

Upper bound on the mass of the type III seesaw triplet in an SU(5) model

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ABSTRACT: We investigate correlation between gauge coupling unification, fermion mass spectrum, proton decay, perturbativity and ultraviolet cutoff within an SU(5) grand unified theory with minimal scalar content and an extra adjoint representation of fermions. We find strong correlation between the upper bound on the mass of both the bosonic and fermionic SU(2) triplets and the cutoff. The upper bound on the mass of fermionic triplet responsible for Type III seesaw mechanism is $10^{2.1}$ GeV for the Planck scale cutoff. In that case both the idea of grand unification and nature of seesaw mechanism could be tested at future collider experiments through the production of those particles. Moreover, the prediction for the proton decay lifetime is at most an order of magnitude away from the present experimental limits. If the cutoff is lowered these predictions change significantly. In the most general scenario, if one does (not) neglect a freedom in the quark and lepton mixing angles, the upper bound on the fermionic triplet mass is at $10^{5.4}$ GeV (10^{10} GeV). Since the predictions of the model critically depend on the presence of the higher-dimensional operators and corresponding cutoff we address the issue of their possible origin and also propose alternative scenarios that implement the hybrid seesaw framework of the original proposal.

KEYWORDS: GUT, Beyond Standard Model.

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1. Introduction

The possibility to have unification of fundamental interactions is one of the main motivations for the physics beyond the Standard Model (SM). Partial realization of this dream is an intrinsic feature of the so-called grand unified theories which are hence considered the most natural extensions of the Standard Model. The simplest grand unified theory (GUT) is the SU(5) model of Georgi and Glashow [1]. One generation of the SM matter is partially unified and the Higgs sector is truly minimal. This theory is very predictive but it is certainly not realistic: one cannot unify experimentally observed gauge couplings at the high scale, neutrinos are massless, and unification of Yukawa couplings of the down quarks and charged leptons at the high scale contradicts experimentally inferred values.

Recently, there have been several efforts to define simple realistic extensions of the Georgi-Glashow (GG) model. In particular, it has been shown [2] that the simplest extension with the SM matter content and an extra **15** dimensional representation in the Higgs sector simultaneously generates neutrino masses via Type II seesaw mechanism [3] and achieves unification. Different phenomenological and cosmological aspects of this proposal have been analyzed and reviewed in subsequent works [4–7]. In short, this theory predicts existence of light scalar leptoquarks and an upper bound on the proton lifetime: $\tau_p \lesssim 1.6 \times 10^{36}$ years. Therefore, this realistic grand unified theory could be tested in future collider experiments, particularly at LHC, through the production of scalar leptoquarks and in the next generation of proton decay experiments.

If, on the other hand, one contemplates extensions of the GG model with extra fermions, there is another simple realistic GUT model with an extra adjoint representation of fermions. This possibility has been recently introduced [8]. The model is very appealing since it generates two massive neutrinos via combination of both Type III [9] and Type I [10] seesaw due to the presence of higher-dimensional operators. That model is the primary focus of our work and we refer to it as the “SU(5) with 24_F ” in what follows for clarity.

In this paper we study the constraints on the spectrum of the adjoint SU(5) coming from gauge unification and perturbativity at the one-loop level. Furthermore, we discuss in detail correlation between the ultraviolet cutoff of the adjoint SU(5), prediction for the fermion masses and proton decay. We find that the upper bound on the mass of the fermionic SU(2) triplet responsible for Type III seesaw mechanism depends strongly on the cutoff. The most exciting scenario is when the cutoff of the theory is identified with the Planck scale as in the original proposal [8]. In that case the discovery of those “SU(2) gauginos” is imminent. If that is not the case, part of the predictivity is lost in terms of both collider and proton decay signatures. In fact, if one lowers the cutoff and does (not) neglect a freedom in the quark and lepton mixing angles, the upper bound on the fermionic triplet mass is at $10^{5.4}$ GeV (10^{10} GeV). Towards the end we also discuss possible origins of the higher-dimensional operators that make the original model [8] realistic and propose some alternative scenarios.

2. SU(5) with 24_F : unification constraints

The SU(5) model of Georgi and Glashow [1] is the simplest GUT. It offers partial matter unification of one SM family a ($a = 1, 2, 3$) in the anti-fundamental $\bar{\mathbf{5}}_a$ and antisymmetric $\mathbf{10}_a$ representations. The Higgs sector comprises the adjoint $\mathbf{24}_H = (\Sigma_8, \Sigma_3, \Sigma_{(3,2)}, \Sigma_{(\bar{3},2)}, \Sigma_{24}) = (\mathbf{8}, \mathbf{1}, 0) + (\mathbf{1}, \mathbf{3}, 0) + (\mathbf{3}, \mathbf{2}, -5/6) + (\bar{\mathbf{3}}, \mathbf{2}, 5/6) + (\mathbf{1}, \mathbf{1}, 0)$ and fundamental $\mathbf{5}_H = (\Psi_D, \Psi_T) = (\mathbf{1}, \mathbf{2}, 1/2) + (\mathbf{3}, \mathbf{1}, -1/3)$ representations. The GUT symmetry is broken down to the SM by the vacuum expectation value (VEV) of the SM singlet in the $\mathbf{24}_H$ ($< \mathbf{24}_H > \cong v/\sqrt{30} \text{diag}(2, 2, 2, -3, -3)$), while the SM Higgs resides in the $\mathbf{5}_H$. The beauty of the model cannot be denied. However, the model itself is not realistic.

We are interested in the predictions of a promising extension of the GG model with an extra fermionic adjoint $\mathbf{24}_F = (\rho_8, \rho_3, \rho_{(3,2)}, \rho_{(\bar{3},2)}, \rho_{24}) = (\mathbf{8}, \mathbf{1}, 0) + (\mathbf{1}, \mathbf{3}, 0) + (\mathbf{3}, \mathbf{2}, -5/6) + (\bar{\mathbf{3}}, \mathbf{2}, 5/6) + (\mathbf{1}, \mathbf{1}, 0)$ that has been introduced only recently [8]. The nice feature of this extension is that neutrino masses are generated using both Type III and Type I seesaw mechanisms. Although it has been argued that ρ_3 could be very light [8] the interplay between perturbativity, unification constraint, fermionic mass spectrum and proton decay has not been studied to corroborate that claim. In this section we study this issue in order to find correct upper bound on the mass of ρ_3 —the field responsible for Type III seesaw mechanism. The possibility to test this theory through proton decay is also discussed. We also suggest possible origins of the higher-dimensional operators that play critical role in the original model [8] and suggest alternative scenarios that implement the hybrid seesaw framework of the original proposal.

2.1 Gauge unification constraints

In order to understand the constraints coming from the unification of gauge couplings we use the well-known relations [11]:

$$\frac{B_{23}}{B_{12}} = \frac{5 \sin^2 \theta_W(M_Z) - \alpha_{em}(M_Z)/\alpha_s(M_Z)}{3/8 - \sin^2 \theta_W(M_Z)}, \quad \ln \frac{M_{\text{GUT}}}{M_Z} = \frac{16\pi}{5} \frac{3/8 - \sin^2 \theta_W(M_Z)}{\alpha_{em}(M_Z) B_{12}}. \quad (2.1)$$

	Higgsless SM	Ψ_D	Ψ_T	V	Σ_8	Σ_3
B_{23}	$\frac{11}{3}$	$\frac{1}{6}$	$-\frac{1}{6}r_{\Psi_T}$	$-\frac{7}{2}r_V$	$-\frac{1}{2}r_{\Sigma_8}$	$\frac{1}{3}r_{\Sigma_3}$
B_{12}	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15}r_{\Psi_T}$	$-7r_V$	0	$-\frac{1}{3}r_{\Sigma_3}$

Table 1: B_{ij} coefficients in the GG model [1].

	ρ_8	ρ_3	$\rho_{(3,2)}$	$\rho_{(\bar{3},2)}$
B_{23}	$-2r_{\rho_8}$	$\frac{4}{3}r_{\rho_3}$	$\frac{1}{3}r_{\rho_{(3,2)}}$	$\frac{1}{3}r_{\rho_{(\bar{3},2)}}$
B_{12}	0	$-\frac{4}{3}r_{\rho_3}$	$\frac{2}{3}r_{\rho_{(3,2)}}$	$\frac{2}{3}r_{\rho_{(\bar{3},2)}}$

Table 2: Extra contributions to B_{ij} coefficients in the SU(5) with 24_F [8].

where the coefficients $B_{ij} = B_i - B_j$ and $B_i = b_i + \sum_I b_{iI} r_I$ are the so-called effective coefficients. Here b_{iI} are the appropriate one-loop β coefficients of the particle I with mass M_I , where $r_I = (\ln M_{\text{GUT}}/M_I)/(\ln M_{\text{GUT}}/M_Z)$ ($0 \leq r_I \leq 1$) is its “running weight”. M_{GUT} is the GUT scale where the SM gauge couplings meet. We find:

$$\frac{B_{23}}{B_{12}} = 0.716 \pm 0.005, \quad (2.2a)$$

$$\ln \frac{M_{\text{GUT}}}{M_Z} = \frac{184.9 \pm 0.2}{B_{12}}, \quad (2.2b)$$

where we use $\sin^2 \theta_W(M_Z) = 0.23120 \pm 0.00015$, $\alpha_{em}^{-1}(M_Z) = 127.906 \pm 0.019$ and $\alpha_s(M_Z) = 0.1176 \pm 0.002$ [12].

Eq. (2.2a) is sometimes referred to as the B -test. It basically shows whether unification takes place or not. Eq. (2.2b), on the other hand, can be referred to as the GUT scale relation since it yields the GUT scale value when eq. (2.2a) is satisfied. The GUT scale relation can also bound M_{GUT} for the given particle content of the theory without any reference to eq. (2.2a).

The B -test fails badly in the SM case: $B_{23}^{SM}/B_{12}^{SM} = 0.53$. Hence the need for extra light particles with suitable B_{ij} coefficients. The B_{ij} coefficients for all the particles in the GG scenario are presented in table 1. Clearly, only Σ_3 can slightly improve unification with respect to the SM case, i.e., $B_{23}/B_{12} = 0.60$ at most. In table 2 we shown extra contributions to the B_{ij} coefficients in the adjoint SU(5) [8]. Notice that the field ρ_3 is the only field that can further improve unification. It is thus clear that it has to be below the GUT scale and that the upper bound on M_{ρ_3} corresponds to the smallest allowed value for M_{Σ_3} .

Before we address the implications of the exact gauge coupling unification within the scenario with the **24** dimensional fermionic representation we need to investigate the question of fermion masses. Clearly, the model must rely on higher-dimensional operators to be realistic which critically affects fermion masses in three different sectors. Firstly, these operators correct the GUT scale relation $Y_D = Y_E^T$, where Y_D and Y_E are Yukawa matrices for the down quarks and charged leptons, respectively. Secondly, they increase the rank of the effective neutrino mass matrix from one to two. As a result two neutrinos are predicted

to be massive [8]. Thirdly, they generate mass splitting between the fermionic fields in $\mathbf{24}_F$ that is crucial in order to achieve unification. Hence, the low-energy predictions of the theory depend critically on the cut-off scale that determines their maximal impact. Clearly, the higher that scale is the more predictive the theory becomes. If the cut-off goes to infinity one recovers renormalizable model which is not consistent with experimental data. Thus, the cut-off scale cannot be arbitrarily large. In the original proposal [8] the cut-off (Λ) is at the Planck scale $\Lambda = M_{\text{Pl}} \cong 1.2 \times 10^{19}$ GeV. This yields, as the most significant result, rather low limit on the mass of ρ_3 which should be at the electroweak scale. We find that this cut-off, at least at the one-loop level, leaves very narrow allowed region within which both SU(2) triplets — fermionic and bosonic — are at the electroweak scale and the proton decay lifetime is an order of magnitude below the current experimental bounds if we neglect the quark and lepton mixings. However, we also find that if one lowers Λ , the upper bound on the mass of ρ_3 relaxes significantly.

To reach these conclusions we rely on the upper bound on the cutoff that comes from the relation between charged fermion masses. In this particular case the least conservative bound reads

$$\Lambda \leq \sqrt{\frac{2}{\alpha_{\text{GUT}}}} \times \frac{M_{\text{GUT}}}{Y_\tau - Y_b}. \tag{2.3}$$

Eq. (2.3) is obtained by considering the difference between Y_D and Y_E at the GUT scale and assuming that the Yukawa coefficients Y_{ij} ($i, j = 1, 2, 3$) that multiply higher-dimensional operators remain perturbativity, i.e., $|Y_{ij}| \leq \sqrt{4\pi}$. See reference [5] for details. We also use the well-known SU(5) relation for the masses of proton decay mediating gauge bosons: $M_{(X,Y)} = \sqrt{5\pi\alpha_{\text{GUT}}/3}v$. The mass splitting between M_X and M_Y is negligible for our purposes and we identified them with the GUT scale, i.e., $M_{(X,Y)} \equiv M_{\text{GUT}}$.

Since b and τ do not unify to a high degree as can happen in supersymmetric SU(5) theory the upper limit on Λ is determined by the mismatch between b and τ masses at the GUT scale. See for example figure 1 in [5] for behavior of Y_b and Y_τ as one goes from low energy to high energy scales. Note that the most conservative limit on Λ is in fact proportional to $(Y_\tau + Y_b)^{-1}$. In any case, one can find Y_τ , Y_b , α_{GUT} and M_{GUT} at any point allowed by unification and check whether inferred Λ from eq. (2.3) is consistent with the initial assumption $\Lambda = M_{\text{Pl}}$.

In order to find the maximal allowed value for Λ we need to maximize M_{GUT} . That turns out to be easy. Namely, there exist a simple procedure to find maximal value for the GUT scale as a function of M_{ρ_8} as allowed by the assumption that Planck scale effects split multiplets of $\mathbf{24}_F$. We will explain all the details later. What is important at this point is the following. If we set $M_{\rho_3} = M_{\Sigma_3} = M_Z$ and $M_{\Sigma_8} = M_{\text{GUT}}$ we get $M_{\text{GUT}} \cong 10^{15.74}$ GeV for $\alpha_{\text{GUT}}^{-1} \cong 35.4$, $M_{\rho_{(3,2)}} = 2.5 \times 10^{13}$ GeV and $M_{\rho_8} = 6.3 \times 10^5$ GeV at the one-loop level. At the same time we find $(Y_\tau - Y_b) = 0.0038 \pm 0.0002$ where the errors are associated to the 1σ variation of the b quark mass at the M_Z scale. We take as input $m_b = 2.89 \pm 0.11$ GeV and $m_\tau = 1.74646_{-0.00026}^{+0.00029}$ GeV at M_Z [5]. This in turn implies via eq. (2.3) that Λ barely reaches the Planck scale ($\Lambda = 1.22 \times 10^{19}$ GeV = M_{Pl}). Since $(Y_\tau - Y_b)$ remains almost constant in the region of interest and M_{GUT} can only be below its maximal value that means that if the cutoff of the theory is the Planck scale the allowed parameter space is

extremely narrow. To illustrate our point we plot the allowed parameter space for M_{Σ_3} and $M_{\rho_{(3,2)}}$ in $M_{\text{GUT}}-M_{\rho_3}$ plane in figure 1. (Note, whenever we refer to $\rho_{(3,2)}$ we also refer to $\rho_{(\bar{3},2)}$ since $M_{\rho_{(3,2)}} = M_{\rho_{(\bar{3},2)}}$.) The region to the left of the dashed line in figure 1 is excluded through the use of eq. (2.3). More precisely, the dashed line is obtained by setting $\Lambda = M_{\text{Pl}}$ and then plotting the smallest possible value of M_{GUT} as allowed by eq. (2.3). The region to the right of the blue line is eliminated by the perturbativity constraints on the spectrum of fermionic particles in the adjoint representation as we discuss later. The only viable parameter space is the strip between the blue and dashed line. This scenario could clearly be tested at the LHC since both the bosonic and fermionic triplets are light. Moreover, proton decay is only factor 3-6 away from the current bound on the proton lifetime. To move the strip to the right one needs to either lower slightly mass of Σ_8 or ρ_8 . This however would make the allowed region disappear once $M_{\text{GUT}} \cong 10^{15.74}$ GeV is reached. Since the prediction for M_{ρ_3} is practically at the present experimental limit on its mass $M_{\rho_3} > 100$ GeV [12], which admittedly is model dependent, the two-loop analysis would probably be in order. We do not attempt that since even the full one-loop treatment would also have to include influence of higher-dimensional operators on the gauge coupling unification conditions [13, 14] which we neglected.

Unfortunately, we do not know what the cutoff(s) is (are) and we also have to see how the predictions of adjoint SU(5) hold if we allow Λ to vary within reasonable range $10M_{\text{GUT}} \leq \Lambda \leq M_{\text{Pl}}$.

Before we answer what happens if we lower Λ let us discuss the correct procedure to obtain exact gauge coupling unification within this scenario in view of the fact that the masses of particles within the fermionic adjoint are related to each other. The masses of relevant particles in $\mathbf{24}_F$ to the leading order in $1/\Lambda$ are [8]:

$$M_{\rho_0} = m_F - \frac{\lambda_F v}{\sqrt{30}} + \frac{v^2}{\Lambda} \left[a_1 + a_2 + \frac{7}{30}(a_3 + a_4) \right], \quad (2.4)$$

$$M_{\rho_3} = m_F - \frac{3\lambda_F v}{\sqrt{30}} + \frac{v^2}{\Lambda} \left[a_1 + \frac{3}{10}(a_3 + a_4) \right], \quad (2.5)$$

$$M_{\rho_8} = m_F + \frac{2\lambda_F v}{\sqrt{30}} + \frac{v^2}{\Lambda} \left[a_1 + \frac{2}{15}(a_3 + a_4) \right], \quad (2.6)$$

$$M_{\rho_{(3,2)}} = m_F - \frac{\lambda_F v}{2\sqrt{30}} + \frac{v^2}{\Lambda} \left[a_1 + \frac{(13a_3 - 12a_4)}{60} \right]. \quad (2.7)$$

It is understood that λ_F and a_i ($i = 1, 2, 3$) should be perturbative. We hence demand that $|a_i| \leq \sqrt{4\pi}$. In that case we obtain

$$a_4 = \frac{2 \Lambda \pi \alpha_{\text{GUT}} \left((M_{\rho_8} - 2M_{\rho_{(3,2)}}) + M_{\rho_3} \right)}{M_{\text{GUT}}^2}, \quad (2.8)$$

where we again use $M_{(X,Y)} = \sqrt{5\pi\alpha_{\text{GUT}}/3}v = M_{\text{GUT}}$. Clearly, M_{ρ_8} , $M_{\rho_{(3,2)}}$ and M_{ρ_3} are not independent from each other. In fact, perturbativity of a_4 implies that there are three regimes:

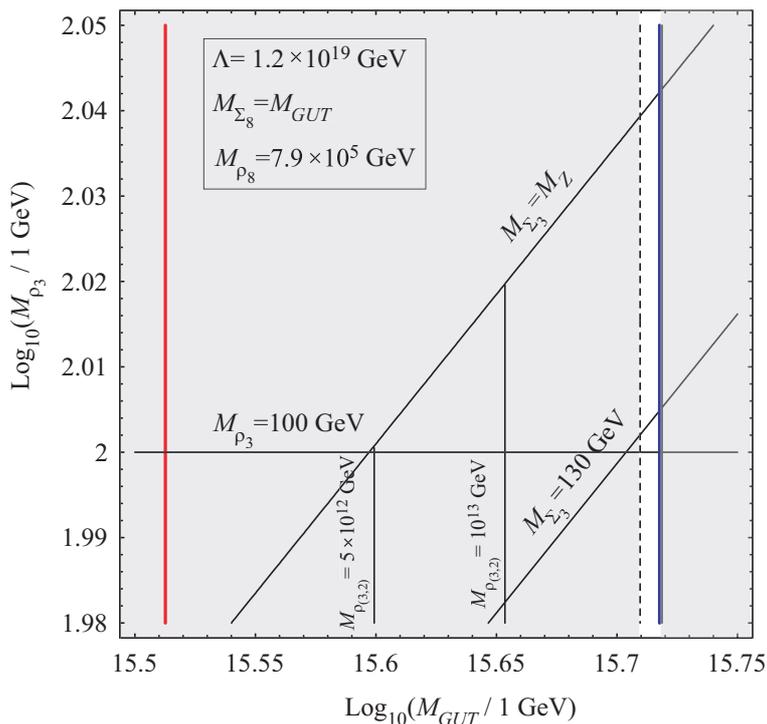


Figure 1: Gauge coupling unification at the one-loop level for central values of low-energy observables. Blue line corresponds to the bound coming from perturbativity constraints on the mass spectrum of the particles in the fermionic adjoint, i.e. $a_4 \geq -\sqrt{4\pi}$. Red line corresponds to the up-to-date experimental bound from proton decay on M_{GUT} . Dashed line corresponds to the minimal value of M_{GUT} if the cutoff is taken to be the Planck scale. In the triangle given by the blue line, $M_{\rho_3} = 100$ GeV line and $M_{\Sigma_3} = 130$ GeV we get $(Y_\tau - Y_b) = 0.0038 \pm 0.0002$.

- In the first regime, M_{ρ_8} and $M_{\rho_{(3,2)}}$ could both be at the GUT scale as long as $M_{\rho_8} \cong 2M_{\rho_{(3,2)}}$. In that case M_{ρ_3} must be of the order of M_{GUT}^2/Λ or smaller where it wants to be anyway in order to fix the B-test. In this regime the GUT scale is too low to be in agreement with the experimental limits on proton decay lifetime unless one allows for special realization of fermion mass matrices [15]. Even though we do not advocate this scenario it is important to realize that this is still allowed by experimental data and the structure of the model. In that case one needs Λ to be around 10^{17} GeV in order to suppress proton decay by adjusting Yukawa couplings of ordinary matter [5]. We show one such example in figure 2.
- In the second regime, M_{ρ_3} is small and M_{ρ_8} and $M_{\rho_{(3,2)}}$ are both of the order of M_{GUT}^2/Λ . The cancellation between the two masses does not have to be as efficient as in the first case and one can easily see that the upper bound on M_{GUT} comes when $M_{\rho_8} < M_{\rho_{(3,2)}} \sim M_{GUT}^2/M_{Pl}$, i.e., $a_4 = -\sqrt{4\pi}$. Recall, light $\rho_{(3,2)}$ spoils unification. Hence, in the large M_{GUT} regime $\rho_{(3,2)}$ has to be as heavy as possible. This scenario is promising since large GUT scale implies light ρ_3 . It is this scenario that is advocated

in [8]. Example given in figure 1 corresponds to this scenario once we fix $\Lambda = M_{P1}$.

- There is however a third scenario that interpolates between the two. The nice feature of this scenario is that it yields correct upper bound on the mass of ρ_3 . It can be best grasped by considering the case when both M_{ρ_8} and M_{ρ_3} are below the GUT scale. In this case the GUT scale is guaranteed to be large. It is then easy to see that if $M_{\rho_{(3,2)}} = M_{GUT}$ and $a_4 \geq -\sqrt{4\pi}$ one obtains

$$\Lambda \leq \frac{M_{GUT}}{2\alpha_{GUT}\sqrt{\pi}}. \tag{2.9}$$

We will next discuss this case to establish an upper bound on M_{ρ_3} .

Clearly, since in this theory $\alpha_{GUT}^{-1} \sim 35$ and proton decay experiments require $M_{GUT} > (2-3) \times 10^{15}$ GeV then eq. (2.9) implies that we need a scenario where $\Lambda \simeq 10M_{GUT} \sim 2-3 \times 10^{16}$ GeV. This then allows us to maximize M_{ρ_3} in order to understand the testability of the model. So, to maximize M_{ρ_3} at the one-loop level is simple. All one needs is to set $M_{\rho_{(3,2)}} = M_{\Sigma_8} = M_{GUT}$, $M_{\Sigma_3} = M_Z$ and solve for M_{ρ_8} , M_{ρ_3} and M_{GUT} using B-test, the GUT scale relation and eq. (2.3) where M_{GUT} must match the most stringent constraint coming from proton decay. We find that $M_{\rho_3} = 2.3 \times 10^5$ GeV, $M_{\rho_8} = 2.0 \times 10^9$ GeV, $M_{GUT} = 3.1 \times 10^{15}$ GeV, $\alpha_{GUT}^{-1} = 38.7$ and accordingly $\Lambda \leq 3.4 \times 10^{16}$ GeV. So, the correct upper bound on M_{ρ_3} at the one-loop level is $M_{\rho_3} = 2.3 \times 10^5$ GeV. One can also check that $v/\Lambda \geq 0.12$ which implies that higher order corrections in $1/\Lambda$ on eq. (2.3) cannot significantly affect our conclusions.

As far as experimental proton decay constraints are concerned we match M_{GUT} with the experimental limit from the dominant proton decay mode in SU(5) which is usually taken to be $p \rightarrow \pi^0 e^+$. The theoretical prediction for this channel can be summarized in the following way: $\tau^{tho} = 1.2 \times 10^{32} (M_{GUT}/10^{16} \text{ GeV})^4 \alpha_{GUT}^2 (\alpha/0.015 \text{ GeV}^3) \text{ years}$. Here, α is a relevant matrix element. So, the current experimental limit $\tau > 1.6 \times 10^{33}$ years [12] translates into the following bound on M_{GUT} : $M_{GUT} > (1.6/1.2)^{1/4} 10^{16} \sqrt{\alpha_{GUT}^{-1}} \text{ GeV}$. Red line in figure 1 is generated using this result.

Let us finally address the case when we allow for suppression of the proton decay through gauge boson mediation [15] in the adjoint SU(5) model. That case would correspond to the first scenario when $M_{\rho_8} - 2M_{\rho_{(3,2)}} \simeq 0$. So, allowed region would be very narrow strip given by the allowed range for a_4 . We show one such scenario in figure 2, where the vertical blue lines correspond to the bounds coming from perturbativity, i.e. $|a_4| \leq \sqrt{4\pi}$. From this plot we can basically find the upper bound on the mass of the fermionic SU(2) triplet ρ_3 responsible for the Type III seesaw mechanism to be $M_{\rho_3} \lesssim 10^{10}$ GeV. This bound clearly reflects the worst case scenario as far as the testability of the model is concerned. The dashed line in figure 2 is the experimental bound on M_{GUT} if $d = 6$ gauge mediated proton decay is suppressed. It corresponds to $M_{GUT} > 5.1 \times 10^{14} \sqrt{\alpha_{GUT}} \text{ GeV}$ [15].

We stress again that the maximal value of the GUT scale depends crucially on the cutoff of the theory. In fact, the highest value that the GUT scale can reach at the one-loop level in this model is basically $M_{GUT} = 2 \times 10^{16}$ GeV which corresponds to setting $B_{12}^{(min)} = 28/5$

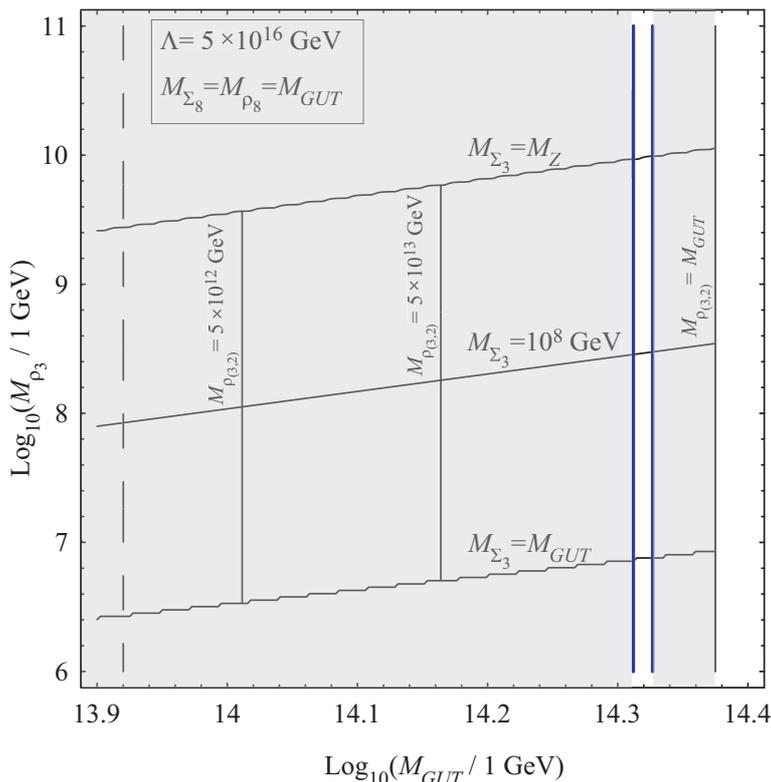


Figure 2: The gauge coupling unification at the one-loop level for central values of low-energy observables. Blue lines correspond to the bounds coming from perturbativity, i.e. $|a_4| \leq \sqrt{4\pi}$. The dashed line in is the experimental bound on M_{GUT} if $d = 6$ gauge mediated proton decay is suppressed.

in eq. (2.2b). With that in mind we make the following two comments regarding proton decay. If we neglect the fermionic mixings we can make a naive estimation of the upper bound on the proton decay lifetime to be $\tau_p \lesssim 10^{36-37}$ years. Of course, the absolute upper bound, if we use the whole freedom in the Yukawa sector, reads as $\tau_p^{24F} \lesssim 5.2 \times 10^{42}$ years, where we set $\alpha_{GUT} = 1/37$ and $\alpha = 0.015 \text{ GeV}^3$ for the matrix element.

2.2 The origin of higher-dimensional operators and alternative scenarios

Before we conclude, let us address the issue of the possible origin of higher-dimensional operators. After all, they play decisive role in making the model realistic.

The original model [8] includes higher-dimensional operators that are suppressed by the Planck scale. We have confirmed that these operators are indeed sufficient, albeit barely, to make the model viable. Operators of this sort are expected to appear on general grounds as harbingers of gravitational physics where the only relevant scale is the Planck scale. As such, they have been extensively used in the grand unified model building ever since they were proposed to correct the Georgi-Glashow fermion mass predictions [17].

If, however, the cutoff scale is below the Planck scale one might ask for the possible renormalizable model that would effectively mimic the original model. To this end we observe that one does not need to invoke nonrenormalizable operators at all nor any “exotic” physical setup. Namely, in order to have the most minimal renormalizable setup that yields the original model, it is sufficient to introduce the following additional matter representations: $\mathbf{1}$, $\mathbf{5}$, $\bar{\mathbf{5}}$ and $\mathbf{24}$. These can clearly have masses above the GUT scale that can be identified with the scale Λ . Once these fields are integrated out the effective model would have exactly the same features as the original model [8]. In particular, the established upper limits on the masses of ρ_3 and Σ_3 would still hold as well as the upper bound on M_{GUT} .

There is another rather different renormalizable realization of the model in question. Namely, if one introduces an additional $\mathbf{45}$ -dimensional Higgs field one can simultaneously generate both the charged [18] and neutrino masses at the renormalizable level [19]. However, in this case the predictions are quite different. This is primarily due to the fact that there are more fields that can potentially contribute to the running of the gauge couplings and proton decay. Moreover, the possible mass spectrum of the fields in 24_F is rather different [19].

The $SU(5)$ scenario that incorporates combination of Type I and Type III seesaw due to the presence of extra fermionic adjoint is tailor-made for applications within the extra-dimensional setup. Here, in particular, we have in mind a five-dimensional nonsupersymmetric framework [20, 21]. In this approach the $SU(5)$ symmetry of the five-dimensional bulk is reduced to the effective SM symmetry on one four-dimensional brane and $SU(5)$ symmetry on the other brane by compactification upon $S^1/(Z_2 \times Z'_2)$. Such a setup would naturally accommodate doublet-triplet splitting if the SM Higgs field originates from the bulk. Also, since the symmetry breaking would be accomplished using the judicious parity assignment under Z_2 and Z'_2 there would not be any need for the adjoint Higgs representation. In other words, there is a possibility to have a rather predictive setup due to potentially very small number of light extra fields with respect to the SM particle content. In addition, it would be possible to completely suppress proton decay for particular locations of matter fields. One particularly nice feature of this framework is that the parity assignment that yields the SM on one of the branes generates same parity properties for $(\mathbf{8}, \mathbf{1}, 0)$, $(\mathbf{1}, \mathbf{3}, 0)$ and $(\mathbf{1}, \mathbf{1}, 0)$ in the fermionic adjoint. This would guarantee, unlike in the ordinary four-dimensional framework, that the least massive fermionic triplet and singlet states are degenerate as long as they originate from the bulk. In addition, this assignment automatically insures that no anomalies are introduced at the branes. One possible scenario would be to have the $\mathbf{5}$ and $\mathbf{45}$ dimensional Higgs representations in the bulk along with the gauge fields. In that case it would be possible to build the model with all the matter fields located on the $SU(5)$ brane.

3. Summary and discussions

We investigated the relation between perturbativity, unification constraint, prediction for fermion masses, proton decay and ultra-violet cutoff Λ within the $SU(5)$ grand unified

theory with minimal scalar content and an extra adjoint representation of fermions. If the cutoff is at the Planck scale the upper bound on the mass of the Type III triplets is practically at the current experimental limit $M_{\rho_3} < 10^{2.1}$ GeV. In that case both the idea of grand unification and nature of seesaw mechanism could be tested at collider experiments through the production of those particles. Moreover, the prediction for the proton decay lifetime is at most an order of magnitude away from the present experimental limits. If the cutoff is below the Planck scale we find that the upper bound on the mass of the fermionic SU(2) triplet responsible for Type III seesaw mechanism is $M_{\rho_3} \lesssim 2 \times 10^5$ GeV if we use the strongest constraints on the GUT scale coming from proton decay, $M_{\text{GUT}} > (2-3) \times 10^{15}$ GeV. Finally, if we allow for suppression of proton decay operators using the full freedom of the model the limit is not relevant for collider physics at all and it reads $M_{\rho_3} < 10^{10}$ GeV. Since the predictions of the model depend critically on the cutoff we have addressed the issue of the possible origin of the higher-dimensional operators and proposed some alternative scenarios.

Let us finally compare our results with the results presented in refs. [8, 22]. Firstly, we show that M_{P1} can be the UV cutoff of the adjoint SU(5). This is in conflict with the results presented in ref. [22] where the authors retract their initial claim [8]. Secondly, the upper bound on the mass of $\rho_{(3,2)}$ — $M_{\rho_{(3,2)}} < M_{\text{GUT}}^2/\Lambda$ —as suggested in refs. [8, 22] depends on the specific assumptions about the cutoff and hence does not reflect the full parameter freedom of the model. See in particular eq. (11) in [8] and eq. (12) in [22]. In fact, we show that $\rho_{(3,2)}$ could be at the GUT scale. As a consequence, we obtain the upper bound on the mass of ρ_3 that is two orders of magnitude above the bounds suggested in refs. [8, 22] if we neglect the quark and lepton mixing angles. Thirdly, we show that the absolute upper bound on the mass of ρ_3 , the field responsible for Type III seesaw, is 10^{10} GeV if we use the full freedom of the model. This freedom has not been accounted for elsewhere.

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