Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 639 (2006) 661–666
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Spontaneous CP violation in a SUSY model with a complex CKM

G.C. Branco [a,b,*], D. Emmanuel-Costa [a], J.C. Romão [a]

[a] *Departamento de Física and Centro de Física Teórica de Partículas (CFTP), Instituto Superior Técnico, Av. Rovisco Pais, P-1049-001 Lisboa, Portugal*
[b] *Physik-Department, Technische Universität München, James-Franck-Strasse, D-85748 Garching, Germany*

Received 26 April 2006; received in revised form 20 June 2006; accepted 22 June 2006

Available online 14 July 2006

Editor: G.F. Giudice

Abstract

It is pointed out that the recent measurement of the angle γ of the unitarity triangle, providing irrefutable evidence for a complex Cabibbo–Kobayashi–Maskawa (CKM) matrix, presents a great challenge for supersymmetric models with spontaneous CP violation. We construct a new minimal extension of the minimal supersymmetric standard model (MSSM), with spontaneous CP breaking, which leads to a complex CKM matrix, thus conforming to present experimental data. This is achieved through the introduction of two singlet chiral superfields and a vector-like quark chiral superfield which mixes with the standard quarks. A Z_3 symmetry is introduced in order to have a potential solution to the strong CP problem.

PACS: 11.30.Er; 11.30.Qc; 12.15.Ff; 12.15.Hh; 12.60.-i

Keywords: Spontaneous symmetry breaking; Supersymmetric models; CP violation; Fermion masses

1. Introduction

Four decades after the experimental discovery of CP violation, the origin of CP breaking remains one of the fundamental open questions in particle physics. There are two basic scenarios for CP violation in the framework of gauge theories, namely explicit CP breaking at the Lagrangian level and spontaneous CP breaking by the vacuum (SCPV) [1,2]. In the standard model (SM), one assumes that CP is explicitly broken at the Lagrangian level through the introduction of complex Yukawa couplings leading to CP violation in the charged-weak interactions, parametrised by a complex Cabibbo–Kobayashi–Maskawa (CKM) matrix [3].

In supersymmetric (SUSY) extensions of the SM there are additional sources of explicit CP violation, arising from complex soft SUSY breaking terms as well as from the complex SUSY conserving μ parameter. The SUSY phases also generate large contributions to the electric dipole moments (EDMs) of the electron, neutron and mercury atom. The non-observation of the EDMs imposes strong constraints on the SUSY phases, forcing them to be very small. This is the so-called SUSY CP problem and many solutions have been proposed to overcome it [4,5].

SCPV is an attractive approach to the SUSY CP problem, since all the couplings of the Lagrangian are real, due to the imposition of CP invariance at the Lagrangian level. The only source of CP violation are the vacuum phases [1,2]. SCPV also provides an appealing solution to the strong CP problem, since in this case one may have naturally vanishing $\bar{\Theta}$ at tree level and calculable at higher orders [6–11].

In this Letter, we construct a minimal supersymmetric extension of the SM, with spontaneous CP breaking, which is in agreement with all the present data, provided by both BaBar and Belle Collaborations [12,13]. The question of compatibility of spontaneous CP breaking in supersymmetric (SUSY) models with recent CP data is highly non-trivial, for the following reason. One of the salient features of most SUSY models with spontaneous CP breaking is the fact that they lead to a real CKM matrix, since the phases in quark mass matrices can be

* Corresponding author.
E-mail addresses: gbranco@cftp.ist.utl.pt (G.C. Branco), david.costa@ist.utl.pt (D. Emmanuel-Costa), jorge.romao@ist.utl.pt (J.C. Romão).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2006.06.079

removed by rephasing of right-handed quark fields, in a manner entirely analogous to the one encountered in models with natural flavour conservation in the Higgs sector [2]. Until recently, SUSY models with spontaneous CP breaking and a real CKM matrix [14,15] could be viable since both ε_K and $a_{J/\psi K_S}$ could be generated by supersymmetric contributions to K^0–$\bar{K}^0$ and B_d–$\bar{B}_d$ mixing, respectively [16]. The novel experimental input is the recent measurement of the angle γ of the unitarity triangle [12,13]. A recent analysis [17] of the present experimental data provides clear evidence for a complex CKM matrix even allowing for the presence of new physics contributions to ε_K, $a_{J/\psi K_S}$, ΔM_{B_d} and ΔM_{B_s}. These experimental findings lead to the question of whether it is possible to have a SUSY extension of the SM with spontaneous CP violation and a complex CKM matrix. In this Letter we will show that the answer to the above question is in the affirmative. Indeed we construct a minimal supersymmetric extension of the SM (MSSM) with spontaneous CP breaking which leads to a complex CKM matrix, thus avoiding conflict with recent data. The crucial point is the introduction of at least one $Q=-1/3$ vector-like quark which mixes with standard quarks and leads to a non-trivial phase in the 3×3 effective CKM matrix connecting standard quarks. The existence of such matter states naturally arises in various extensions of the SM, such as in E_6 grand unified theories as well as in models with extra-dimensions, including some of the superstring inspired models.

The model presented here has another advantage with respect to previously suggested SUSY models with SCPV. This stems from the fact that physical phases only arise in the vacuum expectation values (VEVs) of Higgs singlet phases. As these singlets only couple to the quarks and squarks, they will not show up in the neutralino and chargino sectors, making the solution of the SUSY CP problem easier in this context.

2. The model

We consider a simple extension of the MSSM where in addition to the usual superfield content, we introduce an isosinglet vector-like down quark, D_4, $D_4^{\rm C}$, and in the Higgs sector, two singlet chiral superfields S_1, S_2 which have, under $SU(3)_{\rm C}\times SU(2)_{\rm L}\times U(1)_{\rm Y}$, the following quantum numbers

$$D_4:\quad (\mathbf{3},\mathbf{1})_{-1/3},\qquad D_4^{\rm C}:\quad (\bar{\mathbf{3}},\mathbf{1})_{1/3},\qquad S_1,S_2:\quad (\mathbf{1},\mathbf{1})_0. \tag{1}$$

We will show that the extended Higgs structure can spontaneously break CP through phases in the VEVs of the singlet scalar components s_1, s_2. Furthermore, these phases induce a non-vanishing phase in the CKM matrix, which is not suppressed by the small ratios $|\langle h_{u,d}\rangle/\langle s_{1,2}\rangle| \ll 1$. We emphasise that small values of the later ratios are crucial in order to naturally suppress flavour-changing neutral currents (FCNC), but also the one-loop finite contributions to the parameter $\bar{\Theta}$ associated with strong CP violation [18].

Since we want to achieve spontaneous CP breaking, we impose CP invariance at the Lagrangian level, which implies that all the parameters appearing in the superpotential and in the soft breaking terms are real. Moreover, we introduce a discrete Z_3 symmetry, under which the standard superfields transform trivially, while the new chiral superfields have the following Z_3 assignment

$$\{D_4^{\rm C}, S_1, S_2\} \to e^{\frac{2\pi}{3}i},\qquad D_4 \to e^{\frac{4\pi}{3}i}. \tag{2}$$

The rôle of the Z_3 symmetry is to forbid quark bare mass terms of the form $\bar{d}_{i\rm L} d_{4\rm R}$, thus leading to the vanishing of $\bar{\Theta}$ at the tree level. The most general renormalisable gauge invariant superpotential which is compatible with R-parity invariance and the Z_3 symmetry reads as

$$\begin{aligned} W = \varepsilon_{ab}\big(& Y_u^{ij} Q_i^a H_u^b U_j^{\rm C} + Y_d^{ij} Q_i^b H_d^a D_j^{\rm C} \\ &+ Y_e^{ij} L_i^b H_d^a E_j^{\rm C} + \mu H_u^a H_d^b\big) \\ &+ f^i S_1 D_4 D_i^{\rm C} + g^i S_2 D_4 D_i^{\rm C} + M_D D_4 D_4^{\rm C} \\ &+ \lambda_{11} S_1^3 + \lambda_{12} S_1^2 S_2 + \lambda_{21} S_1 S_2^2 + \lambda_{22} S_2^3, \end{aligned} \tag{3}$$

where the indices $a, b = 1, 2$ are $SU(2)$ indices and ε is a antisymmetric 2×2 matrix, with $\varepsilon_{12}=1$. We have assumed that the singlets superfields S_1, S_2 are even and the isosinglet superfields D_4, $D_4^{\rm C}$ are odd under the extended R-parity. The matrices Y_u, Y_d, Y_e, the vectors f, g and M_D generate the masses of quarks (including the vector-like one) and leptons.

In addition to the superpotential given by Eq. (3), we have to specify the explicit soft-breaking terms, which read as

$$\begin{aligned} -\mathcal{L}_{\rm soft} = {}& -\mathcal{L}_{\rm soft}^{\rm MSSM} + M_{D_4^{\rm C}}^2 \tilde{d}_4^{\rm C} \tilde{d}_4^{\rm C*} + M_{D_4}^2 \tilde{d}_4 \tilde{d}_4^* \\ &+ M_{S_1}^2 s_1^* s_1 + M_{S_2}^2 s_2^* s_2 \\ &+ \big(A_{f^i} f^i s_1 \tilde{d}_4 \tilde{d}_i^{\rm C} + A_{g^i} g^i s_2 \tilde{d}_4 \tilde{d}_i^{\rm C} + A_{\lambda_{11}} \lambda_{11} s_1^3 \\ &+ A_{\lambda_{12}} \lambda_{12} s_1^2 s_2 + A_{\lambda_{21}} \lambda_{21} s_1 s_2^2 + A_{\lambda_{22}} \lambda_{22} s_2^3 + \text{h.c.}\big). \end{aligned} \tag{4}$$

In the above equation, we take the soft-SUSY breaking coefficients $M^2_{Q,\ldots}$, B, $A_{u,\ldots}$ and $M_{1,2,3}$ as free parameters at the weak scale, and choose them real in order to respect CP invariance of the Lagrangian.

3. Analysis and results

3.1. Minimisation of the neutral scalar potential

After spontaneous electroweak symmetry breaking, in the neutral Higgs sector, we have verified that there is a region of parameter space where minimum of the Higgs potential is at:

$$\begin{aligned} &\langle h_d\rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle h_u\rangle = \begin{pmatrix} 0 \\ v_u e^{i\theta} \end{pmatrix}, \\ &\langle s_1\rangle = V_1 e^{i\phi_1}, \qquad \langle s_2\rangle = V_2 e^{i\phi_2}, \end{aligned} \tag{5}$$

where v_d, v_u, V_1 are real, positive and θ, ϕ_1 and ϕ_2 are physically meaningful phases. It will be shown that the minimum of the potential is at $\theta = 0$. On the contrary, the phases ϕ_1, ϕ_2 are in general non-vanishing and lead to SCPV. Furthermore, we will show that these phases lead to an effective down quark mass matrix capable of generating a CP violating phase in the 3×3 sector of the CKM matrix.

From the superpotential and soft SUSY breaking terms, Eqs. (3) and (4), we derive the following CP-invariant neutral scalar potential:

$$\begin{aligned}V_{\text{neutral}} = {} & V_{\text{neutral}}^{\text{MSSM}} + M_{S_1}^2 V_1^2 + M_{S_2}^2 V_2^2 \\ & + \left(9\lambda_{11}^2 + 1\lambda_{12}^2\right)V_1^4 + 4\left(\lambda_{12}^2 + \lambda_{21}^2\right)V_1^2 V_2^2 \\ & + \left(9\lambda_{22}^2 + 1\lambda_{21}^2\right)V_2^4 + 4V_1V_2\left(3\lambda_{11}\lambda_{12}V_1^2\right. \\ & \left. + \lambda_{21}\left(3\lambda_{22}V_2^2 + \lambda_{12}\left(V_1^2 + V_2^2\right)\right)\right)\cos(\phi_1 - \phi_2) \\ & + 6\lambda_{11}\lambda_{21}V_1^2V_2^2\cos\left(2(\phi_1-\phi_2)\right) \\ & + 6\lambda_{12}\lambda_{22}V_1^2V_2^2\cos\left(2(\phi_1-\phi_2)\right) \\ & + 2A_{\lambda_{12}}\lambda_{12}V_1^2V_2\cos(2\phi_1+\phi_2) \\ & + 2A_{\lambda_{21}}\lambda_{21}V_1V_2^2\cos(\phi_1+2\phi_2) \\ & + 2A_{\lambda_{11}}\lambda_{11}V_1^3\cos(3\phi_1) \\ & + 2A_{\lambda_{22}}\lambda_{22}V_2^3\cos(3\phi_2), \end{aligned} \tag{6}$$

where the MSSM part is

$$\begin{aligned}V_{\text{neutral}}^{\text{MSSM}} = {} & \left(M_{H_d}^2 + \mu^2\right)v_d^2 + \left(M_{H_u}^2 + \mu^2\right)v_u^2 \\ & + \frac{1}{8}\left(g^2 + g'^2\right)\left(v_u^2 - v_d^2\right)^2 - 2B\mu v_u v_d \cos\theta. \end{aligned} \tag{7}$$

From Eq. (7) it is clear that the minimisation equations will require $\theta = 0$, for $\beta\mu$ positive. Note that $\beta\mu$ positive is required, as in the MSSM, in order to obtain positive squared masses for the physical CP-odd state contained in h_u, h_d. This is to be expected as the singlets do not mix with the MSSM doublets and it is well known that the MSSM does not to lead spontaneous CP violation [2,19]. Setting $\theta = 0$ we still have six minimisation equations for the four VEVs and two phases. The minimisation of this potential is quite involved. We have done it numerically, following the procedure of Ref. [20]. To explain it briefly, we determined the minimisation equations which correspond to system of equations

$$\frac{\partial V_{\text{neutral}}}{\partial v_i} = 0, \tag{8}$$

where v_i are the VEV parameters v_d, v_u, V_1, V_2, ϕ_1, ϕ_2. This system of equations can be analytically solved for the quadratic soft masses [20] and for the $A_{\lambda_{11}}$ and $A_{\lambda_{22}}$ parameters [15]. The procedure is therefore to scan over all the VEVs, phases and remaining parameters, and obtain $M_{H_d}^2$, $M_{H_u}^2$, $M_{S_1}^2$, $M_{S_2}^2$, $A_{\lambda_{11}}$, $A_{\lambda_{22}}$ from Eq. (8). We are then certain that the extrema equations are satisfied. To assure that we are at a minimum we compute the eigenvalues of the neutral Higgs boson mass matrices and require that they are all positive, except for the Goldstone bosons that, of course, should remain massless. If this is verified we are at a minimum. Finally we check if this minimum is deeper than the minimum that does not violate CP. Following this procedure we easily obtained a very large number of good solutions, which are true minima of the potential and do violate CP. Next we analyse the question of CP violation in the quark sector.

3.2. *The quark mass matrices*

Upon electroweak gauge symmetry breaking, quark mass matrices are generated through the Yukawa terms, Y_u, Y_d, f, g, and the mass term M_D. Note that Y_u, Y_d and M_D are real. The couplings f and g are also real, but the mass terms generated by them are in general complex due to the phases $\phi_i \equiv \arg\langle s_i\rangle$. Thus, the quark mass matrix are encoded in the following Lagrangian:

$$-\mathcal{L}_{\text{MASS}} = u_i^T (m_u)_{ij} u_j^{\text{C}} + d_\alpha^T (M_d)_{\alpha\beta} d_\beta^{\text{C}} + \text{h.c.}, \tag{9}$$

where $i, j = 1, 2, 3$, $\alpha = 1, 2, 3, 4$, and the matrices m_u and M_d are given

$$m_u = v_u Y_u, \qquad M_d = \begin{pmatrix} m_d & 0 \\ M & M_D \end{pmatrix}, \tag{10}$$

with $m_d = v_d Y_d$, and $M_i = f_i V_1 e^{i\phi_1} + g_i V_2 e^{i\phi_2}$. In order to simplify our analysis, and without loss of generality, we perform an orthogonal weak basis transformation that leaves the matrix Y_u in a diagonal form. The complex elements M_i can be written in terms of moduli and phases as

$$M = \left(|M_1|e^{i\varphi_1}, |M_2|e^{i\varphi_2}, |M_3|e^{i\varphi_3}\right), \tag{11}$$

where these moduli and phases are given by

$$|M_i|^2 = f_i^2 V_1^2 + g_i^2 V_2^2 + 2 f_i g_i V_1 V_2 \cos(\phi_1 - \phi_2), \tag{12}$$

and

$$\tan\varphi_i = \frac{f_i V_1 \sin\phi_1 + g_i V_2 \sin\phi_2}{f_i V_1 \cos\phi_1 + g_i V_2 \cos\phi_2}, \tag{13}$$

respectively. Moreover, the phase φ_1 can be set to zero, without loss of generality, by re-definition of the vector-like quark fields. The phases φ_2 and φ_3 are the only source of CP violating that appear encoded on the 3×4 CKM matrix. In what follows, we assume m_d to be a 3×3 real symmetric matrix. Since the mass terms M, M_D are $\Delta I = 0$, they can be much larger than v_u and v_d.

As we have mentioned in the introduction, the strong CP problem is automatically solved at tree level. The parameter $\bar{\Theta}$ associated with strong CP problem can be written as $\bar{\Theta} = \Theta_{\text{QCD}} + \Theta_{\text{QCD}}$, where Θ_{QCD} is the coefficient of $g_s^2 F\tilde{F}/32\pi^2$ which vanishes since CP symmetry is imposed at the Lagrangian level. The parameter Θ_{QCD}, at tree level, is given by

$$\Theta_{\text{QCD}} = \arg\left[\det(m_u)\det(M_d)\right], \tag{14}$$

since the matrix m_u and the determinant of the matrix M_d from Eq. (10) are real,

$$\det(M_d) = M_D \det(m_d), \tag{15}$$

it follows that

$$\bar{\Theta}_{\text{tree}} = 0, \tag{16}$$

thus providing a strong CP problem solution of the Barr–Nelson type [10,11,21]. We stress that in this model, $\bar{\Theta}$ vanishes at tree level in a natural way. Note that the determinant of Eq. (15) is real due to the Z_3 symmetry, which forbids terms like $Q_i H_d D_4^{\text{C}}$.

As a result, higher order contributions to $\bar{\Theta}$ are finite and calculable. An exact evaluation of $\bar{\Theta}$ in the framework of this model, is beyond the scope of this Letter. However, there are generic arguments [21] which indicate that supersymmetric Barr–Nelson models lead to a $\bar{\Theta}$ smaller than their non-supersymmetric versions. Typically one obtains $\bar{\Theta} < 10^{-13}$.

In order to determine the left-handed unitary rotations which encodes the quark-isosinglet mixings and the CP violation information one has to diagonalise the Hermitian matrix $H = M_d M_d^\dagger$ (4×4) written as

$$H = \begin{pmatrix} m_d^2 & m_d M^\dagger \\ M m_d & \bar{M}^2 \end{pmatrix}, \tag{17}$$

where $\bar{M}^2 = |M_1|^2 + |M_2|^2 + |M_3|^2 + M_D^2$. The matrix H can be diagonalised by the following unitary matrix [18]

$$U = \begin{pmatrix} K & R \\ S & T \end{pmatrix}, \tag{18}$$

leading to:

$$U^\dagger H U = \begin{pmatrix} \mathcal{D}_d^2 & 0 \\ 0 & \tilde{M}_D^2 \end{pmatrix}, \tag{19}$$

where $\mathcal{D}_d^2$ is diagonal formed with the light down quark squared masses. From Eqs. (18) and (19) one derives:

$$\tilde{H} \equiv K \mathcal{D}_d^2 K^{-1} = m_d m_d^\dagger - \frac{m_d M^\dagger M m_d^\dagger}{\bar{M}^2}\left(\mathbf{1} - \frac{\tilde{H}}{\bar{M}^2}\right)^{-1}. \tag{20}$$

An effective 3×3 light down quark mass matrix can be easily obtained in the limit where K is an unitary matrix and $\tilde{H} \ll \bar{M}^2$ [18,22]. One obtains:

$$m_{\text{eff}} m_{\text{eff}}^\dagger \approx m_d \left(\mathbf{1} - \frac{M^\dagger M}{\bar{M}^2}\right) m_d, \tag{21}$$

while the mass of the down-type quark isosinglet is given in an excellent approximation by

$$\tilde{M}_D \approx \bar{M}. \tag{22}$$

Note that the matrix K is almost unitary since from Eq. (19) and exact unitarity of U, it follows that

$$K^\dagger K = 1 - S^\dagger S, \tag{23}$$

with $S \approx \mathcal{O}(m_d/\bar{M})$ [18]. Although, the matrix m_d is real, the phases in $M^\dagger M$ in Eq. (21) are sufficient to generate a complex K. The full CKM matrix, which now has dimension 3×4, comes directly from the matrix U and is given by:

$$(V_{\text{CKM}})_{i\alpha} = (\, K \quad R\,)_{i\alpha}\,, \tag{24}$$

which contributes to the electroweak Lagrangian as:

$$\begin{aligned}\mathcal{L}_{\text{W,Z}} = &-\frac{g}{\sqrt{2}}\left(\bar{u}_{Li}\gamma^\mu (V_{\text{CKM}})_{i\alpha} d_{L\alpha} W_\mu^+ + \text{h.c.}\right) \\ &- \frac{g}{2\cos\theta_{\text{W}}}\left(\bar{u}_{Li}\gamma^\mu u_{Li} - \bar{d}_{L\alpha}\gamma^\mu X_{\alpha\beta} d_{L\beta} \right. \\ &\left. - 2\sin^2\theta_{\text{W}} J_{\text{EM}}^\mu\right) Z_\mu,\end{aligned} \tag{25}$$

where

$$X_{\alpha\beta} \equiv \left(V_{\text{CKM}}^\dagger V_{\text{CKM}}\right)_{\alpha\beta} = \delta_{\alpha\beta} - \begin{pmatrix} S^\dagger S & S^\dagger T \\ T^\dagger S & T^\dagger T \end{pmatrix}_{\alpha\beta}, \tag{26}$$

and the electromagnetic current, J_{EM}^μ, is given by

$$J_{\text{EM}}^\mu = \frac{2}{3}\bar{u}_i \gamma^\mu u_i - \frac{1}{3}\bar{d}_\alpha \gamma^\mu d_\alpha. \tag{27}$$

The 3×4 CKM matrix from Eq. (24) encodes three independent phases which violate CP. However in the limit where one neglects the small mixings between the physical heavy quark d_4 and the light quarks (i.e. the limit where K becomes unitary) only one phase appears in K [23].

3.3. Numerical example

In this section we present one concrete numerical example which leads to values of quark masses and mixings in agreement with experiments. It is clear from the effective mass matrix presented in Eq. (21), that the light quarks and their mixing do not depend on the overall scale, which only rescales the heavy vector-like down quark mass. We have verified numerically that this feature still holds, at a very high precision, provided $M_D \gtrsim 1$ TeV, which enforces the goodness of the effective matrix given in Eq. (21). We have then parametrised the input values M_i as a function of an overall scale $M_D = 1$ TeV. Moreover, we fixed the down quark masses to the following values:

$$m_d = 4.38\ \text{MeV}, \qquad m_s = 94.6\ \text{MeV}, \qquad m_b = 3.11\ \text{GeV}. \tag{28}$$

We have verified, using Eq. (20) that there are solutions to the matrix m_d leading to the effective 3×3 CKM matrix K consistent with experimental data. This was done by varying the ratios M_i/M_D and the others parameters in a consistent way with Eq. (20). A particular example is obtained with the following values for the ratios M_i/M_D:

$$\frac{M_1}{M_D} = 0.5, \qquad \frac{M_2}{M_D} = 0.8, \qquad \frac{M_3}{M_D} = 3.8, \tag{29}$$

and the following values for the phases φ_i (in radians):

$$\varphi_2 = 1.1, \qquad \varphi_3 = 1.7. \tag{30}$$

We considered the moduli of matrix K elements as

$$\left|V_{\text{CKM}}^{3\times 3}\right| \equiv |K| = \begin{pmatrix} 0.97 & 0.22 & 0.0038 \\ 0.22 & 0.97 & 0.04 \\ 0.0086 & 0.04 & 0.9992 \end{pmatrix}, \tag{31}$$

that are within the range allowed by experiments [24]. The relevant CP violating weak basis invariants that encodes phases content in K are

$$J \equiv \mathcal{I}m\left(K_{12} K_{13}^* K_{22}^* K_{23}\right) = 3.14 \times 10^{-5}, \tag{32}$$

and

$$\beta \equiv \arg\left(-K_{21} K_{23}^* K_{31}^* K_{33}\right) = 23.8^\circ, \tag{33}$$

which corresponds to:

$$\sin 2\beta = 0.739, \tag{34}$$

in agreement with the most recent experimental data [24]. Note that the quantities J and $\sin 2\beta$ are only relevant to describe CP violation in the quark sector when K is nearly unitarity. The deviation from unitarity of the matrix K is given by

$$\mathbf{1} - K^\dagger K = S^\dagger S \simeq \mathcal{O}(10^{-7}), \tag{35}$$

which confirms our previous assertions about the almost unitarity of K. Thus, the values of the elements of m_d obtained are:

$$m_d = \begin{pmatrix} 0.01 & 0.025 & 0.02 \\ 0.025 & 0.13 & 0.47 \\ 0.02 & 0.47 & 9.67 \end{pmatrix}. \tag{36}$$

Once the matrix m_d is determined, the mass of the heavy vector-like down quark state can be calculated by using Eqs. (17)–(19), giving the following result:

$$\tilde{M}_D \simeq \bar{M} = 4.0\ \text{TeV}, \tag{37}$$

which enforces the excellent approximation given in Eq. (22).

Finally we have to verify if the inputs are compatible with the minimisation of the potential. The only parameters that enter the quark sector and that are relevant for the minimisation of the potential are the VEVs V_1, V_2 and the phases ϕ_1 and ϕ_2. As can be seen from Eqs. (12) and (13) there is a significant freedom in obtaining the values of V_i and ϕ_i from M_i and φ_i. We have numerically verified that there is a large number of possible choices of parameters that give a successful minimisation for a given choice of M_i and φ_i, thus implying that our solutions are fully consistent.

4. Discussion and conclusions

We have presented a new minimal supersymmetric extension of the SM, where CP is spontaneously broken. The novel feature of the model is the fact the CKM matrix is non-trivially complex, in contrast with previous SUSY models with SCPV.

It was emphasised that having a complex CKM matrix is crucial, in view of the recent measurement of the angle γ of the unitarity triangle. Prior to this important experimental result, it was possible to have viable SUSY extensions of SM, with SCPV and a real CKM matrix, where CP violation in the kaon and B-sectors was entirely generated by new SUSY contributions to K^0–$\bar{K}^0$ and B–$\bar{B}$ mixings. This class of models are no longer viable. In order to generate a non-trivial CP-violating phase in the CKM matrix, we have introduced a $Q = -1/3$ heavy isosinglet quark. The mass terms mixing this heavy quark with the standard quarks are responsible for the generation of a complex effective 3×3 CKM matrix, which in turn leads to a complex CKM matrix.

Due to the appearance of naturally suppressed FCNC at tree level, in the down-quark sector, there are in the model new contributions [25] to K^0–$\bar{K}^0$, B_d–$\bar{B}_d$ and B_s–$\bar{B}_s$ mixings. At present, all experimental data on CP violation as well as on rare K and B decays is in impressive agreement with the SM. It seems by now clear that the SM and its CKM mechanism of mixing and CP violation give the dominant contributions to the physical quantities entering the standard unitarity triangle. However, there is still room for New Physics, that can even give a dominant contribution to physical quantities which do not enter directly in the standard unitarity triangle, an example being the invariant phase $\chi \equiv \arg(-V_{ts}V_{tb}^*V_{cs}^*V_{cb})$. This phase has not been measured yet, but it will be measured at LHCb.

We have also emphasised that the class of models presented here, have the potential of alleviating the so-called SUSY CP-problem. In particular, we have pointed out that since CP violation arises entirely from VEVs of the Higgs singlets, they will not show up in the neutralino and chargino sectors.

A complete analysis of the phenomenological implications of this model, including the evaluation of EDMs is outside the scope of this Letter and it will be presented elsewhere.

The fact that a minimal realistic extension of the MSSM, capable of generating spontaneous CP violation, requires the introduction of vector-like quarks, provides further motivation for the search of these heavy fermions which arise in a variety of extensions of the SM.

Acknowledgements

This work was partially supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal), through the projects POCTI/FNU/44409/2002, PDCT/FP/FNU/50250/2003, POCI/FP/63405/2005, POCI/FP/63415/2005, POCTI/FP/FNU/50167/2003, by the CFTP-FCT UNIT 777, which are partially funded through POCTI (FEDER), and by the European Commission Human Potential Program RTN network MRTN-CT-2004-503369. The work of G.C.B. was supported by the Alexander von Humboldt Foundation through a Humboldt Research Award. G.C.B. would like to thank Andrzej J. Buras for the kind hospitality at TUM. D.E.C. is supported by *Fundação para a Ciência e a Tecnologia* (FCT, Portugal) under the grants SFRH/BPD/1598/2000.

References

[1] T.D. Lee, Phys. Rev. D 8 (1973) 1226.
[2] G.C. Branco, Phys. Rev. Lett. 44 (1980) 504;
G.C. Branco, Phys. Rev. D 22 (1980) 2901;
G.C. Branco, A.J. Buras, J.M. Gerard, Nucl. Phys. B 259 (1985) 306.
[3] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531;
M. Kobayashi, T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
[4] S. Abel, S. Khalil, O. Lebedev, Nucl. Phys. B 606 (2001) 151, hep-ph/0103320.
[5] S. Khalil, Int. J. Mod. Phys. A 18 (2003) 1697, hep-ph/0212050.
[6] R.D. Peccei, in: CP Violation, Advanced Series on Directions in High Energy Physics, vol. 3, World Scientific, Singapore, 1989, p. 503;
J.E. Kim, Phys. Rep. 150 (1987) 1;
H.Y. Cheng, Phys. Rep. 158 (1988) 1.
[7] R.N. Mohapatra, G. Senjanovic, Phys. Lett. B 79 (1978) 283.
[8] H. Georgi, Hadronic J. 1 (1978) 155.
[9] S.M. Barr, P. Langacker, Phys. Rev. Lett. 42 (1979) 1654.
[10] A.E. Nelson, Phys. Lett. B 136 (1984) 387.
[11] S.M. Barr, Phys. Rev. Lett. 53 (1984) 329.
[12] B. Aubert, et al., BaBar Collaboration, Phys. Rev. Lett. 93 (2004) 131801;
B. Aubert, et al., BaBar Collaboration, hep-ex/0408088.

[13] Y. Chao, et al., Belle Collaboration, Phys. Rev. Lett. 93 (2004) 191802; K. Abe, et al., Belle Collaboration, hep-ex/0411049.
[14] G.C. Branco, F. Kruger, J.C. Romão, A.M. Teixeira, JHEP 0107 (2001) 027.
[15] C. Hugonie, J.C. Romão, A.M. Teixeira, JHEP 0306 (2003) 020.
[16] G.C. Branco, G.C. Cho, Y. Kizukuri, N. Oshimo, Nucl. Phys. B 449 (1995) 483;
G.C. Branco, G.C. Cho, Y. Kizukuri, N. Oshimo, Phys. Lett. B 337 (1994) 316;
S.A. Abel, J.M. Frere, Phys. Rev. D 55 (1997) 1623;
S. Khalil, T. Kobayashi, A. Masiero, Phys. Rev. D 60 (1999) 075003;
D.A. Demir, A. Masiero, O. Vives, Phys. Rev. D 61 (2000) 075009;
M. Brhlik, L.L. Everett, G.L. Kane, S.F. King, O. Lebedev, Phys. Rev. Lett. 84 (2000) 3041;
S. Baek, J.H. Jang, P. Ko, J.H. Park, Phys. Rev. D 62 (2000) 117701.
[17] F.J. Botella, G.C. Branco, M. Nebot, M.N. Rebelo, Nucl. Phys. B 725 (2005) 155.
[18] L. Bento, G.C. Branco, P.A. Parada, Phys. Lett. B 267 (1991) 95;
L. Bento, G.C. Branco, Phys. Lett. B 245 (1990) 599.
[19] J.C. Romão, Phys. Lett. B 173 (1986) 309.
[20] J.C. Romão, C.A. Santos, J.W.F. Valle, Phys. Lett. B 288 (1992) 311.
[21] S.M. Barr, A. Masiero, Phys. Rev. D 38 (1988) 366;
S.M. Barr, A. Masiero, Phys. Rev. Lett. 58 (1987) 187.
[22] W. Grimus, L. Lavoura, JHEP 0011 (2000) 042.
[23] G.C. Branco, L. Lavoura, Nucl. Phys. B 278 (1986) 738.
[24] S. Eidelman, et al., Particle Data Group, Phys. Lett. B 592 (2004) 1.
[25] G.C. Branco, P.A. Parada, T. Morozumi, M.N. Rebelo, Phys. Rev. D 48 (1993) 1167.