PHYSICAL REVIEW D **73,** 071501(R) (2006)

Looking for $\Delta I = 5/2$ amplitude components in $B \to \pi\pi$ and $B \to \rho\rho$ experiments

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(Received 17 February 2006; published 19 April 2006)

We discuss how experiments measuring $B \to \pi\pi$ and $B \to \rho\rho$ may be used to search for a $\Delta I = 5/2$ amplitude component. This component could be the explanation for a recent (albeit very tentative) hint from $B(\bar{B}) \to \rho\rho$ decays that the isospin triangles do not close.

DOI: 10.1103/PhysRevD.73.071501 PACS numbers: 13.25.Hw, 11.30.Er, 12.60.−i, 14.40.Nd

Within the standard model (SM), CP violation is due to a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. This phase information can be elegantly encoded in the unitarity triangle [1,2], in which the interior CP-violating angles are called α, β and γ. Independent measurements of the sides and angles of the unitarity triangle allow tests of the SM explanation of CP violation.

The canonical decay mode for measuring α is $B^0(t) \to \pi^+\pi^-$. However, due to the fact that this decay receives both tree and penguin contributions, α cannot be extracted cleanly—there is penguin "pollution." On the other hand, if one uses isospin to combine measurements of $B^+ \to \pi^+\pi^0$, $B^0(t) \to \pi^+\pi^-$ and $B^0(t) \to \pi^0\pi^0$, as well as the CP-conjugate decays, then the penguin pollution can be removed, and α obtained cleanly [3].

The isospin analysis goes as follows. Because of Bose statistics and the fact that the final-state pions come from the decay of a spinless state, they must be in a symmetric isospin configuration. As a result, the final states are

$$\begin{aligned} \langle\pi^0\pi^0| &= \sqrt{\frac{2}{3}}\langle 2,0| - \sqrt{\frac{1}{3}}\langle 0,0|, \\ \langle\pi^+\pi^-| &= \sqrt{\frac{1}{3}}\langle 2,0| + \sqrt{\frac{2}{3}}\langle 0,0|, \\ \langle\pi^+\pi^0| &= \langle 2,1|. \end{aligned} \tag{1}$$

In the SM, short-distance diagrams contribute only to the $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions. Thus, the physical decay amplitudes are

$$\begin{aligned} A_{+-} &\equiv \langle\pi^+\pi^-|T|B^0\rangle = -\sqrt{\frac{1}{3}}A_{1/2} + \sqrt{\frac{1}{6}}A_{3/2}, \\ A_{00} &\equiv \langle\pi^0\pi^0|T|B^0\rangle = \sqrt{\frac{1}{6}}A_{1/2} + \sqrt{\frac{1}{3}}A_{3/2}, \\ A_{+0} &\equiv \langle\pi^+\pi^0|T|B^+\rangle = \frac{\sqrt{3}}{2}A_{3/2}, \end{aligned} \tag{2}$$

where A_k ($k = 1/2, 3/2$) are the relevant reduced matrix elements. The parametrization for the CP-conjugate modes is similar, with the isospin amplitudes replaced by $\bar{A}_k$. Because there are two transitions, but three decays, the B decay amplitudes obey a triangle relation:

$$\sqrt{2}A_{+0} = A_{+-} + \sqrt{2}A_{00}. \tag{3}$$

The measurement of the three decays allows one to extract $A_{3/2}$, while the CP-conjugate decays give $\bar{A}_{3/2}$. However, the penguin amplitude contributes only to $A_{1/2}$, so that the relative phase between $A_{3/2}$ and $(q/p)\bar{A}_{3/2}$ is 2α, where q/p describes $B - \bar{B}$ mixing. Thus, the penguin pollution has been removed.

Now, a generic $B \to \pi\pi$ transition contains $\Delta I = 1/2$, $\Delta I = 3/2$, and $\Delta I = 5/2$ terms, which contribute to the physical decay amplitudes as

$$\begin{aligned} A_{+-} &= -\sqrt{\frac{1}{3}}A_{1/2} + \sqrt{\frac{1}{6}}A_{3/2} - \sqrt{\frac{1}{6}}A_{5/2}, \\ A_{00} &= \sqrt{\frac{1}{6}}A_{1/2} + \sqrt{\frac{1}{3}}A_{3/2} - \sqrt{\frac{1}{3}}A_{5/2}, \\ A_{+0} &= \frac{\sqrt{3}}{2}A_{3/2} + \sqrt{\frac{1}{3}}A_{5/2}. \end{aligned} \tag{4}$$

The key point is that, in the presence of a nonzero $A_{5/2}$, the three $B \to \pi\pi$ amplitudes by themselves no longer obey a triangle relation. That relation is modified as follows:

$$\sqrt{2}A_{+0}(1 - z) = A_{+-} + \sqrt{2}A_{00}, \tag{5}$$

with

$$y \equiv \frac{A_{5/2}}{A_{3/2}} = \frac{z}{1 + \frac{2}{3}(1 - z)}. \tag{6}$$

Although isospin symmetry was mentioned above, Eq. (4) already take into account any possible isospin-breaking effects in the decay amplitudes, since the three isospin amplitudes are enough to encode all the information contained in the three experimental amplitudes.

Note also that, although $B \to \pi\pi$ decays were described above, the isospin analysis also holds for each final-state polarization of $B \to \rho\rho$ decays. In addition, it holds for the decay of any neutral isospin-$1/2$ meson. In particular, it applies if the initial meson is K or D.

As noted above, the SM contributes only to the $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions at short distance. The $\Delta I = 5/2$ transitions arise from rescattering effects, such as the combination of $A_{1/2}$ with a $\Delta I = 2$ electromagnetic rescattering of the two pions in the final state. This is naively estimated to be of order $|A_{5/2}| \sim \alpha |A_{1/2}|$, where $\alpha \sim 1/127$ is the electromagnetic coupling constant. There are also strong-interaction isospin-violating effects ($m_u \neq m_d$).

A $\Delta I = 5/2$ contribution was first identified in $K \to \pi\pi$ decays. In this case, $|A_{1/2}| \sim 20|A_{3/2}|$ (known as the $\Delta I = 1/2$ rule), meaning that $|A_{5/2}| \sim 0.1|A_{3/2}|$, thus influencing the decay $K^+ \to \pi^+\pi^0$ [4]. A detailed comparison between theory and experiment is rather involved; a recent analysis within chiral perturbation theory may be found in Ref. [5].

In contrast, in the B system it is expected that $|A_{1/2}| \sim |A_{3/2}|$ and $A_{5/2}$ is normally discarded (as above, in the isospin analysis). (Recent analyses including electromagnetic and strong isospin violation in $B \to \pi\pi$ can be found in Ref. [6]. These detailsed computations agree with our rough estimate that, within the SM, $|A_{5/2}| \sim \alpha |A_{1/2}|$.) Our main purpose is to encourage experiments to scrutinize this assumption very closely, highlighting the fact that current data could be interpreted as showing some hints of $A_{5/2} \neq 0$. This is an important issue since, if $A_{5/2} \neq 0$, the isospin triangles do not close, and the extraction of α will be affected.

If the SM is valid and the arguments leading to $A_{5/2} = 0$ are correct, then four predictions can be made:

(1) as noted above, the triangle in Eq. (3) and its conjugate version close.

(2) all measurements of α will yield the same result. For example, the CP phase β has already been measured very precisely in $B^0(t) \to J/\psi K_S$: $\sin 2\beta = 0.726 \pm 0.037$ [7], which determines β up to a four-fold ambiguity. The phase γ can in principle be cleanly determined through CP violation in decays such as $B \to DK$ [8], or from a fit to a variety of other measurements (the latest analysis gives $\gamma = 58.2^{+6.7}_{-5.4}{}^\circ$ [9]). The phase α is then given by $\alpha_{UT} \equiv \pi - \beta - \gamma$. If $A_{5/2} = 0$, then $\alpha_{\rm fit} = \alpha_{UT}$, where $\alpha_{\rm fit}$ is determined from $B \to \pi\pi$ or $B \to \rho\rho$ decays.

(3) the direct CP asymmetry in $B^+ \to \pi^+\pi^0$ (C_{+0}) vanishes.

(4) because there is one more observable than independent parameters in $B \to \pi\pi$, the interference CP asymmetry parameter in $B^0 \to \pi^0\pi^0$ (S_{00}), may be written as a function of the other observables: $F(S_{00}, C_{00}, B_{00}, S_{+-}, C_{+-}, B_{+-}, C_{+0}, B_{+0}) = 0$. Here B, C, and S represent the CP-averaged branching ratio, the direct CP violation and the interference CP violation, respectively.

TABLE I. Strategies to utilize the experimental observables to distinguish three cases: neglecting isospin-violations in the SM (IC-SM); considering isospin-conserving new physics (NP); and considering $\Delta I = 5/2$ components.

	IC-SM	NP	$\Delta I = 5/2$
triangle	closes	closes	does not close
$\alpha_{\rm fit} - \alpha_{UT}$	$= 0$	$\neq 0$	$\neq 0$
C_{+0}	$= 0$	$\neq 0$	$\neq 0$
$F(S_{00}, \ldots)$	$= 0$	$= 0$	$\neq 0$

Of the four predictions, only the first and fourth are smoking-gun signals of $A_{5/2} \neq 0$; the others can be violated in the presence of physics beyond the SM with $A_{5/2} = 0$. The situation is summarized in Table I.

The most obvious test for a nonzero $A_{5/2}$ is the nonclosure of the isospin triangle. In the following, we examine the present data on $B(\bar{B}) \to \pi\pi$ and $B(\bar{B}) \to \rho\rho$ decays with this in mind. In analyzing the $\rho\rho$ data we assume that these particles are completely longitudinally polarized. This is known experimentally to be an excellent approximation [10].

Note that, since $A_{5/2}$ is expected to be small, it can only be seen in those triangles which are relatively flat. This is the case for the $B(\bar{B}) \to \rho\rho$ triangles, since the branching ratios for $B^0 \to \rho^0\rho^0$ and $\bar{B}^0 \to \rho^0\rho^0$ are much less than those of the other decay channels. It is also, by chance, the case for the $B \to \pi\pi$ triangle, but not for that of $\bar{B} \to \pi\pi$.

The current $B \to \pi\pi$ and $B \to \rho\rho$ experimental measurements are shown in Table II. This data can be turned into measurements of the $B \to f$ (A_f) and $\bar{B} \to f$ ($\bar{A}_f$) decay amplitudes through:

$$|A_f|^2 \propto B_f(1 + C_f), \qquad |\bar{A}_f|^2 \propto B_f(1 - C_f). \quad (7)$$

The proportionality constants involve two ingredients. First, there is the phase-space factor $K(m_B, m_f)$ which is essentially the same for all amplitudes in each channel. The second factor is the lifetime of the decaying B. Thus, B_+ and B_- must be multiplied by $x = \tau(B^0)/\tau(B^+)$, $1/x = 1.076 \pm 0.008$, due to the difference between the charged

TABLE II. Branching ratios B_f, direct CP asymmetries C_f, and interference CP asymmetries S_f (if applicable) for the three $B \to \pi\pi(\rho\rho)$ decay modes. Data comes from Refs. [11–16]; averages (shown) are taken from Ref. [17].

	$B_f[10^{-6}]$	C_f	S_f
$B^+ \to \pi^+\pi^0$	5.5 ± 0.6	-0.01 ± 0.06	
$B^0 \to \pi^+\pi^-$	5.0 ± 0.4	-0.37 ± 0.10	-0.50 ± 0.12
$B^0 \to \pi^0\pi^0$	1.45 ± 0.29	-0.28 ± 0.40	
$B^+ \to \rho^+\rho^0$	26.4 ± 6.4	0.09 ± 0.16	
$B^0 \to \rho^+\rho^-$	26.2 ± 3.7	-0.03 ± 0.17	-0.21 ± 0.22
$B^0 \to \rho^0\rho^0$	≤ 1.1	$(-1, 1)$	

TABLE III. The isospin amplitudes in $B(\bar{B}) \to \pi\pi$ and $B(\bar{B}) \to \rho\rho$ (in arbitrary units).

	$\sqrt{2}\lvert A_{+0}\rvert$	$\lvert A_{+-}\rvert$	$\sqrt{2}\lvert A_{00}\rvert$
$B \to \pi\pi$:	3.2 ± 0.3	1.8 ± 0.2	1.4 ± 0.6
$B \to \rho\rho$:	7.3 ± 1.5	5.0 ± 0.8	$<1.5\sqrt{1+C_{00}}$
	$\sqrt{2}\lvert \bar{A}_{+0}\rvert$	$\lvert \bar{A}_{+-}\rvert$	$\sqrt{2}\lvert \bar{A}_{00}\rvert$
$\bar{B} \to \pi\pi$:	3.2 ± 0.3	2.6 ± 0.2	1.9 ± 0.5
$\bar{B} \to \rho\rho$:	6.7 ± 1.4	5.2 ± 0.8	$<1.5\sqrt{1-C_{00}}$

and neutral B lifetimes [2]. We present the norms $|A_f|$ and $|\bar{A}_f|$ in Table III in arbitrary units (i.e. we include the factor x but not $K(m_B, m_f)$).

We note in passing that, in addition, for the decays of the neutral B mesons in which S_f is measured, we also have access to the relative phase in

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f} = \frac{\pm\sqrt{1 - C_f^2 - S_f^2} + iS_f}{1 - C_f}, \tag{8}$$

where q/p arises from $B - \bar{B}$ mixing. However, we will not use this information.

In order to see if the isospin triangles close, we proceed as follows. In the absence of $A_{5/2}$, the triangle relation of Eq. (3) holds. We therefore have

$$|\sqrt{2}A_{+0}| = |A_{+-} + \sqrt{2}A_{00}| \leq |A_{+-}| + |\sqrt{2}A_{00}|. \tag{9}$$

Thus, if $|\sqrt{2}A_{+0}|$ is larger than $|A_{+-}| + |\sqrt{2}A_{00}|$, the triangle cannot close. The logic is similar for the CP-conjugate triangle.

For the $\pi\pi$ final state we see from the data that the central values do close both the $B \to \pi\pi$ and $\bar{B} \to \pi\pi$ unitarity triangles (but just barely for $B \to \pi\pi$): $|\sqrt{2}A_{+0}| = 3.2$, $|A_{+-}| + |\sqrt{2}A_{00}| = 3.2$; $|\sqrt{2}\bar{A}_{+0}| = 3.2$, $|\bar{A}_{+-}| + |\sqrt{2}\bar{A}_{00}| = 4.5$.

However, the same is not true for $B \to \rho\rho$. Here, the data show that the $B(\bar{B}) \to \rho\rho$ isospin triangles *do not* close (we present a detailed analysis below). This is quite tantalizing: is it simply a statistical flucturation, or is it a signal of a $\Delta I = 5/2$ component at a level larger than naive expectations?

Consider $B \to \rho\rho$. The length $\sqrt{2}|A_{00}|$ depends on the value of C_{00}, but for the purposes of illustration, suppose that $C_{00} = 0$. Then the central values give $|\sqrt{2}A_{+0}| = 7.3$, $|A_{+-}| + |\sqrt{2}A_{00}| < 6.5$, and the triangle does not close. This situation can be rectified by the inclusion of a $\Delta I = 5/2$ piece. For various values of C_{00}, the data require that

$$|y| = \left|\frac{A_{5/2}}{A_{3/2}}\right| \geq \begin{pmatrix} 0.01 \pm 0.19; & C_{00} = 1 \\ 0.04 \pm 0.19; & C_{00} = 0.5 \\ 0.07 \pm 0.19; & C_{00} = 0 \\ 0.11 \pm 0.19; & C_{00} = -0.5 \\ 0.21 \pm 0.19; & C_{00} = -1 \end{pmatrix} \tag{10}$$

For all values of C_{00}, a nonzero $A_{5/2}$ is required by the central values of the present data. However, a study of the errors shows that, at present, the effect is not yet statistically significant—it is at most at the level of 1σ ($C_{00} = -1$).

Turning to $\bar{B} \to \rho\rho$, the present data give

$$|\bar{y}| = \left|\frac{\bar{A}_{5/2}}{\bar{A}_{3/2}}\right| \geq \begin{pmatrix} 0.16 \pm 0.21; & C_{00} = 1 \\ 0.06 \pm 0.21; & C_{00} = 0.5 \\ 0.01 \pm 0.20; & C_{00} = 0 \\ \text{No Bound}; & C_{00} = -0.5 \\ \text{No Bound}; & C_{00} = -1 \end{pmatrix} \tag{11}$$

In this case, a nonzero value of $A_{5/2}$ is required only for certain values of C_{00} (and the effect is not yet statistically significant).

This summarizes the present hint for a $\Delta I = 5/2$ piece in $B \to \rho\rho$ and $\bar{B} \to \rho\rho$ decays, separately. However, the signals go in opposite directions in each decay: the size of $A_{5/2}$ in $B \to \rho\rho$ decays increases as C_{00} goes from $+1$ to -1, while $\bar{A}_{5/2}$ in $\bar{B} \to \rho\rho$ decays increases as C_{00} goes from -1 to $+1$. As a result, we may combine information from both sets of data, using

$$|\sqrt{2}A_{+0}| + |\sqrt{2}\bar{A}_{+0}| \leq |A_{+-}| + |\bar{A}_{+-}| + |\sqrt{2}A_{00}| + |\sqrt{2}\bar{A}_{00}|. \tag{12}$$

The presence of a $\Delta I = 5/2$ piece is implied if this inequality is not satisfied. The current data imply that

$$y \vee \bar{y} \geq \begin{pmatrix} 0.08 \pm 0.13: & C_{00} = 1 \\ 0.04 \pm 0.12: & C_{00} = 0.5 \\ 0.04 \pm 0.12: & C_{00} = 0 \\ 0.04 \pm 0.12: & C_{00} = -0.5 \\ 0.08 \pm 0.13: & C_{00} = -1 \end{pmatrix} \tag{13}$$

As above, the present data suggest a nonzero $A_{5/2}$ piece for all values of C_{00}, but the effect is not yet statistically significant.

In summary, we have shown that if the usual $B(\bar{B}) \to \pi\pi$ or $B(\bar{B}) \to \rho\rho$ isospin triangles do not close, this may be due to a SM $\Delta I = 5/2$ piece ($A_{5/2}$) at a level much larger than expected. This is a crucial question since a $A_{5/2}$ piece can also mimic new-physics contributions to other observables, such as C_{+0} or $\alpha_{\rm fit} - \alpha_{UT}$ (see Table I). We have pointed out some strategies to disentangle $A_{5/2}$ from legitimate new physics.

At present, data on $B(\bar{B}) \to \rho\rho$ decays give a hint—not yet statistically significant—that the isospin triangles do not close. The purpose of this letter is to stress the need for experimental scrutiny of such a signal (and to continue to look for one in $B(\bar{B}) \to \pi\pi$). [A probe with $F(S_{00}, \ldots)$ is also possible (Table I), particularly for $B \to \rho\rho$, and advisable once the data become more precise.] If this signal remains, it may be a sign of a SM $\Delta I = 5/2$ piece.

The work of D. L. is supported by NSERC of Canada. F. J. B. is partially supported by the spanish M. E. C. and FEDER under FPA2005-01678. J. P. S. is supported in part by the Portuguese *Fundação para a Ciência e a Tecnologia* (FCT) under the contract CFTP-Plurianual (777), and through the project POCTI/37449/FNU/2001, approved by the Portuguese FCT and POCTI, and cosupported by FEDER.

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