Stability of the Tree-Level Vacuum in a Minimal S_3 Extension of the Standard Model

D. Emmanuel-Costa*, O. Félix-Beltrán†, M. Mondragón** and E. Rodríguez-Jáuregui‡

*Departamento de Física and Centro de Física Teórica (CFTP), Instituto Superior Técnico, Lisboa, Portugal
†Facultad de Cs. de la Electrónica, BUAP, Puebla, Pue. México
**Instituto de Física, UNAM, México, D.F.
‡Depto. de Física UNISON, Sonora, México

Abstract. In this work we analyzed the stability of the minimal S_3 invariant extension of the Higgs potential. In the S_3 invariant minimal standard model it is assumed that the Higgs fields belong to the three-dimensional reducible representation of the permutation group S_3. We show that in the three S_3 Higgs doublet model at tree-level the potential minimum preserving electric charge and CP symmetries, when it exists, is the global one.

Keywords: Extension SM, Higgs Potential, S_3 symmetry
PACS: 14.80.Cp, 14.80.Bn, ,12.60.Fr ,11.30.Qc ,11.30.Űj ,11.15.Ex.

INTRODUCTION

In the Standard Model each family of fermions enters independently. In order to understand the replication of family fermion generation and reduce the number of free parameters, one is lead to introduce a flavour family fermion group. Recently succesfuly work has been done using a flavour family discrete $S(3)$ group. Although their existence is a fundamental piece of the theory, the least understood part of the Standard Model is the property of Higgs boson. With exact $S(3)$ flavour symmetry the fermion masses are degenerated, and a flavoured Higgs boson is needed to avoid this problem. In this paper we extend the Standard Model Higgs Sector using a flavour discrete symmetry $S(3)$ assuming that Higgs fields belong to the three-dimensional reducible representation of the flavour permutational group $S(3)$. Since the potential and its minimization play a vital part in the successful construction of the model we analyze the $S(3)$-invariant Higgs boson potential and found the conditions under which the potential minimum preserving electric charge and CP symmetries, is the global one.

EXTENSION SM$\otimes S_3$

The most general Higgs potential invariant under exact $SU(2)_L \times U(1)_Y \times S_3$ can be expressed as [1]:

$$
\begin{aligned}
V \;=\;& \mu_1^2(x_1+x_2)+\mu_0^2 x_3 + a x_3^2 + b x_3(x_1+x_2) + c(x_1+x_2)^2 - 4 d x_5^2 + g\left[(x_1-x_2)^2+4x_4^2\right] \\
&+2e\left[2x_4x_6 + x_8(x_1-x_2)\right] + f\left(x_6^2+x_7^2+x_8^2+x_9^2\right) + 2h\left(x_6^2-x_7^2+x_8^2-x_9^2\right),
\end{aligned}
$$

CP917, *Particles and Fields,* edited by H. Castilla Valdez, J. C. D'Olivo, and M. A. Perez

where x_i are the components of a vector $\mathbf{X}$ given by $\mathbf{X}^T = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9)$, following the notation for the Higgs potential given in Ref.[2]:

$$\begin{array}{lll} x_1 = \Phi_1^\dagger \Phi_1 & x_2 = \Phi_2^\dagger \Phi_2 & x_3 = \Phi_S^\dagger \Phi_S \\ x_4 = \mathscr{R}\left(\Phi_1^\dagger \Phi_2\right) & x_6 = \mathscr{R}\left(\Phi_1^\dagger \Phi_S\right) & x_8 = \mathscr{R}\left(\Phi_2^\dagger \Phi_S\right) \\ x_5 = \mathscr{I}\left(\Phi_1^\dagger \Phi_2\right) & x_7 = \mathscr{I}\left(\Phi_1^\dagger \Phi_S\right) & x_9 = \mathscr{I}\left(\Phi_2^\dagger \Phi_S\right) \end{array}$$

Here, the $SU(2)_L$ doublet Higgs fields, Φ_1 and Φ_2 belongs to the $S(3)$ doublet H_D and Φ_S is the $S(3)$ singlet:

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \ \Phi_2 = \begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_8 + i\phi_{11} \end{pmatrix}, \ \Phi_S = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}.$$

Defining a vector $\mathbf{A}$ and the square symmetric matrix $\mathbf{B}$ as

$$\mathbf{A} = \begin{pmatrix} \mu_1 \\ \mu_1 \\ \mu_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 2(c+g) & 2(c-g) & b & 0 & 0 & 0 & 0 & 2e & 0 \\ 2(c-g) & 2(c+g) & b & 0 & 0 & 0 & 0 & -2e & 0 \\ b & b & 2a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8g & 0 & 4e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4e & 0 & 2(f+2h) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2(f-2h) & 0 & 0 \\ 2e & -2e & 0 & 0 & 0 & 0 & 0 & 2(f+2h) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2(f-2h) \end{pmatrix}.$$

we can write the $S(3)$ Higgs potential as:

$$V = \mathbf{A^T X} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{X^T B X}.$$

This potential has three types of minimum or stationary points, that is: the normal minimum with the following field configuration: $\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3$ and $\phi_i = 0\,(i = 1, 2, 3, .., 12)$; the electric charge breaking stationary point, where three of the charged fields ϕ_i has a non zero vev: $\phi_7 = v_1', \phi_8 = v_2', \phi_9 = v_3', \phi_1 = \alpha, \phi_3 = \beta, \phi_5 \neq 0$; and the CP-symmetry breaking stationary point where three of the imaginary neutral component fields ϕ_i gets a non zero vev: $\phi_7 = v_1'', \phi_8 = v_2'', \phi_9 = v_3'', \phi_{10} = \delta, \phi_{11} = \gamma, \phi_{12} \neq 0$.

THE NORMAL, CB AND CP BREAKING MINIMUMS

(A) Normal Minimum: From the definitions given above, the normal minimum corresponds to the following field configuration $x_i = v_i^2$ for $i = 1, 2, 3$, $x_4 = v_1 v_2$, $x_6 = v_1 v_3$, $x_8 = v_2 v_3$, and $x_5 = x_7 = x_9 = 0$, with this notation, the minimization conditions can be written as

$$\frac{\partial V}{\partial v_i} = 0 \leftrightarrow \frac{\partial V}{\partial x_j}\frac{\partial x_j}{\partial v_i} = 0 \quad i = 1, 2, 3 \quad j = 1, 2, .., 9$$

The solution to this coupled equations implies $e = 0$ or ${v_1}^2 = 3{v_2}^2$. Defining the vectors $\mathbf{V}' = \frac{\partial V}{\partial X_i}\,|_{min}$, and $\mathbf{X}_N = \mathbf{X}\,|_{normal\,min}$ we obtain that in the normal minimum

$$\mathbf{V}' = \mathbf{A} + \mathbf{B}\mathbf{X}_N \qquad\qquad \mathbf{X}_N^T \mathbf{V}' = 0. \tag{1}$$

In this notation the potential in the normal minimum can be written as

$$V_N = -\frac{1}{2}\mathbf{X}_N^T\mathbf{B}\mathbf{X}_N = \frac{1}{2}\mathbf{A}^T\mathbf{X}_N. \tag{2}$$

(**B**) *Charge Breaking*: We consider the case where the charge breaking comes from the S_3 Higgs doublet H_D for this configuration the fields with vev's different from zero are $\phi_7 = v_1'$, $\phi_8 = v_2'$, $\phi_9 = v_3'$ and $\phi_1 = \alpha$, $\phi_3 = \beta$. These vevs break charge conservation and would give mass to the photon, but in the CB minimum we can always have a solution with $\alpha = \beta = 0$. In the CB stationary point the vector $\mathbf{X}_{CB} = \mathbf{X}\,|_{CB\,min}$ is:

$$\mathbf{X}_{CB}^T = \left(\alpha^2 + v_1'^2, \beta^2 + v_2'^2, v_3'^2, \alpha\beta + v_1'v_2', 0, v_1'v_3', 0, v_2'v_3', 0\right)$$

and the stationarity conditions can be expressed in vectorial form as follows:

$$\frac{\partial \mathbf{V}}{\partial \mathbf{X}}\,|_{\mathbf{X}=\mathbf{X}_{CB}} = 0 \Leftrightarrow \mathbf{A} + \mathbf{B}\mathbf{X}_{CB} = 0. \tag{3}$$

This is a linear equation, then, if it has a solution for $\mathbf{X}_{CB}$ this is unique. The potential evaluated in CB is:

$$\mathbf{V}_{CB} = -\frac{1}{2}\mathbf{X}_{CB}^T\mathbf{B}\mathbf{X}_{CB}\,. \tag{4}$$

From this equation and equations (1)-(3), we can write

$$\mathbf{V}_{CB} - \mathbf{V}_N = \frac{1}{2}\mathbf{X}_{CB}^T\mathbf{V}'. \tag{5}$$

If the right side is positive the normal minimum is always deeper then the CB minimum.

(**C**) *CP Breaking*: We assume that CP breaking comes from the S_3 Higgs doublet H_D, in this case, the CP breaking stationary point is given by the non-zero vevs $\phi_7 = v_1'$, $\phi_8 = v_2'$, $\phi_9 = v_3'$, $\phi_{10} = \delta$ and $\phi_{11} = \gamma$. Then,defining the vectors $\mathbf{V}'_{CP} = \frac{\partial V}{\partial X_i}\,|_{CP\,min}$, and $\mathbf{X}_{CP} = \mathbf{X}\,|_{CP\,min}$, we obtain that

$$\mathbf{X}_{CP}^T\mathbf{V}'_{CP} = 0 \qquad \text{and} \qquad \mathbf{V}_{CP} = \frac{1}{2}\mathbf{A}^T\mathbf{X}_{CP}.$$

Now, we have all the elements to show that the normal minimum is deeper than CP minimum. As in the previous case, we obtain the expression

$$\mathbf{V}_{CP} - \mathbf{V}_N = \frac{1}{2}\mathbf{X}_{CP}^T\mathbf{V}'. \tag{6}$$

Also, the normal minimum is deeper if $\mathbf{X}_{CP}^T\mathbf{V}'$ is positive.

MASS MATRICES

The second derivatives of the Higgs potential are necessary to know the nature of the stationary points. These derivatives are given by:

$$\frac{\partial^2 V}{\partial\phi_i\partial\phi_j} = \frac{\partial V}{\partial x_l}\frac{\partial^2 x_l}{\partial\phi_i\partial\phi_j} + \frac{\partial^2 V}{\partial x_l\partial x_m}\frac{\partial x_l}{\partial\phi_i}\frac{\partial x_m}{\partial\phi_j}. \tag{7}$$

The corresponding squared Higgs mass matrix has the form

$$[\mathbf{M}^2] = \frac{1}{2}\left([\mathbf{M}_I^2] + \mathbf{C}^T\mathbf{B}\mathbf{C}\right). \tag{8}$$

The derivatives $\partial x_i/\partial \phi_j$ form a 9×12 matrix, which we call $\mathbf{C}$ and the second derivatives in the second term of equation (7) are clearly the matrix elements B_{lm} of $\mathbf{B}$ matrix. Evaluated at each of the different stationary points only the fields ϕ_7, ϕ_8, ϕ_9, ϕ_1, ϕ_3, ϕ_5 and ϕ_{12} appear, and the rest vev's are always zero at the stationary points. In the normal minimum $[\mathbf{M}_I^2] = diag(\mathbf{M}_{11}^2, \mathbf{M}_{12}^2)$ where $\mathbf{M}_{11}^2$ and $\mathbf{M}_{12}^2$ are 6×6 matrices. Then, the squared mass matrix is computed and take the form $diag(\mathbf{M}_C^2, \mathbf{M}_C^2, \mathbf{M}_S^2, \mathbf{M}_P^2)$. In particular, the physics charged Higgs masses can be expressed as

$$m_{H^\pm}^2 = V_1' + V_8' \pm \left(4V_1'^2 - 8V_1'V_8' + 8V_4'V_6' + 16V_3'V_2'\right)^{1/2}.$$

$\mathbf{M}_S^2$ and $\mathbf{M}_P^2$ are block diagonal symmetric scalar and pseudoscalar Higgs mass matrices, respectively.

CONCLUSIONS

We analyzed the S_3 extended Higgs potential of the SM including a Higgs doublet H_D and a Higgs singlet Φ_S of the non-abelian discrete symmetry S_3. In particular, we studied the Higgs potential stationary points: the normal one, charge breaking and CP breaking. We found the conditions in eq. (5)and (6) showing under which conditions the normal minimum, is a global and stable one. A complete analisis of the parameters space for the stationary points will be published else where.

ACKNOWLEDGMENTS

We acknowledge support by CONACyT México, under grant 42026-F, and by PAPIIT-UNAM IN115207. The work of E.R.J. was suported by a SESIC-PROMEP(México) under F-PROMEP-39 grant. We have enjoyed fruitful and enlighten discussions with Alfonso Mondragón B.

REFERENCES

1. S. Pakvasa and H. Sugawara, *Phys. Lett.* **B**73, 61 (1978); S. Pakvasa and H. Sugawara, *Phys. Lett.* **B**82, 105 (1979); J. Kubo, A. Mondragón, M. Mondragón, E. Rodríguez-Jáuregui, O. Félix-Beltrán and E. Peinado, J. *Phys.: Conf. Ser.* **18**, 380-384 (2005); J. Kubo, H. Okada and F. Sakamaki, *Phys. Rev.* **D**70, 036007 (2004) Y. Koide, *Phys. Rev.* **D** 73, 057901 (2006)
2. P. M. Ferreira, R. Santos and A. Barroso, *Phys. Lett.* **B**603, 219 (2004) [Erratum-ibid. **B** 629, 114 (2005)] A. Barroso, P. M. Ferreira and R. Santos, arXiv:hep-ph/0507329; A. Barroso, P. M. Ferreira and R. Santos, *PoS HEP2005*, 337 (2006).