paper is dedicated to the implementation of chiral symmetry in the quark model, here I choose to analyze in detail the chiral partnership conjecture. I also review the calibration problem of chiral quark models. Finally this suggests that the chiral partners of the $D_s(1968)$ and the $D_s^*(2112)$ are quark-antiquark mesons heavier than the $D_s^*(2317)$ and $D_s^*(2460)$.

In Sec. II, starting from a confining and chiral invariant potential, the mass and bound-state equations are derived for the first time for mesons with a quark and an antiquark with different and finite masses. In particular the spin-tensor potentials are studied in detail; they will be important to discuss the calibration of the model. In Sec. III, the bound-state equations are applied to a chiral interpolation, designed to scrutinize chiral partnership. I interpolate from the ideal heavy-light limit in the false chiral invariant vacuum to the true symmetry breaking vacuum and to the D and D_s families with finite current quark masses. In Sec. IV, I discuss the calibration of confining and chiral invariant quark potentials, including the model used here, and show the conclusion on the nature of the new $D_s^*(2317)$ and $D_s^*(2460)$.

II. MASS GAP AND BOUND STATES WITH CHIRAL SYMMETRY AND CONFINEMENT

The first models of chiral symmetry, like the σ model of Gell-Mann and Levy [11], or the Nambu and Jona-Lasinio model [12], were only very accurate for the ground-state pseudoscalar mesons, because they did not address confinement. The ideal chiral framework should access the full phenomenology of the meson spectrum. I submit that this

TABLE I. Matrix elements of the spin-dependent potentials.

${}^{2S+1}L_J$	$\delta_{\mathbf{S}_q,\mathbf{S}_{\bar{q}}}$	$\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}$	$(\mathbf{S}_q + \mathbf{S}_{\bar{q}}) \cdot \mathbf{L}$	$(\mathbf{S}_q - \mathbf{S}_{\bar{q}}) \cdot \mathbf{L}$	tensor
${}^1S_0$	1	$-3/4$	0	0	0
${}^3P_0$	1	$1/4$	-2	0	$-1/3$
${}^3S_1$	1	$1/4$	0	0	0
${}^3D_1$	1	$1/4$	-3	0	$-1/6$
${}^3S_1 \leftrightarrow {}^3D_1$	0	0	0	0	$\sqrt{2}/6$
${}^1P_1$	1	$-3/4$	0	0	0
${}^3P_1$	1	$1/4$	-1	0	$1/6$
${}^1P_1 \leftrightarrow {}^3P_1$	0	0	0	$\sqrt{2}$	0

ideal framework is already under development. When the quarks were discovered, the confining quark model was calibrated with correct confining and spin-dependent potentials. The first matrix elements of the spin-tensor potentials are shown in Table I. However, it was realized that the main difficulty of the confining quark model consisted in understanding the low pion mass. But Nambu and Jona-Lasinio [12] already had shown that the spontaneous dynamical breaking of global chiral symmetry provides a mechanism for the generation of the constituent fermion mass and for the almost vanishing mass of the pion. This mechanism was extended to the confining quark model by Le Yaouanc, Oliver, Ono, Pène, and Raynal with the Salpeter equations in Dirac structure [13] and by P. B. and Ribeiro with the equivalent Salpeter equations in a form [14] identical to the random phase approximation (RPA) equations of Llanes-Estrada and Cotanch [15]. Moreover, these chiral quark models also comply with the PCAC theorems, say the Gell-Mann Oakes and Renner relation [13,16], the Adler zero [17–19], the Goldberger-Treiman relation [17,20], or the Weinberg theorem [17,18,21]. However, the correct fit of the hadronic spectra remains to be fully addressed for confining and chiral invariant quark-antiquark interactions. Nevertheless, I submit that a confining chiral quark model with the correct spin-tensor potentials should eventually reproduce the full spectrum of hadrons, including heavy-light systems [5,14].

For clarity, I now produce for the first time the full mesonic spin-tensor potentials of a confining and chiral invariant quark model for a quark and an antiquark with different and finite masses. This will be applied to study the D and D_s meson families with a quark u, d, or s really lighter than the scale of QCD and an antiquark c much heavier than the scale of QCD. The bound-state equations are exactly solved to study chiral partners in the true vacuum and in the limits of light or heavy quarks.

This can be accomplished in the framework of the simplest confining and chiral invariant quark model [13,14,16]. The Hamiltonian can be approximately derived from QCD,

$$\begin{aligned} H = \int d^3x \Big[& \psi^\dagger(x)(m_0\beta - i\vec{\alpha}\cdot\vec{\nabla})\psi(x) \\ & + \frac{1}{2} g^2 \int d^4y \bar{\psi}(x)\gamma^\mu \frac{\lambda^a}{2}\psi(x) \\ & \times \langle A^a_\mu(x) A^b_\nu(y)\rangle \bar{\psi}(y)\gamma^\nu \frac{\lambda^b}{2}\psi(y) + \cdots \Big] \end{aligned} \tag{1}$$

up to the first cumulant order, of two gluons [22–24], which can be evaluated in the modified coordinate gauge,

$$g^2\langle A^a_\mu(x) A^b_\nu(y)\rangle \simeq -\tfrac{3}{4}\delta_{ab} g_{\mu 0} g_{\nu 0}[K_0^3(\mathbf{x}-\mathbf{y})^2 - U], \tag{2}$$

and this is a simple density-density harmonic effective confining interaction. m_0 is the current mass of the quark, and $K_0 \simeq 0.3$ to 0.4 GeV is the only physical scale in the interaction. Like QCD, this model has only one scale in the interaction. The infrared constant U confines the quarks but the meson spectrum is completely insensitive to it.

The relativistic invariant Dirac-Feynman propagators [13] can be decomposed in the quark and antiquark Bethe-Goldstone propagators [16], close to the formalism of nonrelativistic quark models,

$$\begin{aligned} S_{\rm Dirac}(k_0, \vec{k}) &= \frac{i}{\not{k} - m + i\epsilon} \\ &= \frac{i}{k_0 - E(k) + i\epsilon}\sum_s u_s u_s^\dagger \beta \\ &\quad - \frac{i}{-k_0 - E(k) + i\epsilon}\sum_s v_s v_s^\dagger \beta, \\ u_s(\mathbf{k}) &= \left[\sqrt{\frac{1+S}{2}} + \sqrt{\frac{1-S}{2}}\hat{k}\cdot\vec{\sigma}\gamma_5\right] u_s(0), \\ v_s(\mathbf{k}) &= \left[\sqrt{\frac{1+S}{2}} - \sqrt{\frac{1-S}{2}}\hat{k}\cdot\vec{\sigma}\gamma_5\right] v_s(0), \\ &= -i\sigma_2\gamma_5 u_s^*(\mathbf{k}), \end{aligned} \tag{3}$$

where $S = \sin(\varphi) = \frac{m_c}{\sqrt{k^2+m_c^2}}$, $C = \cos(\varphi) = \frac{k}{\sqrt{k^2+m_c^2}}$ and φ is a chiral angle. In the noncondensed vacuum, φ is equal to $\arctan\frac{m_0}{k}$, but φ is not determined from the onset when chiral symmetry breaking occurs. In the physical vacuum, the constituent quark mass $m_c(k)$, or the chiral angle $\varphi(k) = \arctan\frac{m_c(k)}{k}$, is a variational function which is determined by the mass gap equation. Examples of solutions, for different light current quark masses m_0, are depicted in Fig. 1.

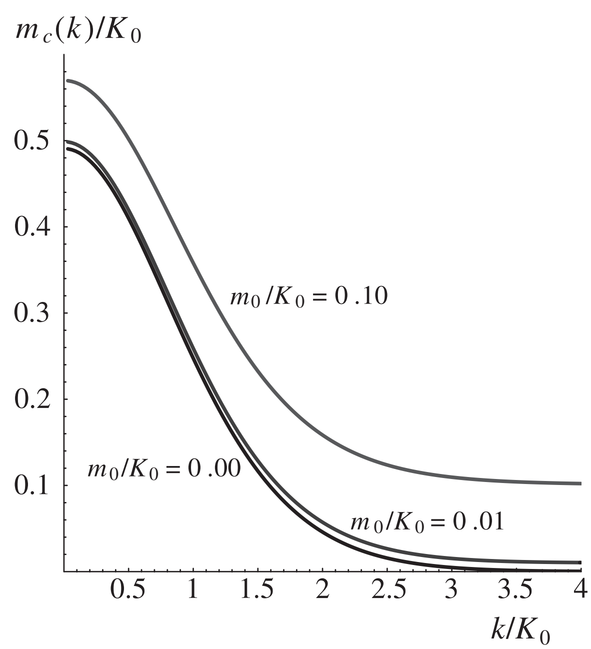

FIG. 1. The constituent quark masses $m_c(k)$, solutions of the mass gap equation, for different current quark masses m_0.

Then there are three equivalent methods to find the true and stable vacuum, where constituent quarks acquire the constituent mass. One method consists in assuming a quark-antiquark $^3P_0$ condensed vacuum and in minimizing the vacuum energy density. A second method consists in rotating the quark and antiquark fields with a Bogoliubov-Valatin canonical transformation to diagonalize the terms in the Hamiltonian with two quark or antiquark second quantized fields. A third method consists in solving the Schwinger-Dyson equations for the propagators. Any of these methods lead to the same mass gap equation and to the quark dispersion relation. Here I replace the propagator of Eq. (3) in the Schwinger-Dyson equation,

$$\begin{aligned}0&=u_s^\dagger(k)\Big\{k\hat{k}\cdot\vec{\alpha}+m_0\beta-\int\frac{dw'}{2\pi}\frac{d^3k'}{(2\pi)^3}iV(k-k')\\&\quad\times\sum_{s'}\Big[\frac{u(k')_{s'}u^\dagger(k')_{s'}}{w'-E(k')+i\epsilon}-\frac{v(k')_{s'}v^\dagger(k')_{s'}}{-w'-E(k')+i\epsilon}\Big]\Big\}v_{s''}(k)\\E(k)&=u_s^\dagger(k)\Big\{k\hat{k}\cdot\vec{\alpha}+m_0\beta-\int\frac{dw'}{2\pi}\frac{d^3k'}{(2\pi)^3}iV(k-k')\\&\quad\times\sum_{s'}\Big[\frac{u(k')_{s'}u^\dagger(k')_{s'}}{w'-E(k')+i\epsilon}-\frac{v(k')_{s'}v^\dagger(k')_{s'}}{-w'-E(k')+i\epsilon}\Big]\Big\}u_s(k),\end{aligned}\tag{4}$$

where, with the simple density-density harmonic interaction [13], the integral of the potential is a Laplacian and the mass gap equation and the quark energy are finally

$$\begin{aligned}\Delta\varphi(k)&=2kS(k)-2m_0C(k)-\frac{2S(k)C(k)}{k^2}\\E(k)&=kC(k)+m_0S(k)-\frac{\varphi'(k)^2}{2}-\frac{C(k)^2}{k^2}+\frac{U}{2}.\end{aligned}\tag{5}$$

Numerically, this equation is a nonlinear ordinary differential equation. It can be solved with the Euler-Runge-Kutta and shooting method. Examples of solutions for the current quark mass $m_c(k)=k\tan\varphi$, for different current quark masses m_0, are depicted in Fig. 1.

The Salpeter-RPA equations for a meson (a color singlet quark-antiquark bound state) can be derived from the Lippman-Schwinger equations for a quark and an antiquark or by replacing the propagator of Eq. (3) in the Bethe-Salpeter equation. In either way, one gets [16]

$$\begin{aligned}\phi^+(k,P)&=\frac{u^\dagger(k_1)\chi(k,P)v(k_2)}{+M(P)-E(k_1)-E(k_2)}\\\phi^{-t}(k,P)&=\frac{v^\dagger(k_1)\chi(k,P)u(k_2)}{-M(P)-E(k_1)-E(k_2)},\\\chi(k,P)&=\int\frac{d^3k'}{(2\pi)^3}V(k-k')[u(k_1')\phi^+(k',P)v^\dagger(k_2')\\&\quad+v(k_1')\phi^{-t}(k',P)u^\dagger(k_2')],\end{aligned}\tag{6}$$

where $k_1=k+\frac{P}{2}$, $k_2=k-\frac{P}{2}$, and P is the total momentum of the meson. Notice that, solving for χ, one gets the Salpeter equations of Yaouanc *et al.* [13].

The Salpeter-RPA equations of P. B. *et al.* [14] and of Llanes-Estrada *et al.* [15] are obtained deriving the equation for the positive energy function ϕ^+ and for the negative energy function ϕ^-. The relativistic equal time equations have double the coupled equations than the Schrödinger equation, although in many cases the negative energy components can be quite small. This results in four potentials $V^{\alpha\beta}$ respectively coupling $\nu^\alpha=r\phi^\alpha$ to ν^β. The Pauli $\vec{\sigma}$ matrices in the spinors of Eq. (3) produce the spin-dependent [25] potentials of Table II.

Notice that both the pseudoscalar and scalar equations have a system with two equations. This is the minimal number of relativistic equal time equations. However the spin-dependent interactions couple an extra pair of equations both in the vector and axial vector channels. While the coupling of the s wave and the d wave are standard in vectors, the coupling of the spin singlet and spin triplet in axial vectors only occurs if the quark and antiquark masses are different, say in heavy-light systems. I now combine the algebraic matrix elements of Table I with the spin-dependent potentials of Table II to derive the full Salpeter-RPA radial bound-state equations (where the infrared U is dropped from now on). I get the $J^P=0^-$, $^1S_0$ pseudoscalar (P) equations,

$$\Big\{\Big(-\frac{d^2}{dk^2}+E_q(k)+E_{\bar{q}}(k)+\frac{\varphi_q'^2+\varphi_{\bar{q}}'^2}{4}+\frac{1-S_qS_{\bar{q}}}{k^2}\Big)\times\begin{bmatrix}1&0\\0&1\end{bmatrix}+\Big(\frac{\varphi_q'\varphi_{\bar{q}}'}{2}+\frac{C_qC_{\bar{q}}}{k^2}\Big)\begin{bmatrix}0&1\\1&0\end{bmatrix}-M\begin{bmatrix}1&0\\0&-1\end{bmatrix}\Big\}\begin{pmatrix}\nu^+_{^1S_0}(k)\\\nu^-_{^1S_0}(k)\end{pmatrix}=0,\tag{7}$$

the $J^P=0^+$, $^3P_0$ scalar (S) equations,

TABLE II. The positive and negative energy spin-independent, spin-spin, spin-orbit and tensor potentials are shown, for the simple density-density harmonic model of Eq. (2). $\varphi'(k)$, $\mathcal{C}(k)$, and $\mathcal{G}(k)=1-S(k)$ are all functions of the constituent quark (antiquark) mass.

	$V^{++}=V^{--}$
spin-indep.	$-\frac{d^2}{dk^2}+\frac{\mathbf{L}^2}{k^2}+\frac{1}{4}(\varphi_q'^2+\varphi_{\bar{q}}'^2)+\frac{1}{k^2}(\mathcal{G}_q+\mathcal{G}_{\bar{q}})-U$
spin-spin	$\frac{4}{3k^2}\mathcal{G}_q\mathcal{G}_{\bar{q}}\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}}$
spin-orbit	$\frac{1}{k^2}[(\mathcal{G}_q+\mathcal{G}_{\bar{q}})(\mathbf{S}_q+\mathbf{S}_{\bar{q}})+(\mathcal{G}_q-\mathcal{G}_{\bar{q}})(\mathbf{S}_q-\mathbf{S}_{\bar{q}})]\cdot\mathbf{L}$
tensor	$-\frac{2}{k^2}\mathcal{G}_q\mathcal{G}_{\bar{q}}[(\mathbf{S}_q\cdot\hat{k})(\mathbf{S}_{\bar{q}}\cdot\hat{k})-\frac{1}{3}\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}}]$
	$V^{+-}=V^{-+}$
spin-indep.	0
spin-spin	$-\frac{4}{3}[\frac{1}{2}\varphi_q'\varphi_{\bar{q}}'+\frac{1}{k^2}\mathcal{C}_q\mathcal{C}_{\bar{q}}]\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}}$
spin-orbit	0
tensor	$[-2\varphi_q'\varphi_{\bar{q}}'+\frac{2}{k^2}\mathcal{C}_q\mathcal{C}_{\bar{q}}][(\mathbf{S}_q\cdot\hat{k})(\mathbf{S}_{\bar{q}}\cdot\hat{k})-\frac{1}{3}\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}}]$

$$\left\{\left(-\frac{d^2}{dk^2}+E_q(k)+E_{\bar{q}}(k)+\frac{\varphi_q'^2+\varphi_{\bar{q}}'^2}{4}+\frac{1+S_qS_{\bar{q}}}{k^2}\right)\begin{bmatrix}1&0\\0&1\end{bmatrix}+\left(\frac{\varphi_q'\varphi_{\bar{q}}'}{2}-\frac{C_qC_{\bar{q}}}{k^2}\right)\begin{bmatrix}0&1\\1&0\end{bmatrix}-M\begin{bmatrix}1&0\\0&-1\end{bmatrix}\right\}\begin{pmatrix}\nu^+_{^3P_0}(k)\\ \nu^-_{^3P_0}(k)\end{pmatrix}=0. \tag{8}$$

t3he $J^P = 1^-$ coupled $^3S_1$ and $^3D_1$ vector (V and V^*) equations,

$$\left\{\left(-\frac{d^2}{dk^2}+E_q(k)+E_{\bar{q}}(k)+\frac{\varphi_q'^2+\varphi_{\bar{q}}'^2}{4}+\frac{7-4S_q-4S_{\bar{q}}+S_qS_{\bar{q}}}{3k^2}\right)\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}+\left(-\frac{\varphi_q'\varphi_{\bar{q}}'}{6}-\frac{C_qC_{\bar{q}}}{3k^2}\right)\right.$$
$$\times\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}+\left(-\frac{d^2}{dk^2}+E_q(k)+E_{\bar{q}}(k)+\frac{\varphi_q'^2+\varphi_{\bar{q}}'^2}{4}+\frac{8+4S_q+4S_{\bar{q}}+2S_qS_{\bar{q}}}{3k^2}\right)\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$
$$+\left(\frac{\varphi_q'\varphi_{\bar{q}}'}{6}-\frac{2C_qC_{\bar{q}}}{3k^2}\right)\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}-\frac{(1-S_q)(1-S_{\bar{q}})}{3k^2}\begin{bmatrix}0&0&\sqrt{2}&0\\0&0&0&\sqrt{2}\\\sqrt{2}&0&0&0\\0&\sqrt{2}&0&0\end{bmatrix}-\left(\frac{\varphi_q'\varphi_{\bar{q}}'}{3}-\frac{C_qC_{\bar{q}}}{3k^2}\right)$$
$$\left.\times\begin{bmatrix}0&0&0&\sqrt{2}\\0&0&\sqrt{2}&0\\0&\sqrt{2}&0&0\\\sqrt{2}&0&0&0\end{bmatrix}-M\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}\right\}\begin{pmatrix}\nu^+_{^3S_1}(k)\\\nu^-_{^3S_1}(k)\\\nu^+_{^3D_1}(k)\\\nu^-_{^3D_1}(k)\end{pmatrix}=0, \tag{9}$$

the $J^P = 1^+$, coupled $^1P_1$ and $^3P_1$ axial vector (A and A^*) equations

$$\left\{\left(-\frac{d^2}{dk^2}+E_q(k)+E_{\bar{q}}(k)+\frac{\varphi_q'^2+\varphi_{\bar{q}}'^2}{4}+\frac{3-S_qS_{\bar{q}}}{k^2}\right)\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}+\left(\frac{\varphi_q'\varphi_{\bar{q}}'}{2}+\frac{C_qC_{\bar{q}}}{k^2}\right)\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}\right.$$
$$\times\left(-\frac{d^2}{dk^2}+E_q(k)+E_{\bar{q}}(k)+\frac{\varphi_q'^2+\varphi_{\bar{q}}'^2}{4}+\frac{2}{k^2}\right)\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}+\left(-\frac{\varphi_q'\varphi_{\bar{q}}'}{2}\right)\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$
$$\left.+\frac{S_q-S_{\bar{q}}}{k^2}\begin{bmatrix}0&0&\sqrt{2}&0\\0&0&0&\sqrt{2}\\\sqrt{2}&0&0&0\\0&\sqrt{2}&0&0\end{bmatrix}-M\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}\right\}\begin{pmatrix}\nu^+_{^1P_1}(k)\\\nu^-_{^1P_1}(k)\\\nu^+_{^3P_1}(k)\\\nu^-_{^3P_1}(k)\end{pmatrix}=0, \tag{10}$$

In the light-light limit of $m_q = m_{\bar{q}} \to 0$ and $\varphi \to 0$, it is clear that Eqs. (7) and (8) become identical. They also possess takyonic solutions [13]. In the same limit, Eq. (9) can be block diagonalized [13], and each block, with mixed s wave and d wave, is identical to one of the two independent blocks of Eq. (10). This checks that the chiral partners P-S and V, V^*-A, A^* are degenerate in the false chiral symmetric vacuum.

Another interesting case is the heavy-light case where, say, the antiquark has a mass $m_{\bar{q}} \simeq m_{0q} \gg K_0$; there are no Tachyons, and the negative energy components nearly vanish, like in nonrelativistic quark models. In the infinite

$m_{\bar{q}}$ limit, $S_{\bar{q}} \rightarrow 1$, and the antiquark spin is irrelevant, see Table II, complying with the Isgur-Wise heavy quark symmetry [26].

For the numerical solution, I change the sign of the second and fourth lines in Eqs. (7)–(10) and, replacing the derivatives of the functions by finite difference matrices, the equations become simple eigenvalue equations.

III. INTERPOLATING FROM THE CHIRAL AND HEAVY LIMIT TO THE D_s AND D

I now compute in detail D and D_s masses, relevant to the conjecture of chiral partnership, respectively, between the scalar meson $D_s^*(2317)$ and the axial vector meson $D_s^*(2460)$, and the standard quark-antiquark pseudoscalar meson $D_s(1968)$ and vector meson $D_s^*(2112)$.

To address the importance of chiral partnership it is convenient to start from the chiral invariant false vacuum where, in the ideal heavy-light limit of a massless quark and infinitely massive antiquark, the ground-state pseudoscalar, the ground-state scalar, the ground-state vector, and the ground-state axial vector are all degenerate. Then, interpolating from this ideal limit to the actual constituent masses of the light quark and of the heavy antiquark, the mass splittings between the $D_s^*(2317)$ and the $D_s(1968)$ and between their chiral partners can be computed. To inspect in detail the contributions to the mass splittings, it is important to decompose this chiral interpolation in three different steps.

In the first step the current quark masses are in the ideal chiral limit of $m_{0q} = 0$ and in the ideal Isgur-Wise limit of $m_{0\bar{q}} = \infty$, and I interpolate the quark mass m_q from 0, corresponding to the false chiral invariant vacuum, to the actual constituent quark mass $m_{cq} = 0$ solution of the mass gap equation (5).

In the second step the current mass $m_{0\bar{q}}$ of the heavy antiquark is interpolated from the ideal Isgur-Wise limit of $m_{0\bar{q}} = \infty$, to its actual value of the order of $m0_{\bar{q}} \simeq 5K_0$, fitted in the J/Ψ spectrum. Notice that in the case of heavy quarks or antiquarks, the constituent quark mass is identical to the current quark mass, the mass gap equation (5) essentially does not change the heavy quark masses.

I leave for the third and final step the interpolation of the current quark mass m_{0q} from the ideal chiral limit to the actual values of the order of $m_{0q} \simeq 0.01K_0$ for the u and d and of $m_{0q} \simeq 0.1K_0$ for the s quark. In chiral models the current masses of light quarks are model dependent. Although these current masses m_0 are smaller than the ones used, say, in chiral Lagrangians, in this model these current quark masses are the ones that lead to the correct experimental masses of the light-light π and K mesons. Therefore, our m_0 are not free parameters.

The results are, respectively, inspected in Sec. III A, in Sec. III B, and in Sec. III C, and are, respectively, depicted in Figs. 2–4. Notice that if the three figures are placed side by side, the interpolations of the studied mesons exactly match. At the end of the three interpolations the D and D_s spectra is computed.

A. From the chiral invariant to the true vacua

In the first step one learns the effect of the spontaneously and dynamically generated constituent quark mass in the D and D_s spectra. This also weights the spin-orbit interaction.

Here the quark current mass is in the ideal chiral limit of $m_{0q} = 0$, the antiquark current mass is in the ideal Isgur-Wise limit of $m_{0\bar{q}} = \infty$, and I interpolate the quark mass m_q from 0, corresponding to the false chiral invariant vacuum, to the actual constituent quark mass $m_{cq} = 0$ solution of the mass gap equation (5).

When the antiquark has an infinite mass, all terms depending on its spin vanish. In Table II it is clear that a quark or antiquark spin always comes with the factor

$$\mathcal{G}(k) = 1 - \frac{m_c}{\sqrt{k^2 + m_c^2}}. \tag{11}$$

Thus $\mathcal{G}(k)$ is maximal and equal to 1 when $m_c = 0$ and $\mathcal{G}(k)$ is minimal and equal to 0 when $m_c = \infty$. Because the spin of the heavy antiquark is irrelevant, the masses of the ground-state pseudoscalar P and vector V are degenerate, and the masses of the ground-state scalar S and axial vector A are also degenerate. Thus I get in the present limit

$$M_A - M_V = M_S - M_P. \tag{12}$$

Moreover, the only spin-dependent term that does not vanish in this case is the spin-orbit term $\frac{2}{k^2}\mathcal{G}_q \mathbf{S}_q \cdot \mathbf{L}$. Thus the mass splittings of Eq. (12) measure the angular repulsive barrier and the spin-orbit term.

Now, in the chiral invariant false vacuum the spin-orbit term simplifies to $\frac{2\mathbf{S}_q \cdot \mathbf{L}}{k^2}$. In this case the spin-orbit term is able to fully compete with the angular repulsive barrier $\frac{l(l+1)}{k^2}$, and the spectrum only depends on the total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}_q$ of the light quark,

$$\frac{1}{k^2}\mathbf{L}^2 + \frac{2}{k^2}\mathbf{S}_q \cdot \mathbf{L} = \frac{1}{k^2}(\mathbf{J}^2 - \mathbf{S}_q^2), \tag{13}$$

independently of $\mathbf{L}$. Thus, in the chiral invariant false vacuum, chiral symmetry induces an extra degeneracy in the states, P, V, SA, in the states A^*, V^* and so on.

In the true vacuum the quark mass is the finite constituent quark mass m_{qc}, and this decreases the spin-orbit interaction $\frac{2}{k^2}\mathcal{G}_q \mathbf{S}_q \cdot \mathbf{L}$, which is no longer able to cancel the mass splittings induced by the angular repulsive barrier $\frac{l(l+1)}{k^2}$. In the limit when this spin-orbit interaction vanishes, the splittings are only due to the repulsive barrier.

The opposite limit of large spin-orbit may occur in the case of very large angular excitations [13,22,27,28], leading to chiral doubles in the spectrum, even when the full constituent mass is used.

Notice that this first step accounts for most of the splitting of Eq. (12). In Fig. 2, this splitting is already of the order of 0.61 K_0. After the three steps it will be of the order of 0.81 K_0. This is smaller, although of a comparable order, than the typical scale of angular splittings of the hadronic spectra.

B. From the heavy quark limit to the *c* quark

In the second step one learns the effect of the large but finite heavy antiquark mass in the D and D_s spectra. This also weights the spin-spin and tensor interactions.

Here the current mass $m_{0\bar{q}}$ of the heavy antiquark is interpolated from the ideal isgurwise limit of $m_{0\bar{q}} = \infty$, to its actual value of the order of $m_{0\bar{q}} \simeq 5K_0$. Notice that in the case of heavy quarks or antiquarks, the constituent quark mass is very close to the current quark mass. In this case, the mass gap equation only changes the quark mass in a negligible way. Thus this also interpolates the constituent antiquark mass from ∞ to $m_{0\bar{q}} \simeq 5K_0$.

In this step the spin-spin and the tensor potentials no longer vanish. In Fig. 3, these spin-dependent potentials are able to split the masses of the pseudoscalar and vector and the masses of the scalar and axial vector. It is remarkable that these two mass splittings are almost identical,

$$M_V - M_P \simeq M_A - M_S, \tag{14}$$

with a precision better than 1 per mil. For this result both

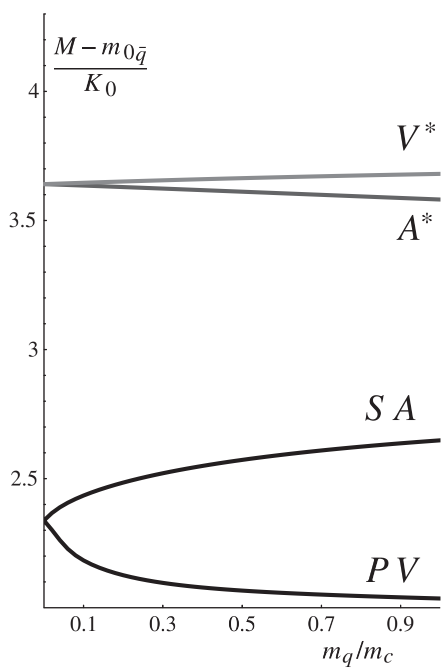

FIG. 2. Heavy-light meson masses. Here $m_{0\bar{q}} = \infty$ and $m_{0q} = 0$. The light constituent quark mass is interpolated from the zero mass of the chiral invariant false vacuum to the solution m_c of the mass gap equation in the true vacuum.

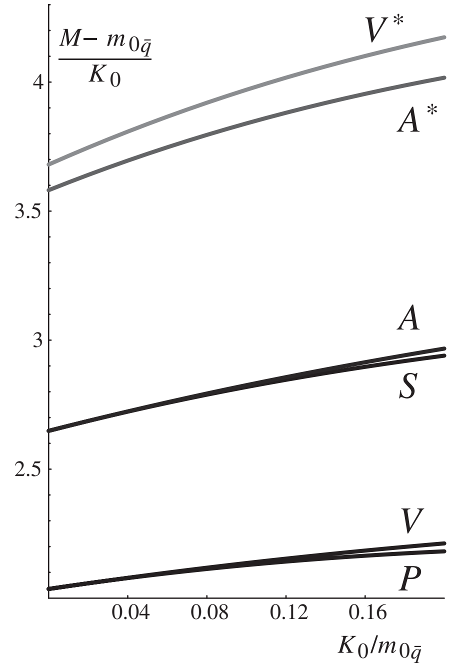

FIG. 3. Heavy-light meson masses. Here, $m_{0q} = 0$ and $m_q(k) = m_c(k)$ remain in the chiral limit. The heavy antiquark masses decrease from the infinite limit of Isgur-Wise to the actual charm mass $m_{0\bar{q}} \simeq 5K_0$.

the spin-spin and tensor interactions have to conspire with a beautiful precision.

Nevertheless, these hyperfine and tensor splittings are too small. This happens because in this model the $\mathcal{G}$ function, defined in Table II, suffers from a steep dependence on the quark mass,

$$\lim_{m\to\infty} \mathcal{G} = \frac{k^2}{2m_c^2}, \tag{15}$$

while it is well known from phenomenology that the spin-spin interaction dependence on the constituent quark masses is much smoother. Thus the splittings in Fig. 3 are more than 1 order of magnitude smaller than the experimental splittings.

C. From the chiral limit to the *u*, *d* and *s* quarks

In the third and final step one learns the effect of the small light current quark masses, the ones that break explicitly chiral symmetry. This discriminates between the D and D_s spectra.

Here the current quark mass m_{0q} is interpolated from the ideal chiral limit to the actual values of the order of $m0_q \simeq 0.01K_0$ for the u and d and of $m0_q \simeq 0.1K_0$ for the s quark. In chiral models the current masses of light quarks are model dependent. Although they are smaller than the ones used, say, in chiral Lagrangians, in this model these current quark masses are the ones that lead to the correct experimental masses of the light-light π and K mesons.

Notice that interpolating from vanishing to finite current quark masses, in this model, essentially does not change the $M_V - M_P$ and $M_A - M_S$ splittings. Essentially the $M_S - M_V$ splitting is slightly increased and the $M_{A^*} - M_A$ splitting is slightly decreased.

IV. DISCUSSION OF CHIRAL MODEL CALIBRATION AND CONCLUSION

For the first time a quark model with a chiral symmetric and confining interaction is applied to compute exactly different D and D_s meson masses for finite u, d, s, and c current quark masses. The different spin-tensor contributions to the meson masses are also analyzed in detail. I now discuss the results both qualitatively and quantitatively, and address the new $D_s^*(2317)$ and $D_s^*(2460)$ resonances.

My qualitative conclusion is that chiral models have the same number of meson states in the spectrum as the normal quark model. The mass splittings can be related, as usual in quark models, to spin-tensor potentials. At the same token the spectrum complies with the chiral relations. For instance, the well-known mass formula, first predicted by the heavy-light chiral papers [4,5],

$$M_A - M_V \simeq M_S - M_P \qquad (16)$$

is correct, in this model, up to the fourth decimal case. This is no coincidence; it is a general result for any chiral symmetric interaction. Analytically, the potentials that create the splitting in the left side and in the right side of Eq. (16) are exactly equal. It is quite remarkable that all the spin and angular momentum tensor potentials precisely conspire to achieve this result. The only difference, affecting only the fourth decimal case, comes from different sizes for the functions. Therefore, I confirm that Eq. (16) must be correct for the standard quark-antiquark mesons $D_s(1968)$, the $D_s(2112)$ and for their scalar and axial vector chiral partners. Moreover, a very similar pattern to the one of Eq. (16) also occurs within the D sector; see Figs. 2–4. The similar pattern of the quark-antiquark, or quenched, spectra for the D and D_s family is expected in confining quark models but here it is mentioned for the first time in a chiral calculation.

Before the quantitative conclusion is presented, notice that, quantitatively, all chiral models, including this simple density-density harmonic confining model of Eq. (2), and the chiral models of Nowak, Rho, and Zahed, and of Bardeen and Hill, suffer from a calibration problem. The present model is confining, so it belongs to a class of models already able to fit the angular and radial excitations of the hadronic spectra. In this sense, the confining models upgrade the nonconfining models like the σ model [11] or the Nambu and Jona-Lasinio model [12], and the related interactions used by Nowak, Rho, and Zahed and by Bardeen and Hill. Nevertheless, the spin-tensor interactions remain to be calibrated, and this is precisely addressed in this paper. This calibration problem is equivalent to the problem of chiral symmetry with scalar confinement recently mentioned, for instance, by Adler [29]. Notice that the calibration problem of chiral quark models is quite important. If this problem was solved, the confining quark model would be further improved, both in accuracy because the pion mass and other particular constraints like Eq. (16) would be correct and in consistency because fewer parameters would be needed to fit the hadron spectra. But I submit that the under development chiral invariant quark models with a confining funnel interaction [15,30] including a short range vector interaction [16,31], and a long-range confining scalar interaction [32,33], can be correctly calibrated. Llanes-Estrada, Cotanch, Szczepaniak, and Swanson showed that the Coulomb potential is crucial to produce correct hyperfine splittings both for light and heavy quark masses. Possibly the scalar confining potential suggested by P. B. and Marques would also suppress the spin-orbit interaction. An important example is provided by quenched lattice QCD calculations with Ginsparg-Wilson or Staggered fermions, which reproduce the spectrum of quark-antiquark mesons, including the low pion mass. Then the correct implementation of chiral symmetry should not affect the broad picture of the quark-model spectrum for $q\bar{q}$ mesons except for particular constraints like the low pion mass and Eq. (16).

Quantitatively, our result for the splittings of Eq. (16), depicted in Fig. 4, may be as large as 325 MeV for the upper bound of 400 MeV for the potential strength K_0. This splitting $M_S - M_P$ is the crucial one for the D_s chiral

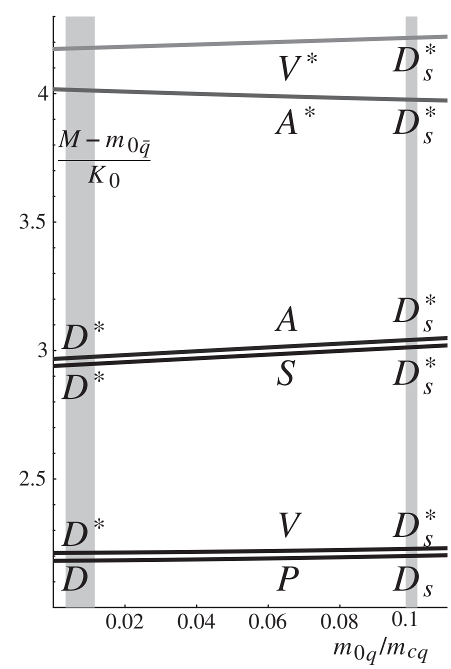

FIG. 4. Heavy-light meson masses. Here $m_{0\bar{q}}$ is the charm mass, and the light quark current quark mass m_0 interpolates from the vanishing mass of the chiral limit, passes by the u and d current quark masses, and ends up at the s quark mass.

conjecture [6,7] (although I also mention here the hyperfine splitting $M_V - M_P$). Notice that the present splitting of 325 MeV is close to the splitting of 350 MeV advocated by the conjecture of chiral partnership [6,7], so apparently the present results confirm the conjecture. However, the educated analysis of this result does not confirm the conjecture of chiral partnership. Notice that the model used here is known to suffer from a calibration problem. It is well known that in the present model the spin-orbit interaction produced by this potential is too large [13] and that the hyperfine interaction is also too small, when compared with the different meson spectra. In a sense, the model is too close to the starting point of the present interpolation, the light quark chiral limit and the infinite Isgur-Wise heavy quark limit, in the false chiral invariant vacuum, where the spin-orbit is so large that it kills the angular splitting and the hyperfine and tensor potentials vanish. If the spin-orbit interaction could be suppressed, the splittings of Eq. (16) would increase. This increase would easily reach the 423 MeV separating the axial vector $D_s^*(2535)$ from the ground-state vector $D_s^*(2112)$ (if the hyperfine splitting could be increased, the splitting between the vector $D_s^*(2112)$ and the pseudoscalar $D_s(1968)$ also would be easily reproduced). I also notice in Fig. 4 that, whatever these splittings turn out to be in a particular chiral quark model, the D and D_s families must have similar patterns. Then the similar [34] experimental 410 to 423 MeV mass splittings of the vector $D^*(2007-2010)$ and axial vector $D(2420)$, and vector $D_s^*(2112)$ and axial vector $D_s^*(2535)$, and the larger lattice splittings [9,10], all suggest that the chiral partners of the $q\bar{q}$ mesons $D_s^*(2112)$ and $D_s(1968)$ are, respectively, the $q\bar{q}$ axial vector $D_s^*(2535)$ and a yet undetected scalar $D_s^*(2392)$. This educated analysis of the present results disagree with the beautiful and important conjecture of chiral partnership for the new $D_s^*(2317)$ and $D_s^*(2460)$ narrow resonances.

Importantly, this suggests that a large non-$q\bar{q}$ component [35,36], say a tetraquark or a hybrid, must be present in the new narrow D_s resonances. Coupled channels [37] or tetraquark [38] explicit calculations, where D and K mesons play a significant role, either as a molecular state or as a coupled meson-meson state, also lead to the $D_s^*(2317)$ and $D_s^*(2460)$, and to the perfect splitting between these mesons and the ground states $D_s(1968)$ and $D_s^*(2112)$.

Nevertheless, once the calibration problem is solved for confining and chiral invariant quark potentials, the techniques developed here should again be applied to the computation of the D and D_s spectra, for a final evaluation of the chiral partnership conjecture for the new D_s^* mesons.

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