International Journal of Modern Physics A
Vol. 27, No. 26 (2012) 1230025 (28 pages)

DOI: 10.1142/S0217751X12300256

World Scientific
www.worldscientific.com

A RESOURCE FOR SIGNS AND FEYNMAN DIAGRAMS OF THE STANDARD MODEL

JORGE C. ROMÃO

Departamento de Física and CFTP, Instituto Superior Técnico,
Technical University of Lisbon, 1049-001 Lisboa, Portugal
jorge.romao@ist.utl.pt

JOÃO P. SILVA

Instituto Superior de Engenharia de Lisboa, 1959-007 Lisboa, Portugal
Centro de Física Teórica de Partículas (CFTP), Instituto Superior Técnico,
Technical University of Lisbon, 1049-001 Lisboa, Portugal
jpsilva@cftp.ist.utl.pt

Received 30 July 2012
Revised 19 September 2012
Accepted 24 September 2012
Published 17 October 2012

When performing a full calculation within the standard model (SM) or its extensions, it is crucial that one utilizes a consistent set of signs for the gauge couplings and gauge fields. Unfortunately, the literature is plagued with differing signs and notations. We present all SM Feynman rules, including ghosts, in a convention-independent notation, and we table the conventions in close to 40 books and reviews.

Keywords: Standard model; Feynman diagrams; electroweak unification.

PACS numbers: 11.15.Bt, 12.10.Dm, 12.15.Lk, 13.40.ks

1. Introduction

Almost every book and review on the standard model (SM) has its own conventions for the signs that enter the definitions of the couplings and fields. Although the signs are irrelevant when a full calculation is made with any given convention, the signs of the various Feynman diagrams are usually different in different conventions. Of course, most papers sidestep writing all Feynman diagrams, with the rationale that these are already contained in several books. Typically, a paper on a model of physics beyond the SM shows only a few Feynman rules, or not even that. As a result, the remaining Feynman rules needed for any given calculation must be derived from first principles or found in books. And this is where the problem resides; which convention was used in the paper? How does it compare with the convention in some specific book?

Here we perform two tasks. We list all Feynman rules with arbitrary signs, allowing one to specify later for any given sign convention being used, and we list the sign conventions of close to 40 known books and reviews.

Section 2 summarizes the SM Lagrangian including the generic signs (represented by parameters $\eta = \pm 1$) necessary to specify the different notations found in the literature. These are listed in table form in Sec. 3, including only those references we consulted which: (i) follow the metric $(+,-,-,-)$; (ii) follow Bjorken and Drell's[1] convention for the propagator, with the explicit i; and (iii) are internally consistent (i.e. we do not include references which make one sign choice in one part of the Lagrangian and a different choice elsewhere). Sections 4 and 5 contain all Feynman rules of the SM, including would-be Goldstone bosons and ghosts in an arbitrary R_ξ gauge, in a convention-independent notation. Consistency remarks due to gauge invariance and invariance under Becchi–Rouet–Stora–Tyutin (BRST) transformations are relegated to App. A.

2. The Standard Model

2.1. *Gauge group $SU(3)_c$*

Here the important conventions are for the field strengths and the covariant derivatives. We have

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - \eta_s g_s f^{abc} G^b_\mu G^c_\nu \quad (a = 1, \dots, 8)\,, \tag{1}$$

where f^{abc} are the group structure constants, satisfying

$$\left[T^a, T^b\right] = i f^{abc} T^c \tag{2}$$

and T^a are the generators of the group. The parameter $\eta_s = \pm 1$ reflects the two usual signs in the literature. The covariant derivative of a (quark) field q in some representation T^a of the gauge group is given by

$$D_\mu q = \left(\partial_\mu + i\eta_s g_s G^a_\mu T^a\right) q\,. \tag{3}$$

In QCD, the quarks are in the fundamental representation and $T^a = \lambda^a/2$, where λ^a are the Gell-Mann matrices. A gauge transformation is given by the matrix

$$U = e^{i\eta_s g_s T^a \beta^a} \tag{4}$$

and the fields transform as

$$\begin{aligned} q &\to e^{i\eta_s g_s T^a \beta^a} q\,, \quad \delta q = i\eta_s g_s T^a \beta^a q\,, \\ G^a_\mu T^a &\to U G^a_\mu T^a U^{-1} + \frac{i}{\eta_s g_s} \partial_\mu U U^{-1}\,, \\ \delta G^a_\mu &= -\partial_\mu \beta^a - \eta_s g_s f^{abc} \beta^b G^c_\mu\,, \end{aligned} \tag{5}$$

where the second column is for infinitesimal transformations. With these definitions, one can verify that the covariant derivative transforms like the field itself,

$$\delta(D_\mu q) = i\eta_s g_s T^a \beta^a (D_\mu q)\,, \tag{6}$$

ensuring the gauge invariance of the Lagrangian. Further consistency checks due to gauge invariance will be relegated to App. A.

2.2. *Gauge group $SU(2)_L \times U(1)_Y$*

For the $SU(2)_L$ group, we have

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - \eta g \epsilon^{abc} W^b_\mu W^c_\nu \quad (a = 1, \ldots, 3)\,, \tag{7}$$

where, for the fundamental representation of $SU(2)_L$, $T^a = \tau^a/2$, where τ^a are the Pauli matrices, ϵ^{abc} is the completely antisymmetric tensor in three dimensions and $\eta = \pm 1$. The covariant derivative for any field ψ_L transforming nontrivially under this group is

$$D_\mu \psi_L = \left(\partial_\mu + i\eta g W^a_\mu T^a\right)\psi_L\,. \tag{8}$$

As for the Abelian $U(1)_Y$ group, we have

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,, \tag{9}$$

with the covariant derivative given by

$$D_\mu \psi = \left(\partial_\mu + i\eta' g' \eta_Y Y B_\mu\right)\psi\,, \tag{10}$$

where Y is the hypercharge of the field, connected to the electric charge through

$$Q = T_3 + \eta_Y Y\,. \tag{11}$$

As before, η', $\eta_Y = \pm 1$. Some authors use

$$Q = T_3 + \eta_Y \frac{Y_{\text{theirs}}}{2} = \frac{\tau_3 + \eta_Y Y_{\text{theirs}}}{2}\,, \tag{12}$$

instead of our Eq. (11). The difference is immaterial for the Feynman rules, which depend only on Q.

It is useful to write the covariant derivative in terms of the mass eigenstates A_μ and Z_μ. These are defined by the relations,[a]

$$\begin{cases} W^3_\mu = \eta_Z Z_\mu \cos\theta_W + A_\mu \eta_\theta \sin\theta_W\,, \\ B_\mu = -\eta_Z Z_\mu \eta_\theta \sin\theta_W + A_\mu \cos\theta_W\,, \end{cases}$$
$$\begin{cases} \eta_Z Z_\mu = W^3_\mu \cos\theta_W - B_\mu \eta_\theta \sin\theta_W\,, \\ A_\mu = W^3_\mu \eta_\theta \sin\theta_W + B_\mu \cos\theta_W\,. \end{cases} \tag{13}$$

[a]One could also include a sign in the photon field A, by substituting $A_\mu \to \eta_A A_\mu$. However, we have found no author who made the choice $\eta_A = -1$.

For a doublet field ψ_L, with hypercharge Y, we get

$$\begin{aligned} D_\mu \psi_L &= \left[\partial_\mu + i\eta \frac{g}{\sqrt{2}}(\tau^+ W^+_\mu + \tau^- W^-_\mu) + i\eta \frac{g}{2}\tau_3 W^3_\mu + i\eta' g' \eta_Y Y B_\mu\right]\psi_L \\ &= \left[\partial_\mu + i\eta \frac{g}{\sqrt{2}}(\tau^+ W^+_\mu + \tau^- W^-_\mu) + i\eta_e e Q A_\mu \right. \\ &\qquad \left. + i\eta \frac{g}{\cos\theta_W}\left(\frac{\tau_3}{2} - Q\sin^2\theta_W\right)\eta_Z Z_\mu\right]\psi_L \,, \end{aligned} \tag{14}$$

where

$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} \,, \tag{15}$$

$$\tau_\pm = \frac{\tau_1 \pm i\tau_2}{\sqrt{2}} \,. \tag{16}$$

The charge operator is defined by

$$Q = \begin{bmatrix} \frac{1}{2} + \eta_Y Y & 0 \\ 0 & -\frac{1}{2} + \eta_Y Y \end{bmatrix} \tag{17}$$

and we have used the relations

$$\eta_e e = (\eta\eta_\theta) g \sin\theta_W = \eta' g' \cos\theta_W \,. \tag{18}$$

Many authors use $\eta_e = +1$. Some authors use $\eta_e = -1$, to account for their other conventions (notably $\eta = \eta' = -1$), and still keep $e = +g'\cos\theta_W = +g\sin\theta_W$. For a singlet of $SU(2)_L$, ψ_R, we have

$$\begin{aligned} D_\mu \psi_R &= \left[\partial_\mu + i\eta' g' \eta_Y Y B_\mu\right]\psi_R \\ &= \left[\partial_\mu + i\eta_e e Q A_\mu - i\eta \frac{g}{\cos\theta_W} Q \sin^2\theta_W \eta_Z Z_\mu\right]\psi_R \,. \end{aligned} \tag{19}$$

We collect in Table 1 the quantum numbers of the SM particles.

Notice that the right-hand sides of Eqs. (14) and (19) only involve Y through Q, where it appears in the combination $\eta_Y Y$. A few authors write Eqs. (14) and (19) directly for each field, sidestepping a precise definition for their η_Y.

Table 1. Values of T_3^f, Q and Y for the SM particles.

Field	ℓ_L	ℓ_R	ν_L	u_L	d_L	u_R	d_R	ϕ^+	ϕ^0
T_3	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
$\eta_Y Y$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$
Q	-1	-1	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	1	0

For each fermion field ψ, one defines $\psi_{R,L} = P_{R,L}\psi$, where

$$P_{R,L} = \frac{1 \pm \gamma_5}{2} \tag{20}$$

and $\psi = \psi_R + \psi_L$.

2.3. *The gauge and fermion fields Lagrangian*

The gauge field Lagrangian is

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\,, \tag{21}$$

where the field strengths are given in Eqs. (1), (7) and (9).

The kinetic terms for the fermions, including the interaction with the gauge fields due to the covariant derivative, is written as

$$\mathcal{L}_{\text{Fermion}} = \sum_{\text{quarks}} i\bar{q}\gamma^\mu D_\mu q + \sum_{\psi_L} i\overline{\psi_L}\gamma^\mu D_\mu \psi_L + \sum_{\psi_R} i\overline{\psi_R}\gamma^\mu D_\mu \psi_R\,, \tag{22}$$

where the covariant derivatives are obtained with the rules in Eqs. (3), (14) and (19).

2.4. *The Higgs Lagrangian*

The SM includes a Higgs doublet with the following assignments:

$$\Phi = \begin{bmatrix} \varphi^+ \\ \dfrac{v + H + i\varphi_Z}{\sqrt{2}} \end{bmatrix}. \tag{23}$$

Since $\eta_Y Y_\Phi = +1/2$, the covariant derivative reads

$$\begin{aligned} D_\mu \Phi &= \left[\partial_\mu + i\eta\frac{g}{\sqrt{2}}\left(\tau^+ W^+_\mu + \tau^- W^-_\mu\right) + i\eta\frac{g}{2}\tau_3 W^3_\mu + i\eta'\frac{g'}{2}B_\mu\right]\Phi \\ &= \left[\partial_\mu + i\eta\frac{g}{\sqrt{2}}\left(\tau^+ W^+_\mu + \tau^- W^-_\mu\right) + i\eta_e e Q A_\mu\right. \\ &\qquad \left. + i\eta\frac{g}{\cos\theta_W}\left(\frac{\tau_3}{2} - Q\sin^2\theta_W\right)\eta_Z Z_\mu\right]\Phi\,, \end{aligned} \tag{24}$$

where, for the doublet field Φ,

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{25}$$

The Higgs Lagrangian is

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\Phi)^\dagger D_\mu \Phi + \mu^2 \Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2\,, \tag{26}$$

leading to the relations

$$v^2 = \frac{\mu^2}{\lambda}\,, \quad m_h^2 = 2\mu^2\,, \quad \lambda = \frac{g^2}{8}\frac{m_h^2}{m_W^2}\,. \tag{27}$$

Expanding this Lagrangian, we find the following terms quadratic in the fields:

$$\begin{aligned}\mathcal{L}_{\text{Higgs}} = \cdots &+ \frac{1}{8}g^2v^2W^3_\mu W^{\mu 3} + \frac{1}{8}g'^2v^2B_\mu B^\mu - \frac{1}{4}\eta\eta' gg'v^2W^3_\mu B^\mu \\ &+ \frac{1}{4}g^2v^2W^+_\mu W^{-\mu} + \frac{1}{2}v\partial^\mu\varphi_Z\left(\eta' g' B_\mu - \eta g W^3_\mu\right) \\ &- \frac{i}{2}\eta g v W^-_\mu\partial^\mu\varphi^+ + \frac{i}{2}\eta g v W^+_\mu\partial^\mu\varphi^- \,. \end{aligned} \tag{28}$$

The first three terms give, after diagonalization, a massless field (the photon) and a massive one (the Z), with the relations given in Eq. (13), while the fourth term gives mass to the charged $W^\pm_\mu$ bosons. Using Eq. (13), we get

$$\begin{aligned}\mathcal{L}_{\text{Higgs}} = \cdots &+ \frac{1}{2}m_Z^2 Z_\mu Z^\mu + m_W^2 W^+_\mu W^{-\mu} \\ &- \eta\eta_Z m_Z Z_\mu\partial^\mu\varphi_Z - i\eta m_W\left(W^-_\mu\partial^\mu\varphi^+ - W^+_\mu\partial^\mu\varphi^-\right), \end{aligned} \tag{29}$$

where

$$m_W = \frac{1}{2}gv\,, \quad m_Z = \frac{1}{\cos\theta_W}\frac{1}{2}gv = \frac{1}{\cos\theta_W}m_W\,. \tag{30}$$

By looking at Eq. (29) we realize that, besides finding a realistic spectra for the gauge bosons, we also get a problem. In fact, the terms in the last line are quadratic in the fields and complicate the definition of the propagators. The gauge fixing terms discussed in Subsec. 2.6 solve this problem.

2.5. *The Yukawa Lagrangian, fermion masses and the CKM matrix*

After spontaneous symmetry breaking, the interaction between the fermions and the Higgs doublet gives masses to the elementary fermions. We have

$$\mathcal{L}_{\text{Yukawa}} = -\bar{L}_L Y_l \Phi \ell_R - \bar{Q}'_L Y_d \Phi\, d'_R - \bar{Q}'_L Y_u \tilde{\Phi} u'_R + \text{h.c.}\,, \tag{31}$$

where a sum over generations is implied by the matrix notation, L_L (Q'_L) are the left-handed lepton (quark) doublets and

$$\tilde{\Phi} = i\sigma_2\Phi^* = \begin{bmatrix} \dfrac{v + H - i\varphi_Z}{\sqrt{2}} \\ -\varphi^- \end{bmatrix}. \tag{32}$$

Y_l, Y_d and Y_u are general complex 3×3 matrices in the respective flavor spaces.

To bring the quarks into the mass basis, Y_d and Y_u are diagonalized through unitary transformations

$$\begin{aligned} \bar{u}'_L &= \bar{u}_L U^\dagger_{uL}\,, \quad \bar{d}'_L = \bar{d}_L U^\dagger_{dL}\,, \\ u'_R &= U_{uR}u_R\,, \quad d'_R = U_{dR}d_R\,, \end{aligned} \tag{33}$$

such that

$$\begin{aligned}\frac{v}{\sqrt{2}}U_{uL}^\dagger Y_u U_{uR} &= M_u = \mathrm{diag}(m_u, m_c, m_t)\,,\\ \frac{v}{\sqrt{2}}U_{dL}^\dagger Y_d U_{dR} &= M_d = \mathrm{diag}(m_d, m_s, m_b)\,.\end{aligned} \tag{34}$$

In this new basis, the Higgs couplings of the quarks become diagonal:

$$-\mathcal{L}_{\mathrm{H}} = \left(1 + \frac{h^0}{v}\right)\left[\bar{u}M_u u + \bar{d}M_d d\right]. \tag{35}$$

The couplings to the photon and the Z remain diagonal. In contrast, the couplings to the W mix the upper and lower components of Q'_L, which transform differently under Eqs. (33). As a result, the couplings to $W^\pm$ become off-diagonal:

$$-\eta\mathcal{L}_{\mathrm{W}} = \frac{g}{\sqrt{2}}\bar{u}_L V\gamma^\mu d_L W_\mu^\dagger + \mathrm{h.c.}\,, \tag{36}$$

where

$$V = U_{uL}^\dagger U_{dL} \tag{37}$$

is the Cabibbo–Kobayashi–Maskawa (CKM) matrix, which also affects the interactions with the charged Goldstone bosons.

In the SM, there are no right-handed neutrinos. As a result, the neutrinos are massless and we are free to rotate them in order to accommodate the transformations of the charged quarks needed to diagonalize Y_l. Thus, without loss of generality, we may take $Y_l = \mathrm{diag}(m_e, m_\mu, m_\tau)$ and $V = 1$ in the leptonic sector.

2.6. *The gauge fixing*

One needs to gauge fix the gauge part of the Lagrangian in order to be able to define the propagators. In the R_ξ gauges, the gauge fixing Lagrangian reads

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi_G}F_G^2 - \frac{1}{2\xi_A}F_A^2 - \frac{1}{2\xi_Z}F_Z^2 - \frac{1}{\xi_W}F_-F_+\,, \tag{38}$$

where

$$\begin{aligned}F_G^a &= \partial^\mu G_\mu^a\,,\\ F_A &= \partial^\mu A_\mu\,,\\ F_Z &= \partial^\mu Z_\mu + \eta\eta_Z\xi_Z m_Z\varphi_Z\,,\\ F_+ &= \partial^\mu W_\mu^+ + i\eta\xi_W m_W\varphi^+\,,\\ F_- &= \partial^\mu W_\mu^- - i\eta\xi_W m_W\varphi^-\,.\end{aligned} \tag{39}$$

One can easily verify that, with these definitions, $\mathcal{L}_{\mathrm{GF}}$ cancels the mixed quadratic terms on the second line of Eq. (29).

2.7. *The ghost Lagrangian*

The last piece needed for the SM Lagrangian is the ghost Lagrangian. For a linear gauge fixing condition, as in Eq. (39), this is given by the Fadeev–Popov prescription:

$$\begin{aligned}\mathcal{L}_{\text{Ghost}} &= \eta_G \sum_{i=1}^{4} \left[\bar{c}_+ \frac{\partial(\delta F_+)}{\partial\alpha^i} + \bar{c}_- \frac{\partial(\delta F_+)}{\partial\alpha^i} + \bar{c}_Z \frac{\partial(\delta F_Z)}{\partial\alpha^i} + \bar{c}_A \frac{\partial(\delta F_A)}{\partial\alpha^i} \right] c_i \\ &\quad + \eta_G \sum_{a,b=1}^{8} \bar{\omega}^a \frac{\partial(\delta F_G^a)}{\partial\beta^b} \omega^b \,, \end{aligned} \tag{40}$$

where we have denoted by ω^a the ghosts associated with the $SU(3)_c$ transformations defined by Eq. (4), and by $c_\pm$, c_A, c_Z the electroweak ghosts associated with the gauge transformations,

$$U = e^{i\eta g T^a \alpha^a} \quad (a = 1, \ldots, 3) \,, \tag{41}$$

and

$$U = e^{i\eta' \eta_Y g' Y \alpha^4} \,. \tag{42}$$

For completeness, we write in App. A the gauge transformations of the gauge fixing terms needed to find the Lagrangian in Eq. (40).

Because ghosts are not external states, the sign $\eta_G = \pm 1$ is immaterial and, although it corresponds to an overall sign affecting all propagators and vertices with ghosts, it drops out in any physical calculation involving ghosts.

2.8. *The complete SM Lagrangian*

Finally, the complete Lagrangian for the SM is obtained by putting together all the pieces. We have

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{Ghost}} \,, \tag{43}$$

where the different terms were given in Eqs. (21), (22), (26), (31), (38) and (40).

3. Notations Found in the Literature

In order to use the results contained in some specific source in the literature, one must find the covariant derivative

$$D_\mu = \partial_\mu + i\eta g \frac{\tau_a}{2} W_\mu^a + i\eta' \eta_Y g' Y B_\mu \tag{44}$$

and Eqs. (11) and (13). This sets the sign convention for η, η', η_Z, η_θ and η_Y. Typically, authors set $\eta_s = \eta$.

The signs and conventions in the literature are shown in Table 2.

Table 2. Sign conventions found in the literature. An asterisk, $*$, on the last column means that such authors have used $Q = (\tau_3 + Y_{\text{theirs}})/2$ instead of our Eq. (11).

Ref.	η	η'	η_Z	η_θ	η_Y	η_e	Y
2–6, 47	$+$	$+$	$+$	$+$	$+$	$+$	
7–17	$+$	$+$	$+$	$+$	$+$	$+$	$*$
18, 19	$-$	$-$	$+$	$+$	$+$	$-$	
20–31	$-$	$-$	$+$	$+$	$+$	$-$	$*$
32, 33	$-$	$-$	$+$	$-$	$+$	$+$	
34	$-$	$-$	$-$	$+$	$+$	$-$	$*$
35	$-$	$+$	$+$	$-$		$+$	
36, 37	$-$	$+$	$+$	$-$	$-$	$+$	
38	$-$	$+$	$+$	$-$	$+$	$+$	
39	$+$	$-$	$+$	$-$	$+$	$-$	$*$

The corresponding Feynman rules are presented in the following sections. A few remarks are in order. As mentioned, since the Feynman rules depend only on Q, authors may choose to sidestep a definition of η_Y; or whether they are using Y, from Eq. (11), or Y_{theirs}, from Eq. (12); or even neglect to mention the hypercharge Y altogether. Similarly, η_θ, η' and g' are absent from the Feynman rules and thus not needed in any calculation. We see that only η_s, in the strong sector, and η, η_e and η_Z, in the electroweak sector, show up in Feynman diagrams.

The fact that authors differ by their η sign, but all keep to the definition of m_W and m_Z in Eq. (30) (assumed positive), means that diagrams proportional to gauge boson masses are also affected by the sign choice. Some conventions lead to peculiar results. For example, the convention in Ref. 32 leads to the unconventional $e = -g' \cos\theta_W$, while keeping the usual $e = g \sin\theta_W$. If one wishes to keep all quantities positive in the relation $m_Z = gv/(2\cos\theta_W)$, then one must assume that g' is negative. This is irrelevant for the Feynman rules, where g' does not show, but is unusual.

The relevant electroweak choices for η, η_e and η_Z may be inferred from any given reference, as long as a few Feynman rules are given. For example, the coupling of the photon with fermions (or W^+W^-, or $\varphi^+\varphi^-$) sets η_e. Similarly, the coupling of the Z with fermions (or W^+W^-, or $\varphi^+\varphi^-$) sets $\eta\eta_Z$. Finally, the coupling of the W^+ with fermions sets η. This sets the notation for all other Feynman rules, even when Goldstone bosons and/or ghosts are included, except for η_G which can be found in any of the propagators or vertices involving ghosts. The sign for η_G is shown in Table 3 for those references including ghosts. A star (*) indicates the references that only include Feynman rules with ghosts for the pure non-Abelian gauge theory or that have an incomplete list of the Feynman rules for the electroweak ghosts. A dagger (†) indicates the references that include all Feynman rules, including electroweak ghosts.

Table 3. Sign convention for η_G found in the literature.

Ref.	η_G	
6, 9, 11, 14–16, 18, 22, 28, 31, 34–36, 39	$+$	$*$
2, 20, 32, 37, 38	$+$	$\dagger$
13, 23, 47	$-$	$*$
3	$-$	$\dagger$

Next we present all Feynman rules, including the generic signs, which have been obtained using the `FeynRules` package.[40] The first Roman letters (a, d, c, d, e) denote group indices; the Roman letters (i, j) refer to the QCD component; the Greek letters (μ, ν, σ, ρ) denote Lorentz indices; while the first Greek letters (α, β), appearing in the CKM matrix V, refer to the flavor indices.

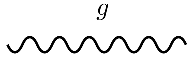
We finish this section by comparing our results with those found in the literature. We only do this comparison for the set of references that have all the Feynman rules for the SM, including ghosts, namely Refs. 2, 3, 20, 32, 37 and 38. We agree with Ref. 2 (including the errata) except for an overall sign in Eqs. (14.66) and (14.67). As for Ref. 3, we disagree with the four gluon vertex on page 572, but we agree when it is written on page 557. We also note that this reference has the complete Feynman rules for the counterterms that we do not include here. Reference 20 has all the Feynman rules, including also those for the counterterms. The conventions of this reference are different from all those that we cite and therefore difficult to compare. However, we have checked a reasonable number of Feynman rules and got agreement in all cases. Reference 37 has all Feynman rules correct, except for an overall sign on the last vertex on page A.16 and the fourth on page A.18. We agree with all Feynman rules contained in Refs. 32 and 38.

4. Feynman Rules for QCD

We give separately the Feynman rules for QCD and the electroweak part of the SM. All moments are incoming, except in the ghost vertices where they are explicitly shown.

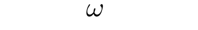
4.1. *Propagators*

g

μ, a  ν, b

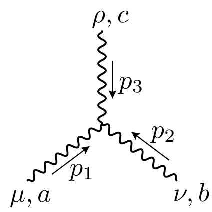
$$-i\delta_{ab}\left[\frac{g_{\mu\nu}}{k^2+i\epsilon}-(1-\xi_G)\frac{k_\mu k_\nu}{(k^2)^2}\right], \tag{45}$$

ω

a  b

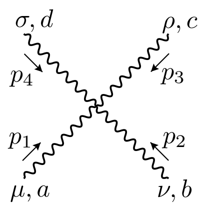
$$\delta_{ab}\frac{i\eta_G}{k^2+i\epsilon}\,. \tag{46}$$

4.2. *Triple gauge interactions*



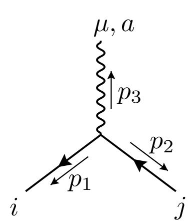
$$-\eta_s g_s f^{abc}[g^{\mu\nu}(p_1 - p_2)^\rho + g^{\nu\rho}(p_2 - p_3)^\mu + g^{\rho\mu}(p_3 - p_1)^\nu]\,. \tag{47}$$

4.3. *Quartic gauge interactions*



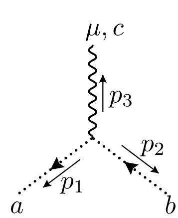
$$\begin{aligned} -ig_s^2\big[&f_{eab}f_{ecd}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) \\ &+ f_{eac}f_{edb}(g_{\mu\sigma}g_{\rho\nu} - g_{\mu\nu}g_{\rho\sigma}) \\ &+ f_{ead}f_{ebc}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma})\big]\,. \end{aligned} \tag{48}$$

4.4. *Fermion gauge interactions*



$$-i\eta_s g_s \gamma^\mu T^a_{ij}\,. \tag{49}$$

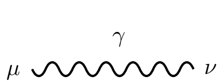
4.5. *Ghost interactions*



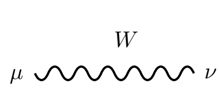
$$-\eta_s \eta_G g_s f^{abc} p_1^\mu\,. \tag{50}$$

5. Feynman Rules for the Electroweak Theory

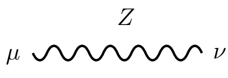
5.1. *Propagators*



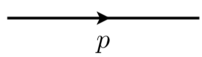
$$-i\left[\frac{g_{\mu\nu}}{k^2 + i\epsilon} - (1 - \xi_A)\frac{k_\mu k_\nu}{(k^2)^2}\right], \tag{51}$$



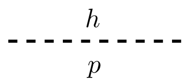
$$-i\frac{1}{k^2 - m_W^2 + i\epsilon}\left[g_{\mu\nu} - (1 - \xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W m_W^2}\right], \tag{52}$$



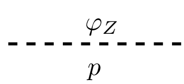
$$-i\frac{1}{k^2-m_Z^2+i\epsilon}\left[g_{\mu\nu}-(1-\xi_Z)\frac{k_\mu k_\nu}{k^2-\xi_Z m_Z^2}\right], \tag{53}$$



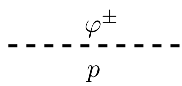
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2+i\epsilon}, \tag{54}$$



$$\frac{i}{p^2-m_h^2+i\epsilon}, \tag{55}$$

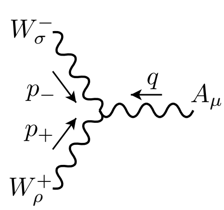


$$\frac{i}{p^2-\xi_Z m_Z^2+i\epsilon}, \tag{56}$$

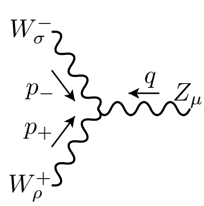


$$\frac{i}{p^2-\xi_W m_W^2+i\epsilon}. \tag{57}$$

5.2. *Triple gauge interactions*

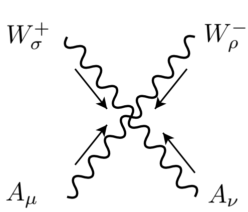


$$\begin{aligned}&-i\eta_e e\big[g_{\sigma\rho}(p_- - p_+)_\mu\\&\qquad + g_{\rho\mu}(p_+ - q)_\sigma + g_{\mu\sigma}(q-p_-)_\rho\big],\end{aligned} \tag{58}$$

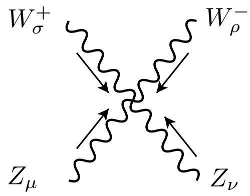


$$\begin{aligned}&-i\eta\eta_Z g\cos\theta_W[g_{\sigma\rho}(p_- - p_+)_\mu\\&\qquad + g_{\rho\mu}(p_+ - q)_\sigma + g_{\mu\sigma}(q-p_-)_\rho].\end{aligned} \tag{59}$$

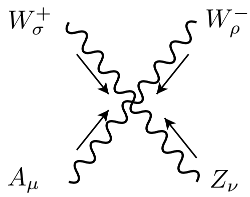
5.3. *Quartic gauge interactions*



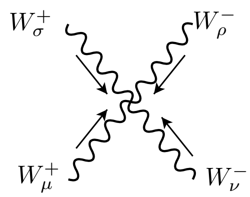
$$-ie^2[2g_{\sigma\rho}g_{\mu\nu}-g_{\sigma\mu}g_{\rho\nu}-g_{\sigma\nu}g_{\rho\mu}], \tag{60}$$



$$-ig^2\cos^2\theta_W[2g_{\sigma\rho}g_{\mu\nu} - g_{\sigma\mu}g_{\rho\nu} - g_{\sigma\nu}g_{\rho\mu}]\,, \tag{61}$$

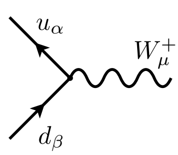


$$-i\eta_e\eta\eta_Z eg\cos\theta_W[2g_{\sigma\rho}g_{\mu\nu} - g_{\sigma\mu}g_{\rho\nu} - g_{\sigma\nu}g_{\rho\mu}]\,, \tag{62}$$

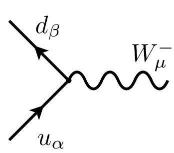


$$ig^2[2g_{\sigma\mu}g_{\rho\nu} - g_{\sigma\rho}g_{\mu\nu} - g_{\sigma\nu}g_{\rho\mu}]\,. \tag{63}$$

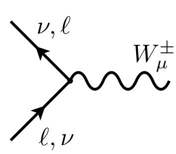
5.4. *Charged current interaction*



$$-i\eta\frac{g}{\sqrt{2}}\gamma_\mu P_L V_{\alpha\beta}\,, \tag{64}$$

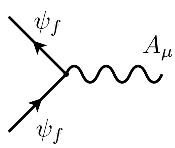


$$-i\eta\frac{g}{\sqrt{2}}\gamma_\mu P_L V^*_{\alpha\beta}\,, \tag{65}$$

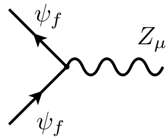


$$-i\eta\frac{g}{\sqrt{2}}\gamma_\mu P_L\,. \tag{66}$$

5.5. *Neutral current interaction*



$$-i\eta_e eQ_f\gamma_\mu\,, \tag{67}$$

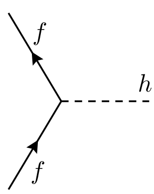


$$-i\eta\eta_Z \frac{g}{\cos\theta_W} \gamma_\mu \left(g_V^f - g_A^f \gamma_5 \right) , \tag{68}$$

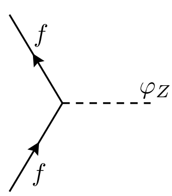
where

$$g_V^f = \frac{1}{2} T_f^3 - Q_f \sin^2\theta_W , \quad g_A^f = \frac{1}{2} T_f^3 . \tag{69}$$

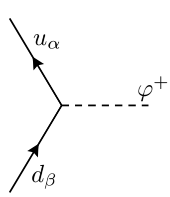
5.6. *Fermion-Higgs and fermion-Goldstone interactions*



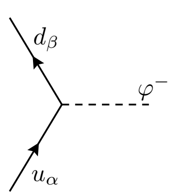
$$-i \frac{g}{2} \frac{m_f}{m_W} , \tag{70}$$



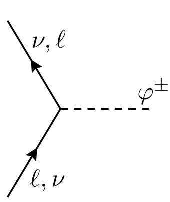
$$-g T_f^3 \frac{m_f}{m_W} \gamma_5 , \tag{71}$$



$$i \frac{g}{\sqrt{2}} \left(\frac{m_{u\alpha}}{m_W} P_L - \frac{m_{d\beta}}{m_W} P_R \right) V_{\alpha\beta} , \tag{72}$$

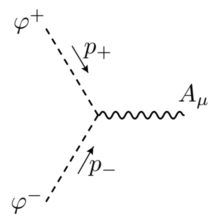


$$i \frac{g}{\sqrt{2}} \left(\frac{m_{u\alpha}}{m_W} P_R - \frac{m_{d\beta}}{m_W} P_L \right) V_{\alpha\beta}^* , \tag{73}$$

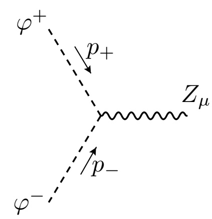


$$-i \frac{g}{\sqrt{2}} \frac{m_\ell}{m_W} P_{R,L} . \tag{74}$$

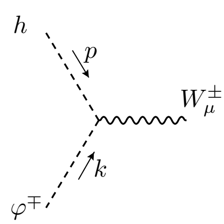
5.7. *Triple Higgs-gauge and Goldstone-gauge interactions*



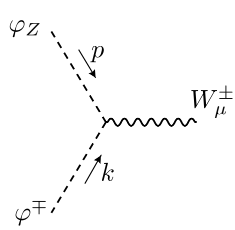
$$-i\eta_e e(p_+ - p_-)_\mu \,, \tag{75}$$



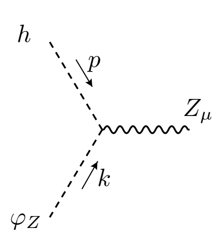
$$-i\eta\eta_Z g\frac{\cos 2\theta_W}{2\cos\theta_W}(p_+ - p_-)_\mu \,, \tag{76}$$



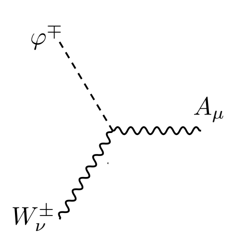
$$\pm\frac{i}{2}\eta g(k-p)_\mu \,, \tag{77}$$



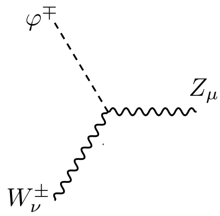
$$-\eta\frac{g}{2}(k-p)_\mu \,, \tag{78}$$



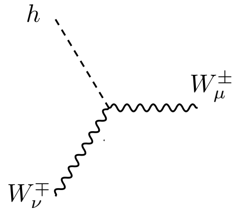
$$-\eta\eta_Z\frac{g}{2\cos\theta_W}(k-p)_\mu \,, \tag{79}$$



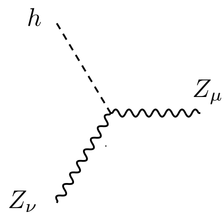
$$i\eta_e\eta e m_W g_{\mu\nu} \,, \tag{80}$$



$$-i\eta_Z g m_Z \sin^2\theta_W g_{\mu\nu}\,, \tag{81}$$

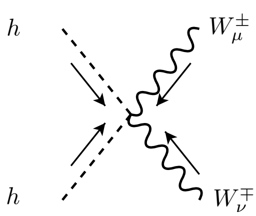


$$i g m_W g_{\mu\nu}\,, \tag{82}$$

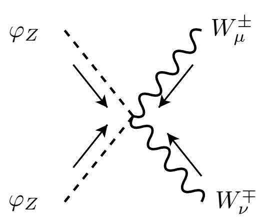


$$i\frac{g}{\cos\theta_W} m_Z g_{\mu\nu}\,. \tag{83}$$

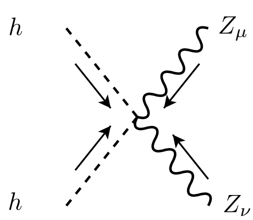
5.8. *Quartic Higgs-gauge and Goldstone-gauge interactions*



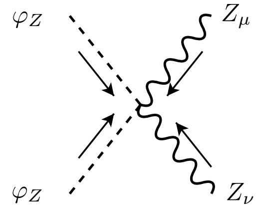
$$\frac{i}{2} g^2 g_{\mu\nu}\,, \tag{84}$$



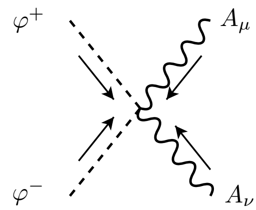
$$\frac{i}{2} g^2 g_{\mu\nu}\,, \tag{85}$$



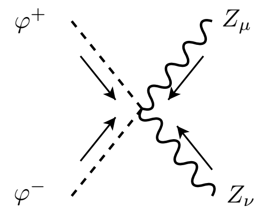
$$\frac{i}{2}\frac{g^2}{\cos^2\theta_W} g_{\mu\nu}\,, \tag{86}$$



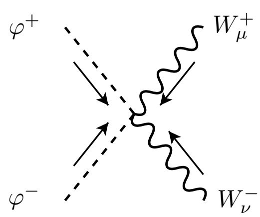
$$\frac{i}{2}\frac{g^2}{\cos^2\theta_W}g_{\mu\nu}\,, \tag{87}$$



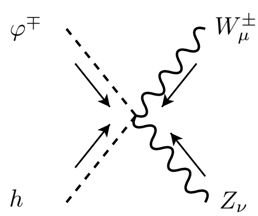
$$2ie^2 g_{\mu\nu}\,, \tag{88}$$



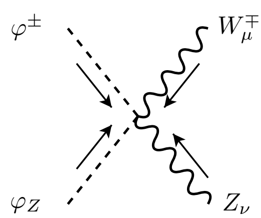
$$\frac{i}{2}\left(\frac{g\cos 2\theta_W}{\cos\theta_W}\right)^2 g_{\mu\nu}\,, \tag{89}$$



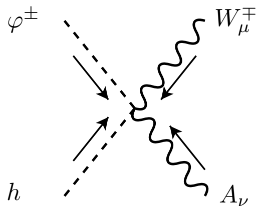
$$\frac{i}{2}g^2 g_{\mu\nu}\,, \tag{90}$$



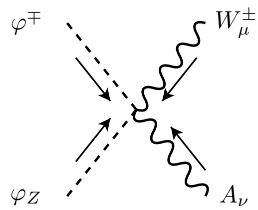
$$-i\eta_Z g^2\frac{\sin^2\theta_W}{2\cos\theta_W}g_{\mu\nu}\,, \tag{91}$$



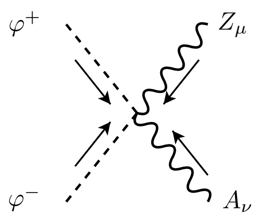
$$\mp\eta_Z g^2\frac{\sin^2\theta_W}{2\cos\theta_W}g_{\mu\nu}\,, \tag{92}$$



$$\frac{i}{2}\eta_e \eta e g g_{\mu\nu}\,, \tag{93}$$

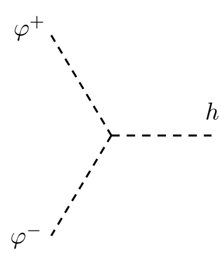


$$\mp\frac{1}{2}\eta_e \eta e g g_{\mu\nu}\,, \tag{94}$$

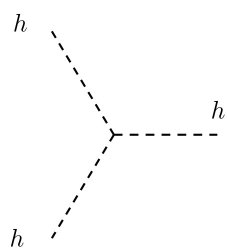


$$i\eta_e \eta \eta_Z e g \frac{\cos 2\theta_W}{\cos\theta_W} g_{\mu\nu}\,. \tag{95}$$

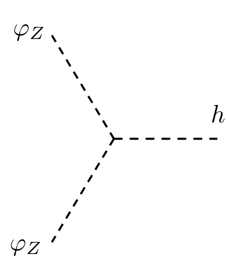
5.9. *Triple Higgs and Goldstone interactions*



$$-\frac{i}{2} g \frac{m_h^2}{m_W}\,, \tag{96}$$

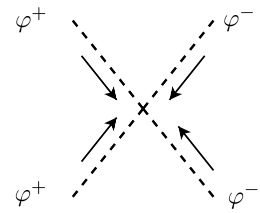


$$-\frac{3}{2} i g \frac{m_h^2}{m_W}\,, \tag{97}$$

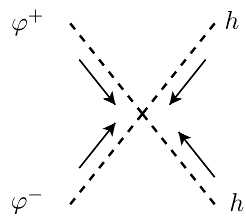


$$-\frac{i}{2} g \frac{m_h^2}{m_W}\,. \tag{98}$$

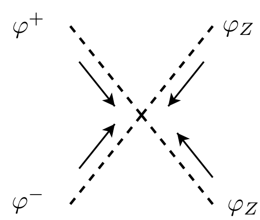
5.10. *Quartic Higgs and Goldstone interactions*



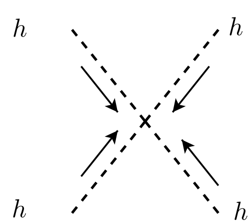
$$-\frac{i}{2}g^2\frac{m_h^2}{m_W^2}\,, \tag{99}$$



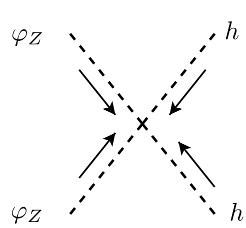
$$-\frac{i}{4}g^2\frac{m_h^2}{m_W^2}\,, \tag{100}$$



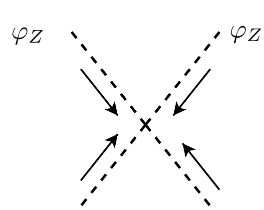
$$-\frac{i}{4}g^2\frac{m_h^2}{m_W^2}\,, \tag{101}$$



$$-\frac{3}{4}ig^2\frac{m_h^2}{m_W^2}\,, \tag{102}$$

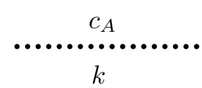


$$-\frac{i}{4}g^2\frac{m_h^2}{m_W^2}\,, \tag{103}$$

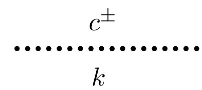


$$-\frac{3}{4}ig^2\frac{m_h^2}{m_W^2}\,. \tag{104}$$

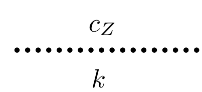
5.11. *Ghost propagators*



$$\frac{\eta_G i}{k^2 + i\epsilon}\,, \tag{105}$$

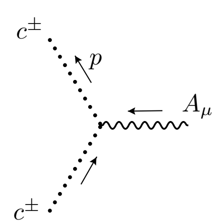


$$\frac{\eta_G i}{k^2 - \xi_W m_W^2 + i\epsilon}\,, \tag{106}$$

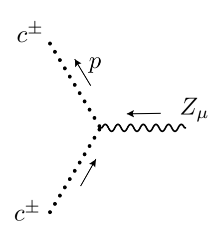


$$\frac{\eta_G i}{k^2 - \xi_Z m_Z^2 + i\epsilon}\,. \tag{107}$$

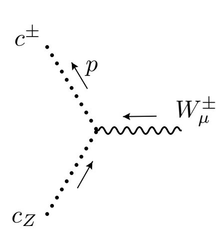
5.12. *Ghost gauge interactions*



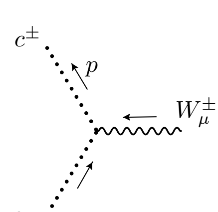
$$\mp i\eta_G \eta_e e p_\mu\,, \tag{108}$$



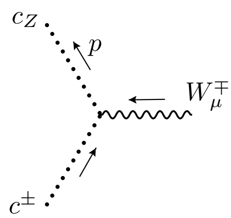
$$\mp i\eta_G \eta \eta_Z g \cos\theta_W p_\mu\,, \tag{109}$$



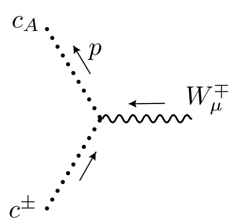
$$\pm i\eta_G \eta \eta_Z g \cos\theta_W p_\mu\,, \tag{110}$$



$$\pm i\eta_G \eta_e e p_\mu\,, \tag{111}$$

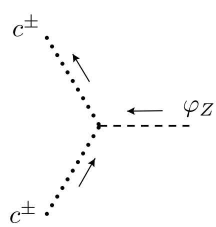


$$\pm i\eta_G \eta g \cos\theta_W p_\mu\,, \tag{112}$$

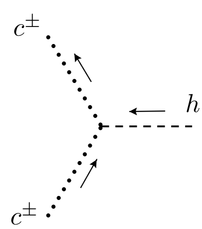


$$\pm i\eta_G \eta_e e p_\mu\,. \tag{113}$$

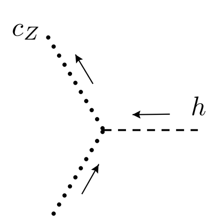
5.13. *Ghost-Higgs and ghost-Goldstone interactions*



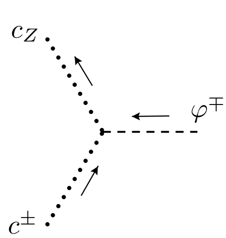
$$\pm \eta_G \frac{g}{2} \xi_W m_W\,, \tag{114}$$



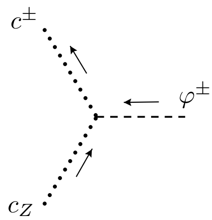
$$-\frac{i}{2}\eta_G g \xi_W m_W\,, \tag{115}$$



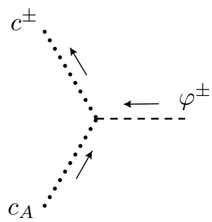
$$-\eta_G \frac{ig}{2\cos\theta_W} \xi_Z m_Z\,, \tag{116}$$



$$\frac{i}{2}\eta_G \eta_Z g \xi_Z m_Z\,, \tag{117}$$



$$-i\eta_G \eta_Z g \frac{\cos 2\theta_W}{2\cos\theta_W} \xi_W m_W \,, \tag{118}$$



$$-i\eta_G \eta_e \eta e \xi_W m_W \,. \tag{119}$$

5.14. *Brief comment on the alternative metric*

As mentioned in Sec. 1, all our calculations and Feynman diagrams have been obtained with the metric $(+,-,-,-)$. A few books use instead the metric $(-,+,+,+)$. Reference 41 differs from ours only in the metric. For example, it uses as we do, $i\bar{\psi}\partial\!\!\!/\psi$, for the fermion kinetic term. Therefore, our results agree with the change $g_{\mu\nu} \to -g_{\mu\nu}$, implying also changes of the type $p^2 \to -p^2$ and $p\!\!\!/ \to -p\!\!\!/$. The comparison is much more involved with respect to Refs. 42 and 43, because in those cases there are many changes besides the metric, involving, in particular, multiple factors of i and 2π in the Feynman rules. As an additional complication, Ref. 43 uses $-\bar{\psi}\partial\!\!\!/\psi$ for the fermion kinetic term, implying also a change in the matrices γ^{μ}, compounded by different gauge fixing terms. A detailed analysis of all such choices lies beyond the scope of this work.

Acknowledgments

We are grateful to A. Barroso and P. Nogueira for useful discussions and to H. Serôdio for reading the manuscript and making suggestions. This work was funded by FCT through the projects CERN/FP/116328/2010, PTDC/FIS/098188/2008 and U777-Plurianual and by the EU RTN project Marie Curie: PITN-GA-2009-237920. J. C. Romão also acknowledges support from project PTDC/FIS/102120/2008.

Appendix A. Gauge and BRST Consistency Checks

A.1. *Gauge transformation and gauge invariance*

For completeness, we write here the gauge transformations of the gauge fixing terms needed to find the Lagrangian in Eq. (40). It is convenient to redefine the parameters as

$$\begin{aligned}\alpha^{\pm} &= \frac{\alpha^1 \mp \alpha^2}{\sqrt{2}}\,,\\ \eta_Z \alpha_Z &= \alpha^3 \cos\theta_W - \eta_\theta \alpha^4 \sin\theta_W\,,\\ \alpha_A &= \eta_\theta \alpha^3 \sin\theta_W + \alpha^4 \cos\theta_W\,.\end{aligned} \tag{A.1}$$

We then get

$$\begin{aligned}\delta F_G^a &= \partial^\mu\big(-\partial_\mu \beta^a - \eta_s g_s f^{abc} \beta^b G_\mu^c\big)\,,\\ \delta F_A &= \partial^\mu(\delta A^\mu)\,,\\ \delta F_Z &= \partial_\mu(\delta Z^\mu) + \eta\eta_Z \xi_Z m_Z \delta\varphi_Z\,,\\ \delta F_+ &= \partial_\mu(\delta W_\mu^+) + i\eta\xi_W m_W \delta\varphi^+\,,\\ \delta F_- &= \partial_\mu(\delta W_\mu^-) - i\eta\xi_W m_W \delta\varphi^-\,.\end{aligned} \tag{A.2}$$

Using the explicit form of the gauge transformations we can finally find the missing pieces:

$$\begin{aligned}\delta A_\mu &= -\partial_\mu \alpha_A - i\eta_e e\big(W_\mu^+ \alpha^- - W_\mu^- \alpha^+\big)\,,\\ \delta Z_\mu &= -\partial_\mu \alpha_Z - i\eta\eta_Z g \cos\theta_W \big(W_\mu^+ \alpha^- - W_\mu^- \alpha^+\big)\,,\\ \delta W_\mu^+ &= -\partial_\mu \alpha^+ - i\eta g\big[\alpha^+(\eta_Z Z_\mu \cos\theta_W + \eta_\theta A_\mu \sin\theta_W)\\ &\quad - (\eta_Z \alpha_Z \cos\theta_W + \eta_\theta \alpha_A \sin\theta_W) W_\mu^+\big]\,,\\ \delta W_\mu^- &= -\partial_\mu \alpha^- + i\eta g\big[\alpha^-(\eta_Z Z_\mu \cos\theta_W + \eta_\theta A_\mu \sin\theta_W)\\ &\quad - (\eta_Z \alpha_Z \cos\theta_W + \eta_\theta \alpha_A \sin\theta_W) W_\mu^-\big]\,.\end{aligned} \tag{A.3}$$

To get the variation of the Goldstone bosons we notice that

$$\delta\Phi = \left[i\eta \frac{g}{\sqrt{2}}\big(\tau^+ \alpha^+ + \tau^- \alpha^-\big) + i\eta \frac{g}{2}\tau_3 \alpha^3 + i\eta' \frac{g'}{2}\alpha^4\right]\Phi \tag{A.4}$$

$$\begin{aligned}&= \left[i\eta \frac{g}{\sqrt{2}}\big(\tau^+ \alpha^+ + \tau^- \alpha^-\big) + i\eta_e e Q \alpha_A\right.\\ &\quad \left. + i\eta \frac{g}{\cos\theta_W}\left(\frac{\tau_3}{2} - Q\sin^2\theta_W\right)\eta_Z \alpha_Z\right]\Phi\,,\end{aligned} \tag{A.5}$$

which we can write as

$$\begin{bmatrix} \delta\varphi^+ \\ \dfrac{\delta(H+i\varphi_Z)}{\sqrt{2}} \end{bmatrix} = -\frac{i}{2}\begin{bmatrix} -\eta g\dfrac{\cos 2\theta_W}{\cos\theta_W}\eta_Z\alpha_Z - 2\eta_e e\alpha_A & -\sqrt{2}\eta g\alpha^+ \\ -\sqrt{2}\eta g\alpha^- & \eta\dfrac{g}{\cos\theta_W}\eta_Z\alpha_Z \end{bmatrix} \times \begin{bmatrix} \varphi^+ \\ \dfrac{v+H+i\varphi_Z}{\sqrt{2}} \end{bmatrix}, \tag{A.6}$$

leading to

$$\begin{aligned} \delta\varphi_Z &= \frac{1}{2}\eta g(\alpha^-\varphi^+ + \alpha^+\varphi^-) - \eta\frac{g}{2\cos\theta_W}\eta_Z\alpha_Z(v+H)\,, \\ \delta\varphi^+ &= i\eta\frac{g}{2}(v+H+i\varphi_Z)\alpha^+ + i\eta\frac{g}{2}\frac{\cos 2\theta_W}{\cos\theta_W}\varphi^+\eta_Z\alpha_Z + i\eta_e e\varphi^+\alpha_A\,, \\ \delta\varphi^- &= -i\eta\frac{g}{2}(v+H-i\varphi_Z)\alpha^- - i\eta\frac{g}{2}\frac{\cos 2\theta_W}{\cos\theta_W}\varphi^-\eta_Z\alpha_Z - i\eta_e e\varphi^-\alpha_A\,, \\ \delta H &= -i\eta\frac{g}{2}(\alpha^+\varphi^- - \alpha^-\varphi^+) + \eta\frac{g}{2\cos\theta_W}\eta_Z\alpha_Z\varphi_Z\,. \end{aligned} \tag{A.7}$$

With the gauge transformations given in Eqs. (A.3) and (A.7), one can easily verify that $\mathcal{L}_{\text{gauge}}$ and $\mathcal{L}_{\text{Higgs}}$ are gauge invariant, independently of the choice of the η's. For instance, for $\mathcal{L}_{\text{Higgs}}$ we have

$$\delta\mathcal{L}_{\text{Higgs}} = \delta(D_\mu\Phi)^\dagger D^\mu\Phi + (D_\mu\Phi)^\dagger\delta(D^\mu\Phi) + \delta\big(\mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2\big) = 0\,. \tag{A.8}$$

To check the fermion part we have to give explicitly the gauge transformations for ψ_L and ψ_R. They can be easily obtained from Eqs. (14) and (19). We get

$$\begin{aligned} \delta\psi_L &= \bigg[i\eta\frac{g}{\sqrt{2}}(\tau^+\alpha^+ + \tau^-\alpha^-) + i\eta_e eQ\alpha_A \\ &\qquad + i\eta\frac{g}{\cos\theta_W}\left(\frac{\tau_3}{2} - Q\sin^2\theta_W\right)\eta_Z\alpha_Z\bigg]\psi_L\,, \\ \delta\psi_R &= \left[i\eta_e eQ\alpha_A - i\eta\frac{g}{\cos\theta_W}Q\sin^2\theta_W\eta_Z\alpha_Z\right]\psi_R\,, \end{aligned} \tag{A.9}$$

supplemented by Eq. (5) for the $SU(3)_c$ transformation of the quarks. Using these transformation laws one can verify that

$$\delta(\mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Yukawa}}) = 0\,, \tag{A.10}$$

completing the proof of the gauge invariance of the classical part of $\mathcal{L}_{\text{SM}}$. This means that, except for $\mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{Ghost}}$ to be discussed later, we have included the various η parameters in the appropriate fashion.

A.2. *Consistency checks and the BRST transformations*

To make the proof of gauge invariance for the complete Lagrangian we have to deal with the gauge fixing and ghost terms. This is more easily done using the BRST transformations of Becchi, Rouet and Stora[44] and Tyutin,[45] and the Slavnov operator. The Slavnov operator is a special kind of gauge transformation on the gauge and matter fields. More specifically, we define

$$s(A^i_\mu) = \frac{\delta A^i_\mu}{\delta\alpha_i} c^i\,, \quad s(\phi_i) = \frac{\delta\phi_i}{\delta\alpha_i} c^i\,, \tag{A.11}$$

where A^i_μ are the gauge fields and ϕ_i represents generically any matter field (fermion or boson). They have the following properties:

(i) For a product of two fields, we have

$$s(XY) = s(X)Y + (-1)^{\mathrm{GN}(X)} X s(Y)\,. \tag{A.12}$$

In this expression the ghost number, GN(X), is defined as zero for gauge and matter fields, +1 for c^i fields (ghosts) and −1 for $\bar{c}^i$ (antighosts).

(ii) s raises the dimension by one unit (in terms of mass).

(iii) s does not change the charge.

(iv) The Slavnov operator is nilpotent, that is, $s^2 = 0$.

To check the last identity we must have, for a non-Abelian group,

$$s(c^i) = -\eta \frac{g}{2} f^{ijk} c^j c^k\,. \tag{A.13}$$

Let us show how the nilpotency of s is obtained for the gauge fields of a non-Abelian theory. From Eq. (A.11) we have

$$s(A^i_\mu) = -\partial_\mu c^i - \eta g f^{ijk} c^j A^k_\mu\,. \tag{A.14}$$

Therefore, using Eq. (A.12) we get

$$\begin{aligned}
s^2 A^i_\mu &= -\partial_\mu s(c^i) - \eta g f^{ijk} s(c^j) A^k_\mu + \eta g f^{ijk} c^j s\big(A^k_\mu\big)\\
&= \eta \frac{g}{2} f^{ijk}\big(\partial_\mu c^j c^k + c^j \partial_\mu c^k\big) + \eta^2 \frac{g^2}{2} f^{ijk} f^{jmn} c^m c^n A^k_\mu\\
&\quad + \eta g f^{ijk} c^j\big(-\partial_\mu c^k - \eta g f^{kmn} c^m A^n_\mu\big)\\
&= \eta g f^{ijk}\big(c^j \partial_\mu c^k - c^j \partial_\mu c^k\big)\\
&\quad + \frac{g^2}{2}(f^{ijk} f^{jmn} + f^{ijm} f^{jnk} + f^{ijn} f^{jkm}) c^m c^n A^k_\mu = 0\,,
\end{aligned} \tag{A.15}$$

where we have used the antisymmetry of the structure constants and of the ghost fields, and the Jacobi identity. This confirms that the assignment of Eq. (A.13) is consistent with Eqs. (A.12) and (A.14). Before proceeding, we should notice that another definition for the product can be used. In particular, Ref. 46 uses

$$s(XY) = (-1)^{\mathrm{GN}(Y)} s(X)Y + X s(Y)\,. \tag{A.16}$$

Then, to verify the nilpotency of s, we must reverse the sign in Eq. (A.13).

To prove the invariance of $\mathcal{L}_{\text{GF+Ghost}}$, we use the BRST technique. This is best explained for a simple group. We have

$$\mathcal{L}_{\text{GF+Ghost}} = -\frac{1}{2\xi}F_i^2 + \eta_G \bar{c}^j \frac{\delta F_j}{\delta \alpha_i} c^i = -\frac{1}{\xi}F_i^2 + \bar{c}^j s(F_j)\,, \tag{A.17}$$

where the last step follows from Eq. (A.11). Now, because of the nilpotency of the Slavnov operator, to ensure the invariance of Eq. (A.17) under BRST transformations, it is enough to require that

$$s(\bar{c}^j) = \eta_G \frac{1}{\xi} F^j\,. \tag{A.18}$$

If the gauge fixing is nonlinear, some subtleties arise, as explained in Ref. 46.

Coming back to the SM, we only have to verify that the Slavnov operator is indeed nilpotent in all the fields. We have verified this explicitly for all the cases. For completeness, we give here the action of the Slavnov operator in all of the SM fields, in a way consistent with our notation. We just give the electroweak part, because, for QCD, they can be read from Eqs. (A.13) and (A.14). We start with the gauge fields

$$\begin{aligned}
s(A_\mu) &= -\partial_\mu c_A - i\eta_e e\big(W_\mu^+ c^- - W_\mu^- c^+\big)\,,\\
s(Z_\mu) &= -\partial_\mu c_Z - i\eta\eta_Z g\cos\theta_W\big(W_\mu^+ c^- - W_\mu^- c^+\big)\,,\\
s(W_\mu^+) &= -\partial_\mu c^+ - i\eta g\big[c^+(\eta_Z Z_\mu \cos\theta_W + \eta_\theta A_\mu \sin\theta_W)\\
&\qquad - (\eta_Z c_Z \cos\theta_W + \eta_\theta c_A \sin\theta_W) W_\mu^+\big]\,,\\
s(W_\mu^-) &= -\partial_\mu c^- + i\eta g\big[c^-(\eta_Z Z_\mu \cos\theta_W + \eta_\theta A_\mu \sin\theta_W)\\
&\qquad - (\eta_Z c_Z \cos\theta_W + \eta_\theta c_A \sin\theta_W) W_\mu^-\big]\,.
\end{aligned} \tag{A.19}$$

For the Higgs we get

$$\begin{aligned}
s(\varphi_Z) &= \frac{1}{2}\eta g(c^-\varphi^+ + c^+\varphi^-) - \eta\frac{g}{2\cos\theta_W}\eta_Z c_Z (v+H)\,,\\
s(\varphi^+) &= i\eta\frac{g}{2}(v+H+i\varphi_Z)c^+ + i\eta\frac{g}{2}\frac{\cos 2\theta_W}{\cos\theta_W}\varphi^+\eta_Z c_Z + i\eta_e e\varphi^+ c_A\,,\\
s(\varphi^-) &= -i\eta\frac{g}{2}(v+H-i\varphi_Z)c^- - i\eta\frac{g}{2}\frac{\cos 2\theta_W}{\cos\theta_W}\varphi^-\eta_Z c_Z - i\eta_e e\varphi^- c_A\,,\\
s(H) &= -i\eta\frac{g}{2}(c^+\varphi^- - c^-\varphi^+) + \eta\frac{g}{2\cos\theta_W}\eta_Z c_Z \varphi_Z\,,
\end{aligned} \tag{A.20}$$

and for the fermions,

$$
\begin{aligned}
s(\psi_L) &= \left[i\eta \frac{g}{\sqrt{2}} (\tau^+ c^+ + \tau^- c^-) + i\eta_e e Q c_A \right. \\
&\qquad \left. + i\eta \frac{g}{\cos\theta_W} \left(\frac{\tau_3}{2} - Q \sin^2\theta_W \right) \eta_Z c_Z \right] \psi_L \,, \\
s(\psi_R) &= \left[i\eta_e e Q c_A - i\eta \frac{g}{\cos\theta_W} Q \sin^2\theta_W \eta_Z c_Z \right] \psi_R \,.
\end{aligned}
\tag{A.21}
$$

Finally, we need the rules for the ghost fields. These are obtained from Eq. (A.13). We get

$$
\begin{aligned}
s(c_A) &= i\eta_e e c^+ c^- \,, \\
s(c_Z) &= i\eta\eta_Z g \cos\theta_W c^+ c^- \,, \\
s(c^+) &= i\eta\eta_Z g \cos\theta_W c_Z c^+ + i\eta_e e c_A c^+ \,, \\
s(c^-) &= -i\eta\eta_Z g \cos\theta_W c_Z c^- - i\eta_e e c_A c^- \,.
\end{aligned}
\tag{A.22}
$$

References

1. J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
2. D. Bailin and A. Love, *Introduction to Gauge Field Theory* (Hilger, Bristol, UK, 1986) [errata available online at http://www.phys.susx.ac.uk/∼mpfg9/fterta.htm].
3. S. Pokorski, *Gauge Field Theories*, 2nd edn. (Cambridge University Press, Cambridge, 2000) [errata available online at http://www.fuw.edu.pl/∼pokorski/errata08.pdf].
4. J. P. Silva, Invited lectures, presented at the *Central European School in Particle Physics*, Faculty of Mathematics and Physics, Charles University, Prague, 14–24 September 2004, arXiv:hep-ph/0410351.
5. A. Lahiri and P. B. Pal, *A First Book of Quantum Field Theory*, 2nd edn. (Alpha Science International, Harrow, UK, 2007).
6. F. Mandl and G. Shaw, *Quantum Field Theory*, 2nd edn. (Wiley, Chichester, UK, 2010).
7. C. Quigg, *Gauge Theories of the Strong, Weak and Electromagnetic Interactions* (Benjamin/Cummings, New York, 1983).
8. F. Halzen and A. D. Martin, *Quarks and Leptons*: *An Introductory Course in Modern Particle Physics* (Wiley, New York, USA, 1984).
9. R. J. Rivers, *Path Integral Methods in Quantum Field Theory* (Cambridge University Press, Cambridge, 1987).
10. I. J. R. Aitchison and A. J. G. Hey, *Gauge Theories in Particle Physics*: *A Practical Introduction*, 2nd edn. (Hilger, Bristol, UK, 1989).
11. J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *The Higgs Hunter's Guide* (Perseus Books, New York, 1990).
12. V. D. Barger and R. J. N. Phillips, *Collider Physics*, updated edition (Westview, New York, 1991).
13. K. Huang, *Quarks, Leptons and Gauge Fields*, 2nd edn. (World Scientific, Singapore, 1992).
14. J. F. Donoghue, E. Golowich and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, 1992).

15. G. F. Sterman, *An Introduction to Quantum Field Theory* (Cambridge University Press, Cambridge, 1993).
16. F. Gross, *Relativistic Quantum Mechanics and Field Theory* (Wiley, New York, 1993).
17. D. Griffiths, *Introduction to Elementary Particles*, revised 2nd edn. (Wiley, Weinheim, Germany, 2008).
18. M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, USA, 1995).
19. L. Alvarez-Gaume and M. A. Vazquez-Mozo, *An Invitation to Quantum Field Theory* (Springer, Berlin, 2012).
20. K. I. Aoki, Z. Hioki, M. Konuma, R. Kawabe and T. Muta, *Prog. Theor. Phys. Suppl.* **73**, 1 (1982).
21. L. B. Okun, *Leptons and Quarks* (North-Holland, Amsterdam, 1982).
22. L. H. Ryder, *Quantum Field Theory* (Cambridge University Press, Cambridge, 1985).
23. T. P. Cheng and L. F. Li, *Gauge Theory of Elementary Particle Physics* (Oxford University Press, Oxford, 1988).
24. R. N. Mohapatra, *Unification and Supersymmetry. The Frontiers of Quark–Lepton Physics*, 2nd edn. (Springer, New York, 1992).
25. Fayyazuddin and Riazuddin, *A Modern Introduction to Particle Physics* (World Scientific, Singapore, 1994).
26. W. B. Rolnick, *The Fundamental Particles and their Interactions* (Addison-Wesley, Reading, USA, 1994).
27. G. L. Kane, *Modern Elementary Particle Physics*, updated edition (Addison-Wesley, Redwood City, USA, 1994).
28. V. P. Nair, *Quantum Field Theory*: *A Modern Perspective* (Springer, New York, 2005).
29. A. Djouadi, *Phys. Rep.* **459**, 1 (2008), arXiv:hep-ph/0503173.
30. A. Djouadi, *Phys. Rep.* **457**, 1 (2008), arXiv:hep-ph/0503172.
31. A. Zee, *Quantum Field Theory in a Nutshell* (Princeton University Press, Princeton, 2010).
32. G. C. Branco, L. Lavoura and J. P. Silva, *CP Violation* (Oxford University Press, Oxford, 1999).
33. W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, *J. Phys. G* **35**, 075001 (2008), arXiv:0711.4022 [hep-ph].
34. C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1986).
35. S. Sakakibara, *Phys. Rev. D* **24**, 1149 (1981).
36. A. Barroso, J. Pulido and J. C. Romao, *Nucl. Phys. B* **267**, 509 (1986).
37. P. Nogueira, Ward identities for $Z\gamma\gamma$ and $ZZ\gamma$ Green's functions, Master thesis, IST, Lisbon (1987).
38. J. C. Romao, Advanced quantum field theory, available online at http://porthos.ist.utl.pt/ftp/textos/tca.pdf.
39. A. Das, *Lectures on Quantum Field Theory* (World Scientific, Singapore, 2008).
40. N. D. Christensen and C. Duhr, *Comput. Phys. Commun.* **180**, 1614 (2009), arXiv:0806.4194.
41. M. Srednicki, *Quantum Field Theory* (Cambridge University Press, UK, 2007).
42. S. Weinberg, *The Quantum Theory of Fields. Vol. 2*: *Modern Applications* (Cambridge University Press, UK, 1996).
43. D. Y. Bardin and G. Passarino, *The Standard Model in the Making*: *Precision Study of the Electroweak Interactions* (Oxford University Press, UK, 1999).
44. C. Becchi, A. Rouet and R. Stora, *Ann. Phys.* **98**, 287 (1976).
45. I. Tyutin, arXiv:0812.0580.
46. J. C. Romao and A. Barroso, *Phys. Rev. D* **35**, 2836 (1987).
47. P. Langacker, *The Standard Model and Beyond* (CRC Press, New York, 2009).