

**Neutrino mass matrices with vanishing determinant**Bhag C. Chauhan<sup>\*,†</sup> and João Pulido<sup>‡</sup>*Centro de Física Teórica das Partículas (CFTP) Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais, P-1049-001 Lisboa, Portugal*Marco Picariello<sup>§</sup>*I.N.F.N.—Milano and Dipartimento di Fisica, Università di Milano and Dipartimento di Fisica, Università di Lecce Via Arnesano, ex Collegio Fiorini, I-73100 Lecce, Italia*  
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We investigate the prospects for neutrinoless double beta decay, texture zeros, and equalities between neutrino mass matrix elements in scenarios with vanishing determinant mass matrices for vanishing and finite  $\theta_{13}$  mixing angles in normal and inverse mass hierarchies. For normal hierarchy and both zero and finite  $\theta_{13}$  it is found that neutrinoless double beta decay cannot be observed by any of the present or next generation experiments, while for inverse hierarchy it is, on the contrary, accessible to experiments. Regarding texture zeros and equalities between mass matrix elements, we find that in both normal and inverse hierarchies with  $\theta_{13} = 0$  no texture zeros nor any such equalities can exist apart from the obvious ones. For  $\theta_{13} \neq 0$  some texture zeros become possible. In normal hierarchy two texture zeros occur if  $8.1 \times 10^{-2} \leq |\sin\theta_{13}| \leq 9.1 \times 10^{-2}$  while in inverse hierarchy three are possible, one with  $|\sin\theta_{13}| \geq 7 \times 10^{-3}$  and two others with  $|\sin\theta_{13}| \geq 0.18$ . All equalities between mass matrix elements are impossible with  $\theta_{13} \neq 0$ .

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**I. INTRODUCTION**

It is a well-known fact that the neutrino mass matrix contains nine parameters while feasible experiments can hope to determine only seven of them in the foreseeable future. This situation can however be overcome, with the number of free parameters being reduced, if physically motivated assumptions are made to restrict the form of the matrix. Among the most common such assumptions and as an incomplete list one may refer to the texture zeros [1], hybrid textures [2], traceless condition [3–5], and vanishing determinant [6–8], the latter two assumptions being basis independent, as shall be seen, and the vanishing determinant one equivalent to one vanishing neutrino mass.

In this paper we perform an investigation on vanishing determinant neutrino mass matrices aimed at neutrinoless double beta decay ( $0\nu\nu\beta\beta$ ), texture zeros and equalities between mass matrix entries. We will assume that neutrinos are Majorana [9], as favored by some experimental evidence [10], and study the neutrino mass matrix  $M$  in the weak basis where all charge leptons are already diagonalized. This is related to the diagonal mass matrix  $D$  through the unitary transformation

$$D = U_{\text{MNS}}^T M U_{\text{MNS}}, \quad (1)$$

where we use the standard parametrization [11]

$$U_{\text{MNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (2)$$

where  $\delta$  is a Dirac  $CP$  violating phase. Equation (1) is equivalent to

$$M = U^* \text{diag}(m_1, m_2 e^{i\phi_1}, m_3 e^{i\phi_2}) U^\dagger, \quad (3)$$

where  $\phi_1, \phi_2$  are two extra  $CP$  violating Majorana phases and  $D = \text{diag}(m_1, m_2 e^{i\phi_1}, m_3 e^{i\phi_2})$ . Applying determinants

properties

$$\begin{aligned} \det M &= \det(U^* D U^\dagger) = \det(U^* U^\dagger D) \\ &= \det U^* \det U^\dagger \det D = \det D (U \text{ real}) \\ &\neq \det D (U \text{ complex}) \end{aligned} \quad (4)$$

because if matrix  $U$  is real,  $U^* U^\dagger = U U^T = 1$ , which is satisfied provided  $\delta = 0$  or  $\theta_{13} = 0$  [see Eq. (2)]. Thus the determinant is not in general basis independent. In order that  $\det D = \det M$  it is necessary and sufficient that there is either no Dirac  $CP$  violation or that it is unobservable. The same arguments hold for the condition  $\text{Tr} D = \text{Tr} M$  [4].

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From Eq. (4) we get that  $\det M = 0$  if and only if  $\det D = 0$ , because  $\det U^\dagger$  and  $\det U^*$  are not zero. The vanishing determinant condition is basis independent, corresponding to a zero eigenvalue of the mass matrix. So requiring  $\det M = 0$  is equivalent to assuming one of the neutrinos to be massless. This is realized for instance in the Affleck-Dine scenario for leptogenesis [12–14] which requires the lightest neutrino to be practically massless ( $m \simeq 10^{-10}$  eV) [15,16]. We will consider separately the cases of vanishing and finite  $\theta_{13}$ .<sup>1</sup> In the first the Dirac phase is unobservable and the usual definition  $U_{\text{MNS}} = U_{23}U_{13}U_{12}$  [18] simplifies to  $U_{\text{MNS}} = U_{23}U_{12}$  with

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha_{22} & \alpha_{23} \\ 0 & \alpha_{32} & \alpha_{33} \end{pmatrix}, \quad U_{12} = \begin{pmatrix} \beta_{11} & \beta_{12} & 0 \\ \beta_{21} & \beta_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (5)$$

where the unitarity condition ( $|\alpha_{22}\alpha_{33} - \alpha_{32}\alpha_{23}| = |\beta_{11}\beta_{22} - \beta_{12}\beta_{21}| = 1$ ) implies  $\alpha_{22}\alpha_{33}\alpha_{32}\alpha_{23} < 0$  and  $\beta_{11}\beta_{22}\beta_{12}\beta_{21} < 0$  with  $\alpha_{22} = \pm \cos\theta_\otimes$ ,  $\beta_{11} = \pm \cos\theta_\otimes$ , the remaining matrix elements being evident. For neutrino masses and mixings we refer to the following  $2\sigma$  ranges [17,19]:

$$\Delta m_\otimes^2 = m_2^2 - m_1^2 = 7.92 \times 10^{-5} (1 \pm 0.09) \text{ eV}^2, \quad (6)$$

$$\Delta m_\otimes^2 = m_3^2 - m_2^2 = \pm 2.4 \times 10^{-3} (1 \pm_{0.61}^{0.21}) \text{ eV}^2, \quad (7)$$

$$\sin^2\theta_\otimes = 0.314 (1 \pm_{0.15}^{0.18}), \quad (8)$$

$$\sin^2\theta_\otimes = 0.44 (1 \pm_{0.22}^{0.41}), \quad (9)$$

obtained from a 3-flavor analysis of all solar and atmospheric data. This favors the widely used form of the  $U_{\text{MNS}}$  matrix [20] (all entries taken in their moduli)

$$U_{\text{MNS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (10)$$

$$M = U_{23}U_{12}DU_{12}^\dagger U_{23}^\dagger = \begin{pmatrix} 3\epsilon e^{i\phi} \beta_{12}^2 & 3\epsilon e^{i\phi} \alpha_{22}\beta_{12}\beta_{22} & 3\epsilon e^{i\phi} \alpha_{32}\beta_{12}\beta_{22} \\ 3\epsilon e^{i\phi} \alpha_{22}\beta_{12}\beta_{22} & 3\epsilon e^{i\phi} \alpha_{22}^2\beta_{22}^2 + m\alpha_{23}^2 & 3\epsilon e^{i\phi} \alpha_{22}\alpha_{32}\beta_{22}^2 + m\alpha_{23}\alpha_{33} \\ 3\epsilon e^{i\phi} \alpha_{32}\beta_{12}\beta_{22} & 3\epsilon e^{i\phi} \alpha_{22}\alpha_{32}\beta_{22}^2 + m\alpha_{23}\alpha_{33} & 3\epsilon e^{i\phi} \alpha_{32}^2\beta_{22}^2 + m\alpha_{33}^2 \end{pmatrix}. \quad (14)$$

Owing to the sign ambiguities of parameters  $\alpha$  and  $\beta$ , four possibilities for matrix  $M$  arise. Suppose entries 12 and 13 in this matrix have  $(+)(+)$  signs. Then  $\alpha_{22}$ ,  $\alpha_{32}$  have the same sign as  $\beta_{12}\beta_{22}$ , that is  $\alpha_{22}\alpha_{32}$  in the (23) entry is  $(+)$ , implying the opposite sign for the coefficient of  $m$

<sup>1</sup>The  $2\sigma$  range recently obtained for this quantity is [17]  $\sin^2\theta_{13} = 0.9 \pm_{0.9}^{2.3} \times 10^{-2}$ , the lower uncertainty being purely formal, corresponding to the positivity constraint  $\sin^2\theta_{13} \geq 0$ .

For finite  $\theta_{13}$  we will work in the approximation [17]

$$U_{13} = \begin{pmatrix} \gamma_{11} & 0 & \gamma_{13} \\ 0 & 1 & 0 \\ \gamma_{31} & 0 & \gamma_{33} \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 & \gamma_{13} \\ 0 & 1 & 0 \\ \gamma_{31} & 0 & 1 \end{pmatrix}, \quad (11)$$

with  $\gamma_{13} = U_{e3} = s_{13}e^{i\delta} = -\gamma_{31}^*$ . This leads to

$$U_{\text{MNS}} = \begin{pmatrix} \beta_{11} & \beta_{12} & \gamma_{13} \\ \alpha_{23}\gamma_{31}\beta_{11} + \alpha_{22}\beta_{21} & \alpha_{23}\gamma_{31}\beta_{12} + \alpha_{22}\beta_{22} & \alpha_{23} \\ \alpha_{33}\gamma_{31}\beta_{11} + \alpha_{32}\beta_{21} & \alpha_{33}\gamma_{31}\beta_{12} + \alpha_{32}\beta_{22} & \alpha_{33} \end{pmatrix}, \quad (12)$$

which generalizes Eq. (10) for small  $s_{13}$ .

The paper is organized as follows: in Sec. II we derive all possible forms of the mass matrix  $M$  in this scenario for both normal and inverse hierarchies and investigate their consequences for  $0\nu\nu\beta\beta$  decay. Since one of the neutrinos is massless, there is only one Majorana phase to be considered. In Sec. III we investigate the prospects for texture zeros and equalities between matrix elements in both hierarchies. In Secs. II and III only  $\theta_{13} = 0$  is considered. In Sec. IV the previous analysis is extended to  $\theta_{13} \neq 0$ , and in Sec. V we briefly expound our main conclusions.

## II. MASS MATRICES WITH VANISHING $\theta_{13}:\beta\beta_{0\nu\nu}$ DECAY

### A. Normal hierarchy (NH)

This is the case where the two mass eigenstates involved in the solar oscillations are assumed to be the lightest so that  $\Delta m_\otimes^2 = \Delta m_{32}^2 > 0$ . We will consider this case as a departure from the degenerate one with  $\Delta m_\otimes^2 = \Delta m_{21}^2 = 0$  and break the degeneracy with a real parameter  $\epsilon$ . Matrix  $D$  with  $m$  and  $\epsilon$  both real is therefore

$$D = \text{diag}(0, 3\epsilon e^{i\phi}, m), \quad (13)$$

where  $\phi$  is the Majorana relative phase between the second and third diagonal matrix elements ( $\phi = \phi_1 - \phi_2$  in the notation of Sec. II) and  $\Delta m_\otimes^2 = 9\epsilon^2$ . Using Eqs. (5) the matrix  $M$  is

( $\alpha_{23}\alpha_{33}$ ). So Eq. (12) has the form

$$M = \begin{pmatrix} \epsilon e^{i\phi} & \epsilon e^{i\phi} & \epsilon e^{i\phi} \\ \epsilon e^{i\phi} & (m/2) + \epsilon e^{i\phi} & -(m/2) + \epsilon e^{i\phi} \\ \epsilon e^{i\phi} & -(m/2) + \epsilon e^{i\phi} & (m/2) + \epsilon e^{i\phi} \end{pmatrix}. \quad (15)$$

Suppose entries 12 and 13 in the matrix have  $(-)(-)$  signs. Then  $\alpha_{22}$ ,  $\alpha_{32}$  have opposite sign to  $\beta_{12}\beta_{22}$ , that is they have the same sign, so  $\alpha_{22}\alpha_{32}$  is  $(+)$  and  $\alpha_{23}\alpha_{33}$  is

(−) so

$$M = \begin{pmatrix} \epsilon e^{i\phi} & -\epsilon e^{i\phi} & -\epsilon e^{i\phi} \\ -\epsilon e^{i\phi} & (m/2) + \epsilon e^{i\phi} & -(m/2) + \epsilon e^{i\phi} \\ -\epsilon e^{i\phi} & -(m/2) + \epsilon e^{i\phi} & (m/2) + \epsilon e^{i\phi} \end{pmatrix}. \quad (16)$$

Suppose entries 12 and 13 in the matrix have (+) (−) signs. Then  $\alpha_{22}$ ,  $\alpha_{32}$  have opposite signs to each other, so  $\alpha_{22}\alpha_{32}$  is (−) and  $\alpha_{23}\alpha_{33}$  is (+). Hence

$$M = \begin{pmatrix} \epsilon e^{i\phi} & \epsilon e^{i\phi} & -\epsilon e^{i\phi} \\ \epsilon e^{i\phi} & (m/2) + \epsilon e^{i\phi} & (m/2) - \epsilon e^{i\phi} \\ -\epsilon e^{i\phi} & (m/2) - \epsilon e^{i\phi} & (m/2) + \epsilon e^{i\phi} \end{pmatrix}. \quad (17)$$

Suppose entries 12 and 13 in the matrix have (−) (+) signs. Then  $\alpha_{22}$ ,  $\alpha_{32}$  have opposite signs to each other, so  $\alpha_{22}\alpha_{32}$  is (−) and  $\alpha_{23}\alpha_{33}$  is (+). Hence the matrix is

$$M = \begin{pmatrix} \epsilon e^{i\phi} & -\epsilon e^{i\phi} & \epsilon e^{i\phi} \\ -\epsilon e^{i\phi} & (m/2) + \epsilon e^{i\phi} & (m/2) - \epsilon e^{i\phi} \\ \epsilon e^{i\phi} & (m/2) - \epsilon e^{i\phi} & (m/2) + \epsilon e^{i\phi} \end{pmatrix}. \quad (18)$$

All matrices (15)–(18) have a vanishing determinant as can be verified easily. For  $0\nu\nu\beta\beta$  decay

$$\langle m_{ee} \rangle = |U_{e1}^2 m_1 + U_{e2}^2 m_2 e^{i\phi_1} + U_{e3}^2 m_3 e^{i\phi_2}|. \quad (19)$$

Hence, for  $m_1 = 0$  and  $U_{e3} = \gamma_{13} = s_{13} e^{i\delta} = 0$

$$\langle m_{ee} \rangle = |U_{e2}^2 m_2 e^{i\phi_1}| = \frac{1}{3} 3\epsilon = \frac{1}{3} \sqrt{\Delta m_{\odot}^2} \simeq 3 \times 10^{-3} \text{ eV}, \quad (20)$$

where we used  $\epsilon = \frac{1}{3} \sqrt{\Delta m_{\odot}^2}$ . So the Majorana phase is not an observable.

There is no commonly accepted evidence in favor of  $0\nu\nu\beta\beta$  decay but there exist reliable upper limits on  $\langle m_{ee} \rangle$ :

$$\langle m_{ee} \rangle \leq (0.3 - 1.2) \text{ eV}, \quad \langle m_{ee} \rangle \leq (0.2 - 1.1) \text{ eV} \quad (21)$$

(the upper limits on  $\langle m_{ee} \rangle$  above are taken from Refs. [10,21], respectively), where the uncertainties follow from the uncertainties in the nuclear matrix elements. The future CUORE experiment [22], of which CUORICINO is a test version [21], is expected to improve this upper bound to  $3 \times 10^{-2}$  eV. Other experiments are also proposed (MAJORANA [23], GENIUS [24], GEM [25] and others) in which the sensitivity of a few  $10^{-2}$  eV is planned to be reached.

*Conclusion.*—Vanishing determinant with vanishing  $\theta_{13}$  and NH implies that  $0\nu\nu\beta\beta$  decay cannot be detected even in the next generation of experiments.

## B. Inverse hierarchy (IH)

We start with matrix  $D$  in the form  $D = \text{diag}\{m, (m + \epsilon)e^{i\phi}, 0\}$  where  $m, \epsilon$  are complex,  $|m| \simeq \sqrt{\Delta m_{\odot}^2}$ ,  $|\epsilon| \simeq \sqrt{\Delta m_{\odot}^2}$  and chosen in such a way that  $m + \epsilon = \tilde{m}$  is real ( $\epsilon = 0$  corresponds to the degenerate case). Alternatively  $D = \text{diag}\{\tilde{m} - \epsilon, \tilde{m}e^{i\phi}, 0\}$  with, of course,  $\tilde{m} - \epsilon$  complex. Multiplying the whole matrix by the inverse phase of  $\tilde{m} - \epsilon$ , it can be redefined as

$$D = \text{diag}\{\tilde{m} - \lambda, \tilde{m}e^{i(\phi-\psi)}, 0\}, \quad (22)$$

with  $\lambda$  real and defined by  $(\tilde{m} - \epsilon)e^{-i\psi} = \tilde{m} - \lambda$  (notice that  $\tilde{m} - \epsilon = |\tilde{m} - \epsilon|e^{i\psi}$  and  $\tilde{m} = \sqrt{\Delta m_{\odot}^2}$ ). There are two solutions for  $\lambda$ . In fact, imposing the solar mass square difference

$$\Delta m_{\odot}^2 = |d_{22}|^2 - |d_{11}|^2 = \tilde{m}^2 - \tilde{m}^2 + 2\lambda\tilde{m} - \lambda^2, \quad (23)$$

and solving the quadratic equation  $\lambda^2 - 2\lambda\tilde{m} + \Delta m_{\odot}^2 = 0$  one gets

$$\lambda = \tilde{m} \pm \sqrt{\tilde{m}^2 - \Delta m_{\odot}^2} = \lambda_{\pm}. \quad (24)$$

Notice that  $\lambda_+$  is large and  $\lambda_-$  is small. To first order in  $\frac{\Delta m_{\odot}^2}{\tilde{m}^2} = \frac{\Delta m_{\odot}^2}{\Delta m_{\odot}^2} \simeq 0.30$  one has

$$\lambda_+ = \tilde{m} \left( 2 - \frac{1}{2} \frac{\Delta m_{\odot}^2}{\Delta m_{\odot}^2} \right) \simeq 1.85\tilde{m}, \quad (25)$$

$$\lambda_- = \frac{\tilde{m}}{2} \frac{\Delta m_{\odot}^2}{\Delta m_{\odot}^2} \simeq \frac{\tilde{m}}{60}. \quad (26)$$

It is straightforward to see that  $D(\lambda_-, \phi + \pi) = -D(\lambda_+, \phi)$  and the same property holds for matrix  $M$ , namely,  $M(\lambda_-, \phi + \pi) = -M(\lambda_+, \phi)$  because  $U_{\text{MNS}}$  is invariant under the transformations  $\lambda_+ \rightarrow \lambda_-$  and  $\phi \rightarrow \phi + \pi$ . So the two solutions for  $\lambda$  are equivalent: one may take either

$$\lambda_+, \psi = 0 \quad (27)$$

or

$$\lambda_-, \psi = \pi. \quad (28)$$

Using  $M = U_{23}U_{12}DU_{12}^T U_{23}^T$  with Eqs. (5) and (22), the matrix  $M$  now has the form

$$M = \begin{pmatrix} \tilde{m}(1 - \frac{1}{3}) - \frac{2}{3}\lambda & (\text{sign})\frac{1}{3}(\tilde{m}t - \lambda) & (\text{sign})\frac{1}{3}(\tilde{m}t - \lambda) \\ (\text{sign})\frac{1}{3}(\tilde{m}t - \lambda) & \tilde{m}(\frac{1}{2} - \frac{1}{3}) - \frac{\lambda}{6} & (\text{sign})[\tilde{m}(\frac{1}{2} - \frac{1}{3}) - \frac{\lambda}{6}] \\ (\text{sign})\frac{1}{3}(\tilde{m}t - \lambda) & (\text{sign})[\tilde{m}(\frac{1}{2} - \frac{1}{3}) - \frac{\lambda}{6}] & \tilde{m}(\frac{1}{2} - \frac{1}{3}) - \frac{\lambda}{6} \end{pmatrix}. \quad (29)$$

which also verifies  $\det M = 0$  as expected. Equation (29) is formally the same for  $\lambda = \lambda_+$  and  $\lambda = \lambda_-$  with the definition  $t = 1 - e^{i\phi}$  for  $\lambda = \lambda_+$  and  $t = 1 + e^{i\phi}$  for  $\lambda = \lambda_-$ , the sign affecting the exponential being related to the  $\psi$  phase. The

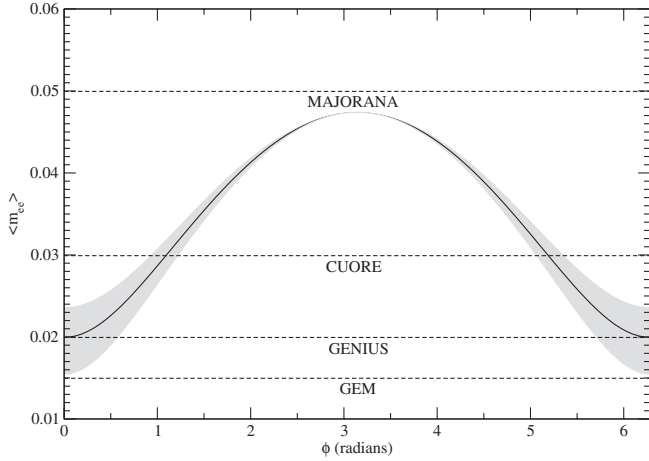


FIG. 1.  $\beta\beta_{0\nu\nu}$  decay effective mass parameter  $\langle m_{ee} \rangle$  as a function of the Majorana phase  $\phi$  showing its accessibility for forthcoming experiments.

structure of (+) and (−) in Eq. (29) is the same as before [(15)–(18)]: equal signs in entries  $M_{12}, M_{13}$  correspond to (+) in both entries  $M_{23}, M_{32}$  while different signs in  $M_{12}, M_{13}$  correspond to (−) in both entries  $M_{23}, M_{32}$ . Equation (29) is the equivalent for IH of (15)–(18) for NH.

For  $\beta\beta_{0\nu\nu}$  decay we have

$$\begin{aligned} \langle m_{ee} \rangle &= |U_{e1}^2 m_1 + U_{e2}^2 m_2 e^{i\phi_1} + U_{e3}^2 m_3 e^{i\phi_2}| \\ &= \left| \frac{2}{3}(\tilde{m} - \lambda_{\pm}) \pm \frac{1}{3}\tilde{m}e^{i(\phi)} \right|. \end{aligned} \quad (30)$$

The quantity  $m_{ee}$  is displayed in Fig. 1 as a function of the phase difference  $\phi$ . The shaded areas correspond to the  $1\sigma$  uncertainties in the solar angle  $\theta_{\odot}$ . It is seen from Eq. (30) and Fig. 1 that for inverse hierarchy (vanishing  $\theta_{13}$  and mass matrix determinant)  $\beta\beta_{0\nu\nu}$  decay is phase dependent and within observational limits of forthcoming experiments. Consequently, our conclusion follows:

*Conclusion.*—Models with vanishing determinant mass matrix and vanishing  $\theta_{13}$  provide, in inverse hierarchy, a Majorana phase dependent  $\beta\beta_{0\nu\nu}$  decay which is physically observable for most values of the phase in the next generation of experiments.

### III. TEXTURE ZEROS AND EQUALITIES BETWEEN $M$ MATRIX ELEMENTS ( $\theta_{13} = 0$ )

#### A. Texture zeros

Here we analyze the possibility of vanishing entries in the mass matrix  $M$ . Taking first NH and recalling Eqs. (15)–(18), it is seen that this implies either  $m/2 = \pm \epsilon e^{i\phi}$  or  $\epsilon = 0$ —both situations being impossible. For IH three cases need to be considered:

(a)  $M_{11} = 0$ . We have in this case  $\tilde{m}(1 - \frac{t}{3}) - \frac{2}{3}\lambda = 0$  implying

$$\tilde{m}(3 - t) = 2\lambda. \quad (31)$$

Replacing  $t \rightarrow 1 - e^{i\phi}$  and  $\lambda \rightarrow \lambda_+$  this leads to

$$e^{i\phi} = 2\sqrt{1 - \frac{\Delta m_{\odot}^2}{\Delta m_{\oplus}^2}}, \quad (32)$$

which is experimentally excluded.

(b)  $M_{12} = 0$ . This gives  $\tilde{m}t - \lambda = 0$ , hence using the same replacement

$$e^{i\phi} = -\sqrt{1 - \frac{\Delta m_{\odot}^2}{\Delta m_{\oplus}^2}}, \quad (33)$$

which is also impossible since  $\Delta m_{\odot}^2 = 0$  is strictly excluded experimentally.

(c)  $M_{22} = 0$ . This gives  $\tilde{m}(\frac{1}{2} - \frac{t}{3}) - \frac{\lambda}{6} = 0$ , hence using the same replacement

$$e^{i\phi} = \frac{1}{2}\sqrt{1 - \frac{\Delta m_{\odot}^2}{\Delta m_{\oplus}^2}}, \quad (34)$$

which is also experimentally excluded. In the former cases (a), (b), (c) the same results are of course obtained with the replacement  $t \rightarrow 1 + e^{i\phi}$  and  $\lambda \rightarrow \lambda_-$ , as can be verified easily. So zero mass textures are not possible in the present scenario.

The same conclusion can be obtained using the results from the literature. In fact the analytical study of various structures of the neutrino mass matrix was presented systematically by Frigerio and Smirnov [26] who also discussed the case of equalities of matrix elements. Here we use a result from [27] where specific relations among the mixing angles were derived for one texture zero and one vanishing eigenvalue. We refer to Table I of [27] and first to NH. Using their definition of parameter  $\chi = |\frac{m_2}{m_3}|$  we have in our model  $\chi = [\frac{\Delta m_{\odot}^2}{\Delta m_{\oplus}^2}]^{1/2} = 0.182$  and so for cases A, B, C, D, E, F, respectively, in their notation

$$\begin{aligned} \chi = 0, & & \chi = 0, & & \chi = 0, & & \chi = 1.50, \\ \chi = 1.50, & & \chi = 1.50, & & & & \end{aligned} \quad (35)$$

For inverse hierarchy, defining  $\eta = \frac{m_1}{m_2} = \frac{|\tilde{m} - \lambda_+|}{\sqrt{\Delta m_{\oplus}^2}} = [1 - \frac{\Delta m_{\odot}^2}{\Delta m_{\oplus}^2}]^{1/2} = 0.983$  we have for cases A, B, C, D, E, F, respectively,

$$\begin{aligned} \eta = 0.50, & & \eta = 1, & & \eta = 1, & & \eta = 2.0, \\ \eta = 2.0, & & \eta = 2.0, & & & & \end{aligned} \quad (36)$$

Notice that  $0.953 < \eta < 0.988$  (using  $1\sigma$  upper and lower values for the solar and atmospheric mass square differences). So one can draw the following:

*Conclusion.*—Both NH and IH cannot work with  $\det D = \det M = 0$ , vanishing  $\theta_{13}$ , and one texture zero. In other words, vanishing determinant scenarios with  $\theta_{13} = 0$  are experimentally excluded, unless they have no texture zeros.

### B. Equalities between matrix elements

First we consider the case of NH. Equations (15)–(18) can be written in the general form

$$M = \begin{pmatrix} \epsilon e^{i\phi} & \text{sign}(\epsilon e^{i\phi}) & \text{sign}(\epsilon e^{i\phi}) \\ \text{sign}(\epsilon e^{i\phi}) & (m/2) + \epsilon e^{i\phi} & \text{sign}[-(m/2) + \epsilon e^{i\phi}] \\ \text{sign}(\epsilon e^{i\phi}) & \text{sign}[-(m/2) + \epsilon e^{i\phi}] & (m/2) + \epsilon e^{i\phi} \end{pmatrix}, \quad (37)$$

and using the same sign conventions as in Eqs. (15)–(18), it is seen that  $|M_{11}| = |M_{12}| = |M_{13}|$  and  $M_{22} = M_{33}$ . The first three equalities provide either  $\epsilon = 0$ , which is excluded by data, or identities. Hence the relations to be investigated are  $M_{11} = M_{22}$ ,  $M_{11} = M_{23}$ ,  $M_{22} = M_{23}$ .

Equation  $M_{11} = M_{22}$  implies  $m = 0$  which is impossible.

Equation  $M_{11} = M_{23}$  yields either

$$(a) \epsilon e^{i\phi} = (-m/2) + \epsilon e^{i\phi}, \quad (38)$$

leading to  $m = 0$ , or

$$(b) \epsilon e^{i\phi} = (+m/2) - \epsilon e^{i\phi}, \quad (39)$$

leading to  $\frac{4}{3} = [1 + \frac{\Delta m_{23}^2}{\Delta m_{22}^2}]^{1/2}$ , which is also experimentally excluded.

Equation  $M_{22} = M_{23}$  yields either

$$(a) (+m/2) + \epsilon e^{i\phi} = (-m/2) + \epsilon e^{i\phi}, \quad (40)$$

leading to  $m = 0$ , or

$$(b) (+m/2) + \epsilon e^{i\phi} = (+m/2) - \epsilon e^{i\phi}, \quad (41)$$

leading to  $\epsilon = 0$ , both experimentally excluded.

Next we consider IH. We use Eq. (29) and note that the matrix is symmetric, so there are at first sight six independent entries. However  $M_{22} = M_{33}$ ,  $|M_{12}| = |M_{13}|$ ,  $|M_{22}| = |M_{23}|$ . So apart from the obvious ones, there are three equalities to be investigated:  $M_{11} = M_{12}$ ,  $M_{11} = M_{23}$ ,  $M_{12} = M_{23}$ .

Equality  $M_{11} = M_{12}$  yields two cases

$$(a) \tilde{m}(1 - \frac{t}{3}) - \frac{2}{3}\lambda = \frac{1}{3}(\tilde{m}t - \lambda), \quad (42)$$

which upon using  $\lambda = \lambda_{\pm}$  for  $t = 1 \mp e^{i\phi}$  gives

$$\tilde{m} - \lambda_{\pm} = \mp 2\tilde{m}e^{i\phi}, \quad (43)$$

which is impossible to satisfy, as seen from Eq. (22), and

$$(b) \tilde{m}(1 - \frac{t}{3}) - \frac{2}{3}\lambda = -\frac{1}{3}(\tilde{m}t - \lambda), \quad (44)$$

leading to

$$\tilde{m} = \lambda, \quad (45)$$

also impossible; see Eq. (24).

Equality  $M_{11} = M_{23}$ . The two cases to be considered are

$$(a) \tilde{m}\left(1 - \frac{t}{3}\right) - \frac{2}{3}\lambda = \tilde{m}\left(\frac{1}{2} - \frac{t}{3}\right) - \frac{\lambda}{6}, \quad (46)$$

from which

$$\tilde{m} = \lambda, \quad (47)$$

which cannot be satisfied [Eq. (24)] and

$$(b) \tilde{m}\left(1 - \frac{t}{3}\right) - \frac{2}{3}\lambda = -\tilde{m}\left(\frac{1}{2} - \frac{t}{3}\right) + \frac{\lambda}{6}, \quad (48)$$

which upon using  $\lambda = \lambda_{\pm}$  for  $t = 1 \mp e^{i\phi}$  gives

$$\tilde{m} - \lambda_{\pm} = \mp \frac{4}{3}\tilde{m}e^{i\phi}, \quad (49)$$

or equivalently

$$5\sqrt{1 - \frac{\Delta m_{23}^2}{\Delta m_{22}^2}} = 4e^{i\phi}, \quad (50)$$

which cannot be satisfied even if  $\phi = 0$ . (Maximizing  $\Delta m_{23}^2$  and minimizing  $\Delta m_{22}^2$  ( $1\sigma$ ) the above square root verifies  $0.953 < [1 - \frac{\Delta m_{23}^2}{\Delta m_{22}^2}]^{1/2} < 0.988$ ).

Equality  $M_{12} = M_{23}$ . The two cases are now

$$(a) \frac{1}{3}(\tilde{m}t - \lambda) = \tilde{m}\left(\frac{1}{2} - \frac{t}{3}\right) - \frac{\lambda}{6}, \quad (51)$$

which gives  $\tilde{m} - \lambda_{\pm} = \pm 4\tilde{m}e^{i\phi}$  or  $\pm[1 - \frac{\Delta m_{23}^2}{\Delta m_{22}^2}]^{1/2} = \pm 4e^{i\phi}$ , again impossible, and

$$(b) \frac{1}{3}(\tilde{m}t - \lambda) = -\tilde{m}\left(\frac{1}{2} - \frac{t}{3}\right) - \frac{\lambda}{6}, \quad (52)$$

or  $\tilde{m} = \lambda$ , also impossible. All these impossibilities mean *experimentally excluded*.

Moreover, it is seen from Eq. (29) that if  $M_{12}$  and  $M_{13}$  have opposite signs, requiring their equality implies they both vanish, leading to two texture zeros which is excluded. The same is true for  $M_{22}$  and  $M_{23}$ . Recall that one texture zero with vanishing determinant cannot work with  $\theta_{13} = 0$  (see Sec. III A). Hence our conclusion:

*Conclusion.*—Equalities between mass matrix elements with  $\theta_{13} = 0$  apart from the obvious ones are experimentally excluded.

### IV. MASS MATRICES WITH $\theta_{13} \neq 0$

Regarding  $\beta\beta_{0\nu\nu}$  decay, it can be seen easily [Eq. (19)] that the conclusions derived in Sec. II for  $\theta_{13} = 0$  remain unchanged here both for normal and inverse hierarchies.

The form of matrix  $U$  can now be derived from Eqs. (5) and (11). We have

$$U = U_{23}U_{13}U_{12} = U_{12}^T U_{13}^T U_{23}^T, \quad (53)$$

For normal and inverse hierarchies, using the notation of Secs. I and II, we have for the mass matrix  $M$  and recalling that  $M$  is symmetric, we have *normal hierarchy*:

$$\begin{aligned} M_{11} &= 3\epsilon\beta_{12}^2 e^{i\phi} + m\gamma_{13}^2, \\ M_{12} &= 3\epsilon(\alpha_{23}\gamma_{31}\beta_{12}^2 + \alpha_{22}\beta_{22}\beta_{12})e^{i\phi} + m\alpha_{23}\gamma_{13}, \\ M_{13} &= 3\epsilon(\alpha_{33}\lambda_{31}\beta_{12}^2 + \alpha_{32}\beta_{22}\beta_{12})e^{i\phi} + m\alpha_{33}\gamma_{13}, \\ M_{22} &= 3\epsilon(\alpha_{23}\gamma_{31}\beta_{12} + \alpha_{22}\beta_{22})e^{i\phi} + m\alpha_{23}^2, \\ M_{23} &= 3\epsilon(\alpha_{23}\gamma_{31}\beta_{12} + \alpha_{22}\beta_{22})(\alpha_{33}\gamma_{31}\beta_{12} \\ &\quad + \alpha_{32}\beta_{22})e^{i\phi} + m\alpha_{23}\alpha_{33}, \\ M_{33} &= 3\epsilon(\alpha_{33}\gamma_{31}\beta_{12} + \alpha_{32}\beta_{22})e^{i\phi} + m\alpha_{33}^2, \end{aligned} \quad (54)$$

and *inverse hierarchy*:

$$\begin{aligned} M_{11} &= (\tilde{m} - \lambda)\beta_{11}^2 + \tilde{m}e^{i(\phi-\psi)}\beta_{12}^2, \\ M_{12} &= (\tilde{m} - \lambda)\beta_{11}(\alpha_{23}\gamma_{31}\beta_{11} + \alpha_{22}\beta_{21}) \\ &\quad + \tilde{m}e^{i(\phi-\psi)}\beta_{12}(\alpha_{23}\gamma_{31}\beta_{12} + \alpha_{22}\beta_{22}), \\ M_{13} &= (\tilde{m} - \lambda)\beta_{11}(\alpha_{33}\gamma_{31}\beta_{11} + \alpha_{32}\beta_{21}) \\ &\quad + \tilde{m}e^{i(\phi-\psi)}\beta_{12}(\alpha_{33}\gamma_{31}\beta_{12} + \alpha_{32}\beta_{22}), \\ M_{22} &= (\tilde{m} - \lambda)(\alpha_{23}\gamma_{31}\beta_{11} + \alpha_{22}\beta_{21})^2 \\ &\quad + \tilde{m}e^{i(\phi-\psi)}(\alpha_{23}\gamma_{31}\beta_{12} + \alpha_{22}\beta_{22})^2, \\ M_{23} &= (\tilde{m} - \lambda)(\alpha_{23}\gamma_{31}\beta_{11} + \alpha_{22}\beta_{21})(\alpha_{23}\gamma_{31}\beta_{11} \\ &\quad + \alpha_{22}\beta_{21}) + \tilde{m}e^{i(\phi-\psi)}(\alpha_{23}\gamma_{31}\beta_{12} + \alpha_{22}\beta_{22}) \\ &\quad \times (\alpha_{33}\gamma_{31}\beta_{12} + \alpha_{32}\beta_{22}), \\ M_{33} &= (\tilde{m} - \lambda)(\alpha_{33}\gamma_{31}\beta_{11} + \alpha_{32}\beta_{21})^2 \\ &\quad + \tilde{m}e^{i(\phi-\psi)}(\alpha_{33}\gamma_{31}\beta_{12} + \alpha_{32}\beta_{22})^2. \end{aligned} \quad (55)$$

Starting with the analysis of texture zeros, we use Eqs. (54) and require  $|s_{13}| \leq 0.16$  (90% CL) or  $|s_{13}| \leq 0.22$  (99.73% C.L.). The NH case is sufficiently simple to be solved analytically. We investigate in turn each of the six independent equations of the form  $M_{ij} = 0$  for NH.

(a)  $M_{11} = 0$ . From the first of Eqs. (54), inserting  $\gamma_{13} = s_{13}e^{i\delta}$ ,  $\beta_{12}^2 = 1/3$  [see Eqs. (10)–(12)] leads to

$$s_{13}^2 m e^{2i\delta} + \epsilon e^{i\phi} = 0, \quad (56)$$

from which

$$\delta = (\phi \pm \pi)/2, \quad s_{13} = \pm \frac{\epsilon}{m}. \quad (57)$$

The last relation gives  $|s_{13}| = 0.25$ . The  $3\sigma$  upper bound on  $|s_{13}|$  is 0.22, so  $M_{11} = 0$  is strongly disfavored.

(b)  $M_{12} = 0$ . Here, as in the following four cases, sign ambiguities must be taken into account. One gets

$$\pm \epsilon e^{i\phi} = \pm \alpha_{23} s_{13} m \left( \pm e^{i\delta} \pm \frac{\epsilon}{m} e^{-i(\delta-\phi)} \right), \quad (58)$$

where the signs are uncorrelated and a condition for  $s_{13}$  follows:

$$s_{13} = \frac{\epsilon}{\alpha_{23} m} \left( 1 \pm O\left(\frac{\epsilon}{m}\right) \right). \quad (59)$$

(c)  $M_{13} = 0$ . In the same way one gets an equation like (58) with the replacement  $\alpha_{23} \rightarrow \alpha_{33}$  and

$$s_{13} = \frac{\epsilon}{\alpha_{33} m} \left( 1 \pm O\left(\frac{\epsilon}{m}\right) \right). \quad (60)$$

Hence both  $M_{12} = 0$  and  $M_{13} = 0$  are allowed with  $|s_{13}|$  oscillating around the average  $|\frac{\epsilon}{\alpha_{23} m}| = |\frac{\epsilon}{\alpha_{33} m}| \approx 0.086$  and in the interval  $8.1 \times 10^{-2} \leq |s_{13}| \leq 9.1 \times 10^{-2}$ . The solution in terms of the parameter spaces  $(s_{13}, \phi)$  and  $(s_{13}, \delta)$  is shown in Fig. 2. Notice that since  $\frac{\epsilon}{m} \approx 6 \times 10^{-2}$ , it follows from Eqs. (59) and (60) that  $s_{13}$  depends quite weakly on the phases  $\phi$ ,  $\delta$  as can be seen in Fig. 2.

(d)  $M_{22} = 0$ . This case leads to

$$\frac{m}{2} + \epsilon \left( \pm 1 \pm \frac{\gamma_{31}}{\sqrt{2}} \right)^2 e^{i\phi} = 0, \quad (61)$$

which is clearly impossible to satisfy.

(e), (f)  $M_{23} = 0$ ,  $M_{33} = 0$ . Comparison of these two matrix elements with  $M_{22}$ , as seen from Eqs. (54), leads to the immediate conclusion that these conditions also cannot be satisfied. Hence, our conclusion:

*Conclusion.*—Two texture zeros are possible for  $\theta_{13} \neq 0$  in NH case, namely,  $M_{12} = 0$  and  $M_{13} = 0$  with  $8.1 \times 10^{-2} \leq |s_{13}| \leq 9.1 \times 10^{-2}$ .

We now turn to texture zeros in IH. In contrast with NH, they cannot be studied analytically in a straightforward way with the exception of  $M_{11} = 0$ . We have in this case

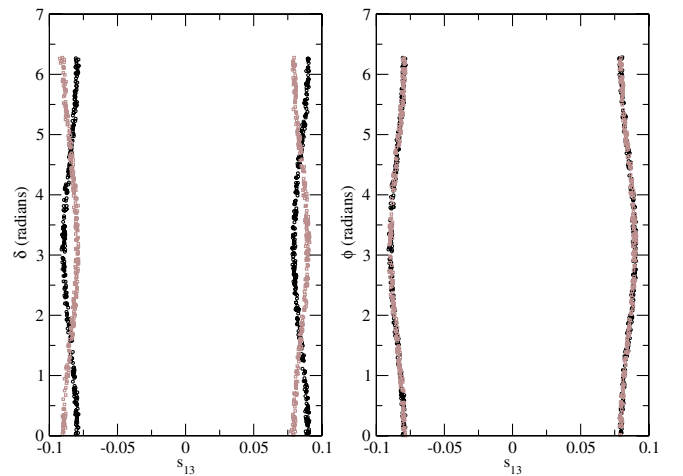


FIG. 2 (color online). Texture zeros in NH: Dirac phase (left panel) and Majorana phase (right panel) vs  $s_{13}$  for solutions to  $M_{12} = 0$  and  $M_{13} = 0$ . Black and gray areas represent the two different sets of parameters that satisfy both textures.

$$\tilde{m}e^{i\phi} = 2\sqrt{\Delta m_{\oplus}^2 - \Delta m_{\odot}^2}, \quad (62)$$

which is clearly impossible to satisfy (recall that  $\tilde{m} = \sqrt{\Delta m_{\oplus}^2}$ ). All other five cases were investigated numerically. We found three possible texture zeros:  $M_{12} = 0$ ,  $M_{22} = 0$ ,  $M_{33} = 0$ . They are displayed in Fig. 3 in the parameter spaces  $(s_{13}, \phi)$  and  $(s_{13}, \delta)$ . Owing to the structure of these three matrix elements, it is seen readily that two possible solutions exist for each texture. They correspond to the black and gray areas of Fig. 3.  $M_{12} = 0$  implies  $|s_{13}| \geq 7 \times 10^{-3}$  whereas  $M_{22} = 0$ ,  $M_{33} = 0$  are excluded up to  $2\sigma$  as  $|s_{13}| \geq 0.18$ . It is seen that no solution exists for  $\theta_{13} = 0$  as derived in Sec. III.

*Conclusion.*—Three texture zeros are possible for  $\theta_{13} \neq 0$  in an IH case, namely,  $M_{12} = 0$ ,  $M_{22} = 0$ ,  $M_{33} = 0$ . The first implies  $|s_{13}| \geq 7 \times 10^{-3}$  and the second and third imply  $|s_{13}| \geq 0.18$ .

As far as equalities between matrix elements of  $M$  are concerned and since only six of these elements are independent, one is lead to 15 possible equalities for each hierarchy. Again, we require  $|s_{13}| \leq 0.16$  (90% C.L.) or  $|s_{13}| \leq 0.22$  (99.73% C.L.) and organize the analysis considering the pairs with  $M_{11}, M_{12}, M_{13}, M_{22}, M_{23}$  as follows:

$$\begin{array}{cccccccccc} M_{11} & M_{12} & M_{12} & M_{13} & M_{13} & M_{22} & M_{22} & M_{23} & M_{23} & M_{33} \\ & M_{13} & & M_{22} & & M_{23} & & M_{33} & & \\ & M_{22} & & M_{23} & & M_{33} & & & & \\ & M_{23} & & M_{33} & & & & & & \\ & M_{33} & & & & & & & & \end{array} .$$

So taking first  $M_{11} = M_{12}$  [see Eq. (54)] and using  $\alpha_{22}$ ,  $\alpha_{23}$ ,  $\beta_{12}$ ,  $\beta_{22}$ ,  $\gamma_{13}$ , as in Eqs. (5), (10), and (11) we have

$$s_{13}e^{2i\delta}m = \frac{1}{\sqrt{2}}e^{i\delta}(\pm\epsilon e^{i(\phi-2\delta)} \pm m), \quad (63)$$

or alternatively,

$$\epsilon e^{i\phi} + s_{13}^2 e^{2i\delta} m = -\epsilon e^{i\phi} \pm \frac{1}{\sqrt{2}} s_{13} e^{-i\delta} \epsilon e^{i\phi} \pm s_{13} e^{i\delta} m, \quad (64)$$

where we eliminated the solution  $s_{13} = 0$  to obtain the first of these. Equating moduli in (63)

$$s_{13}m = \frac{m}{2} \left( 1 \pm \frac{\epsilon}{m} \cos(\phi - 2\delta) + O\left(\frac{\epsilon^2}{m^2}\right) \right), \quad (65)$$

which is clearly impossible to satisfy, owing to the experimental constraints on  $s_{13}$ ,  $m$ , and  $\epsilon$ , as seen above. Equation (64) is also impossible to satisfy as can be checked numerically. Hence we conclude that  $M_{11} = M_{12}$  can be verified only for  $s_{13} = 0$  and with an overall positive sign in front in the term  $3\epsilon\alpha_{22}\beta_{22}\beta_{12}e^{i\phi}$  in  $M_{12}$ . In this case we recover the form of  $M_{11}$  and  $M_{12}$  as in Eqs. (15) and (17). The same arguments can be used to prove that  $M_{11} = M_{13}$  is also impossible unless  $s_{13} = 0$ , since  $M_{13}$  is identical to  $M_{12}$  except for the replacement  $\alpha_{23}, \alpha_{22} \rightarrow \alpha_{33}, \alpha_{32}$  [see Eq. (54)]. It is also apparent that  $M_{11} = M_{22}$ ,  $M_{11} = M_{23}$ , and  $M_{11} = M_{33}$  cannot be satisfied because whereas the coefficients of  $\epsilon (= \frac{1}{3}\sqrt{\Delta m_{\oplus}^2})$  are of the same order in all four matrix entries, those of  $m (= \sqrt{\Delta m_{\oplus}^2})$  in  $M_{11}$  differ by at least 1 order of magnitude from the corresponding ones in  $M_{22}, M_{23}, M_{33}$ . (A similar argument would hold for the above comparison between  $M_{11}, M_{12}$ ). Considering next  $M_{12}$ , the comparison between  $M_{12}$

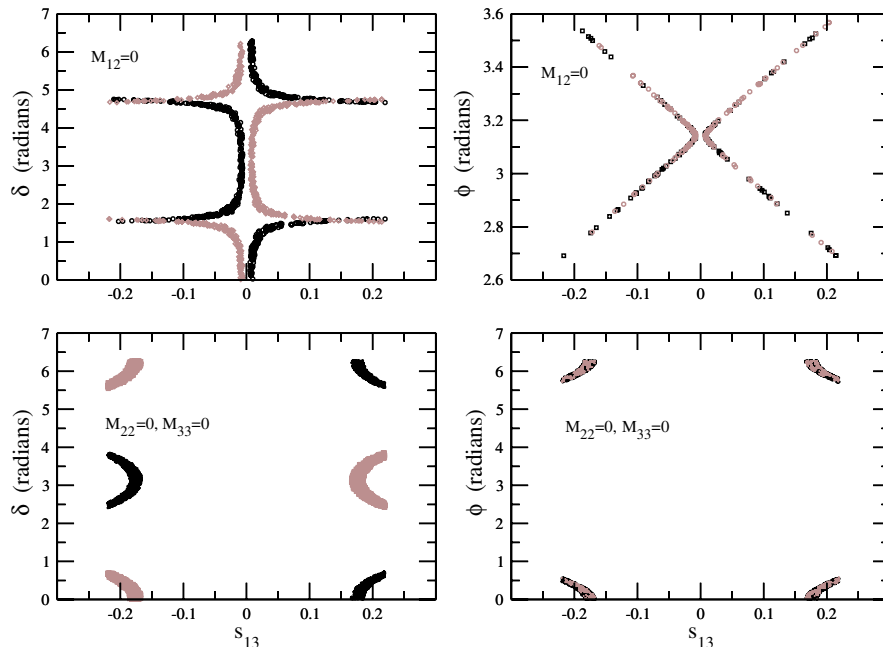


FIG. 3 (color online). The same as Fig. 2 for the three possible texture zeros in IH. As for NH no solution exists with  $\theta_{13} = 0$ .

and  $M_{13}$  (54) shows that although  $\alpha_{23}$  and  $\alpha_{33}$  may be equal, orthogonality of the matrix  $U_{23}$  implies the relative signs of the terms of  $\epsilon e^{i\phi}$  coefficients to be different in the two entries. So one cannot have  $M_{12} = M_{13}$  unless  $s_{13} = 0$ .

Use of the above arguments shows that none of the remaining eight equalities between mass matrix elements can be satisfied for finite  $\theta_{13}$ , a result that can as well be numerically checked.

We finally refer to the equalities in IH. As before, examining first the five possible cases involving  $M_{11}$  [see Eq. (55)], and since the magnitudes of  $\beta_{11}(= \sqrt{\frac{2}{3}})$ ,  $\beta_{12}(= \sqrt{\frac{1}{3}})$  are quite different from all combinations of  $\alpha'$ 's and  $\beta'$ 's that multiply  $\tilde{m} - \lambda$  and  $\tilde{m}e^{i(\phi-\psi)}$ , it follows that all five such equalities are impossible to satisfy. Furthermore, as for NH, the orthogonality of the matrix  $U_{23}$  prevents all remaining 10 equalities involving  $M_{12}$ ,  $M_{13}$ ,  $M_{22}$ ,  $M_{23}$ ,  $M_{33}$  unless  $s_{13} = 0$  in some cases. We are thus lead to the following:

*Conclusion.*—For both NH and IH there are no possible equalities between matrix elements with  $\theta_{13} \neq 0$ . If  $\theta_{13} = 0$  some equalities become possible which are the obvious ones encountered before.

## V. SUMMARY

We have investigated the prospects for neutrino mass matrices with vanishing determinant for vanishing and finite  $\theta_{13}$ . The vanishing determinant condition alone is expressed by two real conditions, so the original nine independent parameters in these matrices are reduced to seven. Hence the undesirable situation of existing and planned experiments not being able to determine all these nine quantities is in this case overcome. Furthermore, as shown in the introduction, the vanishing of  $\theta_{13}$  implies that the  $CP$  violating Dirac phase is unobservable and the mass

matrix can be diagonalized by a real and orthogonal matrix. In such case the mass matrix determinant is basis independent,  $\det M = \det D$ , while the vanishing determinant condition is always basis independent. So  $\det M = 0$  is always equivalent to the lightest neutrino being massless. On the other hand, if  $\theta_{13} \neq 0$  one has in general  $\det M \neq \det D$ , while the vanishing determinant condition remains basis independent.

We considered both the normal and inverse mass hierarchies. Summarizing our main conclusions for vanishing determinant mass matrices gives the following:

(i)  $\theta_{13} = 0$ .—In the case of normal hierarchy there can be no observable  $\beta\beta_{0\nu\nu}$  decay. For inverse hierarchy  $\beta\beta_{0\nu\nu}$  decay depends on the Majorana phase and can be observed in the next generation of experiments for all or most of the possible phase range. Texture zeros and equalities between mass matrix elements besides the obvious ones are incompatible with experimental evidence.

(ii)  $\theta_{13} \neq 0$ .— $\beta\beta_{0\nu\nu}$  decay satisfies the same properties as for  $\theta_{13} = 0$  in both normal and inverse hierarchies whereas texture zeros become possible in this case. In NH for  $\theta_{13} \neq 0$ , one may have  $M_{12} = 0$  and  $M_{13} = 0$  if  $8.1 \times 10^{-2} \leq |s_{13}| \leq 9.1 \times 10^{-2}$  (Fig. 2) and in IH  $M_{12} = 0$  if  $|s_{13}| \geq 7 \times 10^{-3}$  (Fig. 3). Also in IH  $M_{22} = 0$ ,  $M_{33} = 0$  are possible but with rather large  $s_{13}$ , namely,  $|s_{13}| \geq 0.18$  which is excluded at  $2\sigma$  (Fig. 3). All equalities between mass matrix elements both in NH and IH are those which reduce to the obvious ones in the limit  $\theta_{13} = 0$ : there are no such equalities if  $\theta_{13} \neq 0$ .

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- [1] P.H. Frampton, S.L. Glashow, and D. Marfatia, Phys. Lett. B **536**, 79 (2002).
  - [2] S. Kaneko, H. Sawanaka, and M. Tanimoto, J. High Energy Phys. 08 (2005) 073.
  - [3] X.G. He and A. Zee, Phys. Lett. B **560**, 87 (2003).
  - [4] X.G. He and A. Zee, Phys. Rev. D **68**, 037302 (2003).
  - [5] W. Rodejohann, Phys. Lett. B **579**, 127 (2004).
  - [6] G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim, and T. Yanagida, Phys. Lett. B **562**, 265 (2003).
  - [7] G.C. Branco, M.N. Rebelo, and J.I. Silva-Marcos, Nucl. Phys. **B686**, 188 (2004).
  - [8] I. Aizawa, T. Kitabayashi, and M. Yasue, Phys. Rev. D **71**, 075011 (2005).
  - [9] For a recent review see B. Kayser, Phys. Scr. **T121**, 156 (2005).
  - [10] H. V. Klapdor-Kleingrothaus, Nucl. Phys. B, Proc. Suppl. **145**, 219 (2005).
  - [11] S. HeideIman *et al.* (Particle Data Group Collaboration), Phys. Lett. **592**, 1 (2004).
  - [12] I. Affleck and M. Dine, Nucl. Phys. **B249**, 361 (1985).
  - [13] H. Murayama and T. Yanagida, Phys. Lett. B **322**, 349 (1994).
  - [14] M. Dine, L. Randall, and S. Thomas, Nucl. Phys. **B458**, 291 (1996).
  - [15] T. Asaka, M. Fujii, K. Hamaguchi, and T. Yanagida, Phys. Rev. D **62**, 123514 (2000).



- [16] M. Fujii, K. Hamaguchi, and T. Yanagida, *Phys. Rev. D* **64**, 123526 (2001).
- [17] G.L. Fogli, E. Lisi, A. Marrone, and A. Palazzo, hep-ph/0506083.
- [18] See e.g. S.F. King, *Nucl. Phys. B, Proc. Suppl.* **118**, 267 (2003).
- [19] M.C. Gonzalez-Garcia, hep-ph/0410030.
- [20] W.G. Scott, *Nucl. Phys. B, Proc. Suppl.* **85**, 177 (2000).
- [21] C. Arnaboldi *et al.*, *Phys. Rev. Lett.* **95**, 142501 (2005).
- [22] S. Capelli *et al.* (CUORE Collaboration), hep-ex/0505045.
- [23] C.E. Aalseth *et al.* (Majorana Collaboration), *Nucl. Phys. B, Proc. Suppl.* **138**, 217 (2005).
- [24] H.V. Klapdor-Kleingrothaus, L. Baudis, G. Heusser, B. Majorovits, and H. Pas (GENIUS Collaboration), hep-ph/9910205.
- [25] Y.G. Zdesenko, O.A. Ponkratenko, and V.I. Tretyak, *J. Phys. G* **27**, 2129 (2001).
- [26] M. Frigerio and A.Y. Smirnov, *Phys. Rev. D* **67**, 013007 (2003).
- [27] Z.-z. Xing, *Phys. Rev. D* **69**, 013006 (2004).