Neutrino mixing with revamped A_4 flavor symmetry

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We suggest a minimal extension of the simplest A_4 flavor model that can induce a nonzero θ_{13} value, as required by recent neutrino oscillation data from reactors and accelerators. The predicted correlation between the atmospheric mixing angle θ_{23} and the magnitude of θ_{13} leads to an allowed region substantially smaller than indicated by neutrino-oscillation global fits. Moreover, the scheme correlates CP violation in neutrino oscillations with the octant of the atmospheric mixing parameter θ_{23} in such a way that, for example, maximal mixing necessarily violates CP. We briefly comment on other phenomenological features of the model.

I. INTRODUCTION

The historic discovery of neutrino oscillations [1] constitutes a breakthrough in particle physics as it implies the need of new physics beyond the standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ model. The detailed nature of this physics remains elusive, in particular regarding the flavor structure of the mechanism responsible for neutrino mass generation, and its characteristic scale [2]. Early studies establishing the oscillation phenomenon have indicated a very specific pattern for the neutrino mixing angles, vastly different from the Cabibbo-Kobayashi-Maskawa mixing pattern [3,4]: while the atmospheric angle θ_{23} is close to maximal, the solar angle θ_{12} is close to 30 degrees and no evidence was then present for a nonzero θ_{13} value. Although the latter may be accidental, it most likely follows a rationale. This has motivated a strong effort towards the formulation of symmetry-based approaches to address the flavor problem, in terms of an underlying flavor symmetry of leptons and/or quarks, separately or jointly. Indeed, these earlier observations were successfully accounted for in terms of an underlying A_4 flavor symmetry [5,6].

However, the recent accelerator experiments MINOS [7] and T2K [8,9] as well as the measurements reported by the Double CHOOZ [10], Daya Bay [11], and RENO reactor experiments [12] have provided robust indications that θ_{13} is nonzero, opening the door to the possibility of CP violation in neutrino oscillations [13,14]. This finding provides a challenge for many A_4-based schemes [5,6], especially those leading to the so-called tribimaximal (TBM) mixing ansatz proposed by Harrison, Perkins, and Scott [15]. This scheme has effective bimaximal mixing at the atmospheric scale and effective trimaximal mixing at the solar scale.

Here we focus on the model proposed by Babu, Ma, and Valle [5] and studied in detail in Ref. [16]. We present a simple extension of the model that introduces an extra scalar singlet flavon field ζ transforming as a $1'$ of A_4 to the Yukawa sector of the model. We show explicitly how this breaks the remnant symmetry present in the charged lepton sector,[1] so as to induce a nonzero θ_{13} value, hence making the model fully realistic and opening the possibility of CP violation in neutrino oscillation. Both θ_{13} and the CP-violation-invariant J_{CP} correlate with the new term added to the model superpotential. In particular, we show how the model predicts a stringent correlation between the atmospheric and the reactor mixing parameters, substantially more restrictive than the allowed regions that emerge from recent global fits of neutrino oscillations carried out within a generic flavor-blind scheme. We show how the model correlates CP violation in neutrino oscillation with the octant of the atmospheric mixing parameter θ_{23}, and briefly comment on other possible phenomenological implications.

II. THE BMV MODEL

We first recall the basic features of the Babu-Ma-Valle (BMV) model [5]. The particle content is collected in Tables I and II. The model implements an A_4 flavor symmetry within a supersymmetric context. A_4 is a discrete non-Abelian group of even permutations of four objects, it has $\underline{1}$, $\underline{1}'$, $\underline{1}''$ and $\underline{3}$ irreducible representations (irrep), and it is the smallest finite group with a triplet irrep. The decomposition property of the product is

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[1]In Ref. [17] an extra scalar singlet was added in order to modify the mixing in the neutrino sector instead of the charged lepton sector. In contrast, deviations from the TBM ansatz may also arise from the charged lepton sector, as described in Refs. [18,19].

TABLE I. Flavor assignments of the MSSM fields, with $\omega = \exp i2\pi/3$.

	$\hat{Q}$	$\hat{L}$	$\hat{u}_1^c, \hat{d}_1^c, \hat{e}_1^c$	$\hat{u}_2^c, \hat{d}_2^c, \hat{e}_2^c$	$\hat{u}_3^c, \hat{d}_3^c, \hat{e}_3^c$	$\hat{\phi}_{1,2}$
A_4	3	3	1	$1'$	$1''$	1
Z_3	1	1	ω^2	ω^2	ω^2	1

TABLE II. Mirror quark, lepton, and Higgs superfield assignments, with $\omega = \exp i2\pi/3$.

	$\hat{U}$	$\hat{U}^c$	$\hat{D}$	$\hat{D}^c$	$\hat{E}$	$\hat{E}^c$	$\hat{N}^c$	$\hat{\chi}$
A_4	3	3	3	3	3	3	3	3
Z_3	1	1	1	1	1	1	1	ω

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \tag{1}$$

The usual quark $\hat{Q}_i = (\hat{u}_i, \hat{d}_i)$, lepton $\hat{L}_i = (\hat{v}_i, \hat{e}_i)$, and Higgs $\hat{\phi}_i$ transform under A_4 as given in Table I. In addition, one adds the heavy quark, lepton, and Higgs superfields indicated in Table II. These are all SU(2) singlets.

The superpotential of the BMV model is then given by

$$\hat{W} = M_U \hat{U}_i \hat{U}_i^c + f_u \hat{Q}_i \hat{U}_i^c \hat{\phi}_2 + h_{ijk}^u \hat{U}_i \hat{u}_i^c \hat{\chi}_k + M_D \hat{D}_i \hat{D}_i^c$$
$$+ f_d \hat{Q}_i \hat{D}_i^c \hat{\phi}_2 + h_{ijk}^d \hat{D}_i \hat{d}_i^c \hat{\chi}_k + M_E \hat{E}_i \hat{E}_i^c + f_e \hat{L}_i \hat{E}_i^c \hat{\phi}_1$$
$$+ h_{ijk}^e \hat{E}_i \hat{e}_j^c \hat{\chi}_k + \frac{1}{2} M_N \hat{N}_i^c \hat{N}_i^c + f_N \hat{L}_i \hat{N}_i^c \hat{\phi}_2$$
$$+ \mu \hat{\phi}_1 \hat{\phi}_2 + \frac{1}{2} M_\chi \hat{\chi}_i \hat{\chi}_i + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3. \tag{2}$$

The scalar potential involving χ_i is given by

$$V = |M_\chi \chi_1 + h_\chi \chi_2 \chi_3|^2 + |M_\chi \chi_2 + h_\chi \chi_3 \chi_1|^2$$
$$+ |M_\chi \chi_3 + h_\chi \chi_1 \chi_2|^2, \tag{3}$$

which has the supersymmetric solution ($V = 0$)

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u. \tag{4}$$

We assume that the A_4 flavor symmetry is broken softly at some high scale [5].

The Dirac mass matrix linking (e_i, E_i) to (e_j^c, E_j^c) can be written as

$$\mathcal{M}_{eE} = \begin{bmatrix} 0 & 0 & 0 & f_e v_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & f_e v_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_e v_1 \\ h_1^e u & h_2^e u & h_3^e u & M_E & 0 & 0 \\ h_1^e u & h_2^e u\omega & h_3^e u\omega^2 & 0 & M_E & 0 \\ h_1^e u & h_2^e u\omega^2 & h_3^e u\omega & 0 & 0 & M_E \end{bmatrix}$$
$$\equiv \begin{bmatrix} 0 & X_1^D \\ X_2 & Y^D \end{bmatrix}, \tag{5}$$

where $v_1 = \langle \phi_1^0 \rangle$,[2] with similar forms also for the corresponding quark-mass matrices. After performing the block diagonalization of Eq. (5), one finds that the reduced 3×3 Dirac mass matrix for the charged leptons is diagonalized by the magic matrix U_ω,

$$U_\omega = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}. \tag{6}$$

For $f_e v_1 \ll h_i u \ll M_E$ the charged lepton masses are obtained as

$$\tilde{m}_i^2 \simeq \frac{3 f_e^2 v_1^2}{M_E^2} \frac{h_i^{e2} u^2}{1 + 3(h_i^e u)^2/M_E^2}. \tag{7}$$

Turning to the neutral sector, the Majorana mass matrix in the basis (ν_i, N_i^c) and in the basis where charged leptons are diagonal is given by

$$\mathcal{M}_{\nu N} = \begin{bmatrix} 0 & f_N v_2 U_\omega \\ f_N v_2 U_\omega^T & M_N \end{bmatrix}, \tag{8}$$

where $v_2 = \langle \phi_2^0 \rangle$. Hence, the reduced light neutrino mass matrix after seesaw becomes

$$\mathcal{M}_\nu = \frac{f_N^2 v_2^2}{M_N} U_\omega^T U_\omega = m_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = m_0 \boldsymbol{\lambda}, \tag{9}$$

leading to degenerate neutrino masses at this stage. Equation (9) is corrected by the wave-function renormalizations of ν_i, as well as the corresponding vertex renormalizations [5]. Given the structure of the λ_{ij} at the high scale [Eq. (9)], its form at low scale is fixed to first order as

$$\boldsymbol{\lambda} = \begin{bmatrix} 1 + 2\delta_{ee} & \delta_{e\mu} + \delta_{e\tau} & \delta_{e\mu} + \delta_{e\tau} \\ \delta_{e\mu} + \delta_{e\tau} & 2\delta_{\mu\tau} & 1 + \delta_{\mu\mu} + \delta_{\tau\tau} \\ \delta_{e\mu} + \delta_{e\tau} & 1 + \delta_{\mu\mu} + \delta_{\tau\tau} & 2\delta_{\mu\tau} \end{bmatrix}, \tag{10}$$

where all parameters are assumed to be real [5]. By rewriting Eq. (9) with $\delta_0 \equiv \delta_{\mu\mu} + \delta_{\tau\tau} - 2\delta_{\mu\tau}$, $\delta \equiv 2\delta_{\mu\tau}$, $\delta' \equiv \delta_{ee} - \delta_{\mu\mu}/2 - \delta_{\tau\tau}/2$, and $\delta'' \equiv \delta_{e\mu} + \delta_{e\tau}$, one has

$$\begin{bmatrix} 1 + \delta_0 + 2\delta + 2\delta' & \delta'' & \delta'' \\ \delta'' & \delta & 1 + \delta_0 + \delta \\ \delta'' & 1 + \delta_0 + \delta & \delta \end{bmatrix}, \tag{11}$$

so that the eigenvectors and eigenvalues can be determined *exactly*. The effective neutrino mixing matrix is given by

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[2]Here $\phi_{1,2}$ are the usual two Higgs of supersymmetry.

$$U_\nu(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}, \quad (12)$$

while the three light neutrino mass eigenvalues are

$$\begin{aligned} \lambda_1 &= 1 + \delta_0 + 2\delta + \delta' - \sqrt{\delta'^2 + 2\delta''^2}, \\ \lambda_2 &= 1 + \delta_0 + 2\delta + \delta' + \sqrt{\delta'^2 + 2\delta''^2}, \\ \lambda_3 &= -1 - \delta_0, \end{aligned} \quad (13)$$

so that one finds that the BMV model predictions for the neutrino mixing angles are

$$\begin{aligned} \tan^2\theta_{12} &= \frac{\delta''^2}{\delta''^2 + \delta'^2 - \delta'\sqrt{\delta'^2 + 2\delta''^2}}, \\ \sin^2\theta_{13} &= 0, \\ \tan^2\theta_{23} &= 1 \Rightarrow \text{maximal.} \end{aligned} \quad (14)$$

For the other oscillation parameters, namely the squared-mass differences, assuming $\delta', \delta'' \ll \delta$, one has

$$\Delta m_{31}^2 \simeq \Delta m_{32}^2 \simeq 4\delta m_0^2, \quad \Delta m_{21}^2 \simeq 4\sqrt{\delta'^2 + 2\delta''^2}\,m_0^2. \quad (15)$$

One sees that the mixing matrix in the neutrino sector in Eq. (14) has just one free parameter θ, which corresponds to the unpredicted solar mixing angle θ_{12}. One now assumes that radiative corrections lift the neutrino mass degeneracy, as required by the solar neutrino oscillation data. Using the solar angle in Eq. (14) and the squared-mass differences (15), one can estimate the size of some of the wave-function and vertex corrections required in order to fit the observed oscillation parameters. One finds the following relations:

$$\begin{aligned} \frac{\delta}{|\delta'|} &= \xi \frac{\Delta m_{31}^2}{\Delta m_{21}^2}\left(\frac{1}{1 - 2\sin^2\theta}\right) \approx 92.96\xi, \\ \frac{|\delta''|}{|\delta'|} &= \sqrt{\frac{1}{2}\left[\left(\frac{1}{1 - 2\sin^2\theta}\right)^2 - 1\right]} \approx 1.83, \end{aligned} \quad (16)$$

where $\xi = 1(-1)$ corresponds to the case of $\delta' < 0$ ($\delta' > 0$). In order to fit neutrino oscillation data, the threshold parameter δ' must be of the same order as δ'' and also δ', $\delta'' \ll \delta$. With $\delta' < 0$ and $|\delta''/\delta'| = 1.8$ the predicted neutrino mixing pattern is indeed consistent with the oscillation data before the latest T2K, Daya Bay and RENO results for θ_{13}.

III. REVAMPING THE ORIGINAL A_4 MODEL

The main goal of this paper is to accommodate the current neutrino data [20] within a minimally extended A_4-based BMV scenario. In general, the effective mixing in the leptonic sector is given by

$$K = U_\nu(\theta), \quad (17)$$

where we have rotated by the magic matrix U_ω. The idea is now to generate modifications of the mixing in the leptonic sector $U'_\omega = U_\omega U_\delta$, in such a way that the modified lepton mixing matrix is now given by

$$K' = U_\delta^\dagger U_\nu(\theta), \quad (18)$$

where U_δ denotes a correction which may yield a non-vanishing θ_{13} while keeping good predictions for the other neutrino oscillation parameters, in particular, the atmospheric mixing angle θ_{23}.

A. Charged lepton corrections to lepton mixing

As a first attempt we relax the condition used to obtain the charged lepton masses, Eq. (7), by allowing the M_E scale [see Eq. (5)] to lie at the TeV scale.[3] This results in unitarity-violation corrections to the lepton mixing matrix. With M_E in Eq. (5) at the TeV scale, one must take into account not only the first-order terms in the block diagonalization of the mass matrix of the charged lepton sector as in Eq. (7), but also the next-to-leading-order effects. Using the Schechter-Valle procedure [21] for the block diagonalization one finds

$$U = \mathcal{U} \cdot V = \exp(iH) \cdot V, \qquad H = \begin{pmatrix} 0 & S \\ S^\dagger & 0 \end{pmatrix},$$
$$V = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}, \quad (19)$$

where H is an anti-Hermitian operator and V_i are unitary matrices which diagonalize each block. The S matrix is determined at first order by the diagonalization condition $U^\dagger M U = \text{Diag}\{m_i\}$ for a given Hermitian matrix M. In our specific case,

$$\begin{aligned} \mathcal{M}_{eE}&(\mathcal{M}_{eE})^\dagger \\ &= \begin{bmatrix} (f_e v_1)^2 I & M_E f_e v_1 I \\ M_E f_e v_1 I & U_\omega(\text{Diag}\{3(h_i^e u)^2\})U_\omega^\dagger + M_E^2 I \end{bmatrix} \\ &\equiv \begin{bmatrix} m_1^2 & m_2^2 \\ m_2^{2\dagger} & m_3^2 \end{bmatrix}, \end{aligned} \quad (20)$$

the S is given by

$$\begin{aligned} iS &= -m_2^2(m_1^2 - m_3^2)^{-1} \\ &= U_\omega \text{diag}\{-M_E f_e v_1[(f_e v_1)^2 - 3(h_i^e u)^2 - M_E^2]^{-1}\}U_\omega^\dagger, \end{aligned} \quad (21)$$

where the first term in Eq. (20) and the second term in Eq. (21) correspond to our specific case.

In order to calculate the next-to-leading-order terms, one expands the exponential in Eq. (19) in a power series in S.

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[3]This would lead to the existence of flavor-changing neutral currents at the tree level. These would induce sizeable lepton-flavor-violating processes.

The next-to-leading-order terms are combinations of the identity and products of $SS^\dagger$ and S. Given the structure of the S matrix in Eq. (21) is clear that even if we go to higher orders in the expansion, the effective charged lepton mass will always be diagonalized by the magic matrix U_ω. In other words, $U_\delta \equiv 1$.

The origin of the structure of the S matrix in Eq. (21) comes from the fact that in the BMV model the matrices in the upper right corner and the lower right corner in Eq. (5) are proportional to the identity. The net effect is that even allowing for unitarity violation in the charged sector does not change the structure of the lepton mixing matrix. Somehow a remnant symmetry of the A_4 remains that leads to $\theta_{13} \equiv 0$.

B. Minimal flavon extension of the original A_4 model

In order to break the unwanted remnant symmetry present in the charged lepton sector of the model, we now add a scalar singlet flavon field ζ to the superpotential in Eq. (2). The flavon scalar field ζ transforms as a $1'$ under the A_4 flavor symmetry. This leads to a new superpotential term of the form

$$\zeta(EE^c)_{1''}, \tag{22}$$

where we will parametrize the flavon scale as $\langle \zeta \rangle = \beta M_E$. This results in a new mass matrix for the lower right corner of Eq. (5) that now has the structure

$$Y_D = M_E \times I + \beta M_E \times \text{Diag}\{1, \omega, \omega^2\}, \tag{23}$$

so that the corresponding charged lepton matrix in Eq. (20) is now given by

$$\mathcal{M}_{eE}(\mathcal{M}_{eE})^\dagger$$
$$= \begin{bmatrix} (f_e v_1)^2 I & f_e v_1 Y_D^\dagger \\ f_e v_1 Y_D & U_\omega(\text{Diag}\{3(h_i^e u)^2\})U_\omega^\dagger + Y_D Y_D^\dagger \end{bmatrix}, \tag{24}$$

where Y_D is no longer diagonalized by the magic matrix. This changes the structure of the S matrix in Eq. (21) and breaks the unwanted remnant symmetry that leads to $\theta_{13} \equiv 0$. As a consequence, one obtains a corrected matrix U_ω^c that leads to a U_δ matrix of the form

$$U'_\omega = U_\omega U_\delta, \tag{25}$$

with which the effective lepton mixing matrix from Eq. (17) can be calculated. The modified lepton mixing K' is a complex nonunitary 3×3 matrix from which one must extract the three angles and three CP phases that characterize the simplest neutrino mixing parameter set. One finds that, indeed, the proposed flavon extension of the original A_4 model scheme can engender a nonzero value for the reactor mixing angle, as required by recent neutrino oscillation data.

IV. NEUTRINO OSCILLATION PARAMETERS

Using the modifications to the BMV model explained above [e.g., Eq. (24)], we have obtained quantitative results by numerically diagonalizing the charged lepton mixing matrix in Eq. (5). The three mixing angles are obtained directly as

$$\tan\theta_{12} = |K'_{1,2}(\theta)|/|K'_{1,1}(\theta)|,$$
$$\sin\theta_{13} = |K'_{1,3}(\theta)|, \tag{26}$$
$$\tan\theta_{23} = |K'_{2,3}(\theta)|/|K'_{3,3}(\theta)|,$$

where the θ parameter has been varied randomly in the range $0 \leq \sin^2\theta \leq 1$. The scales $f_e v_1$ and M_E have also
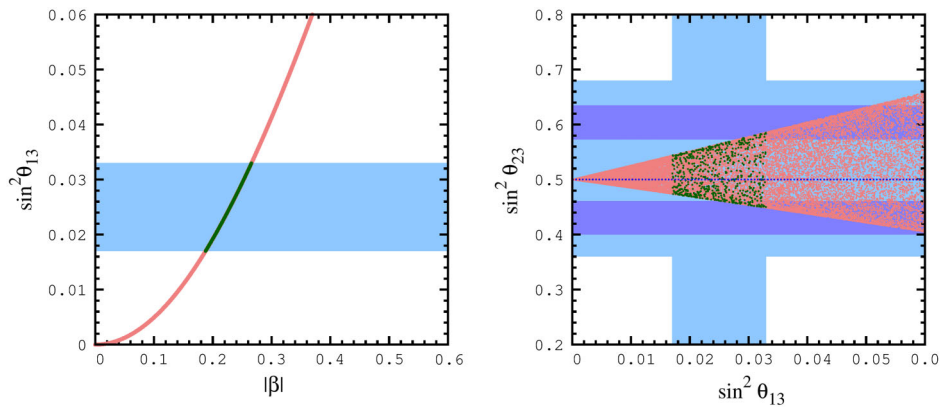

FIG. 1 (color online). (Left) Correlation between the reactor angle $\sin^2\theta_{13}$ and the magnitude of the flavon coupling parameter $|\beta|$. (Right) The "triangle" region gives the predicted correlation between atmospheric and reactor angles for different β parameter choices. The broad vertical (horizontal) bands are the current allowed values for $\sin^2\theta_{13}$ ($\sin^2\theta_{23}$) at 3σ. In both panels the flavon phase ϕ_β has been varied continuously in the range $-\pi/2 \leq \phi_\beta \leq \pi/2$. All points in the "triangle" are allowed by the θ_{12} 3σ solar angle range, but only the green (dark) points are consistent with θ_{13} as well. Finally, the two thin horizontal bands correspond to the 1σ preferred regions in the global oscillation fit of Ref. [20].
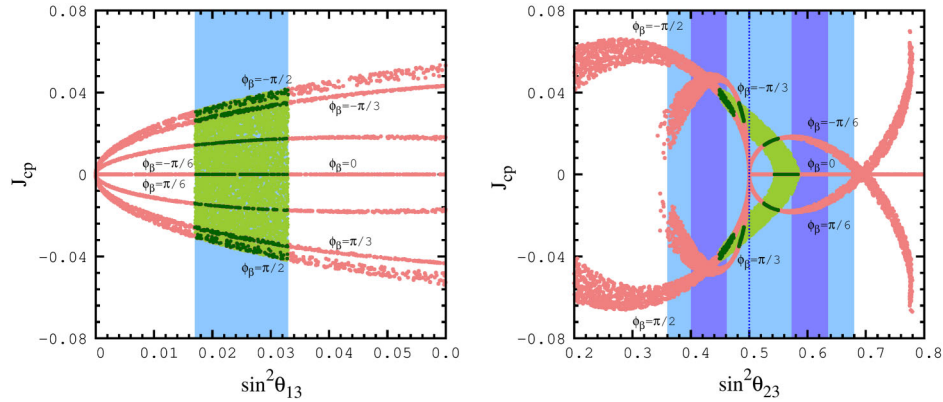
FIG. 2 (color online). Correlation between the magnitude of the *CP*-violation invariant J_{CP} and the two mixing angles, reactor θ_{13} and atmospheric θ_{23}, in left and right panels, respectively. Discrete set of phase values have been used in the range $-\pi/2 \leq \phi_\beta \leq \pi/2$ in steps of $\pi/6$. Random points in pink (grey) are compatible with the current 3σ range of the solar angle θ_{12}, but only the green (dark) points are compatible with the θ_{13} and θ_{23} range at 3σ.

been varied randomly in the range $1 \leq f_e v_1 \leq 10^2$ GeV and $10^4 \leq M_E \leq 10^5$ GeV, leading to the results presented in Fig. 1. As we will see, for such values the mixing matrix K' is well described by a unitary approximation.

As one can see from the left plot of Fig. 1, in order to generate a nonvanishing reactor angle θ_{13} the magnitude in the flavon coupling $|\beta|$ must be nonzero. In principle this result is independent of the phase ϕ_β.

On the other hand, from the right panel of Fig. 1, one sees how the new coupling engenders not only a nonzero θ_{13} value, but also a restricted range for the atmospheric angle θ_{23}. If one takes at face value the hints for nonmaximal θ_{23} at 1σ which follow from global oscillation fits [20] then one finds that the allowed regions for θ_{23} in each octant would be very narrow indeed. However, currently maximal atmospheric mixing remains perfectly consistent [9]. As one sees in the right panel of Fig. 2 for maximal atmospheric mixing, the flavon phase must have a nonzero value.

Note that the allowed region is modulated by the value of the β phase ϕ_β; in other words, as one varies the values of the phase $-\pi/2 \leq \phi_\beta \leq \pi/2$ one sweeps the triangle-shaped region indicated in the right panel of Fig. 1. One finds a linear correlation between the "opening angle" of the triangle and the magnitude of the continuous phase angle ϕ_β. Intermediate ϕ_β values cover the indicated shaded subregion of the vertical strip. While all current neutrino mixing angles, including the reactor angle θ_{13}, are consistent with a real flavon coupling, allowing the latter to be complex results in a determination of the octant for θ_{23} as shown in the right panel of Fig. 2. A measurement of the violating *CP* phase would imply a determination of the octant, or vice versa. Again, continuous phase values in between the extremes lead to the half-moon-like region indicated in the right panel in Fig. 2.

In order to further clarify the issue of leptonic *CP* violation within this model, we now turn to the Dirac phase δ_{CP} associated to *CP* violation in neutrino oscillations.

Rather than trying to extract this phase directly, we have calculated the associated Jarlskog parameter J_{CP},

$$J_{CP} = \mathcal{I}\{K_{e1}^* K_{\mu3}^* K_{e3} K_{\mu1}\}, \qquad (27)$$

which is invariant under any conceivable phase redefinitions. Our numerical result is shown in Fig. 2, in which we have numerically evaluated Eq. (27) for discrete values of the phase $-\pi/2 \leq \phi_\beta \leq \pi/2$ in steps of $\pi/6$. One can see that for $|\beta| > 0$ the invariant J_{CP} is nonzero in correlation with the nonzero value of the phase ϕ_β. By allowing the flavon coupling β to be complex one not only introduces *CP* violation in neutrino oscillations, but also selects the allowed octant of the atmospheric mixing angle θ_{23} in correspondence with the assumed values of the phase ϕ_β, which is clearly seen from the right panel of Fig. 2. This constitutes an important prediction of the model which may be tested in future neutrino oscillation experiments. In contrast, the Majorana phases can hardly be probed within this model since the mass spectrum is almost degenerate, so that there can never be an important destructive interference between different $0\nu\beta\beta$ amplitudes. As a result the $0\nu\beta\beta$ decay rate is expected to be large and should be probed in current and future experiments.

V. ANALYTICAL UNDERSTANDING

In order to gain a better understanding of the proposed scheme, we now turn to an analytic approach. We have fixed the M_E scale to be 10^2 times bigger than the TeV scale and we have obtained the correlations already displayed in Fig. 1. The result in the left panel suggests a simple theoretical relation. Indeed, assuming K' to be nearly unitary, we are within a perturbative limit where we can solve the problem analytically by diagonalizing the effective charged lepton mass matrix at the leading order and keeping only the terms up to second order in $|\beta|$. In this way we find a simple approximate result for the reactor angle given as
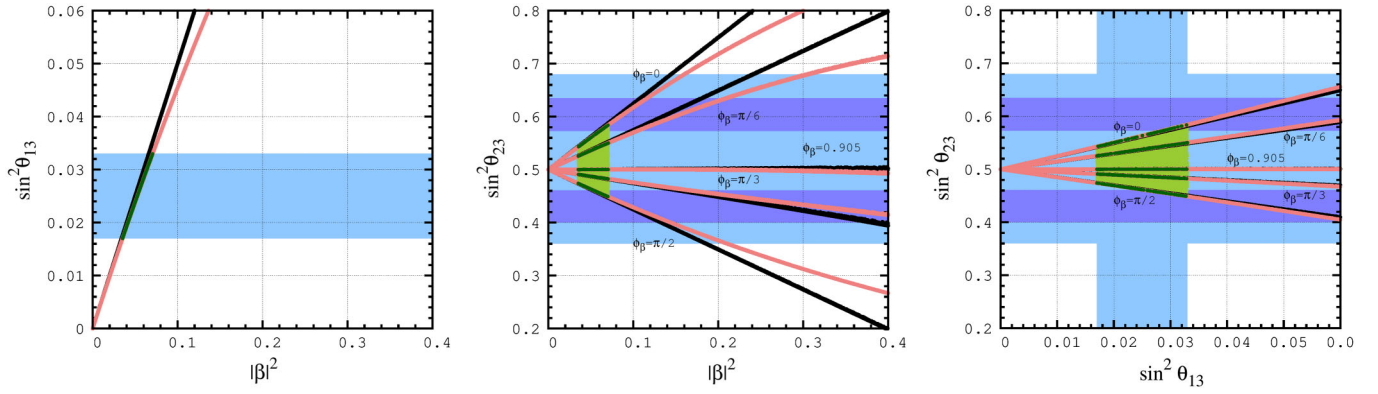
FIG. 3 (color online). Here we give the exact numerically determined predictions for the reactor and atmospheric mixing parameters θ_{13}, θ_{23} in terms of the magnitude of β and its phase ϕ_β varied in steps of $\pi/6$. We also give the results that follow from the approximate expressions in Eqs. (28) and (29). Numerical results are in pink (grey) while analytical ones are in black. There is rather good agreement within the currently allowed 3σ range of the neutrino mixing angles: θ_{12}, θ_{13}, and θ_{23} indicated by the blue (broad) bands. Notice that the negative values of the flavon phase give the same correlation, which is clear from Eqs. (28) and (29).

$$\sin{}^2\theta_{13} = |\beta|^2 \frac{h_1^8 - 2h_1^6 h_3^2 + 2h_1^4 h_3^4 - 2h_1^2 h_2^2 h_3^4 + h_2^4 h_3^4 - 2h_1^2 h_3^2 (h_1^2 - h_2^2)(h_1^2 - h_3^2) \cos 2\phi_\beta}{2[(h_1^2 - h_2^2)(h_1^2 - h_3^2)]^2} \tag{28}$$

in terms of the Yukawa parameters h_i that determine the charged lepton masses through Eq. (7). This way, one can explain analytically the right panel of Fig. 1 and conclude that $\sin\theta_{13}$ can be nonvanishing even if the value of the β phase is zero. In a completely analogous procedure, we have also obtained an approximate relation for the atmospheric angle,

$$\sin^2\theta_{23} = \frac{1}{2} + |\beta| \frac{h_2^2}{h_2^2 - h_3^2} \cos\phi_\beta$$
$$+ |\beta|^2 \frac{(-h_1^8 + 2h_1^6 h_3^2 + h_2^4 h_3^4)(h_2^2 - h_3^2) + 2h_1^2(h_2^2 h_3^6 - h_2^4 h_3^4) + 2h_3^2(h_1^2 - h_2^2)(h_1^2 - h_3^2)[h_1^2(h_2^2 + h_3^2) - 2h_2^2 h_3^2] \cos 2\phi_\beta}{4[(h_1^2 - h_2^2)(h_1^2 - h_3^2)]^2 (h_2^2 - h_3^2)}. \tag{29}$$

Numerically we have checked that the expansions in $|\beta|$ leading to the expressions in Eqs. (28) and (29) reproduce very well the numerical results for the correlations such as, for instance, those given by the curve in the left panel of Fig. 1 within the current allowed range indicated by global neutrino oscillation fits and summarized by the blue bands displayed in Fig. 3. As we have already noted, for special values of ϕ_β the octant of θ_{23} gets determined as shown in Figs. 1 and 3.

Before concluding, let us make one last comment on the size of the corrections in the neutrino sector. Within the revamped BMV model we have now introduced, the mixing predictions have been recalculated through the squared-mass differences given by Eq. (15). As we have already mentioned, the free parameter θ in the mixing corresponds to the solar angle θ_{12} for given values of the underlying radiative corrections. Due to the modified neutrino mixing pattern the correspondence between the free parameter and the solar angle is through the radiative corrections that come from the soft symmetry-breaking

sector and, strictly speaking, these are no longer the same as in the original flavon-less BMV model.

VI. DISCUSSION

We have proposed a minimal extension of the simplest A_4 flavor model of Babu, Ma, and Valle that can induce a nonzero θ_{13} value, as required by recent neutrino oscillation data coming from reactors and accelerators. We have shown how the predicted correlation between the atmospheric mixing angle θ_{23} and the magnitude of θ_{13} leads to an allowed region that is substantially smaller than that indicated by model-independent neutrino oscillation global fits. Moreover, our proposed scheme establishes a correlation between CP violation in neutrino oscillations and the octant of the atmospheric mixing parameter θ_{23}. In particular, one finds that, for example, maximal atmospheric mixing as well as the first octant necessarily violate CP. Currently we find that both are consistent at the 1σ level with the global (including atmospheric data) neutrino

oscillation analysis of Ref. [20]. We also stress that ours is a quasidegenerate neutrino scenario. Recent restrictions on the absolute neutrino mass from the Planck collaboration [22] indicate values for the parameter δ characterizing slepton radiative corrections for which lepton-flavor violation induced by supersymmetric particle exchanges is expected to lie at the limits. This would provide another complementary way to probe this model. This issue will be taken up elsewhere.

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