

Another look at minimal lepton flavour violation, leptogenesis and the ratio $M_\nu/\Lambda_{\text{LFV}}$

Gustavo C. Branco,^{ab} Andrzej J. Buras,^a Sebastian Jäger,^c Selma Uhlig^a and Andreas Weiler^a

^a*Physik Department, Technische Universität München, D-85748 Garching, Germany*

^b*Centro de Física Teórica de Partículas, Departamento de Física,
Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal*

^c*Institut für Theoretische Physik E, RWTH Aachen, D-52056 Aachen, Germany*

E-mail: weiler@lepp.cornell.edu, aburas@ph.tum.de,
jaeger@theorie.physik.uni-muenchen.de, suhlig@ph.tum.de,
gbranco@mail.ist.utl.pt

ABSTRACT: We analyze lepton flavour violation (LFV), as well as generation of the observed baryon-antibaryon asymmetry of the Universe (BAU) within a generalized minimal lepton flavour violation (MLFV) framework where we allow for CP violation both at low and high energies. The generation of BAU is obtained through *radiative resonant leptogenesis* (RRL), where starting with three exactly degenerate right-handed neutrinos at Λ_{GUT} , we demonstrate explicitly within the SM and the MSSM that the splittings between their masses at the see-saw scale M_ν , generated by renormalization group effects, are sufficient for a successful leptogenesis for M_ν even as low as 10^6 GeV. The inclusion of flavour effects plays an important role in this result and can lead to the observed BAU even in the absence of CP violation beyond the PMNS phases. The absence of a stringent lower bound on M_ν in this type of leptogenesis allows to easily satisfy present and near future upper bounds on $\mu \rightarrow e\gamma$ and other charged lepton flavour violating (LFV) processes even for $\Lambda_{\text{LFV}} = \mathcal{O}(1 \text{ TeV})$. We find, that the MLFV framework in the presence of heavy right-handed neutrinos and leptogenesis is not as predictive as MFV in the quark sector and point out that without a specific MLFV model, there is a rich spectrum of possibilities for charged LFV processes and for their correlation with low energy neutrino physics and the LHC physics, even if the constraint from the observed BAU is taken into account. While certain qualitative features of our analysis confirm findings of Cirigliano et al., at the quantitative level we find phenomenologically important differences. We explain the origin of these differences.

KEYWORDS: CP violation, Beyond Standard Model, Baryogenesis, Neutrino Physics.

Contents

1. Introduction	2
2. Basic framework	6
2.1 Preliminaries	6
2.2 Yukawa couplings and Majorana mass terms	7
2.3 Leptonic masses, mixing and CP violations	8
2.4 An useful parametrization	9
2.5 Classification	10
2.6 Final remarks	11
3. Radiative corrections in MLFV	12
3.1 Preliminaries	12
3.2 MLFV with a degeneracy scale	12
3.3 Radiatively generated flavour structure and large logarithms	13
3.4 Renormalization-group evolution: high scales	14
3.5 RGE evolution below M_ν : PMNS matrix and Δ_{ij}	17
4. Numerical analysis: $B(l_i \rightarrow l_j \gamma)$ and CP asymmetries in ν_R decay	17
4.1 Preliminaries	17
4.2 Perturbativity bounds	19
4.3 Lepton flavour violation and $l_i \rightarrow l_j \gamma$	19
4.4 CP asymmetries	21
5. Leptogenesis in the extended MLFV framework	22
5.1 Preliminaries	22
5.2 BAU in the RRL and flavour effects	24
5.3 Two flavour limit	26
5.4 General case	29
5.5 Comparison with [13]	31
6. Summary and conclusions	33
A. Iterative solution of the renormalization group equations	35
B. Basic formulae for Δ_{ij}	36
B.1 Preliminaries	36
B.2 R real and U_ν complex	37
B.3 R and U_ν complex	37

1. Introduction

One of the attractive and predictive frameworks for the description of flavour changing processes in the quark sector is the so-called Minimal Flavour Violation (MFV) hypothesis [1, 2] in which the Standard Model (SM) quark Yukawa couplings are the only sources of flavour changing and in particular CP-violating processes.¹

If only one Higgs doublet is involved in the spontaneous breaking of the underlying gauge symmetry, all flavour changing charged and neutral current processes are governed in the MFV framework by the CKM matrix [4] and the relevant local operators are only those present in the SM. As demonstrated in [5], the existing data on $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing, ε_K , $B \rightarrow X_s \gamma$, $B \rightarrow X_s l^+ l^-$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and the value of the angle β in the unitarity triangle from the mixing induced CP asymmetry in $B \rightarrow \psi K_S$ imply within this framework very stringent bounds on all rare K and B decay branching ratios. Consequently, substantial departures from the SM predictions are not expected if MFV with one Higgs doublet is the whole story.

If two Higgs doublets, like in the MSSM, are involved and the ratio of the corresponding vacuum expectation values $v_2/v_1 \equiv \tan \beta$ is large, significant departures from the SM predictions for certain decays are still possible within the MFV framework [2] in spite of the processes being governed solely by the CKM matrix. The most prominent examples are the decays $B_{d,s} \rightarrow \mu^+ \mu^-$ with a subset of references given in [6]. The prime reason for these novel effects is the appearance of new scalar operators that are usually strongly suppressed within the SM and MFV models at low $\tan \beta$, but can become important and even dominant for large $\tan \beta$. The improved data on $B_{d,s} \rightarrow \mu^+ \mu^-$, expected to come in this decade from Tevatron and LHC, will tell us whether MFV models with large $\tan \beta$ are viable.

One of the important virtues of the MFV in the quark sector are the relations [1, 7] between the ratios of various branching ratios and the CKM parameters measured in low energy processes that have universal character and are independent of the details of the specific MFV model. An example is the universal unitarity triangle common to all MFV models [1]. But also the fact that each branching ratio can be expressed in terms of the CKM parameters and quark masses measured at the electroweak scale or lower energy scales makes this scenario to be a very predictive framework. Moreover, neither fine tuning nor the introduction of unnaturally high scales of new physics are required to make this scenario consistent with the available data.

The MFV scenario in the quark sector in question, although simple and elegant, suffers from the following problem. In the absence of new complex phases beyond the CKM phase, it cannot accommodate the observed size of the baryon asymmetry of the universe (BAU) to be denoted by η_B in what follows. The question then arises, whether one could still explain the right size of η_B within the MFV context by considering simultaneously the lepton sector, where the BAU can in principle be explained with the help of leptogenesis [8, 9]. While this is the most natural possibility, other directions could be explored in principle.

¹For earlier discussions of this hypothesis see [3].

Before addressing this question let us summarize what is known in the literature about the MFV in the lepton sector. The introduction of minimal lepton flavour violation (MLFV) requires additional input from physics at high energies. It is interesting to ask whether a specific MLFV framework still allows for predictions that can be falsified, comparable to those of MFV in the quark sector. Last year, Cirigliano, Grinstein, Isidori and Wise [10] in an interesting paper formulated MFV in the lepton sector both with the minimal field content and with the extended field content, where three degenerate right-handed heavy neutrinos ν_R^i with masses M_ν^i are added to the SM fields and the see-saw mechanism [11] is responsible for the generation of the light neutrino masses with the see-saw scale denoted by M_ν in what follows. Analyzing charged lepton flavour violating (LFV) processes, like $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion in nuclei in these two scenarios in the absence of CP violation, they reached two interesting conclusions:

- Measurable rates for LFV processes within MLFV are only obtained when the scale for total lepton number violation ($\Lambda_{\text{LN}} = \mathcal{O}(M_\nu)$) is by many orders of magnitude, typically a factor $10^7 - 10^9$, larger than the scale of charged lepton flavour violation (Λ_{LFV}).
- Similarly to MFV in the quark sector, the ratios of various LFV rates like $B(\mu \rightarrow e\gamma)/B(\tau \rightarrow \mu\gamma)$ are unambiguously determined in terms of neutrino masses and mixing angles measured in low energy processes.

Various phenomenological aspects of MLFV, as formulated in [10], have been subsequently discussed in [12].

The MLFV framework in [10] does not include CP violation, neither at low energy nor at high energy, being a necessary ingredient in the generation of the BAU. Moreover, possible renormalization group effects between the low energy scale $\mathcal{O}(M_Z)$ and the high energy scales, like the see-saw scale M_ν and the GUT scale Λ_{GUT} , have not been taken into account in [10]. It is then natural to ask:

- whether a successful leptogenesis is at all possible within a MLFV framework in which flavour violation is governed solely by Yukawa couplings,
- how the findings of [10] are modified, when CP violation at low and high energy and the renormalization group effects (RGE) in question are taken into account,
- whether a successful leptogenesis in the MLFV framework puts stringent constraints on charged LFV processes.

The main goal of our paper is to answer these three questions. In fact, as we will demonstrate explicitly in section 5, it is possible to obtain the correct size of η_B in the MLFV framework with three heavy right-handed neutrinos that are assumed to be degenerate in mass at Λ_{GUT} . Other choices for this scale could be considered but Λ_{GUT} seems to be the most natural one. The breakdown of this degeneracy through RGE, that are governed by Yukawa couplings, combined with new sources of CP-violation in the heavy neutrino sector allows to obtain the correct size of η_B in the framework of the resonant

leptogenesis in particular when flavour effects are taken into account. As this type of leptogenesis is generated here radiatively and not put by hand as done in most literature so far, we will call this scheme *radiative resonant leptogenesis* (RRL) in what follows.

The fact that within the MLFV framework one is naturally led to RRL, has significant implications on charged LFV processes, which could in principle be used to distinguish this scenario from other extensions of the SM. In particular, while other types of leptogenesis, with hierarchical right-handed heavy neutrinos, imply generally rather stringent lower bounds for the lightest ν_R^i mass, in the ballpark of $\mathcal{O}(10^8 \text{ GeV})$ or higher, the values of M_ν in RRL are allowed to be by many orders of magnitude lower. As the branching ratios for $l_i \rightarrow l_j \gamma$ are proportional to $M_\nu^2/\Lambda_{\text{LFV}}^4$ [10], it is relatively easy to satisfy the present and in the near future available upper bounds on these processes by simply choosing sufficiently small value of M_ν . Conversely, by choosing M_ν to be larger than say 10^{12} GeV , it is in principle possible to obtain the values of $B(\mu \rightarrow e\gamma)$ close to expected bounds from PSI even if Λ_{LFV} is as high as 100 TeV . This means that non-observation of $\mu \rightarrow e\gamma$ with the rate 10^{-13} at PSI will not necessarily imply within the general MLFV framework that Λ_{LFV} is very high. Conversely, the observation of $\mu \rightarrow e\gamma$ will not necessarily imply LFV physics at scales $\mathcal{O}(1 \text{ TeV})$. In other words, without a specific MLFV model there is a rich spectrum of possibilities for charged LFV processes within the general MLFV framework, even if the constraint from η_B is taken into account.

Thus one of the main messages of our paper is the realization that the MLFV framework in the presence of heavy right-handed neutrinos and leptogenesis is clearly not as predictive as MFV in the quark sector. This is also related to the fact that new physics, even lepton conserving one, that could be present between energy scales M_Z and M_ν , could have in principle an important impact on various observables, like $B(l_i \rightarrow l_j \gamma)$, through RGE.

In the advanced stages of our project a paper by Cirigliano, Isidori and Porretti [13] appeared, in which the idea of the incorporation of leptogenesis into the MLFV framework has been put forward in the literature for the first time and its implications for charged LFV processes have been analyzed in detail. While in agreement with the general predictions of [13] we find that successful leptogenesis is possible for high scales $M_\nu \geq 10^{12} \text{ GeV}$, and contrary to that paper our detailed numerical analysis demonstrates that this is also true for much lower scales, weakening the implications for charged LFV processes found in that paper. Most importantly we do not confirm the lower bound of 10^{12} GeV for M_ν found by these authors which has significant implications for charged LFV processes as stressed above. The inclusion of flavour effects in the leptogenesis in our paper, that has been left out in [13] and the use of approximate formulae in that paper as opposed to a full numerical analysis present here, brings in significant differences in these two analyses for $M_\nu \leq 10^{12} \text{ GeV}$. We will summarize the agreements and differences between [13] and us in section 5.5.

At this stage it is worth also mentioning that there may be other equally reasonable definitions of MLFV. In this paper, we will only consider a conservative generalization of the initial proposal for MLFV [10]. However, it is clear that one may have other well motivated but different proposals for MLFV. In particular one should keep in mind that within the seesaw mechanism neutrinos acquire a mass in a manner which differs significantly from the one in the quark sector. In fact it has been suggested [14] that the fact that neutrino masses

arise from the seesaw mechanism is the key point in understanding why leptonic mixing is large, in contrast with small quark mixing. Therefore a reasonable definition of MLFV may differ from MFV in the quark sector. In our opinion, only in the presence of a theory of lepton flavour, where Yukawa couplings would be constrained by family symmetries, can one define in a unique way what MLFV should be. The question of different definitions of MLFV has been recently addressed in an interesting paper by Davidson and Palorini [15].

Our analysis involves several points and it is useful to list them one by one already at this stage.

- As already stated above, in the framework of the MLFV the right-handed heavy neutrinos are assumed to be degenerate in mass at some high energy scale in order to exclude possible new sources of flavour violation. However, the exact degeneracy of M_ν^i is not RG invariant and can only be true at a single scale which we choose to be $\Lambda_{\text{GUT}} = \mathcal{O}(10^{16} \text{ GeV})$. RGE between Λ_{GUT} and the see-saw scale $M_\nu \ll \Lambda_{\text{GUT}}$ break the degeneracy between M_ν^i at M_ν , a welcome result for leptogenesis that vanishes in the limit of degenerate M_ν^i . This structure is the basis of the so called *radiative leptogenesis* [16, 17] that has been first considered in the case of two degenerate neutrinos in [16–18]. An important ingredient of this framework is the *resonant leptogenesis* [19–21]. Therefore we will call this framework RRL as stated above. Our analysis is one of the first that considers the case of three degenerate neutrinos and includes flavour effects in RRL.
- As the values of the light neutrino masses and of the parameters of the PMNS mixing matrix [22], that enter the formulae for charged LFV processes, are not to be evaluated at the low energy scale but at the high energy scale M_ν , the MLFV relations between neutrino masses, mixing angles and rates for charged LFV processes presented in [10] can be in principle significantly modified through the RGE between the M_Z and M_ν scales, changing the conclusions about the value of the ratio $M_\nu/\Lambda_{\text{LFV}}$ necessary to obtain visible charged LFV rates. While it is conceivable that in certain MLFV scenarios RGE could be neglected, the example of the MSSM with a large $\tan\beta$, presented in this context in [23], shows that the RGE in question could in principle modify $B(l_i \rightarrow l_j \gamma)$ by a few orders of magnitude.
- The requirement of a successful BAU with the help of leptogenesis and in fact in general, necessarily brings into play CP violation. Neglecting flavour effects in the Boltzmann equations, the relevant CP violation is encoded in a complex orthogonal matrix R in the parameterization of Y_ν by Casas and Ibarra [24].

As analyzed already in several papers in the context of supersymmetric models, the size of the imaginary parts of R , crucial for generating the observed BAU in the framework of leptogenesis, can change the rates of charged LFV processes by several orders of magnitude. See, in particular [25], but also [23, 24, 26–28, 31, 34]. We note that when flavour effects are important, the generation of the BAU could be possible without complex phases in R [45, 46].

- The inclusion of CP violation at low energy with the help of the non-vanishing phase δ_{PMNS} [22] has only a moderate impact on the results in [10] but in the presence of a complex matrix R (see above) non-vanishing Majorana phases in the PMNS matrix can modify the results for LFV processes in [10] both directly and indirectly through RGE mentioned above. The numerical studies in [26, 27, 23] show that such effects can be in principle significant.

Our paper is organized as follows. In section 2 we present the generalization of the formulation of MLFV given in [10] by including low energy CP violation in the leptonic sector with the help of the PMNS matrix [22] and the high energy CP violation necessary for the leptogenesis of BAU. The parametrization of the neutrino Yukawa coupling Y_ν of Casas and Ibarra [24] turns out to be useful here.

In section 3 we analyze the issue of the breakdown of the mass degeneracy of heavy neutrinos by radiative corrections. This breakdown is necessary for leptogenesis to work even if R is complex. Assuming then this scale to be the grand unification (GUT) scale, we discuss the renormalization group equations in the SM and the MSSM used to generate the splitting of M_i at scales $\mathcal{O}(M_\nu)$, where the heavy neutrinos are integrated out. The results of this section are a very important ingredient of the leptogenesis that we consider in section 4 and in particular in 5.

In section 4 as a preparation for section 5, we present some numerical aspects of the flavour changing radiative charged lepton decays $l_i \rightarrow l_j \gamma$ and of the CP asymmetries in the right-handed neutrino decays.

In section 5, the most important section of our paper, we describe the scenario of radiative resonant leptogenesis in the case of three quasi-degenerate right-handed Majorana neutrinos. In this context we include in our analysis recently discussed flavour effects that definitely cannot be neglected. The main result of this paper is the demonstration that the right value of η_B can be obtained in this framework. A plot of η_B versus M_ν demonstrates very clearly that already for M_ν as low as 10^6 GeV leptogenesis becomes effective and that flavour effects are important. We compare our results with existing literature and explain why in contrast to [13] we do not find a stringent lower bound on M_ν .

Finally, we return to the $l_i \rightarrow l_j \gamma$ decays and use the knowledge collected in sections 3 and 5 to present a brief numerical analysis of $\mu \rightarrow e \gamma$ that illustrates the points made above. We restrict our analysis to $\tan \beta \leq 10$ so that RGE between M_Z and M_ν are small and other effects can be transparently seen.

In section 5.5 we compare our analysis and our results with [13]. We conclude in section 6.

2. Basic framework

2.1 Preliminaries

The discovery of neutrino oscillations provides evidence for non-vanishing neutrino masses and leptonic mixing, leading to lepton-flavour violation. In the SM, neutrinos are strictly massless, since Dirac masses cannot be constructed due to the absence of right-handed

neutrinos, and left-handed Majorana masses are not generated due to exact $(B - L)$ conservation.

The simplest extension of the SM which allows for non-vanishing but naturally small neutrino masses, consists of the addition of right-handed neutrinos to the spectrum of the SM. This extension has the nice feature of establishing on the one hand a lepton quark symmetry and on the other hand being naturally embedded in a grand unified theory like $SO(10)$. Since right-handed neutrinos are singlets under $U(1) \times SU(2) \times SU(3)$, Majorana neutrino masses M_R should be included, with a mass scale M_ν which can be much larger than the scale v of the electroweak symmetry breaking. Apart from M_R , Dirac neutrino mass terms m_D are generated through leptonic Yukawa couplings upon gauge symmetry breaking. The presence of these two neutrino mass terms leads, through the seesaw mechanism [11], to three light neutrinos with masses of order v^2/M_ν and three heavy neutrinos with mass of order M_ν . The decay of these heavy neutrinos can play a crucial role in the creation of a baryon asymmetry of the universe (BAU) through the elegant mechanism of baryogenesis through leptogenesis [8, 9]. In the presence of neutrino masses and mixing, one has, in general, both CP violation at low energies which can be detected through neutrino oscillations and CP violation at high energies which is an essential ingredient of leptogenesis. The connection between these two manifestations of CP violation can be established in the framework of specific lepton flavour models.

In this paper, we study lepton-flavour violation in this extension of the SM, assuming minimal lepton flavour violation (MLFV) but allowing for CP violation both at low and high energies. The case of no leptonic CP violation either at low or high energies, was considered in [10] where the suggestion of MLFV was first presented. The first discussion of CP violation at low and high energy in a MLFV framework has been presented recently in [13]. We will compare the results of this paper with ours in section 5.5

2.2 Yukawa couplings and Majorana mass terms

We add then three right-handed neutrinos to the spectrum of the SM and consider the following leptonic Yukawa couplings and right-handed Majorana mass terms:

$$\mathcal{L}_Y = -\bar{e}_R Y_E \phi^\dagger L_L - \bar{\nu}_R Y_\nu \tilde{\phi} L_L + h.c. \tag{2.1}$$

$$\mathcal{L}_M = -\frac{1}{2} \bar{\nu}_R^c M_R \nu_R + h.c., \tag{2.2}$$

where Y_E , Y_ν and M_R are 3×3 matrices in the lepton flavour space. In the limit $\mathcal{L}_Y = \mathcal{L}_M = 0$ the Lagrangian of this minimal extension of the SM has a large flavour symmetry

$$SU(3)_L \times SU(3)_E \times SU(3)_{\nu_R} \times U(1)_L \times U(1)_E \times U(1)_{\nu_R}, \tag{2.3}$$

which reflects the fact that gauge interactions treat all flavours on equal footing. This large global symmetry is broken by the Yukawa couplings Y_E , Y_ν and by the Majorana mass terms M_R . A transformation of the lepton fields:

$$L_L \rightarrow V_L L_L, \quad e_R \rightarrow V_E e_R, \quad \nu_R \rightarrow V_{\nu_R} \nu_R \tag{2.4}$$

leaves the full Lagrangian invariant, provided the Yukawa couplings and the Majorana mass terms transform as:

$$Y_\nu \rightarrow Y'_\nu = V_{\nu R} Y_\nu V_L^\dagger, \tag{2.5}$$

$$Y_E \rightarrow Y'_E = V_E Y_E V_L^\dagger, \tag{2.6}$$

$$M_R \rightarrow M'_R = V_{\nu R}^* M_R V_{\nu R}^T, \tag{2.7}$$

which means that there is a large equivalent class of Yukawa coupling matrices and Majorana mass terms, related through (2.5)–(2.7), which have the same physical content. The MLFV proposal [10] consists of the assumption that the physics which generates lepton number violation, leading to M_R , is lepton flavour blind, thus leading to an exactly degenerate eigenvalue spectrum for M_R , at a high-energy scale. As a result, in the MLFV framework, the Majorana mass terms break $SU(3)_{\nu R}$ into $O(3)_{\nu R}$. It is clear that, due to its Majorana nature, M_R is not a weak-basis invariant, even in the limit of exact degeneracy. Our motivation for the above definition of MLFV is to guarantee that the Majorana neutrino mass term is invariant under the maximal possible subgroups of the flavour symmetry $SU(3)_{\nu R}$. This leads to $O(3)_{\nu R}$ which is respected if the Majorana neutrino mass term is proportional to identity at a given scale, which can be chosen at the GUT scale.

2.3 Leptonic masses, mixing and CP violations

Without loss of generality, one can choose a basis for the leptonic fields, where Y_E and M_R are diagonal and real. In this basis, the neutrino Dirac mass matrix $m_D = vY_\nu$ is an arbitrary complex matrix, therefore with nine moduli and nine phases. Three of these phases can be eliminated by a rephasing of L_L . One is then left with six CP violating phases. There are various classes of phenomena which depend on different combinations m_D , m_D^T , m_D^\dagger or equivalently Y_ν , Y_ν^T and Y_ν^\dagger :

A) Leptonic mixing and CP violation at low energies: Since we are working in the basis where the charged lepton mass matrix is diagonal and real, leptonic mixing and CP violation at low energies are controlled by the PMNS matrix U_ν [22], which diagonalizes the effective low energy neutrino mass matrix:

$$U_\nu^T (m_\nu)_{\text{eff}} U_\nu = d_\nu, \tag{2.8}$$

where $d_\nu \equiv \text{diag}(m_1, m_2, m_3)$, with m_i being the masses of the light neutrinos and [11]

$$(m_\nu)_{\text{eff}} = -v^2 Y_\nu^T D_R^{-1} Y_\nu, \tag{2.9}$$

where D_R denotes the diagonal matrix M_R and $v = 174$ GeV. In the case of MLFV, $D_R = M_\nu \mathbb{1}$ and one obtains at $M_\nu \approx \Lambda_{\text{LN}}$

$$(m_\nu)_{\text{eff}} = -\frac{v^2}{M_\nu} Y_\nu^T Y_\nu. \tag{2.10}$$

Consequently $Y_\nu^T Y_\nu$ is the quantity that matters here.

B) Lepton flavour violation: The charged LFV depends on the other hand on the combination $Y_\nu^\dagger Y_\nu$ with Y_ν again normalized at the high energy scale M_ν . We will return to this point in section 4.

C) CP violation relevant for leptogenesis: The generation of BAU through leptogenesis starts by the production of a lepton asymmetry which is proportional to the CP asymmetry in the decays of heavy Majorana neutrinos. This CP asymmetry involves the interference between the tree-level amplitude and the one-loop vertex and self-energy contributions. It has been shown [30] that the CP asymmetry depends on the neutrino Yukawa couplings through the combination $Y_\nu Y_\nu^\dagger$. Again as in classes A and B, Y_ν is evaluated here at the scale M_ν . When flavour effects in the Boltzmann equations become important, the non-summed products $(Y_\nu)_{ik}(Y_\nu)_{jk}^*$ corresponding to different lepton flavours k can attain relevance.

2.4 An useful parametrization

In order to analyze in a systematic way the above phenomena and study the implied relations among low-energy lepton mixing data, lepton flavour violation and leptogenesis in different scenarios classified below, it is convenient to choose an appropriate parametrization for Y_ν . We use the following parametrization [24] of the neutrino Yukawa couplings:

$$(\sqrt{D_R})^{-1} Y_\nu = \frac{i}{v} R \sqrt{d_\nu} U_\nu^\dagger, \tag{2.11}$$

where R is an orthogonal complex matrix ($R^T R = R R^T = \mathbb{1}$), $d_\nu = \text{diag}(m_1, m_2, m_3)$ and $D_R = \text{diag}(M_1, M_2, M_3)$.

It is instructive to count next the number of independent parameters on both sides of (2.11). The left-hand side of (2.11) is an arbitrary 3×3 complex matrix with nine real parameters and six phases, since three of the initial nine phases can be removed by rephasing L_L . It is clear that the right-hand side of (2.11) also has nine real parameters and six phases. Indeed, R , d_ν and U_ν have each three real parameters and moreover R and U_ν have in addition each three phases. We consider now the case where the right-handed neutrinos are exactly degenerate, i.e. $D_R = M_\nu \mathbb{1}$. We will show that three of the real parameters of R can be rotated away. Note that any complex orthogonal matrix can be parametrized as

$$R = e^{A_1} e^{iA_2}, \tag{2.12}$$

with $A_{1,2}$ real and skew symmetric. Now in the degenerate case an orthogonal rotation of $\nu_R \rightarrow O_R \nu_R$ leaves the Majorana mass proportional to the unit matrix and defines a physically equivalent reparametrization of the fields ν_R . Choosing $O_R = e^{A_1}$ we see immediately that

$$Y_\nu \rightarrow O_R^\dagger Y_\nu = \frac{\sqrt{M_\nu}}{v} e^{-A_1} R \sqrt{d_\nu} U_\nu^\dagger = \frac{\sqrt{M_\nu}}{v} e^{iA_2} \sqrt{d_\nu} U_\nu^\dagger, \tag{2.13}$$

which shows that the physically relevant parameterization is given by $R_{\text{deg}} = e^{iA_2}$.

Using the parameterization in (2.11) one finds that the matrix $Y_\nu^T Y_\nu$ which controls low-energy CP-Violation and mixing can be written as follows

$$Y_\nu^T Y_\nu = -\frac{1}{v^2} (U_\nu^\dagger)^T \sqrt{d_\nu} R^T D_R R \sqrt{d_\nu} U_\nu^\dagger = -\frac{M_\nu}{v^2} (U_\nu^\dagger)^T d_\nu U_\nu^\dagger, \quad (2.14)$$

where in the last step we have set $D_R = M_\nu \mathbb{1}$.

On the other hand, the matrix $Y_\nu^\dagger Y_\nu$ which controls charged LFV, can be written as follows (see also [31])

$$Y_\nu^\dagger Y_\nu = \frac{1}{v^2} U_\nu \sqrt{d_\nu} R^\dagger D_R R \sqrt{d_\nu} U_\nu^\dagger = \frac{M_\nu}{v^2} U_\nu \sqrt{d_\nu} R^\dagger R \sqrt{d_\nu} U_\nu^\dagger. \quad (2.15)$$

Finally, the matrix $Y_\nu Y_\nu^\dagger$ which enters in leptogenesis when flavor effects are not relevant is given by (see also [31]):

$$Y_\nu Y_\nu^\dagger = \frac{1}{v^2} \sqrt{D_R} R d_\nu R^\dagger \sqrt{D_R} = \frac{M_\nu}{v^2} R d_\nu R^\dagger. \quad (2.16)$$

We note that $Y_\nu^T Y_\nu$ depends only on U_ν and d_ν , while $Y_\nu Y_\nu^\dagger$ relevant for the leptogenesis only on d_ν and R . This means that CP violation at low energy originating in the complex U_ν and the CP violation relevant for leptogenesis are then decoupled from each other and only the mass spectrum of light neutrinos summarized by d_ν enters both phenomena in a universal way.

In this respect the charged LFV, represented by (2.15), appears also interesting as it depends on d_ν , U_ν and R and consequently can also provide an indirect link between low energy and high energy CP violations and generally a link between low and high energy phenomena.

2.5 Classification

Having the parametrization of Y_ν in (2.11) at hand we can now spell the difference between the analysis of [10] and ours in explicit terms. Indeed, from the above considerations, it follows that possible relations among phenomena A,B,C, discussed in section 2.3, crucially depend on the assumptions one makes about leptonic CP violation at low energies, as well as at high energies. One may consider then separately the following four scenarios:

- **Case 1:** No leptonic CP violation either at low or high energies. The limit that all complex phases vanish leads to

$$-Y_\nu^T Y_\nu = Y_\nu^\dagger Y_\nu. \quad (2.17)$$

This is the case considered in [10], where a close connection is obtained between experimental low energy data on lepton mixing and the pattern of various charged LFV processes, that is the correlation between phenomena A and B in the absence of CP violation. It corresponds to choosing R and U_ν real. However, even in this case the RGE between the low energy scale at which the light neutrino masses and mixings are measured and the scale M_ν at which Y_ν is evaluated could have an impact on the correlation in question.

- **Case 2:** Leptonic CP violation at low energies, but no CP violation relevant for leptogenesis (barring flavour effects). This corresponds to assuming that the leptonic mixing matrix U_ν contains CP violating phases so that $Y_\nu^T Y_\nu$ is complex, but $Y_\nu Y_\nu^\dagger$ is real or equivalently as seen in (2.16) R is real.
- **Case 3:** CP violation relevant for leptogenesis but no low energy leptonic CP violation. This corresponds to having $Y_\nu Y_\nu^\dagger$ and R complex, but U_ν real.
- **Case 4:** There is leptonic CP violation both at low and high energies, that is both U_ν and R are complex quantities. It should be stressed that Case 4 is of course the general case and, in fact, the most “natural” one, since once CP is violated by the Lagrangian the six CP violating phases contained in m_D lead in general to CP violation both at low and high energies.

2.6 Final remarks

It is clear that (2.15), depending on U_ν , R , d_ν and M_ν enables one to analyze separately the four cases considered here. In each case there will be simultaneously implications for lepton flavour violations, leptogenesis and low energy CP violation and mixing with certain correlations between them. These correlations can be affected by RGE between the low energy scale and M_ν .

At this stage the following comments are in order:

- U_ν is relatively well known from oscillation experiments with the exception of s_{13} , the phase δ and the Majorana phases α and β . In order to use it for the calculation of Y_ν it has to be evolved by RG equations to M_ν .
- With the measured two mass differences squared from solar and atmospheric oscillation data, the diagonal matrix d_ν is a function of a single parameter that we choose to be the mass of the lightest neutrino. Again these parameters have to be evaluated at the scale M_ν with the help of renormalization group techniques.
- The matrix R depends on three complex parameters that influence simultaneously lepton flavour violation and leptogenesis as seen in (2.15) and (2.16), respectively. Some constraints on R can then be obtained from these two phenomena but a complete determination of this matrix is only possible in an underlying theory represented usually by special texture zeros of Y_ν .
- Finally, M_ν can be restricted from the BAU in the context of the seesaw mechanism and if the eigenvalues of the right-handed neutrino matrix D_R are hierarchical, the absolute lower bound on the lowest M_i is $\mathcal{O}(10^8)$ or even higher. In the case of the MLFV considered here the right-handed heavy neutrinos have to be quasi-degenerate in order to avoid new flavour violating interactions. In this case BAU can be explained with the help of RRL which combines the resonant leptogenesis considered in [20, 21] and radiative leptogenesis [16–18]. The lower bound on M_ν can be significantly lowered in this case, as we will see explicitly below.

3. Radiative corrections in MLFV

3.1 Preliminaries

Our MLFV scenario defined in the previous section contains no free parameters beyond the neutrino masses, the PMNS matrix, a matrix of form R_{deg} , an initial, universal heavy Majorana neutrino mass, and perhaps additional flavour-blind parameters that depend on the MLFV model. The rates for charged lepton flavour violation thus follow upon computing radiative corrections due to the degrees of freedom between the scales M_Z and Λ_{GUT} , and with suitable washout factors also the baryon asymmetry η_B .

In this section we investigate how the CP- and flavour-violating quantities relevant to leptogenesis and charged lepton flavour violation, respectively, are radiatively generated. Since leptogenesis in the present framework can be considered as a generalization of the setup with two heavy singlets in [18] to the case of three degenerate flavours, we will also clarify what novelties arise in this case. This will be important in comparing our results to the existing literature.

An important point will be that, due to the hierarchy between the GUT/flavour-breaking scale Λ_{GUT} and the neutrino mass scale M_ν , large logarithms appear such that the parameter counting for the coefficients c_i of flavour structures that has been recently presented in [13] should be modified. Rather than being independent, the coefficients of structures containing different powers of Yukawa matrices are related by the renormalization group, while any additional independent effects are suppressed. Although this fact in principle increases the predictivity of MLFV, in our phenomenological sections it will still turn out insufficient to have correlations between high-scale and weak-scale observables.

3.2 MLFV with a degeneracy scale

We have defined our MLFV scenario to have a scale at which the masses of the right-handed neutrinos are exactly degenerate, such that the matrix M_R has no flavour structure at all. In general, there will be additional flavoured particles in the theory. As a specific example, we consider the MSSM. Here the N_i are accompanied by heavy sneutrinos \tilde{N}_i^c , and there are also SU(2) doublet sleptons \tilde{l}_i , transforming as

$$\tilde{l} \rightarrow V_L \tilde{l}, \quad \tilde{N}^c \rightarrow V_{\nu_R}^* \tilde{N}^c \tag{3.1}$$

under the transformation (2.4). The Lagrangian then contains soft SUSY breaking terms

$$\mathcal{L}_{\text{soft}} = -\tilde{N}_i^{c*} \tilde{m}_{\nu_{ij}}^2 \tilde{N}_j^c - \tilde{l}_i^* \tilde{m}_{l_{ij}}^2 \tilde{l}_j + \dots, \tag{3.2}$$

where the ellipsis denotes further scalar mass matrices and trilinear scalar interactions. In general all matrices in $\mathcal{L}_{\text{soft}}$ have non-minimal flavour structure. The simplest generalization of our degenerate scenario is then to extend the requirement of exact degeneracy to all mass matrices, similar to minimal supergravity. To be specific, we require all scalar masses to have the same value m_0 at the high scale and also require the A -terms to have the mSUGRA form $A = aY$ with Y the corresponding Yukawa matrix and a a universal, real parameter of the theory. This example also provides us with a concrete value for the scale

Λ_{LFV} : LFV processes such as $l_i \rightarrow l_j \gamma$ are mediated by loop diagrams involving sleptons and higgsinos or (weak) gauginos, and unless gaugino masses are very large, the scalar particles such as \tilde{l}_i decouple at a scale $\Lambda \sim m_0$. Hence the operators governing charged LFV are suppressed by powers of $m_0 \equiv \Lambda_{\text{LFV}}$. As in the case of the heavy Majorana masses, the generalized degeneracy requirement is not stable under radiative corrections, and for the same reason it is not renormalization scheme independent.

3.3 Radiatively generated flavour structure and large logarithms

As will be discussed in detail in the following section, the CP asymmetries necessary for leptogenesis require mass splittings between the decaying particles. The decaying particles are on their mass shell,² but the degenerate initial conditions are usually specified in a massless scheme³ ($\overline{\text{MS}}$ to be definite [32]).

At one loop, the two mass definitions are related by a formula of the structure

$$M_i^{os} = M_i^{\overline{\text{MS}}}(\mu) + c_i M_i^{\overline{\text{MS}}}(\mu) \ln \frac{M_i}{\mu} + \text{nonlogarithmic corrections}, \quad (3.3)$$

where $\mu \sim \Lambda_{\text{GUT}}$ is the $\overline{\text{MS}}$ renormalization scale, $c_i = 2(Y_\nu Y_\nu^\dagger)_{ii}/(16\pi^2)$ in the standard-model seesaw, and the nonlogarithmic corrections depend on our choice of massless (or any other) renormalization scheme. The resulting scheme dependence cannot be present in physical observables such as the BAU. Since this issue is usually not discussed in the literature on lepton flavour violation, let us elaborate on how it may be resolved.

First, notice that while the nonlogarithmic terms in (3.3) are scheme dependent, the logarithmic corrections proportional to c_i are actually scheme independent. If $\ln \Lambda_{\text{GUT}}/M_\nu \gg 1$, the logarithmic terms must be considered $\mathcal{O}(1)$ and summed to all orders. This is achieved in practice by solving renormalization group equations. Similar resummations must be performed for all other parameters in the theory (such as Yukawa couplings). Correspondingly, the dominant higher-loop corrections to LFV observables and leptogenesis are approximated by using leading-order expressions with one-loop RGE-improved Yukawa couplings and masses. This is the leading-logarithmic approximation. Nonlogarithmic corrections such as those indicated in (3.3) are then sub-leading and should be dropped.

What happens when the logarithms are not large is the following. If the MLFV framework is an effective theory for some fundamental theory where the degeneracy is enforced by a flavour symmetry, for instance the group (2.3), then the degeneracy holds in *any* scheme (that respects the symmetry) in the full theory and the scheme dependence observed in (3.3) must be due to unknown threshold corrections in matching the underlying and effective theories. Since the flavour symmetry in MLFV, by definition, is broken precisely by the Yukawa matrices, this matching introduces all possible terms that are invariant

²We follow the treatment of [20, 21] (see also [42]), where sometimes the on-shell masses are replaced by thermal masses. (We will employ zero-temperature masses.)

³This is likely appropriate if the degeneracy is true to some flavour symmetry of an underlying theory, relating high-energy Lagrangian parameters and broken at the scale Λ_{GUT} .

under transformations (2.4), (2.5), (2.6), (2.7). A list of such structures has recently been given in [13], for instance,

$$M_R = M_\nu \left[1 + c_1(Y_\nu Y_\nu^\dagger + (Y_\nu Y_\nu^\dagger)^T) + c_2(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)^T) + \dots \right]. \quad (3.4)$$

The coefficients c_1 and c_2 have been claimed by these authors to be independent $\mathcal{O}(1)$ coefficients. Indeed these terms contain only non-logarithmic terms and (small) decoupling logs when M_R is taken in the $\overline{\text{MS}}$ scheme, renormalized near the GUT (matching) scale.

However, when computing the (physically relevant) on-shell M_R in the case of $\Lambda_{\text{GUT}} \gg M_\nu$, large logarithms dominate both c_1 and c_2 . The leading logarithmic contributions are not independent, but are related by the renormalization group. c_2 is quadratic in $L \equiv \ln \Lambda_{\text{GUT}}/M_\nu$, while c_1 is linear, and the RGE for M_R implies $c_2|_{L^2} = \frac{1}{2}[c_1|_L]^2$. These logs are summed by RG-evolving $M_R^{\overline{\text{MS}}}$ to a scale $\mu \sim M_\nu$. The additional conversion to on-shell masses is then again a sub-leading correction.

Finally, we note that if there is no underlying symmetry, the degeneracy condition can again be true at most for special choices of scheme/scale, and must be fine-tuned.

Numerically, the logarithms dominate already for mild hierarchies $\Lambda_{\text{GUT}}/M_\nu > 10^2$, as then $2 \ln \Lambda_{\text{GUT}}/M_\nu \approx 10$. Let us now restrict ourselves to hierarchies of at least two orders of magnitude and work consistently in the leading-logarithmic approximation. As explained above, in this case non-logarithmic corrections both of the threshold type (in the coefficients c_i in (3.4) and in physical quantities (on-shell masses, CP asymmetries, etc.) are sub-leading and should be dropped. In this regard our apparently “special” framework of initially degenerate heavy neutrinos turns out to be the correct choice at leading-logarithmic order.

Finally we recall that the positions of the poles of the N_i two-point functions contain an imaginary part related to the widths of these particles. While not logarithmically enhanced, these are also scheme-independent at one loop (as the widths are physical), and it is unambiguous to include them in applications. In fact, these widths effects are often numerically important for the CP asymmetries in N_i decay [20, 21], and we will keep them in our numerical analysis.

3.4 Renormalization-group evolution: high scales

For the running above the seesaw scale the relevant renormalization-group equations have been given in in [51] (in particular, last paper) for the SM and MSSM seesaw models. As the physical quantities studied below, such as leptonic CP asymmetries, involve mass eigenstates, it is convenient to keep the singlet mass matrix diagonal during evolution (see, e.g., appendix B of [52]):

$$M_R(\mu) = \text{diag}(M_1(\mu), M_2(\mu), M_3(\mu)).$$

Defining

$$H = Y_\nu Y_\nu^\dagger, \quad (3.5)$$

and

$$t = \frac{1}{16\pi^2} \ln(\mu/\Lambda_{\text{GUT}}), \quad (3.6)$$

one obtains for the mass eigenvalues in the SM with right handed neutrinos:

$$\frac{dM_i}{dt} = 2H_{ii} M_i \quad (\text{no sum}). \quad (3.7)$$

Note that due to the positivity of the right-hand side of (3.7), the running will always decrease the masses when running from the GUT to the seesaw scale.

The matrix H satisfies the RGEs

$$\frac{dH}{dt} = [T, H] + 3H^2 - 3Y_\nu Y_E^\dagger Y_E Y_\nu^\dagger + 2\alpha H \quad (\text{SM}), \quad (3.8)$$

$$\frac{dH}{dt} = [T, H] + 6H^2 + 2Y_\nu Y_E^\dagger Y_E Y_\nu^\dagger + 2\alpha H \quad (\text{MSSM}), \quad (3.9)$$

where

$$\alpha = \text{Tr}(Y_\nu^\dagger Y_\nu) + \text{Tr}(Y_E^\dagger Y_E) + 3\text{Tr}(Y_u^\dagger Y_u) + 3\text{Tr}(Y_d^\dagger Y_d) - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2 \quad (\text{SM}), \quad (3.10)$$

$$\alpha = \text{Tr}(Y_\nu^\dagger Y_\nu) + 3\text{Tr}(Y_u^\dagger Y_u) - \frac{3}{5}g_1^2 - 3g_2^2 \quad (\text{MSSM}), \quad (3.11)$$

$$T_{ij} = \begin{cases} -\frac{M_j+M_i}{M_j-M_i}\text{Re}H_{ij} - i\frac{M_j-M_i}{M_j+M_i}\text{Im}H_{ij} & (i \neq j, \quad \text{SM}), \\ -2\frac{M_j+M_i}{M_j-M_i}\text{Re}H_{ij} - 2i\frac{M_j-M_i}{M_j+M_i}\text{Im}H_{ij} & (i \neq j, \quad \text{MSSM}), \\ 0 & (i = j), \end{cases} \quad (3.12)$$

and GUT normalization has been employed for g_1 . The matrix T satisfies $\dot{U} = TU$, where $M_R^{(0)}(\mu) = U(\mu)^T M_R(\mu) U(\mu)$ and $M_R^{(0)}$ satisfies the unconstrained RGEs given in [51]. Note that α is real and has trivial flavour structure. Note the different relative signs in (3.8) and (3.9); we will return to this point below.

We now turn to a qualitative analysis of these equations and their impact on leptogenesis and flavour violation. Ignoring flavour effects in the Boltzmann evolution of charged leptons, the baryon asymmetry η_B is approximately proportional to the combinations $\text{Im}((H_{ij})^2) = 2\text{Re}H_{ij}\text{Im}H_{ij}$ ($i \neq j$), evaluated in the mass eigenbasis. At the scale Λ_{GUT} , degeneracy of M_R allows the use of an $\text{SO}(3)$ transformation to make the off-diagonal elements of $\text{Re}H$ vanish.⁴ As explained above, we should RG-evolve all parameters to the scale $\mu \sim M_\nu$ to avoid large logarithms. Let us first consider the formal limit of vanishing charged lepton Yukawa couplings Y_E for the SM case. It is instructive to split (3.8) into real and imaginary parts. The former satisfies

$$\frac{d\text{Re}H}{dt} = [\text{Re}T, \text{Re}H] - [\text{Im}T, \text{Im}H] + 3\left\{(\text{Re}H)^2 - (\text{Im}H)^2\right\} + 2\alpha\text{Re}H. \quad (3.13)$$

To investigate how a nondiagonal $\text{Re}H$ can be generated radiatively, assume that it is zero at some scale (initial or lower). Then (3.13) reduces to

$$\frac{d\text{Re}H}{dt} = -[\text{Im}T, \text{Im}H] - 3(\text{Im}H)^2. \quad (3.14)$$

⁴To see this, notice that H is hermitian, so $\text{Re}H$ is real symmetric. That is, it can be diagonalized by a real orthogonal (and hence unitary) transformation of the right-handed neutrinos. Now if all three neutrinos are degenerate, such a rotation affects no term in the Lagrangian besides Y_ν .

(At $t = 0$, an extra term proportional to the offdiagonal part of $(\text{Im}H)^2$ appears on the right-hand side of (3.14).) Now evaluate this for the $(2, 1)$ element and notice that $T_{ij} = 0$ and $\text{Im}H_{ij} = 0$ for $i = j$. If there were only two heavy singlets in the theory, each term in each matrix product would require one $(2, 1)$ element and one $(1, 1)$ or $(2, 2)$ element from the two matrix factors. For example,

$$(\text{Im}T \text{Im}H)_{21} = \text{Im}T_{21} \underbrace{\text{Im}H_{11}}_0 + \underbrace{\text{Im}T_{22}}_0 \text{Im}H_{21} = 0, \quad (3.15)$$

and similarly for the other terms. Consequently,

$$\text{Re}H_{21} = 0 \Rightarrow \frac{d\text{Re}H_{21}}{dt} = 0. \quad (3.16)$$

We see that there is no radiative leptogenesis in the two-flavour case when $Y_E = 0$. This is consistent with the approximate equation (12) in [18], where $\text{Re}H_{21}$ was found to be proportional to y_τ^2 . It is easy to see that the argument breaks down in the three-flavour case. For instance,

$$((\text{Im}H)^2)_{21} = \text{Im}H_{21}\text{Im}H_{11} + \text{Im}H_{22}\text{Im}H_{21} + \text{Im}H_{23}\text{Im}H_{31} = \text{Im}H_{23}\text{Im}H_{31}, \quad (3.17)$$

which is in general not zero. The other terms in (3.14) are also proportional to $\text{Im}H_{23}\text{Im}H_{31}$. We see that three generations of heavy neutrinos are necessary and sufficient to generate leptogenesis without help from charged lepton Yukawas.

Once we restore the charged lepton Yukawas, they will also contribute. The important qualitative difference is that, whereas the contribution involving the charged-lepton Yukawas is only logarithmically dependent of the seesaw scale (as seen in eqs. (5.10)–(5.12) below for the two-flavour case, or from [21] for the three-flavour case), the pure Y_ν contribution to the radiatively generated $\text{Re}H_{ij}$ scales with M_ν because it contains two extra powers of Y_ν as observed in the three flavour scenario studied in [13].

In summary, we expect the following qualitative behavior for the BAU as a function of M_ν :

- For small Y_ν (small M_ν), the dominant contribution to $\text{Re}H_{ij}$ and hence to η_B should be due to Y_E . η_B turns out to be weakly dependent on M_ν .
- For large Y_ν (large M_ν), in the three-flavour case there is a relevant contribution proportional to $((\text{Im}H)^2)_{ij}$. Since it contains two extra powers of Y_ν with respect to the contribution proportional to y_τ^2 , η_B scales linearly with M_ν .
- In the case of only two heavy flavours, η_B is weakly dependent on M_ν over the whole range of M_ν . We will therefore include an “effective” two-flavour scenario in our numerical analysis.

Let us stress that we reached these qualitative conclusions only upon neglecting flavour effects in the Boltzmann evolution of the products of the N_i decays. We will return to these points in section 4 and in section 5, where we perform a detailed quantitative analysis.

Finally, let us briefly discuss $l_i \rightarrow l_j \gamma$. In MLFV these radiative lepton decays are governed by $\Delta_{ij} \equiv Y_\nu^\dagger Y_\nu$ (and structures involving more powers of Yukawa matrices). In the case of the SM, the rates are known to be essentially zero due to a near perfect GIM cancellation among the tiny neutrino masses. From the point of view of MLFV, this smallness can be traced to the fact that, in the SM, the LFV scale is equal to the LNV scale $\sim M_\nu$.

On the other hand, in the more generic case of the MSSM, there are additional contributions mediated by slepton-higgsino or slepton-gaugino loops suppressed only by a scale $\Lambda_{\text{LFV}} \sim m_{\tilde{t}}$, of order TeV, as discussed in section 3.2. Linearizing the RG evolution, the charged slepton soft mass matrix acquires the form [50]

$$\tilde{m}_l^2(M_\nu) = m_0^2 \mathbf{1} - L \frac{Y_\nu^\dagger Y_\nu}{16\pi^2} (6m_0^2 + 2a_0^2) + \dots, \quad (3.18)$$

where the dots denote terms governed by charged lepton Yukawa couplings Y_E or conserving lepton flavour. Note that the flavour structure in the soft terms is generated at a high scale and that, unlike the case of CP asymmetries in N_i decay, the necessary flavour structure Δ is already present at the initial scale Λ_{GUT} . Hence the RGE running of Δ merely gives a correction. Note also that there is dependence on the MLFV model beyond the choice of LFV scale due to the (in general unknown) RGE coefficients in (the relevant analog of) (3.18).

3.5 RGE evolution below M_ν : PMNS matrix and Δ_{ij}

So far we have ignored renormalization effects in equations such as (2.11), identifying d_ν and U_ν with the physical (light) neutrino masses and mixing matrix, while the objects Y_ν and D_R are defined at a high scale. However, to be orthogonal the matrix R has to be defined with all objects given at the same scale. Now it is well known that using low-energy inputs in d_ν can be a bad approximation because there are significant radiative corrections between the weak and GUT scales. However, as investigated in [52], both in the SM and in the MSSM with small $\tan\beta$ the main effect below M_ν is an approximately universal rescaling of the light neutrino masses. This results in larger magnitudes of the elements of Y_ν extracted by means of (2.11) but in a weak running of the matrix U_ν . Above the scale M_ν , even though the heavy singlets are now dynamical, one can still *define* an effective neutrino mass matrix through the seesaw relation (2.9). However, the evolution becomes more involved, as in the presence of heavy singlets there are additional contributions to the running involving Y_ν . To deal with this situation, where some of our inputs are specified at the weak scale, while the matrix R_{deg} is defined at the scale Λ_{GUT} , we employ an iterative procedure detailed in appendix A. As was the case for the evolution above M_ν , also the RGE effects below M_ν , and consequently the relation of e.g $Y_\nu^\dagger Y_\nu$ to the input parameters necessarily depends on the details of the MLFV model.

4. Numerical analysis: $B(l_i \rightarrow l_j \gamma)$ and CP asymmetries in ν_R decay

4.1 Preliminaries

For our numerical analysis we take our input parameters at the weak scale, except for the

matrix R_{deg} , which has to be defined at the scale Λ_{GUT} . From these inputs we find a consistent set of parameters at the seesaw scale M_ν , where the CP asymmetries as well as $B(l_i \rightarrow l_j \gamma)$ are calculated, through the iterative procedure given in appendix A. For the running we use the package REAP [53].

For the PMNS matrix we use the convention:

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot V \quad (4.1)$$

and $V = \text{Diag}(e^{i\alpha/2}, e^{i\beta/2}, 1)$ where α and β denote the Majorana phases and δ denotes the Dirac phase. We parameterize the complex orthogonal matrix R as follows:

$$R = \begin{pmatrix} \hat{c}_{12} & \hat{s}_{12} & 0 \\ -\hat{s}_{12} & \hat{c}_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \hat{c}_{23} & \hat{s}_{23} \\ 0 & -\hat{s}_{23} & \hat{c}_{23} \end{pmatrix} \begin{pmatrix} \hat{c}_{13} & 0 & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}_{13} & 0 & \hat{c}_{13} \end{pmatrix}, \quad (4.2)$$

with $\hat{s}_{ij} \equiv \sin \hat{\theta}_{ij}$, with $\hat{\theta}_{ij}$ in general complex:

$$\hat{\theta}_{ij} = x_{ij} + i y_{ij}. \quad (4.3)$$

In the degenerate case, the angles x_{ij} can be made to vanish by a redefinition of the right-handed neutrinos, i.e. a matrix of the form R_{deg} is parameterized by three real numbers y_{ij} .

In the following, we use maximal atmospheric mixing $c_{23} = s_{23} = 1/\sqrt{2}$ and a solar mixing angle $\theta_{\text{sol}} = 33^\circ$, with corresponding values for its sine $s \equiv s_{12}$ and cosine $c \equiv c_{12}$. For the sine of the CHOOZ angle s_{13} and the phases we allow the ranges

$$0 \leq s_{13} \leq 0.25, \quad 0 < \alpha, \beta, \delta < 2\pi, \quad (4.4)$$

and for the light neutrinos we use the low energy values

$$\Delta m_{\text{sol}}^2 = m_2^2 - m_1^2 = 8.0 \cdot 10^{-5} \text{ eV}^2 \quad (4.5)$$

$$\Delta m_{\text{atm}}^2 = |m_3^2 - m_2^2| = 2.5 \cdot 10^{-3} \text{ eV}^2 \quad (4.6)$$

$$0 \leq m_\nu^{\text{lightest}} \leq 0.2 \text{ eV} \quad (4.7)$$

with $m_\nu^{\text{lightest}} = m_1(m_3)$ for normal (inverted) hierarchy, respectively. See [33] for a detailed discussion of the neutrino masses and mixing. For the heavy neutrino mass scale, we consider a wide range

$$10^6 \text{ GeV} < M_\nu < 10^{14} \text{ GeV}, \quad (4.8)$$

and the CP violating parameters y_{ij} are all taken in the range $[-1, 1]$ if not otherwise stated.

4.2 Perturbativity bounds

In the MLFV framework the magnitudes of the Yukawa couplings Y_ν are very sensitive to the choice of M_ν , m_ν^{lightest} and the angles in the matrix R_{deg} , as is evident from (2.11). To render the framework perturbative, we impose the constraint

$$\frac{y_{\text{max}}^2}{4\pi} < 0.3, \tag{4.9}$$

where y_{max}^2 is the largest eigenvalue of $Y_\nu^\dagger Y_\nu$. By means of (2.15), it translates into a bound on $R^\dagger R = R^2$ and the angles y_{ij} that scales with M_ν^{-1} and hence is most severe for a large lepton-number-violating scale. Analogous bounds apply to other dimensionless couplings whose number depends on the precise MLFV model. For instance, in the SM there is also the Higgs self coupling λ_H , whereas in the MSSM there is no such additional coupling.

4.3 Lepton flavour violation and $l_i \rightarrow l_j \gamma$

Following Cirigliano *et al.* [10] we consider the normalized branching fractions defined as

$$B(l_i \rightarrow l_j \gamma) = \frac{\Gamma(l_i \rightarrow l_j \gamma)}{\Gamma(l_i \rightarrow l_j \nu_i \bar{\nu}_j)} \equiv r_{ij} \hat{B}(l_i \rightarrow l_j \gamma), \tag{4.10}$$

where $\hat{B}(l_i \rightarrow l_j \gamma)$ is the true branching ratio and $r_{\mu e} = 1.0$, $r_{\tau e} = 5.61$ and $r_{\tau \mu} = 5.76$. Assuming first the heavy right-handed neutrinos to be degenerate but not making the assumptions of $R = \mathbb{1}$ and U_ν being real as done in [10], the straightforward generalization of (29) in [10] is

$$B(l_i \rightarrow l_j \gamma) = 384\pi^2 e^2 \frac{v^4}{\Lambda_{\text{LFV}}^4} |\Delta_{ij}|^2 |C|^2. \tag{4.11}$$

Here $v = 174 \text{ GeV}$ is the vacuum expectation value of the SM Higgs doublet,⁵ Λ_{LFV} is the scale of charged lepton flavour violation, and C summarizes the Wilson coefficients of the relevant operators that can be calculated in a given specific model. They are naturally of $\mathcal{O}(1)$ but can be different in different MLFV models. As we would like to keep our presentation as simple as possible, we will set $|C| = 1$ in what follows, bearing in mind that in certain scenarios C may differ significantly from unity. Thus the true $B(l_i \rightarrow l_j \gamma)$ can be different from our estimate in a given MLFV model, but as C is, within MLFV, independent of external lepton flavours, the ratios of branching ratios take a very simple form

$$\frac{B(l_i \rightarrow l_j \gamma)}{B(l_m \rightarrow l_n \gamma)} = \frac{|\Delta_{ij}|^2}{|\Delta_{mn}|^2}. \tag{4.12}$$

The most important objects in (4.11) and (4.12) are

$$\Delta_{ij} \equiv (Y_\nu^\dagger Y_\nu)_{ij} = \frac{1}{v^2} (U_\nu \sqrt{d_\nu} R^\dagger D_R R \sqrt{d_\nu} U_\nu^\dagger)_{ij}, \tag{4.13}$$

⁵ $v = \sqrt{v_1^2 + v_2^2}$ for two-Higgs-doublet models such as the MSSM. Powers of $\sin \beta$ can be absorbed into C or into a redefinition $\Lambda_{\text{LFV}} \rightarrow \Lambda_{\text{LFV}}^{\text{eff}}$

which in the limit of $R = \mathbb{1}$, $D_R = M_\nu \mathbb{1}$, and U_ν being real reduce to Δ_{ij} as given in (14) of [10].

With the formula (4.13) at hand we can generalize the expressions for Δ_{ij} in (24) of [10] to the general case of complex R and U_ν . To this end we will use the standard parametrization of the PMNS matrix U_ν in (4.1) and the parametrization of R in (4.2). As the general expressions for Δ_{ij} in terms of x_{ij} and y_{ij} are very complicated, we give in appendix B explicit formulae setting all $x_{ij} = y_{ij} = 0$ except for $y_{12} \neq 0$. We will discuss in our numerical analysis also the cases for which y_{13} and y_{23} are non-vanishing. As mentioned above, setting $x_{ij} = 0$ is in accord with the degeneracy of the right-handed neutrinos. Once this degeneracy is broken by RG effects, the x_{ij} become non-zero.

Recall from section 3 that Δ_{ij} evolves above the scale M_ν and the flavour structures it affects, such as the slepton mass matrix $m_\tilde{l}^2$, also evolve between M_ν and Λ_{LFV} (and the resulting effective operators below Λ_{LFV} also evolve). Moreover, the flavour-violating piece in, for example, $m_\tilde{l}^2$ is not exactly proportional to Δ at the scale M_ν beyond leading order because these objects satisfy different RGEs between M_ν and Λ_{GUT} . All this running depends, beyond the operator, also on the details of the model. Below the seesaw scale the flavour-non-universal contributions are governed by Y_E (although trilinear couplings such as the A -terms in the MSSM can also contribute), which is analogous to the case of the PMNS matrix. Based on the experience that the running of the PMNS angles is weak in the SM and the MSSM unless $\tan \beta$ (and hence y_τ) is large, we ignore all these details and evaluate Δ_{ij} at the scale M_ν .

That Δ_{ij} has to be evaluated at the high energy scale M_ν , and hence U_ν and d_ν have to be evaluated at M_ν by means of renormalization group equations with the initial conditions given by their values at M_Z , has recently been stressed in particular in [31]. The dominant contributions to the flavour-violating pieces in the charged slepton masses matrix in the MSSM that is relevant for $l_i \rightarrow l_j \gamma$ are proportional to $Y_\nu^\dagger Y_\nu$ and come from scales above M_ν , as seen for instance in equation (30) of [34] (where charged lepton Yukawas and A -terms have been dropped and only contribute at higher orders) and the fact that right-handed neutrinos and their Yukawa couplings are absent below that scale.

All other parameters of a given MLFV model, hidden in the Wilson coefficient C in (4.11), like slepton and chargino masses in the MSSM, would have to be evaluated at the electroweak scale and lower scales if a concrete value for C was desired.

The ratio $B(\mu \rightarrow e\gamma)/B(\tau \rightarrow \mu\gamma)$ is shown for the case of the MSSM with $\tan \beta = 2$ in figure 1 (left). All other parameters are varied in the ranges given above. We see that this ratio varies over about six orders of magnitude and $B(\mu \rightarrow e\gamma)$ can be a factor 10^3 larger than $B(\tau \rightarrow \mu\gamma)$ in qualitative agreement with [26, 29]. We have checked that the leptogenesis constraint, as discussed in section 5, has no significant impact. This contradicts the findings of [13]. Even when constraining the Dirac and Majorana phases in the PMNS matrix to zero and allowing only for a single non-vanishing angle y_{12} at the scale Λ_{GUT} , we can still have $B(\mu \rightarrow e\gamma) \gg B(\tau \rightarrow \mu\gamma)$. This is again in agreement with [26, 29]. We will consider the single ratio $B(\mu \rightarrow e\gamma)$ together with the leptogenesis constraint in section 5.

It is also interesting to compare our elaborate iterative procedure of matching high- and low-energy parameters to a simpler procedure where we simply impose the weak-scale

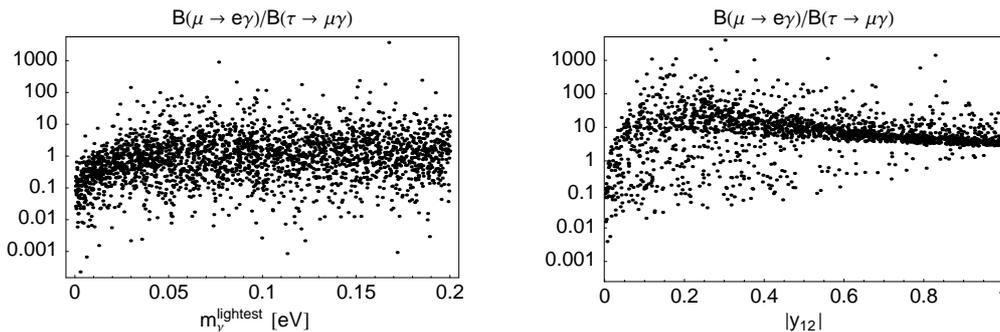


Figure 1: Double ratios of $l_i \rightarrow l_j \gamma$ for the MSSM with $\tan \beta = 2$. Left plot: All parameters varied, right plot: no phases and only $y_{12} \neq 0$. For a discussion, see the text.

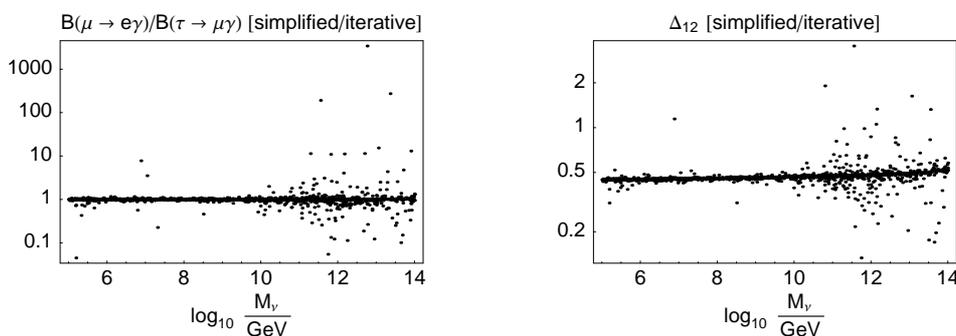


Figure 2: Impact of iterative vs simplified procedure. Left plot: Simplified result for the ratio of branching ratios $B(\mu \rightarrow e \gamma)/B(\tau \rightarrow \mu \gamma)$, normalized to the one obtained with the iterative procedure. Right plot: Similarly for Δ_{12} .

PMNS and neutrino mass parameters at the scale Λ_{GUT} (figure 2 (left)), corresponding to the MSSM with $\tan \beta = 2$). It turns out that both procedures agree well for small scales M_ν . (This agreement is slightly worse for $\tan \beta = 10$.) For large values $M_\nu > 10^{11}$ GeV, deviations up to a few orders of magnitude can occur for some choices of parameters. It appears that this is usually due accidentally small branching ratios in one of the approaches. This is supported by the right plot in the figure 2, which shows a good agreement for the more fundamental flavour-violating quantity Δ_{12} up to the (expected) different overall normalization.

4.4 CP asymmetries

We are also in a position to illustrate and check numerically our qualitative discussion in section 3 of the CP asymmetries relevant for leptogenesis. A thorough investigation of the baryon asymmetry follows in the next section. Figure 3 shows the sum of the three CP asymmetries $|\sum_i \epsilon_i|$ defined below (5.2), for the generic three-flavour case (left plot) and the CP asymmetry ϵ_1 for the effective two-flavour case where only $y_{12} \neq 0$ (right plot). One can see clearly that in the latter case the dependence on M_ν is weak and slightly

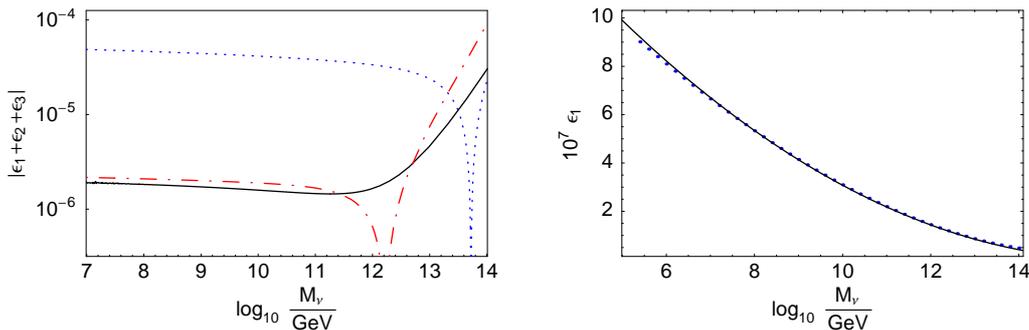


Figure 3: Left plot: M_ν dependence of $|\sum_i \epsilon_i|$ for the generic (3-flavour) case. Right plot: effective 2-flavour case. Normal hierarchy, $m_\nu^{\text{lightest}} = 0.02 \text{ eV}$; $y_{12} = 0.8$, $y_{13} = 0.2$, $y_{23} = 0.6$ (3-flavour case), $y_{12} = 1$ and $y_{13} = y_{23} = 0$ (effective 2-flavour case). The PMNS phases have been taken to be $\delta = \alpha = \beta = \pi/10$. Right plot: Effective two-flavour case; only ϵ_1 is shown, on a linear scale.

reciprocal. In fact this dependence is approximately proportional to $\ln^2 \Lambda_{\text{GUT}}/M_\nu$ (black solid line) in agreement with expectations. The generic case is shown in the left plot for the SM (black solid) as well as the MSSM for $\tan \beta = 2$ (red dot-dashed) and $\tan \beta = 10$ (blue dotted), with the remaining parameters given in the figure caption. In contrast to the two-flavour case, there is strong dependence on M_ν for $M_\nu > 10^{12} \text{ GeV}$, when the contribution due to Y_ν alone starts to dominate the RGEs (3.8), (3.9). The precise form of the M_ν dependence is quite sensitive to the “angles” y_{ij} , but the roughly linear growth of $|\sum_i \epsilon_i|$ in the regime of large M_ν appears to be general. However, the figure also clearly shows a strong dependence on the MSSM parameter $\tan \beta$ particularly for small M_ν . Indeed already for relatively small $\tan \beta = 10$ the CP asymmetries can be more than an order of magnitude larger than in the SM. Moreover, in the case of the MSSM we observe a sign change at some scale $M_\nu \gtrsim 10^{12} \text{ GeV}$, which can be traced to the different relative signs between the terms on the right-hand sides of (3.8) and (3.9). This example clearly demonstrates a rather dramatic dependence on details of the model. Finally, as in the case of the double ratios above, we investigated the impact of the iterative procedure compared to the simplified approach and found it to be generically small. Hence we feel justified to use the simplified procedure in section 5 in order to save computer time.

5. Leptogenesis in the extended MLFV framework

5.1 Preliminaries

One of the most plausible mechanisms for creating the observed matter-antimatter asymmetry in the universe is leptogenesis, where a CP asymmetry generated through the out-of-equilibrium L -violating decays of the heavy Majorana neutrinos leads to a lepton asymmetry which is subsequently transformed into a baryon asymmetry by $(B + L)$ -violating sphaleron processes [8, 9, 35].

Unfortunately, even in its simplest realization through the well-known seesaw mechanism [11], the theory has too many parameters. Indeed, as recalled in section 2.4 in the

framework of the standard model (SM) extended with three heavy Majorana neutrinos N_i ($i = 1, 2, 3$), the high-energy neutrino sector, characterized by the Dirac neutrino (m_D) and the heavy Majorana neutrino (M_R) mass matrices, has eighteen parameters. Of these, only nine combinations enter into the seesaw effective neutrino mass matrix $m_D^T M_R^{-1} m_D$, thus making difficult to establish a direct link between leptogenesis and low-energy phenomenology [36]. Furthermore, there are six CP-violating phases which are physically relevant at high energies, while only three combinations of them are potentially observable at low energies. Therefore, no direct link between the sign of the baryon asymmetry and low-energy leptonic CP violation can be established, unless extra assumptions are introduced.

Furthermore, additional assumptions are usually required to completely determine the high-energy neutrino sector from low-energy observables. Typical examples are the introduction of texture zeros in the Yukawa matrices or the imposition of symmetries to constrain their structure [37]. On the other hand, the heavy Majorana neutrino masses can range from the TeV region to the GUT scale, and the spectrum can be hierarchical, quasi-degenerate or even exactly degenerate [38]. Despite this arbitrariness, the heavy Majorana neutrino mass scale turns out to be crucial for a successful implementation of the leptogenesis mechanism. In particular, the standard thermal leptogenesis scenario with hierarchical heavy Majorana neutrino masses ($M_1 \ll M_2 < M_3$) requires $M_1 \gtrsim 4 \times 10^8$ GeV [39], if N_1 is in thermal equilibrium before it decays, or the more restrictive lower bound $M_1 \gtrsim 2 \times 10^9$ GeV [40] for a zero initial N_1 abundance. Since this bound also determines the lowest reheating temperature allowed after inflation, it could be problematic in supersymmetric theories due to the overproduction of light particles like the gravitino [41].

It should be emphasized, that the above bounds are model dependent in the sense that they can be avoided, if the heavy Majorana neutrino spectrum is no longer hierarchical. For example, if at least two of the N_i are quasi-degenerate in mass, i.e. $M_1 \simeq M_2$, then the leptonic CP asymmetry relevant for leptogenesis exhibits the resonant behavior $\varepsilon_1 \sim M_1/(M_2 - M_1)$ [20, 21]. In this case, it is possible to show that the upper bound on the CP asymmetry is independent of the light neutrino masses and successful leptogenesis simply requires $M_{1,2}$ to be above the electroweak scale for the sphaleron interactions to be effective. The quasi-degeneracy may also be achieved in soft leptogenesis where a small splitting is induced by the soft supersymmetry breaking terms [42].

Another possibility which has been recently explored [16, 17] relies on the fact that radiative effects, induced by the renormalization group (RG) running from high to low energies, can naturally lead to a sufficiently small neutrino mass splitting at the leptogenesis scale. In the latter case, sufficiently large CP asymmetries are generated.

In the minimal seesaw scenario with only two heavy neutrinos the resulting baryon asymmetry in the SM turns out to be below the observed value [16]. On the other hand, this mechanism can be successfully implemented in its minimal supersymmetric extension (MSSM) [17].

It has been shown [18] that the above problems in the SM can be overcome in a more realistic scenario where the effects of a third heavy neutrino are also taken into account. In [18], leptogenesis was studied in the framework of a model where there are three right-

handed neutrinos, with masses $M_1 \approx M_2 \ll M_3$. We will discuss this scenario below as a special limit of the MLFV framework.

In view of the above, it is important to analyze leptogenesis in the extended MLFV framework, where CP violation is allowed both at high and low energies. In the MLFV scenario, right-handed neutrinos are assumed to be *exactly* degenerate at a high energy scale. In the limit of exact degeneracy, no lepton-asymmetries can be generated. However, as previously emphasized, even if exact degeneracy is assumed at a high energy scale, renormalization group effects lead to a splitting of right-handed neutrino masses at the scale of leptogenesis, thus offering the possibility of viable leptogenesis in the extended MLFV framework.

5.2 BAU in the RRL and flavour effects

In leptogenesis scenarios the baryon asymmetry of the universe η_B arises due to non-perturbative sphaleron interactions that turn a lepton asymmetry into a baryon asymmetry. The predicted value of η_B has to match the results of WMAP and the BBN analysis for the primordial deuterium abundance [43]

$$\eta_B = \frac{n_B}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10}. \tag{5.1}$$

The lepton asymmetry is generated by out-of-equilibrium decays of heavy right-handed Majorana neutrinos N_i and is proportional to the CP asymmetry ε_i^l with

$$\varepsilon_i^l = \frac{\Gamma(N_i \rightarrow L_l \phi) - \Gamma(N_i \rightarrow \bar{L}_l \bar{\phi})}{\sum_l [\Gamma(N_i \rightarrow L_l \phi) + \Gamma(N_i \rightarrow \bar{L}_l \bar{\phi})]}, \tag{5.2}$$

and l denoting the lepton flavour, that arises at one-loop order due to the interference of the tree level amplitude with vertex and self-energy corrections.

A characteristic of the MLFV framework is that only admissible BAU with the help of leptogenesis is radiative and thereby resonant leptogenesis. The mass splittings of the right-handed neutrinos induced by the RGE are of similar size $\Delta M \sim \mathcal{O}(M_\nu Y_\nu Y_\nu^\dagger)$ as the decay widths $\Gamma \sim \mathcal{O}(M_\nu Y_\nu Y_\nu^\dagger)$. This is the condition of resonant leptogenesis. The CP asymmetry is for the lepton flavour l given by

$$\varepsilon_i^l = \frac{1}{(Y_\nu Y_\nu^\dagger)_{ii}} \sum_j \Im((Y_\nu Y_\nu^\dagger)_{ij} (Y_\nu)_{il} (Y_\nu^\dagger)_{lj}) g(M_i^2, M_j^2, \Gamma_j^2) \tag{5.3}$$

where $g(M_i^2, M_j^2, \Gamma_j^2)$ is an abbreviation for the full result given in [21]. The total CP asymmetries ε_i are obtained summing over the lepton flavours l .

The baryon to photon number ratio η_B can then be calculated solving the Boltzmann equations for the lepton asymmetry and converting it into η_B using suitable dilution and sphaleron conversion factors. Which Boltzmann equation to use depends on the temperature scale at which leptogenesis takes place. We will follow a simplistic approach ignoring all subtleties generically coming into play in the intermediate regime between different mechanisms at work. Our main conclusions, however, will not be affected by this omission. We will simply divide the temperature scale into a region up to which all three

lepton flavours have to be taken into account and a region above which the single flavour approximation works.

Below some temperature⁶ $T_{\text{eq}}^\mu \simeq 10^9$ GeV [44–46], muon and tau charged lepton Yukawa interactions are much faster than the expansion H rendering the μ and τ Yukawa couplings in equilibrium. The correct treatment in this regime requires the solution of lepton flavour specific Boltzmann equations. In the strong washout regime η_B is independent of the initial abundances and an estimate including flavour effects is given by [19]

$$\eta_B \simeq -10^{-2} \sum_{i=1}^3 \sum_{l=e,\mu,\tau} e^{-(M_i-M_1)/M_1} \varepsilon_i^l \frac{K_i^l}{K^l K_i}, \quad (5.4)$$

with

$$K_i^l = \frac{\Gamma(N_i \rightarrow L_l \phi) + \Gamma(N_i \rightarrow \bar{L}_l \bar{\phi})}{H(T = M_i)} \quad (5.5)$$

$$K_i = \sum_{l=e,\mu,\tau} K_i^l, \quad K^l = \sum_{i=1}^3 K_i^l, \quad H(T = M_i) \simeq 17 \frac{M_i^2}{M_{\text{Pl}}} \quad (5.6)$$

where $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV and K_i^l is the washout factor due to the inverse decay of the Majorana neutrino N_i into the lepton flavour l . The impact of lepton flavour effects on η_B is discussed in [44, 19, 45–47]. As we shall also see below, the inclusion of flavour effects generally leads to an enhancement of the resulting η_B . This is due to two effects: (1) the washout gets reduced since the interaction with the Higgs is with the flavour eigenstates only and (2) an additional source of CP violation arises due to lepton flavour specific CP asymmetries.

For higher values of $T \gtrsim 10^{12}$ GeV the charged lepton Yukawa couplings do not break the coherent evolution of the lepton doublets produced in heavy neutrino decays anymore. In this regime flavour effects can be ignored and an order of magnitude estimate is given by

$$\eta_B \simeq -10^{-2} \sum_{i=1}^3 e^{-(M_i-M_1)/M_1} \frac{1}{K} \sum_{l=e,\mu,\tau} \varepsilon_i^l, \quad (5.7)$$

with $K = \sum_i K_i$. This agrees with a recent analytical estimate by [48] up to factors of $\mathcal{O}(1)$ for the region of interest in parameter space, where the estimate of [48] generally leads to a smaller efficiency and smaller η_B . We have also compared the analytical estimate of [48] and (5.7) with the numerical solution of the Boltzmann equations using the LeptoGen code [19]⁷. For the relevant ranges of the input parameters the analytical estimate of (5.7) and the full numerical solution agree quite well, whereas the estimate of [48] leads to an efficiency and η_B generally smaller by a factor of 5 to 10. This is shown in figure 8. These estimates, however, do not take into account the potentially large lepton flavour effects included in (5.4).

⁶We will chose $T_{\text{eq}}^\mu \simeq 10^{10}$ GeV in our analysis as an effective boundary between the unflavoured and ‘fully flavoured’ regimes, where we (respectively) neglect flavour and distinguish all three flavours. The main conclusions are, however, not affected by the precise choice.

⁷<http://www.ippp.dur.ac.uk/~teju/leptogen/>

Let us remark in passing that in the flavour independent region we are always in the strong washout regime, since

$$K = K_1 + K_2 + K_3 = \frac{1}{m^*} \text{tr} \left(R d_\nu R^\dagger \right) \geq \frac{(\Delta m_{\text{atm}}^2)^{1/2}}{m^*} \simeq 50, \quad (5.8)$$

where $m^* = \mathcal{O}(10^{-3})$. This inequality holds since the trace is linear function of the neutrino masses with positive coefficients, which reaches its minimum for $y_{ij} = 0$. We also made sure that the estimate (5.4) including flavour effects is applicable [19] and checked that the inequality

$$K_i^l \gtrsim 1 \quad (5.9)$$

is always satisfied for the points considered in the plots. Since both (5.8) and (5.9) are satisfied, a simple decay-plus-inverse decay picture is a good description and the estimates (5.4) and (5.7) independent of the initial abundances give a good approximation of the numerical solution of the full Boltzmann equations.

We have performed the leptogenesis analysis specifically for the SM. We do not expect large deviations in the MSSM from the SM if the same $Y_\nu(M_\nu)$ and $M_\nu^i(M_\nu)$ are given. The main differences come (1) from the CP-asymmetries, which now include contributions from the supersymmetric particles, (2) from the washout, and (3) from conversion and dilution factors. The supersymmetric CP asymmetries have the same flavour structure as in the SM and using [30] one can show that $\epsilon^{\text{MSSM}} \simeq 2 \epsilon^{\text{SM}}$ for quasi-degenerate heavy neutrinos. We also expect the correction by the decay widths to be similar in size. Next, the washout in the strong washout regime is about a factor of $\sqrt{2}$ larger [49] in the MSSM, whereas the dilution and sphaleron conversion factors stay almost unchanged. Concluding, we find that in the scenario considered the predicted values roughly satisfy $\eta_B^{\text{MSSM}} \simeq 1.5 \eta_B^{\text{SM}}$ for the same set of input parameters $Y_\nu(M_\nu)$ and $M_\nu^i(M_\nu)$. The RGE induced values of $Y_\nu(M_\nu)$ and $M_\nu^i(M_\nu)$, however, are model dependent and lead to in general different $Y_\nu(M_\nu)$ and $M_\nu^i(M_\nu)$ for the same boundary conditions at the GUT and low-energy scale, as discussed in section 3.4. Especially sensitive is the region $M_\nu \lesssim 10^{12}$ GeV where the CP asymmetries are dominantly generated by the tau Yukawa coupling, which is enhanced by a factor of $\tan \beta$ in the MSSM. Note also that in the MSSM, T_{eq}^μ and T_{eq}^τ should be rescaled by a factor $(1 + \tan^2 \beta)$ to take account of the larger Yukawa couplings [59], which should make flavour effects even more prominent.⁸

5.3 Two flavour limit

As a first step we discuss the special case of y_{12} being non-vanishing at the GUT scale and all other $y_{ij} = 0$. This corresponds approximately to one of the scenarios considered in a recent study of radiative leptogenesis [18] with two right-handed neutrinos quasi-degenerate and a third right-handed neutrino decoupled $M_1 \simeq M_2 \ll M_3$. If only $y_{12} \neq 0$ the calculation of η_B proceeds in the same way since to a good approximation only ν_R^1 and ν_R^2 contribute to the CP asymmetry. The only difference comes from the enhanced wash-out. Since the third heavy neutrino is now also contributing, the lower bound on the washout K in (5.8) is in

⁸We thank S. Antusch for drawing our attention to this point.

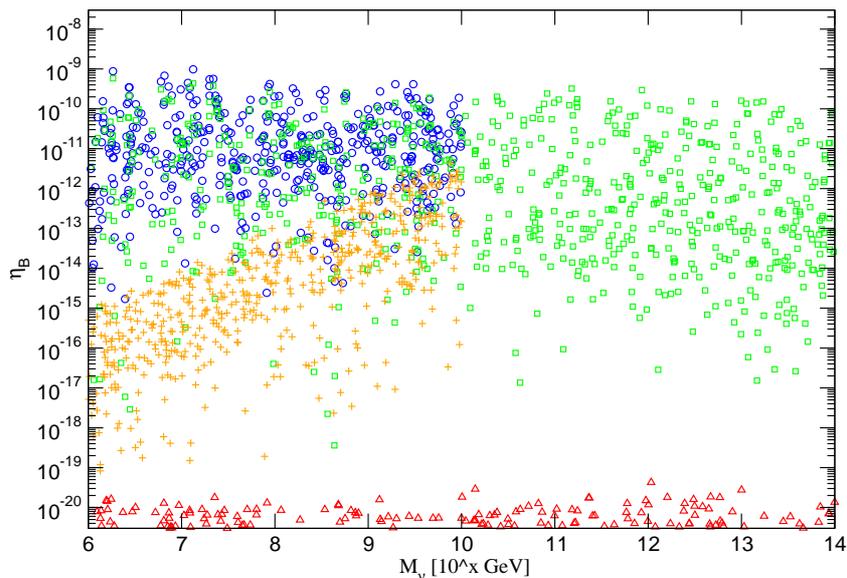


Figure 4: Resulting η_B for the case in which only $y_{12} \neq 0$ (effective two flavour case) as a function of M_ν for the normal hierarchy of light neutrinos: the *orange crosses* and *red triangles* show the unphysical limit setting the charged lepton Yukawas $Y_e = 0$ in the renormalization group evolution with and without including lepton flavour effects in the calculation of η_B , respectively. Setting the charged lepton Yukawas to their physical values, the *blue circles* and the *green squares* correspond to including and ignoring lepton flavour effects in the calculation, respectively.

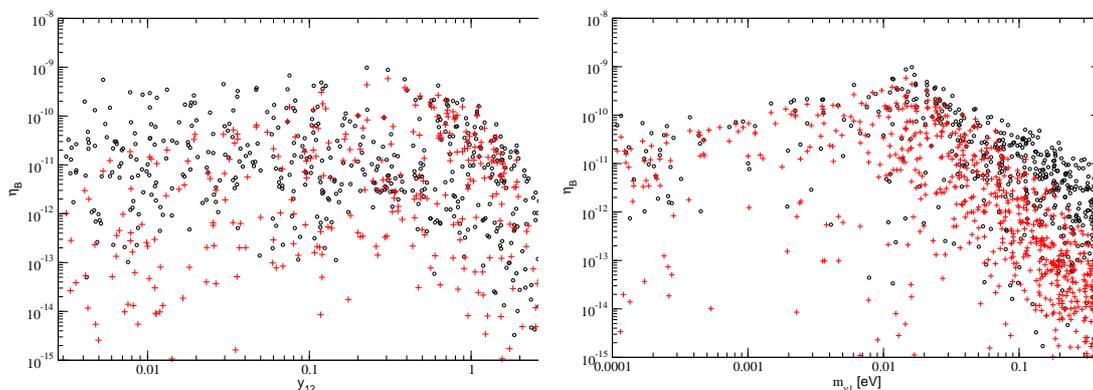


Figure 5: η_B for the case in which only $y_{12} \neq 0$ (effective two flavour case) as a function of y_{12} (left) and $m_{\nu 1}$ (right) for the normal hierarchy. The *black circles* are obtained including lepton flavour effects and the *red crosses* are calculated ignoring them.

our case relatively enhanced by a factor $(\Delta m_{\text{atm}}^2)^{1/2}/(\Delta m_{\text{sol}}^2)^{1/2} \simeq 4 - 5$. We have checked this correspondence for η_B also numerically. Ignoring flavour subtleties in leptogenesis for a moment, the CP violating effects due to renormalization group effects are induced only by the charged lepton yukawa couplings, see section 3.4, and the total CP asymmetries for

each heavy Majorana neutrino take the form [18]

$$\varepsilon_{1,2} \simeq \frac{\bar{\varepsilon}_{1,2}}{1 + D_{2,1}}, \quad \varepsilon_3 \simeq 0, \quad (5.10)$$

and

$$\begin{aligned} \bar{\varepsilon}_j &\simeq \frac{3y_\tau^2}{32\pi} \frac{\text{Im}(H_{21}) \text{Re}[(Y_\nu)_{23}^* (Y_\nu)_{13}]}{H_{jj}(H_{22} - H_{11})} \\ &= \frac{3y_\tau^2}{64\pi} \frac{m_j(m_1 + m_2) \sqrt{m_1 m_2} \sinh(2y_{12}) \text{Re}(U_{\tau 2}^* U_{\tau 1})}{(m_1 - m_2)(m_j^2 \cosh^2 y_{12} + m_1 m_2 \sinh^2 y_{12})}, \end{aligned} \quad (5.11)$$

$$\begin{aligned} D_j &\simeq \frac{\pi^2}{4} \frac{H_{jj}^2}{(H_{22} - H_{11})^2 \ln^2(M_\nu/M_{\text{GUT}})} \\ &= \left[\frac{\pi}{2} \frac{m_j^2 \cosh^2 y_{12} + m_2 m_1 \sinh^2 y_{12}}{m_j(m_2 - m_1) \ln(M_\nu/M_{\text{GUT}})} \right]^2. \end{aligned} \quad (5.12)$$

where D_j are regularization factors coming from the heavy Majorana decay widths. We immediately see that the total CP asymmetries only bare a very mild dependence on the heavy Majorana scale. The almost negligible dependence on M_ν has to be compared with the power-suppression in M_ν in the hierarchical case ($M_1 \ll M_2 < M_3$). We find this expectation confirmed in figure 4, where the resulting η_B is shown as a function of M_ν .

Figure 4 also nicely illustrates the relative importance of flavour effects in leptogenesis. If no cancellations occur, we find, that flavour effects generate an η_B which is of the same order of magnitude (blue circles), however almost always larger than the one calculated ignoring flavour effects (green squares).

If we now consider the unphysical limit of setting $Y_e = 0$ in the renormalization group running only, we find that the *total* CP asymmetries and η_B should vanish since no CP violation effects are induced by the RGE, see section 3.4. We confirm this behavior in figure 4 (red triangles). A very different picture emerges once we include flavour effects. The relevant quantity for leptogenesis is then $\Im((Y_\nu Y_\nu^\dagger)_{ij} (Y_\nu)_{il} (Y_\nu^\dagger)_{lj})$ with no summation over the charged lepton index l . Although no *total* CP asymmetries are generated via the RG evolution in the limit $Y_e = 0$, the CP asymmetries for a specific lepton flavour are non-vanishing. Additionally, the resulting η_B now shows a M_ν dependence which stems from the RGE contributions due to Y_ν only, which are absent in the total CP asymmetries in the two flavour limit (orange crosses).

All plots have been generated assuming a normal hierarchy of the light neutrino masses. We have checked that the results for the inverted hierarchy are similar, although η_B turns out to be generally smaller and below the observed value, in accordance with the findings of [18]. Including flavour effects it is however still possible to generate a η_B of the correct order of magnitude. In figure 5 we additionally show the dependence of η_B on y_{12} and $m_{\nu 1}$. We find that flavour effects enlarge the y_{12} range where successful baryogenesis is possible and slightly soften the upper bound on the light neutrino mass scale. The left panel even demonstrates that leptogenesis in the MLFV scenario is possible for a real R matrix. Then lepton flavour effects are essential for a successful leptogenesis [45, 46].

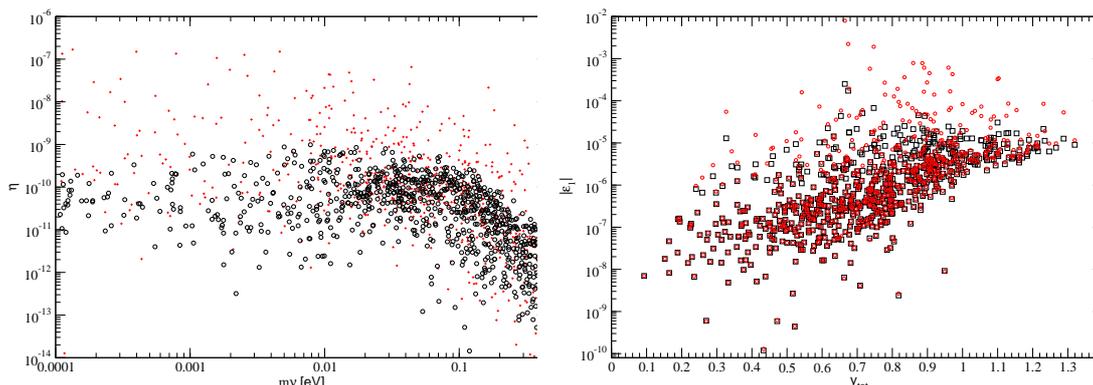


Figure 6: (left) η_B for the general case with $0.01 < |y_{ij}| < 1$ as a function of $m_{\nu 1}$ (right). The *black circles* are obtained including lepton flavour effects whereas the *red crosses* are calculated ignoring them. The flavour blind results (*red crosses*) reach higher values due to the CP asymmetries growing as M_ν gets bigger in this regime. (right) The total CP asymmetry $|\epsilon_1|$ for the general case with $0.01 < |y_{ij}| < 0.8$ as a function of $y_{\text{tot}} = (y_{12}^2 + y_{13}^2 + y_{23}^2)^{\frac{1}{2}}$ for input values that result in the right order of magnitude of η_B . The *red circles* are obtained using the uncorrected CP asymmetries and the *black squares* include the corrections by the decay widths

5.4 General case

Now we consider the general case with all three phases y_{ij} non-vanishing. We have varied the parameters as described in section 4. The regularization of the resonant CP asymmetry by the D_i turns out to be important for values of $\epsilon_i \gtrsim 10^{-6}$, see figure 6. As seen there, in the regime where flavour effects are important we find an upper bound on the light neutrino mass of $m_{\nu 1} \lesssim 0.2 \text{ eV}$ in order to generate the right amount of η_B . Beyond the temperature scale where flavour effects play a role, no relevant bound can be found. This is due to the enhancement of the CP asymmetry which approximately grow linearly for values of $M_\nu \gtrsim 10^{12} \text{ GeV}$, see the discussion in section 4.4

In figure 8, we compare different calculations of η_B :

- the flavour independent estimate of [48] used in Cirigliano et al. [13] (red boxes),
- the numerical solution of the flavour independent Boltzmann equations using the LeptoGen package (black circles),
- the recent estimate by Blanchet and Di Bari [47] that includes flavour effects (green triangles),
- the approximate expression of [19] given in (5.4) that also includes flavour effects (brown crosses).

We find that

- the flavour blind estimate of [48] used in Cirigliano et. al. [13] lies consistently below the numerical solution of the flavour independent Boltzmann equations. For

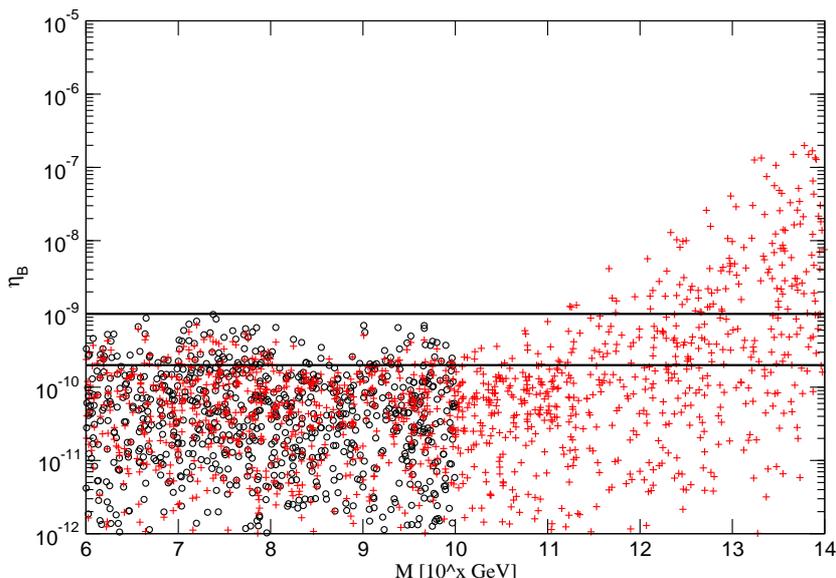


Figure 7: η_B for the general case with $0.01 < |y_{ij}| < 1$ as a function of M_ν . The *circles* are obtained including lepton flavour effects and the *red crosses* are calculated ignoring them.

$M_\nu \geq 10^{12}$ GeV this turns out to be unimportant as flavour effects in this region are small and we confirm the increase of η_B with M_ν in this region found by these authors.

- Potentially large flavour effects that have been left out in [13] generally enhance the predicted η_B , in particular for $M_\nu \leq 10^{12}$ GeV, in accordance with the existing literature.
- Both flavour estimates and the numerical solution of the flavour independent Boltzmann equations show solutions with η_B of the measured order of magnitude without imposing a stringent lower bound on the value of M_ν .

The last finding is in contrast to the analysis of Cirigliano et. al. [13] which using the flavour independent estimate of [48] finds a lower bound on M_ν of $\mathcal{O}(10^{12}\text{GeV})$ as clearly represented by the red boxes in figure 8. The same qualitative conclusion holds for η_B using the RGE induced CP asymmetries in the MSSM. The $\tan \beta$ enhancement of the CP asymmetries as discussed in section 4.4 even facilitates the generation of an η_B of the right size.

Our analysis that includes flavour effects demonstrates that baryogenesis through leptogenesis in the framework of MLFV is a stable mechanism and allows a successful generation of η_B over a wide range of parameters. The absence of a lower bound on M_ν found here has of course an impact on the LFV processes, which we will discuss next.

In figure 9 we show $B(\mu \rightarrow e\gamma)$ vs. M_ν for the parameter ranges described above and a lepton flavor violation scale of 1 TeV. We highlighted the points where successful baryogenesis is possible (black squares). We find that $B(\mu \rightarrow e\gamma)$ can be made small

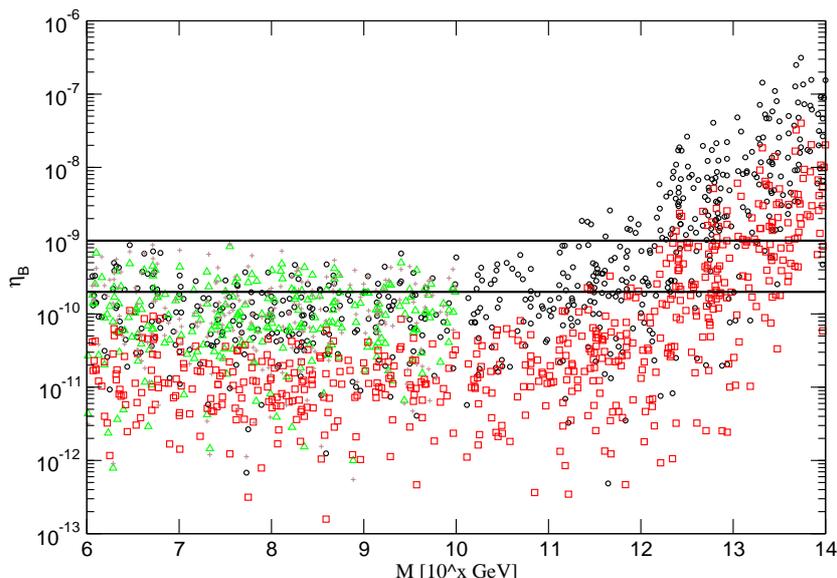


Figure 8: Different determinations of η_B for the general case with $0.01 < |y_{ij}| < 1$ as a function of M_ν . The *black circles* are obtained numerically solving the flavour independent Boltzmann equations using the LeptoGen package, the *green triangles* and the *brown crosses* show estimates including flavour effects of [47] and (5.4), respectively. Finally the *red boxes* show the estimate of [48] used in Cirigliano et. al [13] which ignores flavour effects.

enough to evade bounds from current and future experiments and one can have successful baryogenesis through leptogenesis at the same time. This is another finding of our paper which is in contrast to a recent analysis [13]. We will summarize the differences to [13] in the next paragraph.

5.5 Comparison with [13]

Recently in an independent analysis Cirigliano, Isidori and Porretti [13] generalized MLFV formulation in [10] to include CP violation at low and high energy. Similarly to us they found it convenient to use for Y_ν the parametrization of Casas and Ibarra. They have also pointed out that in the MFLV framework the most natural is the resonant leptogenesis.

On the other hand, these authors neglected flavour dependent effects in the evaluation of η_B , that we find in agreement with other authors to be important [44, 19, 45–47]. This has important consequences already at the qualitative level. Their qualitative discussion of the splittings of the M_ν^i at the see-saw scale is similar to ours and we agree with the main physical points made by these authors in this context. On the other hand, while we have demonstrated explicitly by means of a renormalization group analysis that a successful RRL can be achieved, Cirigliano et al confined their analysis to parametrizing possible radiative effects in terms of a few parameters. In this context a new point made by us (see discussion section 3.3) is that the coefficients c_i in (3.3) are in fact not independent of each other. Indeed the leading logarithmic contribution to c_i are related by the renormalization

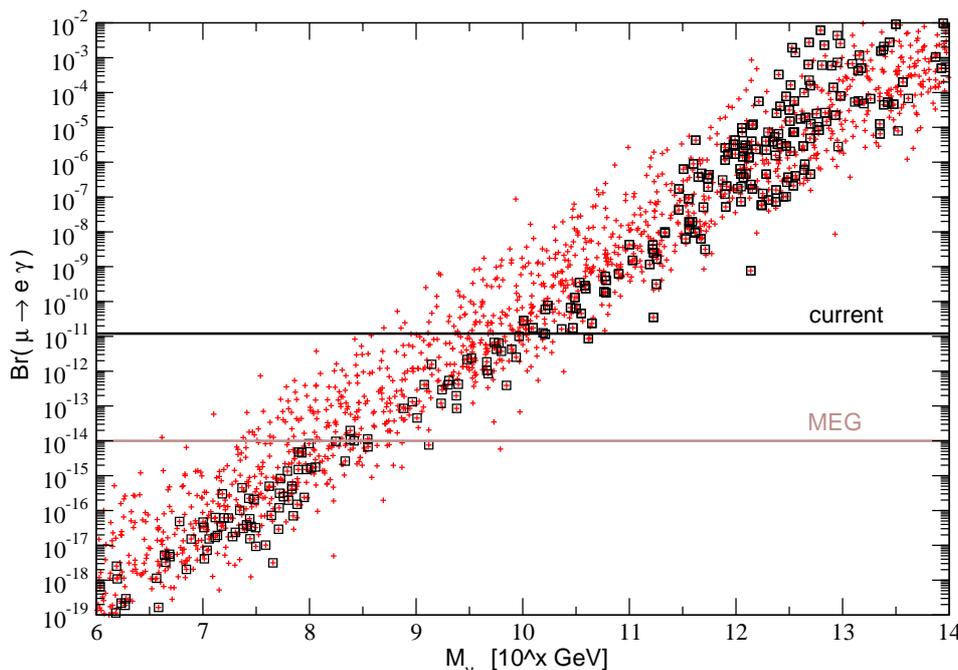


Figure 9: $B(\mu \rightarrow e\gamma)$ as a function of M_ν for $\Lambda_{\text{LFV}} = 1 \text{ TeV}$. The *black squares* show points where a baryon asymmetry in the range $2 \cdot 10^{-10} < \eta_B < 10 \cdot 10^{-10}$ is possible.

group. This can in principle increase the predictivity of MLFV. The three most interesting messages of [13] are

- A successful resonant leptogenesis within the MLFV framework implies a lower bound $M_\nu \geq 10^{12} \text{ GeV}$,
- With $\Lambda_{\text{LFV}} = \mathcal{O}(1 \text{ TeV})$, this lower bound implies the rate for $\mu \rightarrow e\gamma$ close to the present exclusion limit,
- MLFV implies a specific pattern of charged LFV rates: $B(\mu \rightarrow e\gamma) < B(\tau \rightarrow \mu\gamma)$.

For $M_\nu \geq 10^{12} \text{ GeV}$, in spite of some differences in the numerics as discussed above, we basically confirm these findings. Unfortunately, for lower values of M_ν our results differ from theirs. In particular, as we have demonstrated in figure 8, the observed value of η_B can be obtained for M_ν by several orders of magnitude below the bound in [13], in accordance with other analyses of leptogenesis. Once M_ν is allowed to be far below 10^{12} GeV , $\Lambda_{\text{LFV}} = \mathcal{O}(1 \text{ TeV})$ does not imply necessarily $B(\mu \rightarrow e\gamma)$ close to the inclusion limit as clearly seen in figure 9.

One of the reasons for the discrepancy between our result with regard to M_ν and the one of [13] is the neglect of flavour effects in leptogenesis in the latter paper. Figure 8 illustrates that flavour effects in leptogenesis matter.

Concerning $B(\mu \rightarrow e\gamma) < B(\tau \rightarrow \mu\gamma)$, we confirm the result of [13] in the limit of very small y_{12} , but as shown in figure 1, this is not true in general, as also found in [26, 29].

Consequently, this hierarchy of charged LFV rates cannot be used as model independent signature of MLFV.

6. Summary and conclusions

In this paper we have generalized the proposal of minimal flavour violation in the lepton sector of [10] to include CP Violation at low and high energy. While the definition proposed in [10] could be considered to be truly minimal, it appears to us too restrictive and not as general as the one in the quark sector (MFV) in which CP violation at low energy is automatically included [1] and in fact all flavour violating effects proceeding through SM Yukawa couplings are taken into account [2]. The new aspect of MLFV in the presence of right-handed neutrinos, when compared with MFV, is that the driving source of flavour violation, the neutrino Yukawa matrix Y_ν , depends generally also on physics at very high scales. This means also on CP violating sources relevant for the generation of baryon-antibaryon asymmetry with the help of leptogenesis. The first discussion of CP violation at low and high energy has been presented in [13]. Our conclusions for $M_\nu \geq 10^{12}$ agree basically with these authors. However, they differ in an essential manner for lower values of M_ν .

The main points of our paper have been already summarized in the introduction. Therefore it suffices to conclude our paper with the following messages:

- A new aspect of our paper is the realization that in the context of MLFV the only admissible BAU with the help of leptogenesis is the one through radiative resonant leptogenesis (RRL). Similar observations have been made in [13]. In this context our analysis benefited from the ones in [16, 17, 20, 21]. The numerous analyses of leptogenesis with hierarchical right-handed neutrinos present in the literature are therefore outside the MLFV framework and the differences between the results presented here and the ones found in the literature for $M_1 \ll M_2 \ll M_3$ can be used to distinguish MLFV from these analyses that could be affected by new flavour violating interactions responsible for hierarchical right-handed neutrinos.
- We have demonstrated explicitly within the SM and the MSSM at low $\tan\beta$ that within a general MLFV scenario the right size of η_B can indeed be obtained by means of RRL. The important property of this type of leptogenesis is the very weak sensitivity of η_B to the see-saw scale M_ν so that for scales as low as 10^6 GeV but also as high as 10^{13} GeV, the observed η_B can be found.
- Flavour effects, as addressed by several authors recently in the literature [44, 19, 45–47], play an important role for $M_\nu \lesssim 10^{10}$ GeV as they generally enhance η_B . Moreover, they allow for a successful leptogenesis within MLFV even when the R -matrix is real (left panel of figure 5).
- As charged LFV processes, like $\mu \rightarrow e\gamma$ are sensitive functions of M_ν , while η_B is not in the RRL scenario considered here, strong correlations between the rates for these

processes and η_B , found in new physics scenarios with other types of leptogenesis can be avoided.

- Except for this important message, several of the observations made by us with regard to the dependence of charged LFV processes on the complex phases in the matrix R and the Majorana phases have been already made by other authors in the rich literature on LFV and leptogenesis. But most of these analyses were done in the context of supersymmetry. Here we would like to emphasize that various effects and several patterns identified there are valid also beyond low energy supersymmetry, even if supersymmetry allows a definite realization of MLFV provided right-handed neutrinos are degenerate in mass at the GUT scale.
- One of the important consequences of the messages above is the realization that the relations between the flavour violating processes in the charged lepton sector, the low energy parameters in the neutrino sector, the LHC physics and the size of η_B are much richer in a general MLFV framework than suggested by [10, 13]. Without a specific MLFV model no general clear cut conclusions about the scale Λ_{LFV} on the basis of a future observation or non-observation of $\mu \rightarrow e\gamma$ with the rate $\mathcal{O}(10^{-13})$ can be made in this framework.
- On the other hand we fully agree with the point made in [10] that the observation of $\mu \rightarrow e\gamma$ with the rate at the level of 10^{-13} , is much easier to obtain within the MLFV scenario if the scales Λ_{LFV} and M_ν are sufficiently separated from each other. We want only to add that the necessary size of this separation is sensitive to the physics between M_Z and Λ_{GUT} , Majorana phases and CP violation at high energy. In this manner the lepton flavour violating processes, even in the MLFV framework, probe scales well above the scales attainable at LHC, which is not necessarily the case within MFV in the quark sector.
- Finally, but very importantly, MLFV being very sensitive to new physics at high energy scales, does not generally solve possible CP and flavour problems. This should be contrasted with the MFV in the quark sector, where the sensitivity to new physics at scales larger than 1 TeV is suppressed by the GIM mechanism.

Note: during the preparation of this revised version, one of us (S.U.) has investigated parametric dependences in the present scenario for the case of a real R in more detail [60].

Acknowledgments

We would like to thank P. Di Bari, M. Beneke, R.G. Felipe, F.R. Joaquim, M. Plümacher, R. Rückl, M.A. Schmidt, F. Schwab, T. Underwood and E. Wyzomirski for useful discussions. We are grateful to S. Antusch and K. Turzyski for helpful comments. This work has been supported by Bundesministerium für Bildung und Forschung under the contracts 05HT4WOA/3 and 05HT6WOA, the GIF project G-698-22.7/2002, the Humboldt foundation (G.C.B), and the DFG Sonderforschungsbereich/Transregio 9 “Computergestützte

Theoretische Teilchenphysik” (S.J., in part). The work of GCB is also supported by Fundacao para a Ciencia e a Tecnologia (FCT Portugal) through CFTP-FCT UNIT 777 and POCTI/FNU/44409/2002, POCI/FP/63415/2005. GCB would like to thank Andrzej Buras and his group for kind hospitality at TUM.

A. Iterative solution of the renormalization group equations

The goal of our numerical analysis of section 4 is to determine the neutrino Yukawa matrix Y_ν and the masses of the right-handed neutrinos at the scale M_ν taking into account the constraints on the masses and mixings of light neutrinos measured at low energies and imposing the GUT condition characteristic for the MLFV

$$M_1(\Lambda_{\text{GUT}}) = M_2(\Lambda_{\text{GUT}}) = M_3(\Lambda_{\text{GUT}}). \quad (\text{A.1})$$

As discussed in section 2 the latter condition implies

$$\text{Re}(R(\Lambda_{\text{GUT}})) = 0, \quad (\text{A.2})$$

but $\text{Im}(R(\Lambda_{\text{GUT}}))$ must be kept non-zero in order to have CP-violation at high energy.

The RG evolution from Λ_{GUT} down to M_ν generates small splittings between $M_i(M_\nu)$ and a non-vanishing $\text{Re}(R(M_\nu))$, both required for the leptogenesis. As the splittings between $M_i(M_\nu)$ turn out to be small, we integrate the right-handed neutrinos simultaneously at $\mu = M_\nu$ imposing, up to their splittings,

$$M_1(M_\nu) \approx M_2(M_\nu) \approx M_3(M_\nu) \approx M_\nu. \quad (\text{A.3})$$

In view of various correlations and mixing under RG between different variables we reach the goal outlined above by means of the following recursive procedure:

Step 1. We associate the values for the solar and atmospheric neutrino oscillation parameters given in (4.5)–(4.6) with the scale $\mu = M_Z$ and set θ_{13} and the smallest neutrino mass m_ν^{lightest} to particular values corresponding to $\mu = M_Z$.

Step 2. For a chosen value of M_ν , the RG equations, specific to a given MLFV model, are used to find the values of the parameters of Step 1 at $\mu = M_\nu$. For instance we find $m_i^\nu(M_\nu)$ and similarly for other parameters.

Step 3. We choose a value for Λ_{GUT} and set first $m_i^\nu(\Lambda_{\text{GUT}}) = m_i^\nu(M_\nu)$ and similarly for other parameters evaluated in Step 2. Setting next

$$M_1(\Lambda_{\text{GUT}}) = M_2(\Lambda_{\text{GUT}}) = M_3(\Lambda_{\text{GUT}}) = M_\nu \quad (\text{A.4})$$

and choosing the matrix R , that satisfies (A.2), allows also to construct $Y_\nu(\Lambda_{\text{GUT}})$ by means of the parametrization in (2.11).

Step 4. Having determined the initial conditions for all the parameters at $\mu = \Lambda_{\text{GUT}}$ we use the full set of the RG equations [53] to evaluate these parameters at M_ν . In the range $M_\nu \leq \mu \leq \Lambda_{\text{GUT}}$ we use

$$m_\nu(\mu) = -v^2 Y_\nu^T(\mu) M^{-1}(\mu) Y_\nu(\mu). \quad (\text{A.5})$$

The RG effects between Λ_{GUT} and M_ν will generally shift $m_i^\nu(M_\nu)$ to new values

$$\tilde{m}_i^\nu(M_\nu) = m_i^\nu(M_\nu) + \Delta m_i^\nu \quad (\text{A.6})$$

with similar shifts in other low energy parameters. If these shifts are very small our goal is achieved and the resulting $Y_\nu(M_\nu)$ and $M_i(M_\nu)$ can be used for lepton flavour violating processes and leptogenesis. If the shifts in question are significant we go to Step 5.

Step 5. The initial conditions at $\mu = \Lambda_{\text{GUT}}$ are adjusted in order to obtain the correct values for low energy parameters at $\mu = M_\nu$ as obtained in Step 2. In particular we set

$$m_i^\nu(\Lambda_{\text{GUT}}) = m_i^\nu(M_\nu) - \Delta m_i^\nu \quad (\text{A.7})$$

with Δm_i^ν defined in (A.6). Similar shifts are made for other parameters. If the condition (A.3) is not satisfied in Step 4, the corresponding shift in (A.4) should be made. Choosing R as in Step 3 allows to construct an improved $Y_\nu(\Lambda_{\text{GUT}})$. Performing RG evolution with new input from Λ_{GUT} to M_ν we find new values for the low energy parameters at M_ν that should now be closer to the values found in Step 2 than it was the case in Step 4. If necessary, new iterations of this procedure can be performed until the values of Step 2 are reached. The resulting $Y_\nu(M_\nu)$ and $M_i(M_\nu)$ are the ones we were looking for.

B. Basic formulae for Δ_{ij}

B.1 Preliminaries

In what follows we will present two generalizations of the formulae for Δ_{ij} in [10] in the approximation of degenerate right-handed neutrinos. We have checked that the splitting of M_i^ν by RGE has very small impact on these formulae. All the expressions for Δ_{ij} are meant to be valid at M_ν . Similar formulae have been given for instance in [23, 26, 27, 25, 31], but we think that the formulae given below are more transparent.

In order to obtain transparent expressions for Δ_{ij} it is useful to introduce the mass differences

$$\delta_{21} = m_{\nu 2} - m_{\nu 1}, \quad \delta_{31} = m_{\nu 3} - m_{\nu 1}, \quad (\text{B.1})$$

$$\tilde{\delta}_{21}(y_{12}) = \delta_{21} \cosh(2y_{12}), \quad \tilde{\delta}_{31}(y_{12}) = \delta_{31} + m_{\nu 1}(1 - \cosh(2y_{12})) \quad (\text{B.2})$$

and collect the dependence on Majorana phases in the following two functions

$$F_1(\alpha, \beta) = e^{-i(\alpha-\beta)} + 2ic^2 \sin(\alpha - \beta), \quad (\text{B.3})$$

$$F_2(\alpha, \beta, \delta) = s_{13}c^2 \cos(\alpha - \beta + \delta) + ic s \sin(\alpha - \beta) + s_{13}s^2 \cos(\alpha - \beta - \delta). \quad (\text{B.4})$$

B.2 R real and U_ν complex

In this case we find

$$\Delta_{\mu e} = \frac{M_\nu}{\sqrt{2}v^2} (s c \delta_{21} + e^{-i\delta} s_{13} \delta_{31}), \quad (\text{B.5})$$

$$\Delta_{\tau e} = \frac{M_\nu}{\sqrt{2}v^2} (-s c \delta_{21} + e^{-i\delta} s_{13} \delta_{31}), \quad (\text{B.6})$$

$$\Delta_{\tau\mu} = \frac{M_\nu}{2v^2} (-c^2 \delta_{21} + \delta_{31}), \quad (\text{B.7})$$

where we have neglected terms $\mathcal{O}(s_{13})$, whenever it was justified.

For the CP conserving cases $\delta = 0, \pi$, these formulae reduce to the formulae (24) of [10] which represent the case where both matrices, R and U_ν are real. We note that in the presence of a real matrix R , the Δ_{ij} do not depend on the Majorana phases and $\Delta_{\tau\mu}$ does not depend on δ .

B.3 R and U_ν complex

Allowing for one additional phase y_{12} in R , we find the generalization of (B.5)–(B.7) that includes CP violation both at low and high energies represented by $\delta \neq 0, \pi$ and $x_{12}, y_{12} \neq 0$, respectively

$$\Delta_{\mu e} = \frac{M_\nu}{\sqrt{2}v^2} \left(s c \tilde{\delta}_{21}(y_{12}) + e^{-i\delta} s_{13} \tilde{\delta}_{31}(y_{12}) + i\sqrt{m_{\nu 1} m_{\nu 2}} \sinh(2y_{12}) F_1(\alpha, \beta) \right), \quad (\text{B.8})$$

$$\Delta_{\tau e} = \frac{M_\nu}{\sqrt{2}v^2} \left(-s c \tilde{\delta}_{21}(y_{12}) + e^{-i\delta} s_{13} \tilde{\delta}_{31}(y_{12}) - i\sqrt{m_{\nu 1} m_{\nu 2}} \sinh(2y_{12}) F_1(\alpha, \beta) \right), \quad (\text{B.9})$$

$$\Delta_{\tau\mu} = \frac{M_\nu}{2v^2} \left(-c^2 \tilde{\delta}_{21}(y_{12}) + \tilde{\delta}_{31}(y_{12}) + 2i\sqrt{m_{\nu 1} m_{\nu 2}} \sinh(2y_{12}) F_2(\alpha, \beta, \delta) \right). \quad (\text{B.10})$$

For $y_{12} = 0$ (B.8)–(B.10) reduce to (B.5)–(B.7). We note that relative to (B.5)–(B.7) there is an additional dependence on the difference of Majorana phases $\alpha - \beta$, collected in the functions F_1 and F_2 that disappears for $y_{12} = 0$. This means that for y_{12} very close to zero Majorana phases in $l_i \rightarrow l_j \gamma$ decays do not matter but can be important already for small y_{12} .

Indeed, the Δ_{ij} 's are very sensitive to y_{12} and the values of Δ_{ij} can be enhanced by several orders of magnitude [25, 24, 28, 26, 27, 23, 31, 34] relative to the case of $R = 1$, even for $y_{12} = \mathcal{O}(1)$. Indeed as seen in (B.8)–(B.10), the Δ_{ij} depend exponentially on the y_{12} and moreover for $y_{12} \neq 0$, they do not only depend on the neutrino mass differences but also on $\sqrt{m_{\nu 1} m_{\nu 2}}$ which can be much larger than Δm_{ij} . Thus including a non-vanishing phase in R can have in principle a very strong impact on the analysis of [10] as also discussed in [13].

The large enhancement of Δ_{ij} in the case of a complex R is analogous to the large enhancement of $B(B_{d,s} \rightarrow \mu^+ \mu^-)$ for large $\tan \beta$. In the latter case the presence of new scalar operators lifts the helicity suppression of the branching ratios in question. In the case of Δ_{ij} the appearance of a new mass dependence $m_i m_j$ in addition to $m_i - m_j$ has a similar effect provided $\sqrt{m_i m_j} \gg m_i - m_j$.

References

- [1] A.J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, *Universal unitarity triangle and physics beyond the standard model*, *Phys. Lett.* **B 500** (2001) 161 [[hep-ph/0007085](#)];
A.J. Buras, *Minimal flavor violation*, *Acta Phys. Polon.* **B34** (2003) 5615 [[hep-ph/0310208](#)].
- [2] G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavour violation: an effective field theory approach*, *Nucl. Phys.* **B 645** (2002) 155 [[hep-ph/0207036](#)].
- [3] R.S. Chivukula and H. Georgi, *Composite technicolor standard model*, *Phys. Lett.* **B 188** (1987) 99;
L.J. Hall and L. Randall, *Weak scale effective supersymmetry*, *Phys. Rev. Lett.* **65** (1990) 2939.
- [4] N. Cabibbo, *Unitary symmetry and leptonic decays*, *Phys. Rev. Lett.* **10** (1963) 531;
M. Kobayashi and K. Maskawa, *CP-violation in the renormalizable theory of weak interaction*, *Prog. Theor. Phys.* **49** (1973) 652.
- [5] C. Bobeth et al., *Upper bounds on rare K and B decays from minimal flavor violation*, *Nucl. Phys.* **B 726** (2005) 252 [[hep-ph/0505110](#)].
- [6] K.S. Babu and C.F. Kolda, *Higgs-mediated $B^0 \rightarrow \mu^+ \mu^-$ in minimal supersymmetry*, *Phys. Rev. Lett.* **84** (2000) 228 [[hep-ph/9909476](#)];
A. Dedes, H.K. Dreiner and U. Nierste, *Correlation of $B_s \rightarrow \mu^+ \mu^-$ and $(g-2)(\mu)$ in minimal supergravity*, *Phys. Rev. Lett.* **87** (2001) 251804 [[hep-ph/0108037](#)];
C. Bobeth, T. Ewerth, F. Kruger and J. Urban, *Analysis of neutral Higgs-boson contributions to the decays $anti-B_s \rightarrow \ell^+ \ell^-$ and $anti-B \rightarrow K \ell^+ \ell^-$* , *Phys. Rev.* **D 64** (2001) 074014 [[hep-ph/0104284](#)]; *Enhancement of $B(anti-B_d \rightarrow \mu^+ \mu^-)/B(anti-B_s \rightarrow \mu^+ \mu^-)$ in the MSSM with minimal flavour violation and large $\tan \beta$* , *Phys. Rev.* **D 66** (2002) 074021 [[hep-ph/0204225](#)];
G. Isidori and A. Retico, *$B_{s,d} \rightarrow \ell^+ \ell^-$ and $k_l \rightarrow \ell^+ \ell^-$ in SUSY models with non-minimal sources of flavour mixing*, *JHEP* **09** (2002) 063 [[hep-ph/0208159](#)]; *Scalar flavour-changing neutral currents in the large- $\tan \beta$ limit*, *JHEP* **11** (2001) 001 [[hep-ph/0110121](#)];
A.J. Buras, P.H. Chankowski, J. Rosiek and L. Slawianowska, *$\Delta(M_s)/\Delta(M_d)$, $\sin 2\beta$ and the angle γ in the presence of new $\Delta(f) = 2$ operators*, *Nucl. Phys.* **B 619** (2001) 434 [[hep-ph/0107048](#)]; *Correlation between $\Delta M(s)$ and $B_{s,d}^0 \rightarrow \mu^+ \mu^-$ in supersymmetry at large $\tan \beta$* , *Phys. Lett.* **B 546** (2002) 96 [[hep-ph/0207241](#)]; *$\Delta(M_{d,s})$, $B_{d,s}^0 \rightarrow \mu^+ \mu^-$ and $B \rightarrow X_s \gamma$ in supersymmetry at large $\tan \beta$* , *Nucl. Phys.* **B 659** (2003) 3 [[hep-ph/0210145](#)];
A. Dedes and B.T. Huffman, *Bounding the MSSM Higgs sector from above with the Tevatron's $B_s \rightarrow \mu^+ \mu^-$* , *Phys. Lett.* **B 600** (2004) 261 [[hep-ph/0407285](#)];
A. Dedes and A. Pilaftsis, *Resummed effective lagrangian for Higgs-mediated FCNC interactions in the CP-violating MSSM*, *Phys. Rev.* **D 67** (2003) 015012 [[hep-ph/0209306](#)];
J. Foster, K.-i. Okumura and L. Roszkowski, *New Higgs effects in B physics in supersymmetry with general flavour mixing*, *Phys. Lett.* **B 609** (2005) 102 [[hep-ph/0410323](#)]; *Probing the flavour structure of supersymmetry breaking with rare B-processes: a beyond leading order analysis*, *JHEP* **08** (2005) 094 [[hep-ph/0506146](#)];
C. Kolda, *Minimal flavor violation at large $\tan \beta$* , [hep-ph/0409205](#);
M.S. Carena, A. Menon, R. Noriega-Papaqui, A. Szykman and C.E.M. Wagner, *Constraints on B and Higgs physics in minimal low energy supersymmetric models*, *Phys. Rev.* **D 74** (2006) 015009 [[hep-ph/0603106](#)];
J. Foster, K.-i. Okumura and L. Roszkowski, *New constraints on SUSY flavour mixing in light of recent measurements at the Tevatron*, *Phys. Lett.* **B 641** (2006) 452 [[hep-ph/0604121](#)].

- [7] M. Blanke, A.J. Buras, D. Guadagnoli and C. Tarantino, *Minimal flavour violation waiting for precise measurements of ΔM_s , $s_{\psi\phi}$, A_{sl}^s , $|V_{ub}|$, γ and $B_{s,d}^0 \rightarrow \mu^+\mu^-$* , *JHEP* **10** (2006) 003 [[hep-ph/0604057](#)].
- [8] M. Fukugita and T. Yanagida, *Baryogenesis without grand unification*, *Phys. Lett.* **B 174** (1986) 45.
- [9] W. Buchmuller, R.D. Peccei and T. Yanagida, *Leptogenesis as the origin of matter*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 311 [[hep-ph/0502169](#)].
- [10] V. Cirigliano, B. Grinstein, G. Isidori and M.B. Wise, *Minimal flavor violation in the lepton sector*, *Nucl. Phys.* **B 728** (2005) 121 [[hep-ph/0507001](#)].
- [11] P. Minkowski, *$\mu \rightarrow e\gamma$ at a rate of one out of 1-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421;
M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen et al., North-Holland (1979);
S.L. Glashow, in *Quarks and leptons*, Cargèse, eds. M. Lévy et al., Plenum (1980);
T. Yanagida, in *Proceedings of the workshop on the unified theory and the baryon number in the universe*, eds. O. Sawada et al., KEK Report 79-18, Tsukuba (1979);
R.N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912.
- [12] V. Cirigliano and B. Grinstein, *Phenomenology of minimal lepton flavor violation*, *Nucl. Phys.* **B 752** (2006) 18 [[hep-ph/0601111](#)].
- [13] V. Cirigliano, G. Isidori and V. Porretti, *CP-violation and leptogenesis in models with minimal lepton flavour violation*, *Nucl. Phys.* **B 763** (2007) 228 [[hep-ph/0607068](#)].
- [14] G. Altarelli, F. Feruglio and I. Masina, *Large neutrino mixing from small quark and lepton mixings*, *Phys. Lett.* **B 472** (2000) 382 [[hep-ph/9907532](#)];
E.K. Akhmedov, G.C. Branco, F.R. Joaquim and J.I. Silva-Marcos, *Neutrino masses and mixing with seesaw mechanism and universal breaking of extended democracy*, *Phys. Lett.* **B 498** (2001) 237 [[hep-ph/0008010](#)].
- [15] S. Davidson and F. Palorini, *Various definitions of minimal flavour violation for leptons*, *Phys. Lett.* **B 642** (2006) 72 [[hep-ph/0607329](#)].
- [16] R. González Felipe, F.R. Joaquim and B.M. Nobre, *Radiatively induced leptogenesis in a minimal seesaw model*, *Phys. Rev.* **D 70** (2004) 085009 [[hep-ph/0311029](#)];
F.R. Joaquim, *Radiative leptogenesis in minimal seesaw models*, *Nucl. Phys.* **145** (*Proc. Suppl.*) (2005) 276 [[hep-ph/0501221](#)].
- [17] K. Turzyski, *Degenerate minimal see-saw and leptogenesis*, *Phys. Lett.* **B 589** (2004) 135 [[hep-ph/0401219](#)].
- [18] G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim and B.M. Nobre, *Enlarging the window for radiative leptogenesis*, *Phys. Lett.* **B 633** (2006) 336 [[hep-ph/0507092](#)].
- [19] A. Pilaftsis and T.E.J. Underwood, *Electroweak-scale resonant leptogenesis*, *Phys. Rev.* **D 72** (2005) 113001 [[hep-ph/0506107](#)];
T.E.J. Underwood, *Lepton flavour effects and resonant leptogenesis*, [hep-ph/0605232](#).
- [20] A. Pilaftsis, *CP-violation and baryogenesis due to heavy Majorana neutrinos*, *Phys. Rev.* **D 56** (1997) 5431 [[hep-ph/9707235](#)].

- [21] A. Pilaftsis and T.E.J. Underwood, *Resonant leptogenesis*, *Nucl. Phys.* **B 692** (2004) 303 [[hep-ph/0309342](#)].
- [22] B. Pontecorvo, *Mesonium and antimesonium*, *Sov. Phys. JETP* **6** (1957) 429;
Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, *Prog. Theor. Phys.* **28** (1962) 870.
- [23] S.T. Petcov, T. Shindou and Y. Takanishi, *Majorana CP-violating phases, RG running of neutrino mixing parameters and charged lepton flavour violating decays*, *Nucl. Phys.* **B 738** (2006) 219 [[hep-ph/0508243](#)].
- [24] J.A. Casas and A. Ibarra, *Oscillating neutrinos and $\mu \rightarrow e, \gamma$* , *Nucl. Phys.* **B 618** (2001) 171 [[hep-ph/0103065](#)].
- [25] S. Pascoli, S.T. Petcov and C.E. Yaguna, *Quasi-degenerate neutrino mass spectrum, $\mu \rightarrow e + \gamma$ decay and leptogenesis*, *Phys. Lett.* **B 564** (2003) 241 [[hep-ph/0301095](#)].
- [26] S. Kanemura et al., *Phase effects from the general neutrino Yukawa matrix on lepton flavor violation*, *Phys. Rev.* **D 72** (2005) 055012 [[hep-ph/0507264](#)].
- [27] S. Kanemura et al., *Enhancement of lepton flavor violation in a model with bi-maximal mixing at the grand unification scale*, *Phys. Rev.* **D 72** (2005) 093004 [[hep-ph/0501228](#)].
- [28] J.R. Ellis, M. Raidal and T. Yanagida, *Observable consequences of partially degenerate leptogenesis*, *Phys. Lett.* **B 546** (2002) 228 [[hep-ph/0206300](#)];
J.R. Ellis and M. Raidal, *Leptogenesis and the violation of lepton number and CP at low energies*, *Nucl. Phys.* **B 643** (2002) 229 [[hep-ph/0206174](#)].
- [29] S.T. Petcov and T. Shindou, *Lfv radiative decays and leptogenesis in the SUSY seesaw model*, [hep-ph/0605204](#).
- [30] L. Covi, E. Roulet and F. Vissani, *CP-violating decays in leptogenesis scenarios*, *Phys. Lett.* **B 384** (1996) 169 [[hep-ph/9605319](#)].
- [31] S.T. Petcov, W. Rodejohann, T. Shindou and Y. Takanishi, *The see-saw mechanism, neutrino Yukawa couplings, LFV decays $l(i) \rightarrow l(j) + \gamma$ and leptogenesis*, *Nucl. Phys.* **B 739** (2006) 208 [[hep-ph/0510404](#)].
- [32] W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, *Deep inelastic scattering beyond the leading order in asymptotically free gauge theories*, *Phys. Rev.* **D 18** (1978) 3998.
- [33] A. Strumia and F. Vissani, *Implications of neutrino data circa 2005*, *Nucl. Phys.* **B 726** (2005) 294 [[hep-ph/0503246](#)].
- [34] F. Deppisch, H. Pas, A. Redelbach and R. Ruckl, *Constraints on SUSY seesaw parameters from leptogenesis and lepton flavor violation*, *Phys. Rev.* **D 73** (2006) 033004 [[hep-ph/0511062](#)].
- [35] T. Hambye, *Various realizations of leptogenesis and neutrino mass constraints*, [hep-ph/0412053](#), talk given at *SEESAW25: International Conference on the Seesaw Mechanism and the Neutrino Mass*, Paris, France, 10-11 Jun (2004).
- [36] G.C. Branco, T. Morozumi, B.M. Nobre and M.N. Rebelo, *A bridge between CP-violation at low energies and leptogenesis*, *Nucl. Phys.* **B 617** (2001) 475 [[hep-ph/0107164](#)];
G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim and M.N. Rebelo, *Leptogenesis, CP-violation and neutrino data: what can we learn?*, *Nucl. Phys.* **B 640** (2002) 202 [[hep-ph/0202030](#)];

- M.N. Rebelo, *Leptogenesis without CP-violation at low energies*, *Phys. Rev. D* **67** (2003) 013008 [[hep-ph/0207236](#)];
- G.C. Branco et al., *Minimal scenarios for leptogenesis and CP-violation*, *Phys. Rev. D* **67** (2003) 073025 [[hep-ph/0211001](#)].
- [37] S. Kaneko and M. Tanimoto, *Neutrino mass matrix with two zeros and leptogenesis*, *Phys. Lett. B* **551** (2003) 127 [[hep-ph/0210155](#)];
- S. Pascoli, S.T. Petcov and W. Rodejohann, *On the connection of leptogenesis with low energy CP-violation and LFV charged lepton decays*, *Phys. Rev. D* **68** (2003) 093007 [[hep-ph/0302054](#)];
- E.K. Akhmedov, M. Frigerio and A.Y. Smirnov, *Probing the seesaw mechanism with neutrino data and leptogenesis*, *JHEP* **09** (2003) 021 [[hep-ph/0305322](#)];
- L. Velasco-Sevilla, *Hierarchical neutrino mass matrices, CP-violation and leptogenesis*, *JHEP* **10** (2003) 035 [[hep-ph/0307071](#)];
- A. Ibarra and G.G. Ross, *Neutrino phenomenology: the case of two right handed neutrinos*, *Phys. Lett. B* **591** (2004) 285 [[hep-ph/0312138](#)];
- P.H. Chankowski, J.R. Ellis, S. Pokorski, M. Raidal and K. Turzyski, *Patterns of lepton flavor violation motivated by decoupling and sneutrino inflation*, *Nucl. Phys. B* **690** (2004) 279 [[hep-ph/0403180](#)];
- M. Bando, S. Kaneko, M. Obara and M. Tanimoto, *Symmetric mass matrix with two zeros in SUSY so(10) GUT, lepton flavor violations and leptogenesis*, [hep-ph/0405071](#);
- R.N. Mohapatra and S. Nasri, *Leptogenesis and $\mu - \tau$ symmetry*, *Phys. Rev. D* **71** (2005) 033001 [[hep-ph/0410369](#)];
- Z.-Z. Xing and S. Zhou, *Seesaw invariance of Fritzsche-like neutrino mass matrices, leptogenesis and lepton flavor violation*, *Phys. Lett. B* **606** (2005) 145 [[hep-ph/0411044](#)];
- N. Sahu and S. Uma Sankar, *Heavy neutrino mass hierarchy from leptogenesis in left-right symmetric models with spontaneous CP-violation*, *Nucl. Phys. B* **724** (2005) 329 [[hep-ph/0501069](#)];
- G.C. Branco, M.N. Rebelo and J.I. Silva-Marcos, *Leptogenesis, Yukawa textures and weak basis invariants*, *Phys. Lett. B* **633** (2006) 345 [[hep-ph/0510412](#)].
- [38] R. Gonzalez Felipe and F.R. Joaquim, *Is right-handed neutrino degeneracy compatible with the solar and atmospheric neutrino data?*, *JHEP* **09** (2001) 015 [[hep-ph/0106226](#)].
- [39] K. Hamaguchi, H. Murayama and T. Yanagida, *Leptogenesis from sneutrino-dominated early universe*, *Phys. Rev. D* **65** (2002) 043512 [[hep-ph/0109030](#)];
- T. Hambye, *Leptogenesis at the TeV scale*, *Nucl. Phys. B* **633** (2002) 171 [[hep-ph/0111089](#)];
- S. Davidson and A. Ibarra, *A lower bound on the right-handed neutrino mass from leptogenesis*, *Phys. Lett. B* **535** (2002) 25 [[hep-ph/0202239](#)];
- W. Buchmuller, P. Di Bari and M. Plumacher, *Cosmic microwave background, matter-antimatter asymmetry and neutrino masses*, *Nucl. Phys. B* **643** (2002) 367 [[hep-ph/0205349](#)];
- W. Buchmuller, P. Di Bari and M. Plumacher, *The neutrino mass window for baryogenesis*, *Nucl. Phys. B* **665** (2003) 445 [[hep-ph/0302092](#)];
- T. Hambye, Y. Lin, A. Notari, M. Papucci and A. Strumia, *Constraints on neutrino masses from leptogenesis models*, *Nucl. Phys. B* **695** (2004) 169 [[hep-ph/0312203](#)].
- [40] G.F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, *Towards a complete theory of thermal leptogenesis in the SM and MSSM*, *Nucl. Phys. B* **685** (2004) 89 [[hep-ph/0310123](#)].
- [41] M.Y. Khlopov and A.D. Linde, *Is it easy to save the gravitino?*, *Phys. Lett. B* **138** (1984) 265;

- J.R. Ellis, J.E. Kim and D.V. Nanopoulos, *Cosmological gravitino regeneration and decay*, *Phys. Lett.* **B 145** (1984) 181;
- M. Kawasaki and T. Moroi, *Gravitino production in the inflationary universe and the effects on big bang nucleosynthesis*, *Prog. Theor. Phys.* **93** (1995) 879 [[hep-ph/9403364](#)].
- [42] G. D'Ambrosio, G.F. Giudice and M. Raidal, *Soft leptogenesis*, *Phys. Lett.* **B 575** (2003) 75 [[hep-ph/0308031](#)];
Y. Grossman, T. Kashti, Y. Nir and E. Roulet, *Leptogenesis from supersymmetry breaking*, *Phys. Rev. Lett.* **91** (2003) 251801 [[hep-ph/0307081](#)].
- [43] WMAP collaboration, D.N. Spergel et al., *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: determination of cosmological parameters*, *Astrophys. J. Suppl.* **148** (2003) 175 [[astro-ph/0302209](#)];
SDSS collaboration, M. Tegmark et al., *Cosmological parameters from SDSS and WMAP*, *Phys. Rev. D* **69** (2004) 103501 [[astro-ph/0310723](#)].
- [44] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, *Baryogenesis through leptogenesis*, *Nucl. Phys.* **B 575** (2000) 61 [[hep-ph/9911315](#)].
- [45] A. Abada, S. Davidson, F.-X. Josse-Michaux, M. Losada and A. Riotto, *Flavour issues in leptogenesis*, *JCAP* **04** (2006) 004 [[hep-ph/0601083](#)];
A. Abada et al., *Flavour matters in leptogenesis*, *JHEP* **09** (2006) 010 [[hep-ph/0605281](#)].
- [46] E. Nardi, Y. Nir, E. Roulet and J. Racker, *The importance of flavor in leptogenesis*, *JHEP* **01** (2006) 164 [[hep-ph/0601084](#)].
- [47] S. Blanchet and P. Di Bari, *Flavor effects on leptogenesis predictions*, *JCAP* **03** (2007) 018 [[hep-ph/0607330](#)].
- [48] S. Blanchet and P. Di Bari, *Leptogenesis beyond the limit of hierarchical heavy neutrino masses*, *JCAP* **06** (2006) 023 [[hep-ph/0603107](#)].
- [49] P. Di Bari, *Leptogenesis, neutrino mixing data and the absolute neutrino mass scale*, [hep-ph/0406115](#).
- [50] F. Borzumati and A. Masiero, *Large muon and electron number violations in supergravity theories*, *Phys. Rev. Lett.* **57** (1986) 961;
J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, *Lepton flavor violation in the supersymmetric standard model with seesaw induced neutrino masses*, *Phys. Lett.* **B 357** (1995) 579 [[hep-ph/9501407](#)];
J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Lepton-flavor violation via right-handed neutrino Yukawa couplings in supersymmetric standard model*, *Phys. Rev. D* **53** (1996) 2442 [[hep-ph/9510309](#)].
- [51] P.H. Chankowski and Z. Pluciennik, *Renormalization group equations for seesaw neutrino masses*, *Phys. Lett.* **B 316** (1993) 312 [[hep-ph/9306333](#)];
K.S. Babu, C.N. Leung and J.T. Pantaleone, *Renormalization of the neutrino mass operator*, *Phys. Lett.* **B 319** (1993) 191 [[hep-ph/9309223](#)];
S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, *Neutrino mass operator renormalization revisited*, *Phys. Lett.* **B 519** (2001) 238 [[hep-ph/0108005](#)]; *Neutrino mass operator renormalization in two Higgs doublet models and the MSSM*, *Phys. Lett.* **B 525** (2002) 130 [[hep-ph/0110366](#)];
S. Antusch and M. Ratz, *Supergraph techniques and two-loop beta-functions for renormalizable and non-renormalizable operators*, *JHEP* **07** (2002) 059 [[hep-ph/0203027](#)];

- P.H. Chankowski and S. Pokorski, *Quantum corrections to neutrino masses and mixing angles*, *Int. J. Mod. Phys. A* **17** (2002) 575 [[hep-ph/0110249](#)];
S. Antusch, J. Kersten, M. Lindner and M. Ratz, *Neutrino mass matrix running for non-degenerate see-saw scales*, *Phys. Lett. B* **538** (2002) 87 [[hep-ph/0203233](#)].
- [52] S. Antusch, J. Kersten, M. Lindner and M. Ratz, *Running neutrino masses, mixings and CP phases: analytical results and phenomenological consequences*, *Nucl. Phys. B* **674** (2003) 401 [[hep-ph/0305273](#)].
- [53] S. Antusch, J. Kersten, M. Lindner, M. Ratz and M.A. Schmidt, *Running neutrino mass parameters in see-saw scenarios*, *JHEP* **03** (2005) 024 [[hep-ph/0501272](#)].
- [54] W. Buchmuller, P. Di Bari and M. Plumacher, *Cosmic microwave background, matter-antimatter asymmetry and neutrino masses*, *Nucl. Phys. B* **643** (2002) 367 [[hep-ph/0205349](#)].
- [55] T. Hambye, Y. Lin, A. Notari, M. Papucci and A. Strumia, *Constraints on neutrino masses from leptogenesis models*, *Nucl. Phys. B* **695** (2004) 169 [[hep-ph/0312203](#)].
- [56] A. Anisimov, A. Broncano and M. Plumacher, *The CP-asymmetry in resonant leptogenesis*, *Nucl. Phys. B* **737** (2006) 176 [[hep-ph/0511248](#)].
- [57] P. Di Bari, *Seesaw geometry and leptogenesis*, *Nucl. Phys. B* **727** (2005) 318 [[hep-ph/0502082](#)].
- [58] S. Davidson and A. Ibarra, *A lower bound on the right-handed neutrino mass from leptogenesis*, *Phys. Lett. B* **535** (2002) 25 [[hep-ph/0202239](#)].
- [59] S. Antusch, S.F. King and A. Riotto, *Flavour-dependent leptogenesis with sequential dominance*, *JCAP* **11** (2006) 011 [[hep-ph/0609038](#)];
S. Antusch and A.M. Teixeira, *Towards constraints on the SUSY seesaw from flavour-dependent leptogenesis*, *JCAP* **02** (2007) 024 [[hep-ph/0611232](#)].
- [60] S. Uhlig, *Minimal lepton flavour violation and leptogenesis with exclusively low-energy CP-violation*, [hep-ph/0612262](#).