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CP violation and flavor mixings in orbifold grand unified theories

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We address the flavor problem by incorporating the hypothesis of universal strength of Yukawa couplings in the framework of a 5D GUT model compactified on an $S^1/(Z_2 \times Z_2')$ orbifold. We show that a quantitatively successful picture of fermion masses and mixings emerges from the interplay between the bulk suppression factors of geometric origin and the phases of the Yukawa matrices. We give an explicit example, where we obtain a good fit for both the Cabibbo-Kobayashi-Maskawa and Pontecorvo-Maki-Nakagawa-Sakata matrices.

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I. INTRODUCTION AND MOTIVATION

The origin of the observed pattern of fermion masses and flavor mixings is a central and open question in particle physics. In an attempt to seek further insight into the flavor problem, we propose a framework which links two main hypotheses:

(i) The SU(5) gauge symmetry of a 5-dimensional (5D) grand unified theory (GUT) is broken by orbifold boundary conditions. Some matter multiplets are confined at the two orbifold fixed points, while others access the bulk [1–3]. The scalar multiplets which acquire vacuum expectation values (VEVs) are in the bulk.

(ii) Apart from the suppression factors of Yukawa couplings determined solely by the relative locations of the fermions, there is a universal strength of Yukawa couplings (USY) [4]. However, in order to incorporate CP violation, we introduce complex Yukawa couplings with flavor dependent phases.

In this paper, we elaborate on these two ideas in some detail, and illustrate how they lead to a qualitative and quantitative understanding of fermion masses and mixings.

Higher dimensional orbifold GUTs possess some advantages over the conventional ones, such as providing a simpler realization of doublet-triplet splitting which is possible as a consequence of GUT gauge symmetry breaking by boundary conditions. Embedding supersymmetry in such a higher dimensional GUT further helps in achieving gauge coupling unification. Following the initial attempts [1], a series of investigations [2] has been launched aiming at constructing a supersymmetric SU(5) gauge theory using an $S^1/(Z_2 \times Z_2')$ orbifold. Such scenarios allow for a novel handling of the flavor problem by assuming that flavor hierarchies may arise from a suppression mechanism of geometrical origin, where the suppression depends on the relative location of the fields in the extra dimension (y). The physical region stretches on a quarter-circle $y = [0, \pi R/2]$, where R is the radius of compactification. Our matter fields consist of SU(5) multiplets of different generations ($\bar{\mathbf{5}}_\mathbf{i}$, $\mathbf{10}_\mathbf{i}$), and the right-handed singlet neutrinos ($\mathbf{N}_\mathbf{i}$). The orbifold allows three locations where these matter fields can be placed: the SU(5) preserving 5D bulk, and the two fixed points at $y = 0$ (the O brane) and at $y = \pi R/2$ (the O' brane). The O brane has the full SU(5) symmetry, while at the O' brane the unbroken gauge symmetry is that of the Standard Model (SM). If a given matter field spreads into the bulk, its 4D zero mode has a wave function normalization factor, given by ϵ (see the next section for details), attached to its Yukawa coupling. This is in fact a suppression factor (typically, $\epsilon \sim 0.1$) of pure geometric origin. On the other hand, there is no such suppression when the concerned field is localized at one of the two branes. Thus, besides an overall Yukawa strength, the entries of the mass (or, Yukawa) matrices are either 1 or powers of ϵ depending on the relative localization of the matter fields.

An important feature of the previous analyses [2] is that all Yukawa couplings have been assumed to be real, thus neglecting CP violation. Moreover, for generating quantitatively successful Cabibbo-Kobayashi-Maskawa (CKM) and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices, one further needed to exploit the $\mathcal{O}(1)$ uncertainties of the coefficients of ϵ-dependent terms. In fact, it has been shown that with real Yukawa couplings, or even with complex Yukawa couplings where the phases are also decided by the location of the fields, unless one exploits these $\mathcal{O}(1)$ uncertainties, it is not possible to simultaneously satisfy even the gross qualitative features of the CKM and PMNS matrices, no matter what locations are chosen for the matter multiplets [3]. The introduction of these uncertainties, which cannot be calculated, adds a gross arbitrariness to the framework. It is not in fact

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surprising that their exact values have not been spelled out in the existing literature [2].

Clearly, the whole framework would be more appealing if the qualitative and quantitative features of both quark and lepton masses and mixings could be reproduced without having to make use of those uncertainties. This constitutes the main motivation of the present paper, where we attempt to generate a quantitatively successful pattern of fermion masses and mixings, implementing the USY hypothesis in the orbifold GUT scenarios. We take into account the phases associated with USY and the volume suppression factors related to orbifolding, without using the above mentioned $\mathcal{O}(1)$ uncertainties.

Another motivation for the present work stems from recent results from experiment. First, after the recent measurement of the angle γ of the unitarity triangle, it is now mandatory to consider complex quark mass matrices leading to a *complex* CKM matrix. Prior to this measurement, one could have had a real CKM matrix and still explain the observed CP violation in the K and B systems by invoking new physics with nonstandard phases contributing via loops to the $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixings. However, the extraction of γ from experimental data involves tree level process where new physics is unlikely to compete with the SM contributions. Thus the measurement of γ provides an irrefutable evidence for a complex CKM matrix [5], which in the present framework implies complex Yukawa couplings. Second, although the USY hypothesis in the SM has been shown to reproduce the correct pattern of quark masses and mixings, it leads in general to a rather small value for the strength of CP violation, measured by the imaginary part of any rephasing-invariant CKM matrix quartet. We will show that when incorporated in a higher dimensional orbifold framework, the USY hypothesis can lead to a sufficiently large strength of CP violation.

II. FORMALISM OF GUT ORBIFOLDING

Our formalism is based on a 5D GUT scenario with $N = 1$ supersymmetry where the gauge symmetry is SU(5). The extra dimension (y) is compactified on an orbifold $S^1/(Z_2 \times Z_2')$, and the inverse radius R^{-1} is chosen to be of the order of 10^{16} Gev. S^1 is first divided by Z_2 ($y \leftrightarrow -y$) and then further by Z_2' ($y' \leftrightarrow -y'$ with $y' = y + \pi R/2$), restricting the physical spacetime within the interval $[0, \pi R/2]$. The O and O' branes are located at the fixed points $y = 0$ and $y = \pi R/2$ respectively. Now, let us consider a generic 5D bulk field $\phi(x^\mu, y)$. The Z_2 and Z_2' parities (called P and P', respectively) act on this field as

$$\begin{aligned} \phi(x^\mu, y) &\rightarrow \phi(x^\mu, -y) = P\phi(x^\mu, y), \\ \phi(x^\mu, y') &\rightarrow \phi(x^\mu, -y') = P'\phi(x^\mu, y'). \end{aligned} \tag{1}$$

The 4D Kaluza-Klein (KK) towers with $(P, P') = (\pm, \pm)$ are the following: $\phi_{++}^{(2n)}$ with a mass $2n/R$, $\phi_{+-}^{(2n+1)}$ and $\phi_{-+}^{(2n+1)}$ with a mass $(2n+1)/R$ and $\phi_{--}^{(2n+2)}$ with a mass $(2n+2)/R$. Thus only $\phi_{++}^{(2n)}$ have massless modes. Also, only ϕ_{++} and ϕ_{+-} have nonvanishing components at the O brane. From a 4D perspective we have two different $N = 1$ supersymmetries. Now assigning suitable (P, P') quantum numbers to the fields, it is possible to project out one $N = 1$ supersymmetry leaving the other unbroken.

The choice of $P = (+ + + + +)$ and $P' = (- - - + +)$ or $(+ + + - -)$ acting on a **5** keeps SU(5) intact at O but breaks SU(5) to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ at O'. With the above P' assignments it is not possible to fill up a complete SU(5) multiplet with zero modes. We need to introduce $\bar{\mathbf{5}}'$ and $\mathbf{10}'$ with P' assignments opposite to those in $\bar{\mathbf{5}}$ and $\mathbf{10}$ in order to obtain the correct low energy matter content.

We include all Yukawa couplings that are consistent with gauge symmetry and R-parity. We assume that the Higgs multiplets always reside in the bulk. If we denote a Yukawa coupling involving three brane superfields by λ, then the one where one of the three fields is a bulk field is $\lambda/\sqrt{M_* R}$, where M_* is the ultraviolet cutoff of the 5D theory. The appearance of M_* is related to the canonical normalization of the zero mode kinetic terms. It has been shown in Ref. [2] that $(M_* R) \sim 10^{2-3}$ is a good choice for gauge coupling unification. So $\epsilon = 1/\sqrt{M_* R} \sim 0.1$ acts as a suppression factor. The number of such factors multiplying λ is given by the number of bulk zero modes in a given interaction, each bulk field contributing with one ϵ.

III. QUARK AND LEPTON MASS MATRICES

We write the fermion mass matrices in the convention that the fields on the left are left-handed and those on the right are right-handed. The up quark mass terms are given by $\overline{\mathbf{10}}_i (M_u)_{ij} \mathbf{10}_j^{\mathbf{c}}$, with M_u symmetric. The down quark mass terms are written as $\overline{\mathbf{10}}_i (M_d)_{ij} \mathbf{5}_j^{\mathbf{c}}$, while the charged lepton mass matrix M_l is simply $(M_d)^\dagger$. The effective light neutrino Majorana mass matrix terms are given by $\bar{\mathbf{5}}_i (M_\nu^{\mathrm{Maj}})_{ij} \bar{\mathbf{5}}_j$, after integrating out the $\mathbf{N_i}$ states.

In order to obtain the flavor structure of the fermion mass matrices, we have to specify the matter multiplets of the different generations. Although there is no strict criteria in this regard, let us be guided by some phenomenological considerations: (i) we place $\bar{\mathbf{5}}_{\mathbf{1}}$ and $\mathbf{10}_{\mathbf{1}}$ in SU(5) bulk, then the first generation zero mode quarks and leptons come from different SU(5) multiplets, which prevents proton decay to occur at leading order; (ii) we place $\bar{\mathbf{5}}_{\mathbf{3}}$ and $\mathbf{10}_{\mathbf{3}}$ at the SU(5) preserving O brane, which helps to ensure $m_b = m_\tau$ at the GUT scale; (iii) we choose to put $\bar{\mathbf{5}}_{\mathbf{2}}$ in the bulk and $\mathbf{10}_{\mathbf{2}}$ at the O brane; finally (iv) all $\mathbf{N}_i$ are placed in the bulk, as it is difficult to confine on a brane a particle which does not have any gauge charge.

With the above assignments, we can write the different mass matrices as:

$$M_u = \lambda_u \begin{bmatrix} \epsilon_u^2 & \epsilon_u & \epsilon_u \\ \epsilon_u & 1 & e^{ia_u} \\ \epsilon_u & e^{ia_u} & e^{ib_u} \end{bmatrix}; \qquad M_d = \lambda_d \begin{bmatrix} \epsilon_d^2 & \epsilon_d^2 & \epsilon_d \\ \epsilon_d & \epsilon_d e^{ia_d} & 1 \\ \epsilon_d & \epsilon_d e^{ib_d} & e^{ic_d} \end{bmatrix}; \tag{2}$$

$$M_\nu = \lambda_\nu \begin{bmatrix} \epsilon_\nu^2 & \epsilon_\nu^2 e^{ia_\nu} & -\epsilon_\nu \\ \epsilon_\nu^2 e^{ia_\nu} & \epsilon_\nu^2 e^{ib_\nu} & -\epsilon_\nu \\ -\epsilon_\nu & -\epsilon_\nu & 1 \end{bmatrix}; \qquad M_l = \rho M_d^\dagger. \tag{3}$$

A few comments are in order. As shown in [4], within USY, in general in each charged sector, only four phases contribute to the spectrum of fermion masses. By making appropriate weak-basis transformations, one can make some changes in the location of the phases, without altering the physical consequences of the model. In Eq. (2) we have just made one such convenient choice. Since the ϵ_f's are by definition positive, the negative signs in some entries in the neutrino mass matrix correspond to a phase choice of π. In this paper we do not scan the entire set of phase choices, but just provide an example which leads to the right mixing and mass spectrum, both in the quark and lepton sectors. The geometric suppression factors and the USY phases are in general flavor dependent. The factor ρ appearing before the charged lepton mass matrix deserves some mention. We recall that zero mode bulk matter fields of a given generation arise from different SU(5) multiplets. Yet, for simplicity and predictability, we maintain the usual relationship between M_l and $M_d^\dagger$ modulo an overall ρ factor. The deviation of ρ from unity may accrue due to induced effects from the O' brane. Renormalization group running effects too may partly account for such deviation, although a detailed study is beyond the scope of the present paper. From a more practical point of view, for numerical fitting of the charged lepton mass matrix, we require $\rho \sim 0.6$.

Notice that each of these matrices is of type $M = \lambda D_1 M^{\mathrm{USY}} D_2$, where $D_{1,2} = \mathrm{diag}\ (\epsilon^n, \epsilon^m, 1)$ for n, $m = 0, 1$. The matrix $M^{\mathrm{USY}} = (e^{ia_{ij}})$ has a USY-type texture, i.e. all Yukawa couplings in a given matrix have identical moduli but differ in some complex phases. As an example, one can write the down quark mass matrix of Eq. (2) as

$$M_d = \lambda_d \,\mathrm{diag}(\epsilon_d, 1, 1) \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{ia_d} & 1 \\ 1 & e^{ib_d} & e^{ic_d} \end{bmatrix} \mathrm{diag}(\epsilon_d, \epsilon_d, 1). \tag{4}$$

To study the parameter space, it is instructive to calculate some invariants of $H_f = M_f M_f^\dagger$. We first define the dimensionless matrix $H_f' \equiv H_f/t$, where $t = \mathrm{Tr}(H_f)$. Then noting that

$$\det(H_f') = \frac{(\frac{m_1}{m_3})^2(\frac{m_2}{m_3})^2}{(1 + (\frac{m_1}{m_3})^2 + (\frac{m_2}{m_3})^2)^3}, \tag{5}$$

and using the known fermion mass hierarchies, one obtains in leading order of ϵ_u, ϵ_d and ϵ_ν:

$$\begin{aligned} \frac{1}{2}\epsilon_u^2 \sin^2\Big(\frac{a_u}{2}\Big) &= \Big(\frac{m_u}{m_t}\Big)\Big(\frac{m_c}{m_t}\Big); \\ \sqrt{2}\epsilon_d^3 \Big|\sin\Big(\frac{a_d}{2}\Big)\sin\Big(\frac{c_d}{2}\Big)\Big| &\simeq \Big(\frac{m_d}{m_b}\Big)\Big(\frac{m_s}{m_b}\Big); \\ 4\epsilon_\nu^4 \sin^2\Big(\frac{a_\nu}{2}\Big) &= \frac{(\frac{m_{\nu 1}}{m_{\nu 3}})(\frac{m_{\nu 2}}{m_{\nu 3}})}{(1 + (\frac{m_{\nu 1}}{m_{\nu 3}})^2 + (\frac{m_{\nu 2}}{m_{\nu 3}})^2)^{3/2}}; \end{aligned} \tag{6}$$

where (except for the neutrinos) we kept only the leading order terms in the masses. Just to have a numerical feel, taking e.g. $\epsilon_u = 0.1$ leads to $|a_u| = (4 - 9) \times 10^{-3}$. Now we construct the second invariant of H_f', given by $\chi(H_f') = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$, where λ_i's denote the H_f' eigenvalues. In terms of fermion mass ratios,

$$\chi(H') = \Big(\frac{m_2}{m_3}\Big)^2 \frac{1 + (\frac{m_1}{m_2})^2 + (\frac{m_1}{m_3})^2}{(1 + (\frac{m_1}{m_3})^2 + (\frac{m_2}{m_3})^2)^2}. \tag{7}$$

Using Eqs. (2), (3), and (7), one obtains the following relations in leading order of ϵ_u, ϵ_d and ϵ_ν:

$$\begin{aligned} \chi(H_u') &= \frac{1}{4}\sin^2\Big(a_u - \frac{b_u}{2}\Big) = \Big(\frac{m_c}{m_t}\Big)^2; \\ \chi(H_d') &= \epsilon_d^2\Big[\sin^2\Big(\frac{c_d}{2}\Big) + \sin^2\Big(\frac{a_d + c_d - b_d}{2}\Big)\Big] = \Big(\frac{m_s}{m_b}\Big)^2; \\ \chi(H_\nu') &= 4\epsilon_\nu^2\Big[2\sin^2\Big(\frac{a_\nu}{2}\Big) + \sin^2\Big(\frac{b_\nu}{2}\Big)\Big] \\ &= \Big(\frac{m_{\nu 2}}{m_{\nu 3}}\Big)^2 \frac{1 + (\frac{m_{\nu 1}}{m_{\nu 2}})^2 + (\frac{m_{\nu 1}}{m_{\nu 3}})^2}{(1 + (\frac{m_{\nu 1}}{m_{\nu 3}})^2 + (\frac{m_{\nu 2}}{m_{\nu 3}})^2)^2}; \end{aligned} \tag{8}$$

where we have kept only the leading order of the mass ratios of the quarks and charged leptons, while for the neutrinos the right-hand side is exact.

The above matrices can be diagonalized through:

$$\begin{aligned} V_u^\dagger M_u M_u^\dagger V_u &= D_u^2, & V_d^\dagger M_d M_d^\dagger V_d &= D_d^2, \\ V_l^\dagger M_l M_l^\dagger V_l &= D_l^2, & V_\nu^{\mathrm{T}} M_\nu^{\mathrm{Maj}} V_\nu &= D_\nu, \end{aligned} \tag{9}$$

where the D_f are diagonal matrices containing the masses of the fermions for each $f = u, d, l, \nu$. Notice that the diagonalization of the neutrino mass matrix is somewhat different, because it is a symmetric matrix obtained after the integration of the bulk $\mathbf{N_i}$ states. The physical mixing matrices in the quark and lepton sectors are given by:

$$V_{\mathrm{CKM}} = V_u^\dagger V_d; \qquad V_{\mathrm{PMNS}} = V_l^\dagger V_\nu. \tag{10}$$

In the next section, we provide an explicit example where the correct spectrum of fermion masses and mixing is obtained, including $J = \mathrm{Im}(V_{us}V_{cb}V_{ub}^* V_{cs}^*)$, which controls the strength of CP violation in the quark sector. In order to obtain a sufficiently large value of J, it is crucial to have ϵ_d not exactly equal to ϵ_u. To understand the requirement $\epsilon_d \neq \epsilon_u$, let us first recall the identity $\mathrm{Tr}\,([H_u, H_d]^3) = 6\Delta J$, where $\Delta = (m_t^2 - m_u^2) \times (m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)$. For simplicity of presentation, we take the limit $m_u = 0$ setting $a_u = 0$. If we now take $\epsilon_d = \epsilon_u = \epsilon$, and employ Eqs. (6) and (8) we obtain

$$|J| \approx \frac{1}{\sqrt{2}}\epsilon^5\left(\frac{m_d}{m_s}\right)\left|\sin\left(\frac{a_d}{2}\right)\right| \times \left|2\sin^2\left(\frac{a_d}{2}\right) - \sin^2\left(\frac{a_d - b_d}{2}\right)\right|. \quad (11)$$

Putting $\epsilon = 0.1$ leads to $|J| \approx 10^{-7}$, which is too small. On the other hand, if $\epsilon_d \neq \epsilon_u$, one finds in leading order,

$$|J| \approx \frac{1}{\sqrt{2}}|\epsilon_d - \epsilon_u|\left(\frac{m_d}{m_s}\right)\left|\sin\left(\frac{a_d - b_d}{2}\right)\cos\left(\frac{b_d}{2}\right)\right| \quad (12)$$

which for $|\epsilon_d - \epsilon_u| = 4 \times 10^{-3}$ can be as large as 10^{-5}. We checked that these estimates are very close to the exact numbers obtained without any approximation.

We recall that, in the conventional 4D USY case, $|J|$ is quite small because all phases have to be small due to the strong quark mass hierarchy and due to the smallness of $|V_{ub}|$ and $|V_{cb}|$. Here, with the embedding in a 5D orbifold GUT, some of the phases, e.g. in the down quark sector, may be quite large because the mass hierarchy is now ensured through powers of ϵ entering the mass ratio relations, as can be seen from Eqs. (6) and (8). This is essentially the main reason why in the present model a sufficiently large value of $|J|$ can be obtained. As we have just seen, the effect is magnified even by a slight variation from the equality between ϵ_d and ϵ_u, namely $|\epsilon_d - \epsilon_u|/(\epsilon_d + \epsilon_u) = \mathcal{O}(1\%)$.

IV. AN EXPLICIT NUMERICAL EXAMPLE

In this example, the charged lepton quark matrix M_l has the same structure as the Hermitian conjugate of the down quark mass matrix M_d. However, for a good numerical fit, we have to choose different values for the phases. This is to avoid exact mass ratio relations between the charged leptons and down quarks. Thus we choose,

$$M_l = \rho(M_d')^\dagger \quad (13)$$

where

$$M_d' = \begin{bmatrix} \epsilon_d^2 & \epsilon_d^2 & \epsilon_d \\ \epsilon_d & \epsilon_d e^{ia_d'} & 1 \\ \epsilon_d & \epsilon_d e^{ib_d'} & e^{ic_d'} \end{bmatrix}.$$

Using Eqs. (2), (3), and (13) and with the input

$$\begin{gathered} \lambda_u = 89.55\ \text{GeV} \qquad \epsilon_u = 0.1 \qquad a_u = -0.004\,41 \\ b_u = 0.007 \qquad \lambda_d = 2.09\ \text{GeV} \qquad \epsilon_d = 0.1046 \\ a_d = 0.87 \qquad b_d = 0.495 \qquad c_d = 0.084; \\ \rho = 0.59 \qquad a_d' = 0.06 \qquad b_d' = -0.1 \\ c_d' = 0.8 \qquad \lambda_\nu = 0.0387\ \text{eV} \qquad \epsilon_\nu = 0.45 \\ a_\nu = 0.2 \qquad b_\nu = 0.43 \end{gathered} \quad (14)$$

one obtains the following output for the quark masses (at the weak scale):

$$\begin{gathered} m_u = 1.1\ \text{MeV} \qquad m_c = 709\ \text{MeV} \qquad m_t = 180\ \text{GeV}; \\ m_d = 3.42\ \text{MeV} \qquad m_s = 72.4\ \text{MeV} \qquad m_b = 3.0\ \text{GeV}; \end{gathered} \quad (15)$$

CKM mixing:

$$\begin{gathered} |V_{\text{CKM}}| = \begin{bmatrix} 0.9755 & 0.2201 & 0.0039 \\ 0.2199 & 0.9748 & 0.0374 \\ 0.0102 & 0.0362 & 0.9993 \end{bmatrix}; \\ \left|\frac{V_{ub}}{V_{cb}}\right| = 0.104 \qquad |J| = 2.91 \times 10^{-5}; \\ \sin(2\beta) = 0.65; \end{gathered} \quad (16)$$

charged lepton and neutrino masses:

$$\begin{gathered} m_e = 0.50\ \text{MeV} \qquad m_\mu = 105.2\ \text{MeV} \\ m_\tau = 1.77\ \text{GeV}; \qquad \Delta m_{21}^2 = 7.09 \times 10^{-5}\ \text{eV}^2 \\ \Delta m_{31}^2 = 2.89 \times 10^{-3}\ \text{eV}^2 \qquad m_{\nu 3} = 0.0504\ \text{eV}; \end{gathered} \quad (17)$$

and PMNS mixing:

$$\begin{gathered} |V_{\text{PMNS}}| = \begin{bmatrix} 0.8379 & 0.5396 & 0.0821 \\ 0.3925 & 0.6759 & 0.6337 \\ 0.3792 & 0.5020 & 0.7773 \end{bmatrix}; \\ \begin{matrix} \sin^2(\theta_{12}) = 0.293 \\ \sin^2(\theta_{23}) = 0.392 \\ |U_{13}|^2 = 0.0067; \end{matrix} \quad ; \qquad |J'| = 0.0126, \end{gathered} \quad (18)$$

where J' measures the strength of CP violation in the lepton sector, which is expected to be large because this sector involves two large mixing angles.

V. CONCLUSIONS

We have shown that the ansatz of universality of strength of Yukawa couplings, when implemented on a 5D orbifold GUT model, can accommodate the observed pattern of quark and lepton masses and mixings. This is achieved by taking into account the geometric suppression factors, arising from the relative locations of the fermion fields, which appear as powers of ϵ, and allowing for complex phases in the Yukawa couplings. We reiterate that in view of recent experimental data, the CKM matrix is necessarily complex, even if one allows for the presence of physics beyond the SM. This renders mandatory the introduction of complex Yukawa couplings which, apart from the geometric suppression factors, have all the same modulus within the framework considered. It is remarkable that a good fit of the fermion masses and mixings is obtained, without having to invoke the order one uncertainties in the moduli of the Yukawa matrix elements.

Another important point is the fact that embedding USY in a 5D orbifold GUT enables one to obtain a correct value for the rephasing invariant J in the quark sector, which is too small in the conventional 4D USY picture. *CP* violation in the lepton sector [6] is not measured yet, but it is expected to be large due to large leptonic mixing.

In addition, the higher dimensional embedding permits large mixing in the neutrino mass matrix even with hierarchical neutrinos. Note that in the 4D context, when one imposes USY on the effective light neutrino mass matrix, neutrinos have to be necessarily quasidegenerate in order to achieve large mixing [7].

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