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Supersymmetric adjoint *SU*(5)

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Recently we have proposed a renormalizable grand unified theory, based on the $SU(5)$ gauge symmetry, where the neutrino masses are generated through the type I and type III seesaw mechanisms. In this article we study the supersymmetric version of this theory. As in the nonsupersymmetric version it is possible to generate all fermion masses with the minimal number of Higgses, the theory predicts one massless neutrino, and the leptogenesis mechanism can be realized. All contributions to the decay of the proton and the properties of neutralinos are discussed. This theory can be considered as the simplest renormalizable supersymmetric grand unified theory based on the $SU(5)$ gauge symmetry since it has the minimal number of superfields and free parameters.

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I. INTRODUCTION

The so-called grand unified theories (GUTs) can be considered as one of the most appealing candidates for physics beyond the standard model (SM). These theories predict the unification of the electromagnetic, weak, and strong interactions at the high scale, $M_{\rm GUT} \approx 10^{14-16}$ GeV, the quantization of the electric charge, the value of $\sin^2\theta_W(M_{\rm GUT}) = 3/8$ at the GUT scale, the decay of the proton and the existence of vector and scalar leptoquarks. The simplest grand unified theory was proposed in Ref. [1]. This theory is based on $SU(5)$ gauge symmetry and one standard model family is partially unified in the antifundamental $\bar{\mathbf{5}}$ and antisymmetric $\mathbf{10}$ representations. The Higgs sector is composed of $\mathbf{24_H}$ and $\mathbf{5_H}$, while the GUT symmetry is broken down to the standard model by the vacuum expectation value of the Higgs singlet field in $\mathbf{24_H}$, and the SM Higgs resides in $\mathbf{5_H}$. As is well known, this theory is ruled out since in this case the unification of gauge couplings is in disagreement with the values of α_s, $\sin\theta_W$, and $\alpha_{\rm em}$ at the electroweak scale. Recently, several efforts has been made in order to define the simplest realistic extension of the Georgi-Glashow model. See Refs. [2–4] for different minimal realistic nonsupersymmetric grand unified theories.

The minimal supersymmetric $SU(5)$ theory was discussed for the first time in Ref. [5]. In this case one generation of matter of the minimal supersymmetric standard model (MSSM) is unified in two chiral superfields $\hat{\bar{\mathbf{5}}} = (\hat{d}^C, \hat{L})$ and $\hat{\mathbf{10}} = (\hat{u}^C, \hat{Q}, \hat{e}^C)$, while the Higgs sector is composed of $\hat{\mathbf{5}}_{\mathbf{H}} = (\hat{T}, \hat{H}_1)$, $\hat{\bar{\mathbf{5}}}_{\mathbf{H}} = (\hat{\bar{T}}, \hat{\bar{H}}_1)$, and $\hat{\mathbf{24}}_{\mathbf{H}}$. In our notation the SM decomposition of the adjoint Higgs superfield reads as $\hat{\mathbf{24}}_{\mathbf{H}} = (\hat{\Sigma}_8, \hat{\Sigma}_3, \hat{\Sigma}_{(3,2)}, \hat{\Sigma}_{(\bar{3},2)}, \hat{\Sigma}_{24}) = (8, 1, 0) \bigoplus (1, 3, 0) \bigoplus (3, 2, -5/6) \bigoplus (\bar{3}, 2, 5/6) \bigoplus (1, 1, 0)$. As is well known, the renormalizable version of this theory is ruled out since the relation between Y_E and Y_D, $Y_E = Y_D^T$, is in disagreement with the experimental values of the fermion masses at the low scale and the neutrinos are massless if the so-called R-parity is conserved. See Ref. [6] for the most general constraints coming from unification and [7] for all possible dimension-five contributions to the decay of the proton in this context.

In this article we write down and study the supersymmetric version of the theory proposed in Ref. [4]. In this renormalizable theory the fermion masses are generated with the minimal set of Higgs bosons, $\mathbf{5_H}$ and $\mathbf{45_H}$. The neutrino masses are generated through the type I [8] and type III [9] seesaw mechanisms using the fermionic $\mathbf{24}$ representation. We conclude that if we want R-parity as a symmetry of the theory we have to introduce one matter chiral superfield $\hat{\mathbf{24}}$. We refer to this model as "Supersymmetric Adjoint $SU(5)$." As in the nonsupersymmetric version the theory predicts one massless neutrino and the leptogenesis mechanism can be realized. We discuss the LLLL and RRRR contributions to proton decay and the properties of neutralinos. This theory can be considered as the simplest renormalizable grand unified theory based on $SU(5)$ since it has the minimal number of chiral superfields and free parameters.

II. SUPERSYMMETRIC ADJOINT *SU*(5)

Recently, a realistic renormalizable grand unified theory based on $SU(5)$ gauge symmetry has been proposed, where the Higgs sector is composed of $\mathbf{5_H}$, $\mathbf{24_H}$, and $\mathbf{45_H}$ [4]. In this case an extra matter multiplet in the adjoint representation has been added in order to generate neutrino masses through the type I and type III seesaw mechanisms. This model is consistent with all constraints coming from proton decay, predicts one massless neutrino at tree level, and the leptogenesis mechanism can be realized [4]. Let us discuss in this section the supersymmetric version of this model.

As is well known in the minimal supersymmetric $SU(5)$ [5] the MSSM chiral superfields are unified in $\hat{\bar{\mathbf{5}}}$ and $\hat{\mathbf{10}}$, while its Higgs sector comprises $\hat{\mathbf{5}}_{\mathbf{H}}$, $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$, and $\hat{\mathbf{24}}_{\mathbf{H}}$. Now, in order to write down the supersymmetric version of the realistic grand unified theory proposed in Ref. [4] we have to introduce three extra chiral superfields, $\hat{\mathbf{45}}_{\mathbf{H}}$, $\hat{\overline{\mathbf{45}}}_{\mathbf{H}}$, and $\hat{\mathbf{24}}$. Therefore, our Higgs sector will be composed of $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$, $\hat{\mathbf{5}}_{\mathbf{H}}$, $\hat{\mathbf{24}}_{\mathbf{H}}$, $\hat{\mathbf{45}}_{\mathbf{H}} = (\hat{\Phi}_1, \hat{\Phi}_2, \hat{\Phi}_3, \hat{\Phi}_4, \hat{\Phi}_5, \hat{\Phi}_6, \hat{H}_2) = (8, 2, 1/2) \bigoplus (\bar{6}, 1, -1/3) \bigoplus (3, 3, -1/3) \bigoplus (\bar{3}, 2, -7/6) \times \bigoplus (3, 1, -1/3) \bigoplus (\bar{3}, 1, 4/3) \bigoplus (1, 2, 1/2)$, and $\hat{\overline{\mathbf{45}}}_{\mathbf{H}} = (\hat{\bar{\Phi}}_1, \hat{\bar{\Phi}}_2, \hat{\bar{\Phi}}_3, \hat{\bar{\Phi}}_4, \hat{\bar{\Phi}}_5, \hat{\bar{\Phi}}_6, \hat{\bar{H}}_2) = (8, 2, -1/2) \bigoplus (6, 1, 1/3) \times \bigoplus (\bar{3}, 3, 1/3) \bigoplus (3, 2, 7/6) \bigoplus (\bar{3}, 1, 1/3) \bigoplus (3, 1, -4/3) \times \bigoplus (1, 2, -1/2)$. The fields in the 45 representation satisfy the following conditions: $(45)^{\alpha\beta}_{\delta} = -(45)^{\beta\alpha}_{\delta}$, $\sum_{\alpha=1}^{5} (45)^{\alpha\beta}_{\alpha} = 0$, $v_{45} = \langle 45_H \rangle^{15}_{1} = \langle 45_H \rangle^{25}_{2} = \langle 45_H \rangle^{35}_{3}$, and $v_{\overline{45}} = \langle \overline{45}_H \rangle^{1}_{15} = \langle \overline{45}_H \rangle^{2}_{25} = \langle \overline{45}_H \rangle^{3}_{35}$.

In this model the Yukawa superpotential for charged fermions reads as

$$\mathcal{W}_0 = \hat{10}\,\hat{\bar{5}}(Y_1 \hat{\bar{5}}_H + Y_2 \hat{\overline{45}}_H) + \hat{10}\,\hat{10}(Y_3 \hat{5}_H + Y_4 \hat{45}_H), \tag{1}$$

and the relation between the masses for charged leptons and down quarks is given by

$$M_D - M_E^T = 8 Y_2 v_{\overline{45}} \tag{2}$$

where Y_2 is an arbitrary 3×3 matrix. As is well known, the relation between the masses of τ lepton and b quark, $m_b(M_{\text{GUT}}) = m_\tau(M_{\text{GUT}})$, is in agreement with the experiment. Therefore, the Y_2 matrix only must modify the relation between the masses of quarks and leptons of the first and second generation. See Ref. [10] for a recent study of the relation between fermion masses in supersymmetric scenarios.

There are three different possibilities to generate the neutrino masses at tree level in the context of supersymmetric (SUSY) $SU(5)$ models:

(i) We can add at least two fermionic superfields for the singlets and generate neutrino masses through the type I seesaw [8] mechanism.

(ii) We can add two Higgs chiral superfields $\hat{\mathbf{15}}_{\mathbf{H}}$ and $\hat{\overline{\mathbf{15}}}_{\mathbf{H}}$ to generate neutrino masses through the type II seesaw [11] mechanism.

(iii) One can generate neutrino masses through the type III [9] and type I seesaw mechanisms adding just one fermionic $\hat{\mathbf{24}}$ chiral superfield. The last possibility has been realized at the renormalizable level in the model proposed in Ref. [4]. Therefore, in order to realize the mechanism a new chiral supermultiplet, $\hat{\mathbf{24}} = (\hat{\rho}_8, \hat{\rho}_3, \hat{\rho}_{(3,2)}, \hat{\rho}_{(\bar{3},2)}, \hat{\rho}_0) = (8, 1, 0) \bigoplus (1, 3, 0) \bigoplus (3, 2, -5/6) \bigoplus (\bar{3}, 2, 5/6) \bigoplus (1, 1, 0)$, has to be introduced [12].

The idea of using extra matter in the adjoint representation to generate neutrino masses through the type I and type III seesaw mechanisms was pointed out for the first time in Ref. [12] in the context of SUSY $SU(5)$ and in Ref. [3] in the context of non-SUSY $SU(5)$. It is important to say that this possibility is very appealing since we have to introduce only one extra chiral matter superfield and there is no need to introduce $SU(5)$ singlets.

Since in this article we are interested in the supersymmetric version of the model proposed in Ref. [4] a new matter chiral superfield has to be introduced only if we want to have the so-called matter-parity as a symmetry of the theory. Matter-parity is defined as $M = (-1)^{3(B-L)} = (-1)^{2S} R$, where $M = -1$ for all matter superfields and $M = 1$ for the Higgses and gauge superfields. In the case that matter-parity is not conserved, the neutrino masses can be generated through the matter-parity violating interactions $\epsilon_i \hat{\bar{5}}_i \hat{5}_H$ and $\eta_i \hat{\bar{5}}_i \hat{24}_H \hat{5}_H$. Particularly, in the second term we have an $SU(2)$ fermionic triplet needed for the type III seesaw mechanism. In this article we want to keep matter-parity as a symmetry of the theory to avoid the dimension-four contributions to the decay of the proton coming from $\lambda_{ijk} \hat{10}_i \hat{\bar{5}}_j \hat{\bar{5}}_k$ and have the lightest neutralino as a good candidate for the cold dark matter of the universe.

The new superpotential relevant for neutrino masses in this context is given by

$$\mathcal{W}_1 = c_i \hat{\bar{5}}_i \hat{24} \hat{5}_H + p_i \hat{\bar{5}}_i \hat{24} \hat{45}_H. \tag{3}$$

Notice from Eqs. (1) and (3) the possibility to generate all fermion masses, including the neutrino masses, with the Higgs chiral superfields $\hat{\mathbf{5}}_{\mathbf{H}}$, $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$, $\hat{\mathbf{45}}_{\mathbf{H}}$, and $\hat{\overline{\mathbf{45}}}_{\mathbf{H}}$. As in the non-SUSY model the Higgses in the **45** representation play a crucial role to generate masses for charged fermions and neutrinos as well.

There are also new relevant interactions between $\hat{\mathbf{24}}$ and $\hat{\mathbf{24}}_{\mathbf{H}}$ in this model:

$$\mathcal{W}_2 = m_\Sigma \operatorname{Tr} \hat{24}_H^2 + \lambda_\Sigma \operatorname{Tr} \hat{24}_H^3 + m \operatorname{Tr} \hat{24}^2 + \lambda \operatorname{Tr}(\hat{24}^2 \hat{24}_H). \tag{4}$$

Notice that there are only two extra terms since matter-parity is conserved. Once 24_H gets the expectation value, $\langle 24_H \rangle = 2 m_\Sigma \operatorname{diag}(2, 2, 2, -3, -3)/3\lambda_\Sigma$, the masses of the fields living in 24 are given by

$$M_{\rho_0} = m - \frac{2 m_\Sigma \lambda}{3 \lambda_\Sigma}, \tag{5}$$

$$M_{\rho_3} = m - \frac{2 \lambda m_\Sigma}{\lambda_\Sigma}, \tag{6}$$

$$M_{\rho_8} = m + \frac{4 \lambda m_\Sigma}{3 \lambda_\Sigma}, \tag{7}$$

and

$$M_{\rho_{(3,2)}} = M_{\rho_{(\bar{3},2)}} = m - \frac{\lambda m_{\Sigma}}{3\lambda_{\Sigma}}. \tag{8}$$

From the above equations we can see that when the fermionic triplet ρ_3, responsible for the type III seesaw mechanism, is very light the rest of the fields living in $\hat{\mathbf{24}}$ have to be heavy if we do not assume a very small value for the λ parameter. The GUT symmetry is broken as usual, and $\hat{\mathbf{24}}$ does not get expectation value.

Since our Higgs sector is composed of $\hat{\mathbf{5}}_{\mathbf{H}}$, $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$, $\hat{\mathbf{45}}_{\mathbf{H}}$, $\hat{\overline{\mathbf{45}}}_{\mathbf{H}}$, and $\hat{\mathbf{24}}_{\mathbf{H}}$ there are also additional interactions between the different Higgs chiral superfields in the theory:

$$\begin{aligned}\mathcal{W}_3 = {} & m_H \hat{\bar{5}}_H \hat{5}_H + \lambda_H \hat{\bar{5}}_H \hat{24}_H \hat{5}_H + c_H \hat{\bar{5}}_H \hat{24}_H \hat{45}_H \\ & + b_H \hat{\overline{45}}_H \hat{24}_H \hat{5}_H + m_{45} \hat{\overline{45}}_H \hat{45}_H \\ & + a_H \hat{\overline{45}}_H \hat{45}_H \hat{24}_H. \end{aligned} \tag{9}$$

Notice the simplicity of the model. Unfortunately, the scalar sector of the nonsupersymmetric grand unified theory proposed in Ref. [4] is not very simple since there are many possible interactions between $\mathbf{5_H}$, $\mathbf{24_H}$, and $\mathbf{45_H}$. We have the same problem in any renormalizable nonsupersymmetric grand unified model. Supersymmetric Adjoint $SU(5)$, the model proposed in this article, can be considered as the simplest supersymmetric grand unified theory based on $SU(5)$ since it has the minimal number of chiral superfields and free parameters.

III. PHENOMENOLOGICAL ASPECTS: PROTON DECAY, NEUTRINO MASSES, NEUTRALINOS, AND LEPTOGENESIS

In this section we will discuss the most relevant phenomenological and cosmological aspects of this proposal. However, the detailed analysis of those issues is beyond the scope of this article. As is well known, the most important prediction coming from the unification of fundamental forces is proton decay. See Ref. [13] for a review and [14] for future proton decay experiments. In this model there are several multiplets that mediate proton decay. We have the usual gauge $d = 6$ contributions, mediated by the superheavy gauge bosons $V = (3, 2, -5/6) \bigoplus (\bar{3}, 2, 5/6)$, and Higgs $d = 6$ contributions mediated by the fields T, $\bar{T}$, Φ_3, $\bar{\Phi}_3$, Φ_5, $\bar{\Phi}_5$, Φ_6, and $\bar{\Phi}_6$. The most important contributions to the decay of the proton in supersymmetric scenarios are the dimension-five contributions if the so-called matter-parity is conserved. In our model the most important proton decay contributions are mediated by the superpartners of the above fields: $\tilde{T}$, $\tilde{\bar{T}}$, $\tilde{\Phi}_3$, $\tilde{\bar{\Phi}}_3$, $\tilde{\Phi}_5$, $\tilde{\bar{\Phi}}_5$, $\tilde{\Phi}_6$, and $\tilde{\bar{\Phi}}_6$. Let us discuss the different LLLL and RRRR contributions. The so-called LLLL effective operators, $\hat{Q}\hat{Q}\hat{Q}\hat{L}$, are generated once we integrate out the fields $\tilde{T}$, $\tilde{\bar{T}}$, $\tilde{\Phi}_3$, $\tilde{\bar{\Phi}}_3$, $\tilde{\Phi}_5$, and $\tilde{\bar{\Phi}}_5$. The RRRR contributions, $\hat{U}^C\hat{E}^C\hat{U}^C\hat{D}^C$, are due to the presence of the fields $\tilde{T}$, $\tilde{\bar{T}}$, $\tilde{\Phi}_5$, $\tilde{\bar{\Phi}}_5$, $\tilde{\Phi}_6$, and $\tilde{\bar{\Phi}}_6$. As is well known, those fields have to be very heavy in order to satisfy the experimental bounds on the proton decay lifetime. There are also new contributions to nucleon decay in this context. Once we compute the F-terms of the fields in the adjoint representation we find new dimension-five contributions. However, these contributions are suppressed since they are proportional to m_W/M_T^2 [15].

Let us analyze how we could suppress the LLLL and RRRR contributions mentioned above. The different dimension-five contributions are obtained through the mixings between $\hat{\mathbf{5}}_{\mathbf{H}}$ and $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$ (the usual contributions in minimal $SU(5)$), $\hat{\mathbf{5}}_{\mathbf{H}}$ and $\hat{\overline{\mathbf{45}}}_{\mathbf{H}}$ (proportional to b_H), $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$ and $\hat{\mathbf{45}}_{\mathbf{H}}$ (proportional to Y_4), and through the mixing between $\hat{\mathbf{45}}_{\mathbf{H}}$ and $\hat{\overline{\mathbf{45}}}_{\mathbf{H}}$ (proportional to Y_4). Notice that in the case when Y_4 and b_H are very small the only relevant LLLL and RRRR contributions to the decay of the proton are due to the mixing between $\hat{\mathbf{5}}_{\mathbf{H}}$ and $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$ since all others are suppressed. Now, without assuming large masses for sfermions one can satisfy the experimental bounds on the proton decay lifetime if the unification scale and the mass of the triplets $\tilde{T}$ and $\tilde{\bar{T}}$ is around 10^{17} GeV. In this model it is easy to realize this scenario. The complete numerical analysis of the proton decay issue in this model is beyond the scope of this article and will be studied in detail in a future publication [15].

As in the nonsupersymmetric version of the model [4], integrating out the singlet ρ_0 and the neutral component of the $SU(2)$ fermionic triplet ρ_3 in $\mathbf{24}$, the neutrino mass matrix reads as

$$M_{ij}^{\nu} = \frac{a_i a_j}{M_{\rho_3}} + \frac{b_i b_j}{M_{\rho_0}}, \tag{10}$$

with

$$a_i = c_i v_5 - 3 p_i v_{45}, \tag{11}$$

and

$$b_i = \frac{\sqrt{15}}{2}\left(\frac{c_i v_5}{5} + p_i v_{45}\right). \tag{12}$$

The theory predicts one massless neutrino at tree level—this is one of the main predictions. Therefore, we could have a *normal neutrino mass hierarchy*: $m_1 = 0$, $m_2 = \sqrt{\Delta m_{\text{sun}}^2}$ and $m_3 = \sqrt{\Delta m_{\text{sun}}^2 + \Delta m_{\text{atm}}^2}$ or *the inverted neutrino mass hierarchy*: $m_3 = 0$, $m_2 = \sqrt{\Delta m_{\text{atm}}^2}$ and $m_1 = \sqrt{\Delta m_{\text{atm}}^2 - \Delta m_{\text{sun}}^2}$. $\Delta m_{\text{sun}}^2 \approx 8 \times 10^{-5}$ eV^2 and $\Delta m_{\text{atm}}^2 \approx 2.5 \times 10^{-3}$ eV^2 are the mass-squared differences of solar and atmospheric neutrino oscillations, respectively. The Higgs sector is composed of two pairs of Higgs chiral superfields, $\hat{H}_1$, $\hat{\bar{H}}_1$, $\hat{H}_2$, and $\hat{\bar{H}}_2$. See Ref. [16] for phenomenological aspects of supersymmetric models with several chiral Higgs superfields.

In this model the neutralino states are defined as $\tilde{\Psi}_i^0 = N_{i1}\tilde{B} + N_{i2}\tilde{W}_3^0 + N_{i3}\tilde{\bar{H}}_1^0 + N_{i4}\tilde{H}_1^0 + N_{i5}\tilde{\bar{H}}_2^0 + N_{i6}\tilde{H}_2^0$, and the mass matrix for them reads as

$$\begin{pmatrix} M_1 & 0 & -\frac{1}{2}g'v_{\bar{5}} & \frac{1}{2}g'v_5 & -\frac{1}{2}g'v_{\overline{45}} & \frac{1}{2}g'v_{45} \\ 0 & M_2 & \frac{1}{2}gv_{\bar{5}} & -\frac{1}{2}gv_5 & \frac{1}{2}gv_{\overline{45}} & -\frac{1}{2}gv_{45} \\ -\frac{1}{2}g'v_{\bar{5}} & \frac{1}{2}gv_{\bar{5}} & 0 & -\mu_1 & 0 & -\mu_2 \\ \frac{1}{2}g'v_5 & -\frac{1}{2}gv_5 & -\mu_1 & 0 & -\mu_3 & 0 \\ -\frac{1}{2}g'v_{\overline{45}} & \frac{1}{2}gv_{\overline{45}} & 0 & -\mu_3 & 0 & -\mu_4 \\ \frac{1}{2}g'v_{45} & -\frac{1}{2}gv_{45} & -\mu_2 & 0 & -\mu_4 & 0 \end{pmatrix} \qquad (13)$$

where $v_5 = \langle 5_H \rangle$, and $v_{\bar{5}} = \langle \bar{5}_H \rangle$. The μ_i parameters in the above matrix are given by $\mu_1 = -m_H + 2m_\Sigma \lambda_H/\lambda_\Sigma$, $\mu_2 = -10c_H m_\Sigma/\lambda_\Sigma$, $\mu_3 = 10b_H m_\Sigma/\lambda_\Sigma$, and $\mu_4 = 12m_{45} + 22a_H m_\Sigma/\lambda_\Sigma$. At low energy we have just one pair of light Higgsinos with mass μ_{eff}.

Also as in the nonsupersymmetric version of the model it is possible to realize the leptogenesis mechanism in this context. (For a review see [17].) In this case a net B-L asymmetry can be generated through the out-of-equilibrium decays of the fields ρ_0 and ρ_3 and their superpartners in the adjoint representation.

Let us now compare our model with the unrealistic minimal renormalizable SUSY $SU(5)$. In our model we have three extra chiral superfields, two Higgs chiral superfields $\hat{\mathbf{45}}_{\mathbf{H}}$ and $\hat{\overline{\mathbf{45}}}_{\mathbf{H}}$, and one matter chiral superfield $\hat{\mathbf{24}}$. All those fields could modify the predictions coming from the unification of gauge couplings. In $\hat{\mathbf{24}}$ we have four superfields, $\hat{\rho}_8$, $\hat{\rho}_3$, $\hat{\rho}_{(3,2)}$, and $\hat{\rho}_{(\bar{3},2)}$, which contribute to the running of gauge couplings. However, only $\hat{\rho}_3$ could help us to improve the unification in agreement with the values of $\alpha_s(M_Z)$, $\alpha_{\text{em}}(M_Z)$, and $\sin\theta_W(M_Z)$ since it has positive (negative) contribution to $b_2 - b_3$ $(b_1 - b_2)$. Here b_i stands for the different beta functions. Notice that in the limit when $\lambda \rightarrow 0$, see Eq. (4), the mass splitting between the fields in the adjoint representation is very small; they do not modify the running of the gauge couplings at one-loop level, and still we can generate mass for two neutrinos. In the case of the $\hat{\mathbf{45}}_{\mathbf{H}}$ and $\hat{\overline{\mathbf{45}}}_{\mathbf{H}}$ there are four fields, $\hat{\Phi}_3$, $\hat{\bar{\Phi}}_3$, $\hat{H}_2$, and $\hat{\bar{H}}_2$, with positive (negative) contributions to $b_2 - b_3$ $(b_1 - b_2)$. However, as we have discussed above the fields in $\hat{\Phi}_3$ and $\hat{\bar{\Phi}}_3$ mediate proton decay; therefore they have to be at the GUT scale if we do not suppress their contributions. A detailed numerical analysis of the unification of gauge couplings in this model is beyond the scope of this article.

In minimal unrealistic SUSY $SU(5)$ the most important contributions to the decay of the proton are mediated by $\tilde{T}$ and $\tilde{\bar{T}}$. Now, in the so-called super-KM (Kobayashi-Maskawa) basis, in which the mixing angles of fermion and sfermions are equal, the most important mode is $p \rightarrow K^+\bar{\nu}$. For more details see Ref. [13]. As we have mentioned above, in our model there are extra dimension-five contributions to proton decay mediated by $\tilde{\Phi}_3$, $\tilde{\bar{\Phi}}_3$, $\tilde{\Phi}_5$, $\tilde{\bar{\Phi}}_5$, $\tilde{\Phi}_6$, and $\tilde{\bar{\Phi}}_6$. In this case, working in the super-KM basis, $p \rightarrow K^+\bar{\nu}$ is the dominant mode, but we have to take into account the contributions of all those fields mentioned above, and typically the decay rate is much smaller than in the case of minimal SUSY $SU(5)$. Since there are several fields mediating proton decay it is possible to suppress the supersymmetric contributions assuming a possible cancellation between the different sources. See Ref. [18] for this possibility. All the phenomenological and cosmological aspects mentioned above will be studied in detail in a future publication [15].

IV. SUMMARY AND OUTLOOK

In this article we have written and studied the minimal supersymmetric version of the renormalizable grand unified theory based on the $SU(5)$ gauge symmetry with extra matter in the adjoint representation proposed in Ref. [4]. We refer to this model as "Supersymmetric Adjoint $SU(5)$." As in the non-SUSY version of the theory it is possible to generate all fermion masses, including the neutrino masses, with the minimal number of Higgses. The theory predicts one massless neutrino at tree level and a net $B - L$ asymmetry can be generated in the early universe through the out-of-equilibrium decays of the fermions responsible for type I and type III seesaw mechanisms and their superpartners in the adjoint matter chiral superfield. The LLLL and RRRR contributions to proton decay and the properties of neutralinos have been discussed. A detailed analysis of the predictions for neutrino masses, the constraints coming from leptogenesis, the numerical analysis for proton decay, and the unification of gauge couplings will be published in a future publication. The theory presented in this work can be considered as the simplest renormalizable supersymmetric grand unified theory based on the $SU(5)$ gauge symmetry since it has the minimal number of superfields.

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