PHYSICAL REVIEW D **77**, 091504(R) (2008)

Study of the gluon-quark-antiquark static potential in SU(3) lattice QCD

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(Received 17 April 2007; revised manuscript received 20 September 2007; published 21 May 2008)

We study the long distance interaction for hybrid hadrons, with a static gluon, a quark and an antiquark with lattice QCD techniques. A Wilson loop adequate to the static hybrid three-body system is developed and, using a $24^3 \times 48$ periodic lattice with $\beta = 6.2$ and $a \sim 0.072$ fm, two different geometries for the gluon-quark segment and the gluon-antiquark segment are investigated. When these segments are perpendicular, the static potential is compatible with confinement realized with a pair of fundamental strings, one linking the gluon to the quark and another linking the same gluon to the antiquark. When the segments are parallel and superposed, the total string tension is larger and agrees with the Casimir scaling measured by Bali. This can be interpreted with a type-II superconductor analogy for the confinement in QCD, with repulsion of the fundamental strings.

DOI: 10.1103/PhysRevD.77.091504 PACS numbers: 12.38.Gc, 12.38.Aw, 12.39.Mk

I. INTRODUCTION

Here we explore the static potential of the hybrid three-body system composed of a gluon, a quark and an antiquark using lattice QCD methods. The Wilson loop method was devised to extract from pure-gauge QCD the static potential for constituent quarks and to provide detailed information on the confinement in QCD. In what concerns gluon interactions, the first lattice studies were performed by Michael [1,2] and Bali extended them to other SU(3) representations [3]. Recently Okiharu and colleagues [4,5] extended the Wilson loop for three-quark baryons to tetraquarks and to pentaquarks. Our study of hybrids continues the lattice QCD mapping of the static potentials for exotic hadrons.

The interest in hybrid three-body gluon-quark-antiquark systems is increasing in anticipation to the future experiments BESIII at IHEP in Beijin, GLUEX at JLab and PANDA at GSI in Darmstadt, dedicated to study the mass range of the charmonium, with a focus on its plausible hybrid excitations. Moreover, several evidences of a gluon effective mass of 600–1000 MeV from the lattice QCD gluon propagator in Landau gauge [6,7], from Schwinger-Dyson and Bogoliubov-Valatin solutions for the gluon propagator in Landau gauge [8], from the analogy of confinement in QCD to superconductivity [9], from the lattice QCD breaking of the adjoint string [1], from the lattice QCD gluonic excitations of the fundamental string [10] from constituent gluon models [11–13] compatible with the lattice QCD glueball spectra [14–17], and with the Pomeron trajectory for high energy scattering [18,19], may be suggesting that the static interaction for gluons is relevant.

Importantly, an open question has been residing in the potential for a hybrid system, where the gluon is a color octet, and where the quark and antiquark are combined to produce a second color octet. While the constituent quark (antiquark) is usually assumed to couple to a fundamental string, in constituent gluon models the constituent gluon is usually assumed to couple to an adjoint string. Notice that in lattice QCD, using the adjoint representation of SU(3), Bali [3] found that the adjoint string is compatible with the Casimir scaling, where the Casimir invariant $\lambda_i \cdot \lambda_j$ produces a factor of $9/4$ from the $q\bar{q}$ interaction to the gg interaction. Thus we already know that the string tension, or energy per unit length, of the adjoint string is 1.125 times larger than the sum of the string tension of two fundamental strings. How can these two pictures, of one adjoint string and of two fundamental strings, with different total string tensions, match? This question is also related to the superconductivity model for confinement, is QCD similar to a type-I or type-II superconductor? Notice that in type type-II superconductors the flux tubes repel each other while in type-I superconductors they attract each other and tend to fuse in excited vortices [20]. This is sketched in Fig. 1. String-string interactions have also been studied in 2 + 1 dimension lattice QCD

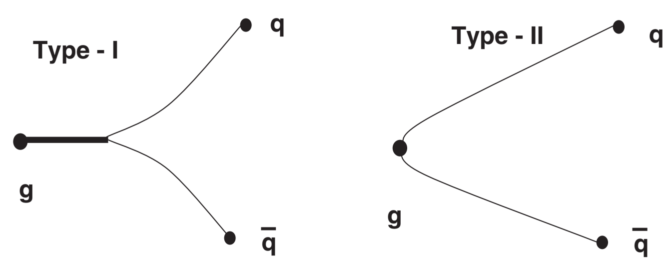

FIG. 1. String attraction and fusion, and string repulsion, respectively, in type-I and type-II superconductors.

[21–24]. The understanding of the hybrid potential will answer these questions for 3 + 1 dimension lattice QCD.

In Sec. II we produce a Wilson loop adequate to study the static hybrid potential. In Sec. III we present the results of our Monte Carlo simulation. In Sec. IV we interpret the results and conclude.

II. HYBRID WILSON LOOP

In principle, any Wilson loop with a geometry similar to the one in Fig. 2 and describing correctly the quantum numbers of the hybrid is adequate, although the signal to noise ratio may depend on the choice of the Wilson loop. A correct Wilson loop must include a SU(3) octet, the gluon, a SU(3) triplet, the quark and a SU(3) antitriplet, the antiquark. It must also include the connection between the three links of the gluon, the quark and the antiquark.

In the limit of infinite quark mass, a nonrelativistic potential V can be derived from the large time behavior of Euclidean time propagators. Typically, one has a meson operator $\mathcal{O}$ and computes the Green function,

$$\langle 0|\mathcal{O}(t)\mathcal{O}(0)|0\rangle \longrightarrow \exp\{-Vt\} \quad (1)$$

for large t. Different types of operators allow the definition

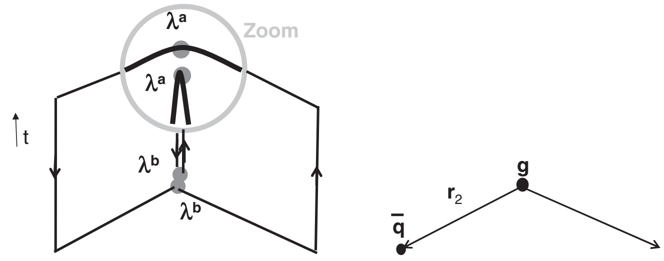

FIG. 2. Wilson loop for the $q\bar{q}g$ potential, and equivalent position of the static antiquark, gluon, and quark.

of different potentials. In the static gluon-quark-antiquark interaction, the static gluon can be replaced by a static quark-antiquark pair in a color octet representation. In this way, we can construct the gluon-quark-antiquark Wilson loop starting from the mesonic operator,

$$\mathcal{O}(x) = \tfrac{1}{4}[\bar{q}(x)\lambda^a\Gamma_1 q(x)][\bar{q}(x)\lambda^a\Gamma_2 q(x)], \quad (2)$$

where Γ_i are spinor matrices. Using the lattice links to comply with gauge invariance, the second operator in Eq. (2) can be made nonlocal to separate the quark and the antiquark from the gluon,

$$\mathcal{O}(x) = \tfrac{1}{4}[\bar{q}(x)\lambda^a\Gamma_1 q(x)][\bar{q}(x - r_1\hat{\mu}_1)U_{\mu_1}(x - r_1\hat{\mu}_1)\cdots U_{\mu_1}(x - \hat{\mu}_1)\lambda^a\Gamma_2 U_{\mu_2}(x)\cdots U_{\mu_2}(x + (r_2 - 1)\hat{\mu}_2)q(x + r_2\hat{\mu}_2)]. \quad (3)$$

The nonrelativistic potential requires the computation of the Green functions present in Eq. (1). Assuming that all quarks are of different nature, the contraction of the quark field operators gives rise to the gluon operator,

$$\begin{aligned} W_O = \frac{1}{16}&\mathrm{Tr}\{U_4^\dagger(t-1,x)\cdots U_4^\dagger(0,x)\lambda^b U_4(0,x)\cdots U_4(t-1,x)\lambda^a\}\times \mathrm{Tr}\{U_{\mu_2}(t,x)\cdots U_{\mu_2}(t,x+(r_2-1)\hat{\mu}_2) \\ &\times U_4^\dagger(t-1,x+r_2\hat{\mu}_2)\cdots U_4^\dagger(0,x+r_2\hat{\mu}_2)U_{\mu_2}^\dagger(0,x+(r_2-1)\hat{\mu}_2)\cdots U_{\mu_2}^\dagger(0,x)\lambda^b \\ &\times U_{\mu_1}^\dagger(0,x-\hat{\mu}_1)\cdots U_{\mu_1}^\dagger(0,x-r_1\hat{\mu}_1)U_4(0,x-r_1\hat{\mu}_1)\cdots U_4(t-1,x-r_1\hat{\mu}_1) \\ &\times U_{\mu_1}(t,x-r_1\hat{\mu}_1)\cdots U_{\mu_1}(t,x-\hat{\mu}_1)\lambda^a\}. \end{aligned} \quad (4)$$

Gauge invariance of (4) can be proven with the help of the relation

$$\sum_a\left(\frac{\lambda^a}{2}\right)_{ij}\left(\frac{\lambda^a}{2}\right)_{kl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{6}\delta_{ij}\delta_{kl}. \quad (5)$$

How does our operator relate with the operators used so far to investigate the gluon interactions on the lattice? The gluonic timelike links used by Michael and colleagues [2,25] to study the glue lump are the real 8×8 matrices,

$$U_4^{\mathrm{Adj}\alpha\beta} = \tfrac{1}{2}\mathrm{Tr}\{U_4\lambda^\alpha U_4^\dagger\lambda^\beta\}, \quad (6)$$

built from the usual SU(3) fundamental representation links U_i, whereas in the investigation of Casimir scaling by Bali [3], the author worked directly with adjoint links, i.e. with the 8×8 matrix SU(3) representation. If one now compares the Wilson loop in Eq. (4) with Eq. (6), it follows that, when t corresponds to a single lattice spacing, then the gluonic trace, i.e. the first trace, of Eq. (4) is a "Michael link." Notice that when the quark and antiquark are superposed, they become equivalent to a gluon. Then all three operators (our operator, the operator of Michael and colleagues and the operator of Bali) couple to the same quantum numbers and thus, variationally, the respective results should be equivalent.

III. THE STATIC HYBRID POTENTIAL

In this paper we consider two possible hybrid geometries: $\perp$ with the quark-gluon segment perpendicular ($\widehat{\mathbf{r}_1, \mathbf{r}_2} = \pi/2$) to the antiquark-gluon segment and $\|$ with the quark-gluon segment parallel ($\widehat{\mathbf{r}_1, \mathbf{r}_2} = 0$) to the antiquark-gluon segment. We denote the potentials, respectively, $V_\perp(r_1, r_2)$ and $V_\|(r_1, r_2)$, where r_1 (r_2) is the quark-gluon (antiquark-gluon) distance in lattice units, defined in Fig. 2 and in Eq. (4).

We now discuss the results of our simulation with 142 configurations of a $24^3 \times 48$ lattice with a $\beta = 6.2$ pure-gauge Wilson SU(3) action. The configurations are generated with the version 6 of the MILC code [26], via a combination of Cabbibo-Mariani and over-relaxed updates. In order to improve the signal to noise ratio, the links are replaced by "fat links" [27]

$$U_\mu(\mathbf{s}) \to \frac{U_\mu(\mathbf{s}) + w \sum_{\nu \neq \pm\mu} U_\nu(\mathbf{s}) U_\mu(\mathbf{s}+\nu) U_\nu^\dagger(\mathbf{s}+\mu)}{1+6w} \tag{7}$$

followed by a projection into SU(3). We simply with $w = 0.2$ and, then, one iteration in the time with $w = 1.0$. The temporal smearing slightly reduces the short-range Coulomb potential but produces a clearer signal for the long-range potential, the one we are interested in. Furthermore, to improve the quality of the signal, we explore the symmetry $r_1 \leftrightarrow r_2$ when computing $V_\perp(r_1, r_2)$ and $V_\parallel(r_1, r_2)$.

Using Eq. (1), the static potentials are extracted from the fit of minus the log of the Wilson loop, $-\log W$, for large Euclidean time t. This fit provides us with the potential, and we estimate the respective error bar with the jackknife method. In our results for the static hybrid potentials $V_\perp$ and $V_\parallel$ are displayed in Fig. 3. To illustrate the fit of $-\log W$ as a function of t, an example is displayed in Fig. 4.

We are interested in large distances, to compare the different possible string tensions. With 24^3 (space) $\times 48$ (time) lattices with periodic boundary conditions, the maximum distance we reach is $12a$. With a lattice spacing $a \sim 0.072$ fm, our maximal distance is still comfortably shorter than the string breaking distance [28], which is larger than 1 fm for 3 + 1 dimension lattice QCD, and comfortably longer than the perturbative distance of say, 0.3 fm. To get the string tensions σ, we fit the potentials values with funnel potentials, including only the points with $4 \leq r_1, r_2 \leq 12$.

The string tension is the coefficient of the linear component of the potential. Assuming a fundamental $q\bar{q}$ string tension $\sqrt{\sigma_0} = 440$ MeV as in Bali and Schilling [29], we get an inverse lattice spacing of $a^{-1} = 2718 \pm 32$ MeV. To study the onset of two fundamental strings, we plot in Fig. 5 the perpendicular geometry potential $V_\perp$ as a function of the sum of the two distances in lattice spacing units, r_1 between the quark and the gluon and r_2 between the antiquark and the gluon, as in Eq. (4). Indeed the potential is linear in the sum of the distances, if we subtract the Coulomb terms. $V_\perp(r_1, r_2)$ to $c_0 - \gamma(\frac{1}{r_1} + \frac{1}{r_2}) + \sigma(r_1 + r_2)$, this ansatz corresponds to having two independent strings linking the quark or the antiquark to gluon, with σ being the string tension and γ the coefficient a Luscher-like term. Since our aim here is the string tension, we leave more detailed studies of the short-range potential for future studies.

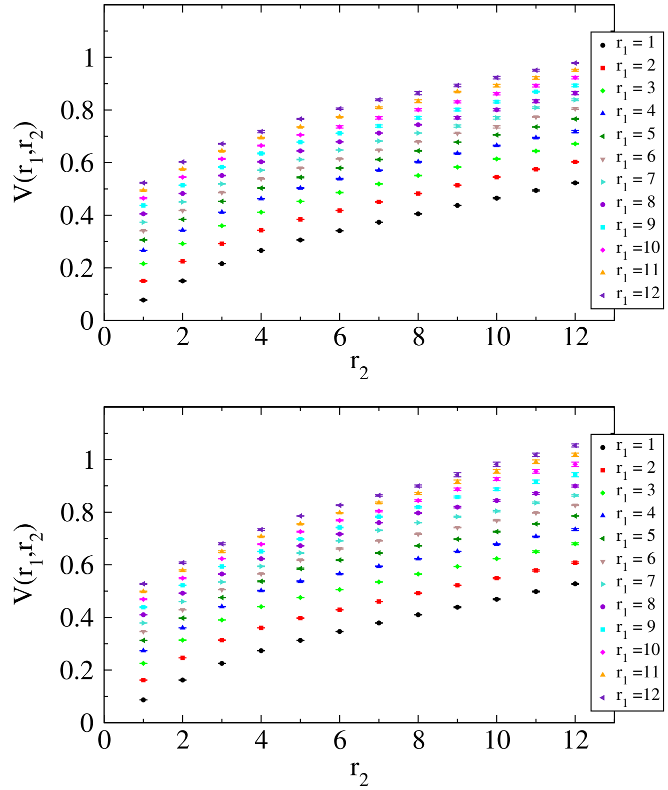

FIG. 3 (color online). Potential in lattice spacing units, for the system $q\bar{q}g$, respectively (top) for the $\perp$ and (bottom) for the $\parallel$ geometries.

We get

$$\sigma = (1.03 \pm 0.03)\sigma_0(\chi^2/dof = 1.54), \tag{8}$$

which is consistent with $\sigma = \sigma_0$, reinforcing the picture that when the quark and antiquark are distant we have two

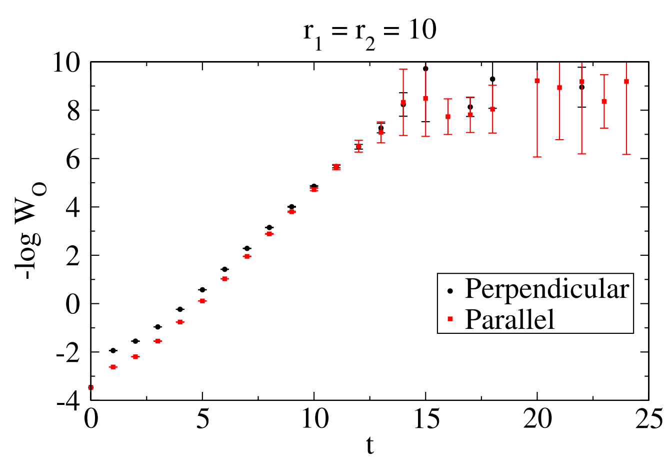

FIG. 4 (color online). Graph of $-\log W_O$ as a function of t, in lattice units, for $r_1 = r_2 = 10$. Notice that the fit to calculate $V(r_1, r_2, t)$ used the interval from $t = 5$ to $t = 15$.

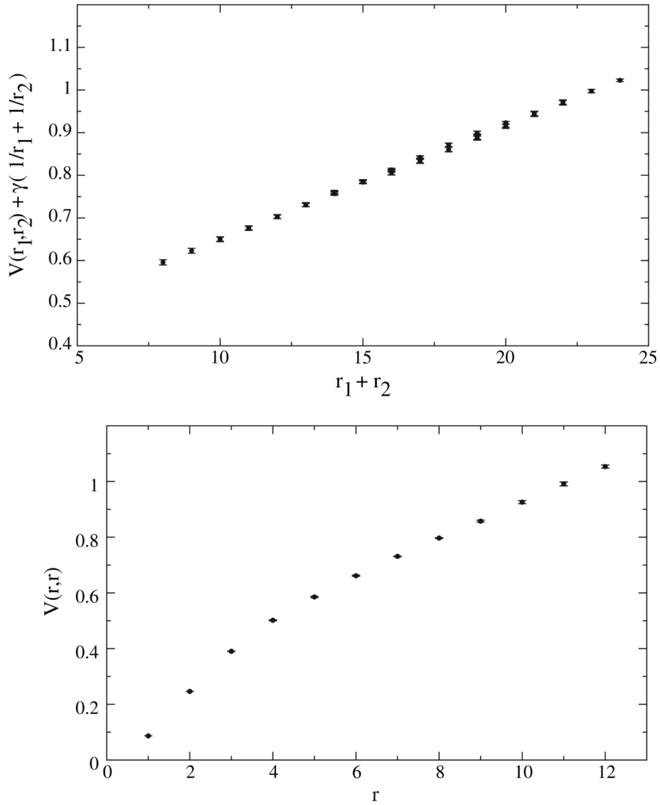

FIG. 5. Potential for the system $q\bar{q}g$, in lattice spacing units, respectively (top) for the $\perp$ geometry (with the Coulomb contributions subtracted) as a function of $r_1 + r_2$ and (bottom) for the $\parallel$ geometry as a function of $r_1 = r_2 = r$.

fundamental strings, one linking the quark to the gluon and the other linking the antiquark to the gluon.

To compare with the Casimir scaling result found by Bali [3] we now consider the case where the quark and antiquark are superposed. In this case the static quark and antiquark are equivalent to a static gluon, and therefore our potential is equivalent to a static gluon-gluon potential. This is the case of the parallel geometry potential $V_\parallel$ when the two distances r_1 between the quark and the gluon and r_2 between the antiquark and the gluon, as in Eq. (4), are identical, $r_1 = r_2$. This is plotted in Fig. 5 and indeed we find a linear behavior. Fitting the static potential of the parallel geometry for distances $r_1 = r_2 = r$, by $V_\parallel(r, r) = c_0 - \frac{\gamma}{r} + \sigma r$, we get

$$\sigma = (2.21 \pm 0.06)\sigma_0, \qquad (\chi^2/\mathrm{dof} = 0.29), \qquad (9)$$

which is consistent with Casimir scaling and agrees with the result obtained by Bali [3]. In general, the energy of the string-antistring system includes their interaction, and also the coupling to all the bound states and resonances [21–24]. The increase in the static hybrid potential when the quark and antiquark are superposed can be interpreted with a repulsive energy between the two fundamental strings. A similar repulsive energy exists in type-II superconductors, when all possible resonances and bound states are situated above the energy of the pair of fundamental strings.

Moreover, in the parallel geometry, we consider an antiquark quite distant from the quark-gluon pair, to study whether they are linked by a fundamental string. When we fix the distance r_1, the funnel potential fit in r_2 yields

$$r_1 = 1\colon \sigma = (1.01 \pm 0.04)\sigma_0, \qquad (\chi^2/\mathrm{dof} = 0.62),$$
$$r_1 = 2\colon \sigma = (1.04 \pm 0.05)\sigma_0, \qquad (\chi^2/\mathrm{dof} = 1.33), \qquad (10)$$
$$r_1 = 3\colon \sigma = (1.04 \pm 0.05)\sigma_0, \qquad (\chi^2/\mathrm{dof} = 1.04),$$

consistent with a fundamental string tension.

IV. CONCLUSION

We explore, in $24^3 \times 48$ periodic lattices with $\beta = 6.2$ and $a \sim 0.072$, two different geometries for the gluon-quark segment and the gluon-antiquark segment. When these segments are perpendicular, the static potential is consistent with confinement realized with a pair of fundamental strings, one linking the gluon to the quark and one linking the same gluon to the antiquark. When the segments are parallel and superposed, the total string tension is larger and is compatible with a repulsive energy between the two fundamental strings. Notice that when the two segments are parallel and superposed, the total string tension is also compatible with the Casimir scaling measured by Bali.

This can be interpreted with a type-II superconductor analogy for the confinement in QCD, with repulsion of the fundamental strings and with the string tension of the first topological excitation of the string (the adjoint string) larger than twice the fundamental string tension. Nevertheless, because the energy of two fundamental strings plus the repulsive energy measured here is quite similar to the energy of the adjoint string measured by Bali [3], this shows that the pure-gauge QCD is similar to a type-II superconductor quite close to the phase transition to a type-I superconductor [20].

Our results are important for constituent models of hybrids and glueballs. In the three-body hybrid, with one quark, one antiquark and one gluon, our results suggest that the best potential model has only two fundamental strings, plus a repulsion acting only when the two fundamental

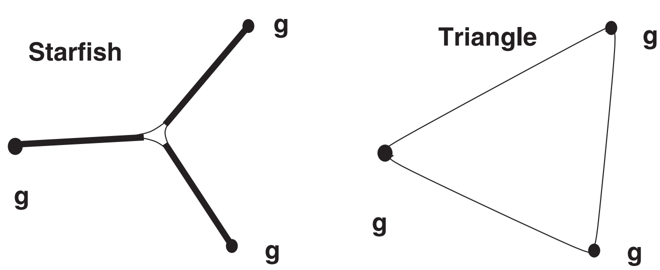

FIG. 6. The starfishlike and trianglelike possible geometries for the strings in the static three-gluon system.

strings are close. In the two-body gluon-gluon glueball, our results suggest that the string tension is similar to the one of the Casimir scaling model, with a factor of the order of $\frac{9}{4}$ when compared with the quark-antiquark potential. We can also extrapolate our result for three-body glueballs, relevant for the odderon problem [30]. With three gluons, a triangle formed by three fundamental strings costs less energy than three adjoint strings with a starfishlike geometry, depicted in Fig. 6. Thus we anticipate that the three-gluon potential is similar to a sum of three mesonic quark-antiquark interactions, plus a repulsion acting only when there is superposition of the fundamental strings.

ACKNOWLEDGMENTS

This work was financed by the FCT Contracts No. POCI/FP/63436/2005 and No. POCI/FP/63923/2005.

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