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Oblique corrections from triplet quarks

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Abstract: We present general formulas for the oblique-correction parameters S, T, U, V, W, and X in an extension of the Standard Model having arbitrary numbers of singlet, doublet, and triplet quarks with electric charges $-4/3$, $-1/3$, $2/3$, and $5/3$ that mix with the standard quarks of the same charge.

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1 Introduction and notation

In this paper we consider an extension of the standard SU(2) × U(1) gauge model with

h-type quarks, i.e. quarks with electric charge $Q_h = 5/3$,

u-type quarks, i.e. quarks with electric charge $Q_u = 2/3$,

d-type quarks, i.e. quarks with electric charge $Q_d = -1/3$,

and l-type quarks, i.e. quarks with electric charge $Q_l = -4/3$.

The total number of h-type quarks is n_h. The specific h-type quarks h and h' have masses m_h and $m_{h'}$, respectively. Similar notations are utilized for the u-type, d-type, and l-type quarks.

The mass of the gauge bosons $W^\pm$ is m_W. The mass of the gauge boson Z is m_Z. We define $c_w \equiv m_W/m_Z$ and $s_w \equiv \sqrt{1-c_w^2}$.

In our model there are arbitrary numbers of the following gauge-SU(2) multiplets of quarks [1]:

SU(2) singlets with weak hypercharge[1] 2/3

$$\sigma_{0,4,\aleph}; \tag{1.1}$$

SU(2) singlets with weak hypercharge $-1/3$

$$\sigma_{0,-2,\aleph}; \tag{1.2}$$

SU(2) doublets with weak hypercharge 7/6

$$\begin{pmatrix} \delta_{1,7,\aleph} \\ \delta_{-1,7,\aleph} \end{pmatrix}; \tag{1.3}$$

SU(2) doublets with weak hypercharge 1/6

$$\begin{pmatrix} \delta_{1,1,\aleph} \\ \delta_{-1,1,\aleph} \end{pmatrix}; \tag{1.4}$$

SU(2) doublets with weak hypercharge $-5/6$

$$\begin{pmatrix} \delta_{1,-5,\aleph} \\ \delta_{-1,-5,\aleph} \end{pmatrix}; \tag{1.5}$$

SU(2) triplets with weak hypercharge 2/3

$$\begin{pmatrix} \tau_{2,4,\aleph} \\ \tau_{0,4,\aleph} \\ \tau_{-2,4,\aleph} \end{pmatrix}; \tag{1.6}$$

SU(2) triplets with weak hypercharge $-1/3$

$$\begin{pmatrix} \tau_{2,-2,\aleph} \\ \tau_{0,-2,\aleph} \\ \tau_{-2,-2,\aleph} \end{pmatrix}. \tag{1.7}$$

In eqs. (1.1)–(1.7),

the letter σ denotes singlets of gauge SU(2), the letter δ stands for doublets, and the letter τ means triplets;

the first number in the subscript is two times the third component of weak isospin;

the second number in the subscript is six times the weak hypercharge;

the letter $\aleph$ stands for either L, in the case of left-handed quarks, or R, in the case of right-handed quarks.

[1] We use the normalization $Y = Q - T_3$, where Y is the weak hypercharge, Q is the electric charge, and T_3 is the third component of weak isospin.

The numbers of multiplets (1.1)–(1.7) in our generic model are $n_{\sigma,4,\aleph}$, $n_{\sigma,-2,\aleph}$, $n_{\delta,7,\aleph}$, $n_{\delta,1,\aleph}$, $n_{\delta,-5,\aleph}$, $n_{\tau,4,\aleph}$, and $n_{\tau,-2,\aleph}$, respectively. Clearly,

$$\begin{aligned} n_h &= n_{\delta,7,L} + n_{\tau,4,L} \\ &= n_{\delta,7,R} + n_{\tau,4,R}, \end{aligned} \tag{1.8a}$$

$$\begin{aligned} n_u &= n_{\sigma,4,L} + n_{\delta,7,L} + n_{\delta,1,L} + n_{\tau,4,L} + n_{\tau,-2,L} \\ &= n_{\sigma,4,R} + n_{\delta,7,R} + n_{\delta,1,R} + n_{\tau,4,R} + n_{\tau,-2,R}, \end{aligned} \tag{1.8b}$$

$$\begin{aligned} n_d &= n_{\sigma,-2,L} + n_{\delta,1,L} + n_{\delta,-5,L} + n_{\tau,4,L} + n_{\tau,-2,L} \\ &= n_{\sigma,-2,R} + n_{\delta,1,R} + n_{\delta,-5,R} + n_{\tau,4,R} + n_{\tau,-2,R}, \end{aligned} \tag{1.8c}$$

$$\begin{aligned} n_l &= n_{\delta,-5,L} + n_{\tau,-2,L} \\ &= n_{\delta,-5,R} + n_{\tau,-2,R}. \end{aligned} \tag{1.8d}$$

The purpose of this paper is to compute the oblique parameters in this generic model. The oblique parameters are defined as [2][2,3]

$$S = \frac{16\pi c_w^2}{g^2} \left[\frac{A_{ZZ}\left(m_Z^2\right) - A_{ZZ}\left(0\right)}{m_Z^2} + \frac{c_w^2 - s_w^2}{c_w s_w}\, \frac{\partial A_{\gamma Z}\left(q^2\right)}{\partial q^2}\bigg|_{q^2=0} - \frac{\partial A_{\gamma\gamma}\left(q^2\right)}{\partial q^2}\bigg|_{q^2=0} \right], \tag{1.9a}$$

$$T = \frac{4\pi}{g^2 s_w^2} \left[\frac{A_{WW}\left(0\right)}{m_W^2} - \frac{A_{ZZ}\left(0\right)}{m_Z^2} \right], \tag{1.9b}$$

$$\begin{aligned} U = \frac{16\pi}{g^2} \Bigg[& \frac{A_{WW}\left(m_W^2\right) - A_{WW}\left(0\right)}{m_W^2} - c_w^2\, \frac{A_{ZZ}\left(m_Z^2\right) - A_{ZZ}\left(0\right)}{m_Z^2} \\ & + 2 c_w s_w\, \frac{\partial A_{\gamma Z}\left(q^2\right)}{\partial q^2}\bigg|_{q^2=0} - s_w^2\, \frac{\partial A_{\gamma\gamma}\left(q^2\right)}{\partial q^2}\bigg|_{q^2=0} \Bigg], \end{aligned} \tag{1.9c}$$

$$V = \frac{4\pi}{g^2 s_w^2} \left[\frac{\partial A_{ZZ}\left(q^2\right)}{\partial q^2}\bigg|_{q^2=m_Z^2} - \frac{A_{ZZ}\left(m_Z^2\right) - A_{ZZ}\left(0\right)}{m_Z^2} \right], \tag{1.9d}$$

$$W = \frac{4\pi}{g^2 s_w^2} \left[\frac{\partial A_{WW}\left(q^2\right)}{\partial q^2}\bigg|_{q^2=m_W^2} - \frac{A_{WW}\left(m_W^2\right) - A_{WW}\left(0\right)}{m_W^2} \right], \tag{1.9e}$$

$$X = \frac{4\pi c_w}{g^2 s_w} \left[\frac{\partial A_{\gamma Z}\left(q^2\right)}{\partial q^2}\bigg|_{q^2=0} - \frac{A_{\gamma Z}\left(m_Z^2\right) - A_{\gamma Z}\left(0\right)}{m_Z^2} \right], \tag{1.9f}$$

where g is the SU(2) gauge coupling constant. The $A_{VV'}\left(q^2\right)$ are the coefficients of the metric tensor $g^{\mu\nu}$ in the vacuum-polarization tensor

$$\Pi^{\mu\nu}_{VV'}\left(q^2\right) = g^{\mu\nu}\, A_{VV'}\left(q^2\right) + q^\mu q^\nu\, B_{VV'}\left(q^2\right) \tag{1.10}$$

between gauge bosons V_μ and V'_ν carrying four-momentum q. In $A_{VV'}\left(q^2\right)$

[2]We use the sign conventions in ref. [8]. Those conventions differ from the ones used in many other papers, viz. in ref. [2]. For a resource paper on sign conventions, see ref. [9]; using the notation of that paper, our convention has $\eta_e = \eta_Z = 1$ and $\eta = -1$.

[3]The definitions (1.9) build on, and generalize, previous work in refs. [3–7]. They are appropriate for the case where the functions $A_{VV'}\left(q^2\right)$ are not linear in the range $0 < q^2 < m_Z^2$, viz. where New Physics is not much above the Fermi scale.

- one only takes into account the dispersive part — one discards the absorptive part;
- one subtracts the Standard-Model contribution from the New-Physics-model one.

This paper generalizes the results of ref. [10], where only the multiplets (1.1), (1.2), and (1.4) existed, hence no SU(2) triplets, and neither h-type nor l-type quarks were present. It also generalizes recent partial results that appeared in refs. [11–13].

The outline of this paper is as follows. In section 2 we present our notation for the gauge interactions. In section 3 we present our notation for the Passarino-Veltman (PV) functions. In section 4 we display the results for the oblique parameters. Thereafter, three appendices deal with technical issues: appendix A gives technical details of the computations, appendix B gives analytic formulas for the PV functions, and appendix C demonstrates the cancellation of the ultraviolet divergences in S, T, and U. The reader does not need to read the appendices in order to fully understand the scope and results of this paper.

2 Notation for the gauge interactions

The interactions of the quarks with the photon field A_μ are given by

$$\mathcal{L}_A = -g s_w A_\mu \left(Q_h \sum_h \bar{h}\gamma^\mu h + Q_u \sum_u \bar{u}\gamma^\mu u + Q_d \sum_d \bar{d}\gamma^\mu d + Q_l \sum_l \bar{l}\gamma^\mu l \right). \tag{2.1}$$

The interactions of the quarks with the gauge bosons $W^\pm$ are given by

$$\begin{aligned}\mathcal{L}_W = \frac{g}{\sqrt{2}}\, W_\mu^+ \Bigg\{ & \sum_{h,u} \bar{h}\,\gamma^\mu \left[(N_L)_{hu}\,\gamma_L + (N_R)_{hu}\,\gamma_R \right] u \\ & + \sum_{u,d} \bar{u}\,\gamma^\mu \left[(V_L)_{ud}\,\gamma_L + (V_R)_{ud}\,\gamma_R \right] d \\ & + \sum_{d,l} \bar{d}\,\gamma^\mu \left[(Q_L)_{dl}\,\gamma_L + (Q_R)_{dl}\,\gamma_R \right] l \Bigg\} + \text{H.c.},\end{aligned} \tag{2.2}$$

where $\gamma_L = (1-\gamma_5)/\,2$ and $\gamma_R = (1+\gamma_5)/\,2$ are the projectors of chirality. Note the presence of the

- $n_h \times n_u$ mixing matrices $N_\aleph$,
- $n_u \times n_d$ mixing matrices $V_\aleph$,
- and $n_d \times n_l$ mixing matrices $Q_\aleph$.

The matrix V_L is the generalized Cabibbo-Kobayashi-Maskawa matrix.

The interactions of the quarks with the gauge boson Z are given by

$$\begin{aligned}\mathcal{L}_Z = \frac{g}{2c_w} Z_\mu \Bigg\{ & \sum_{h,h'} \bar{h}\,\gamma^\mu \left[\left(\bar{H}_L\right)_{hh'} \gamma_L + \left(\bar{H}_R\right)_{hh'} \gamma_R\right] h' \\ & + \sum_{u,u'} \bar{u}\,\gamma^\mu \left[\left(\bar{U}_L\right)_{uu'} \gamma_L + \left(\bar{U}_R\right)_{uu'} \gamma_R\right] u' \\ & - \sum_{d,d'} \bar{d}\,\gamma^\mu \left[\left(\bar{D}_L\right)_{dd'} \gamma_L + \left(\bar{D}_R\right)_{dd'} \gamma_R\right] d' \\ & - \sum_{l,l'} \bar{l}\,\gamma^\mu \left[\left(\bar{L}_L\right)_{ll'} \gamma_L + \left(\bar{L}_R\right)_{ll'} \gamma_R\right] l' \Bigg\}, \end{aligned} \tag{2.3}$$

with Hermitian mixing matrices $\bar{H}_\aleph$, $\bar{U}_\aleph$, $\bar{D}_\aleph$, and $\bar{L}_\aleph$. Note the minus signs in the third and fourth lines of eq. (2.3). Since the Z couples to a current proportional to $(g/c_w)\,(T_3 - Q s_w^2)$, those matrices are of the form

$$\bar{H}_\aleph = H_\aleph - 2\,|Q_h|\, s_w^2\, \mathbb{1}, \tag{2.4a}$$

$$\bar{U}_\aleph = U_\aleph - 2\,|Q_u|\, s_w^2\, \mathbb{1}, \tag{2.4b}$$

$$\bar{D}_\aleph = D_\aleph - 2\,|Q_d|\, s_w^2\, \mathbb{1}, \tag{2.4c}$$

$$\bar{L}_\aleph = L_\aleph - 2\,|Q_l|\, s_w^2\, \mathbb{1}. \tag{2.4d}$$

where $\mathbb{1}$ always is the unit matrix of the appropriate dimension.

Because of the SU(2) algebra relation $T_3 = [T_+,\ T_-]$, where T_+ and T_- are the SU(2) raising and lowering operators, respectively, there are relations between the mixing matrices appearing in eq. (2.3) and the ones in eq. (2.2), viz.

$$H_\aleph = N_\aleph N_\aleph^\dagger, \tag{2.5a}$$

$$U_\aleph = V_\aleph V_\aleph^\dagger - N_\aleph^\dagger N_\aleph, \tag{2.5b}$$

$$D_\aleph = V_\aleph^\dagger V_\aleph - Q_\aleph Q_\aleph^\dagger, \tag{2.5c}$$

$$L_\aleph = Q_\aleph^\dagger Q_\aleph. \tag{2.5d}$$

Thus, the matrices $N_\aleph$, $V_\aleph$, and $Q_\aleph$ are the fundamental ones, while the matrices $H_\aleph$, $U_\aleph$, $D_\aleph$, and $L_\aleph$ are derived ones.

3 Notation for the Passarino-Veltman functions

Our notation for the relevant PV functions [14] is the one of `LoopTools` [15, 16]:

$$\mu^\epsilon \int \frac{\mathrm{d}^d k}{(2\pi)^d}\, \frac{1}{k^2 - I} = \frac{i}{16\pi^2}\, A_0(I), \tag{3.1a}$$

$$\mu^\epsilon \int \frac{\mathrm{d}^d k}{(2\pi)^d}\, \frac{1}{k^2 - I}\, \frac{1}{(k+q)^2 - J} = \frac{i}{16\pi^2}\, B_0(Q,I,J), \tag{3.1b}$$

$$\mu^\epsilon \int \frac{\mathrm{d}^d k}{(2\pi)^d}\, k^\theta\, \frac{1}{k^2 - I}\, \frac{1}{(k+q)^2 - J} = \frac{i}{16\pi^2}\, q^\theta B_1(Q,I,J), \tag{3.1c}$$

$$\mu^\epsilon \int \frac{\mathrm{d}^d k}{(2\pi)^d}\, k^\theta k^\psi\, \frac{1}{k^2 - I}\, \frac{1}{(k+q)^2 - J} = \frac{i}{16\pi^2} \left[g^{\theta\psi} B_{00}(Q,I,J) + q^\theta q^\psi B_{11}(Q,I,J)\right], \tag{3.1d}$$

where $Q \equiv q^2$ and I and J have mass-squared dimensions. The quantities Q, I, and J are assumed to be non-negative. In eqs. (3.1), μ is an arbitrary quantity with mass dimension and $d = 4 - \epsilon$ (where eventually $\epsilon \to 0^+$) is the dimension of space-time. We also define

$$B_0'(Q, I, J) \equiv \frac{\partial B_0(Q, I, J)}{\partial Q}, \tag{3.2a}$$

$$B_1'(Q, I, J) \equiv \frac{\partial B_1(Q, I, J)}{\partial Q}, \tag{3.2b}$$

$$B_{00}'(Q, I, J) \equiv \frac{\partial B_{00}(Q, I, J)}{\partial Q}. \tag{3.2c}$$

All the functions in this section may be computed through softwares like `LoopTools` [15, 16] or `COLLIER` [17, 18]. They may as well be computed analytically; the results of that computation are presented in appendix B.

4 Results for the oblique paramaters

4.1 T

We have

$$\begin{aligned} T = \frac{N_c}{4\pi c_w^2 s_w^2} \Bigg\{ & 2 \sum_h \sum_u F\left[(N_L)_{hu}, (N_R)_{hu}, m_h^2, m_u^2\right] \\ & + 2 \sum_u \sum_d F\left[(V_L)_{ud}, (V_R)_{ud}, m_u^2, m_d^2\right] \\ & + 2 \sum_d \sum_l F\left[(Q_L)_{dl}, (Q_R)_{dl}, m_d^2, m_l^2\right] \\ & - \sum_{h,h'} F\left[(H_L)_{hh'}, (H_R)_{hh'}, m_h^2, m_{h'}^2\right] \\ & - \sum_{u,u'} F\left[(U_L)_{uu'}, (U_R)_{uu'}, m_u^2, m_{u'}^2\right] \\ & - \sum_{d,d'} F\left[(D_L)_{dd'}, (D_R)_{dd'}, m_d^2, m_{d'}^2\right] \\ & - \sum_{l,l'} F\left[(L_L)_{ll'}, (L_R)_{ll'}, m_l^2, m_{l'}^2\right] \Bigg\} \\ & - \text{SM value}, \end{aligned} \tag{4.1}$$

where $N_c = 3$ is the number of quark colors,

$$F(x, y, I, J) \equiv \left(|x|^2 + |y|^2\right) \frac{4\, B_{00}(0, I, J) - I - J}{4 m_Z^2} - \text{Re}(x y^*)\, \frac{\sqrt{IJ}}{m_Z^2}\, B_0(0, I, J), \tag{4.2}$$

and the last line of eq. (4.1) means that, in the end, one should not forget to subtract from T the same quantity computed in the context of the Standard Model.

4.2 Simplified notation

In order to present the expressions for the oblique parameters in a compact way, we introduce a new notation wherein *all* the quarks are denoted by letters a and/or b. The symbol $\sum_a$ means a sum over all the quarks. The symbol "$\sum_{a,a'}$" means firstly a sum over the h-type quarks h and h', then a sum over the u-type quarks u and u', ..., and finally a sum over the l-type quarks l and l'. The matrices $A_\aleph$ and $\bar{A}_\aleph$ correspond to the quarks a just as the matrices $H_\aleph$ and $\bar{H}_\aleph$ correspond to the quarks h, ..., and the matrices $L_\aleph$ and $\bar{L}_\aleph$ correspond to the quarks l. We also use the symbol "$\sum_a \sum_b$" when we sum both over the quarks a and over the quarks b such that the electric charge Q_a of the quarks a is equal to the electric charge Q_b of the quarks b plus one unit: $Q_a = Q_b + 1$; in this case, we have to deal with charged-current mixing matrices $M_\aleph$ that are

- $N_\aleph$ when $a = h$ and $b = u$;
- $V_\aleph$ when $a = u$ and $b = d$;
- $Q_\aleph$ when $a = d$ and $b = l$.

In this way, the expression for T in eq. (4.1) gets shortened to

$$T = \frac{N_c}{4\pi c_w^2 s_w^2} \left\{ 2 \sum_a \sum_b F\left[(M_L)_{ab}, (M_R)_{ab}, m_a^2, m_b^2 \right] - \sum_{a,a'} F\left[(A_L)_{aa'}, (A_R)_{aa'}, m_a^2, m_{a'}^2 \right] \right\} - \text{SM value}. \tag{4.3}$$

4.3 *S* and *U*

We have

$$S = -\frac{N_c}{2\pi} \left\{ \sum_{a,a'} G\left[\left(\bar{A}_L\right)_{aa'}, \left(\bar{A}_R\right)_{aa'}, m_Z^2, m_a^2, m_{a'}^2 \right] + 2\left(s_w^2 - c_w^2\right) \sum_a |Q_a| \left(\bar{A}_L + \bar{A}_R\right)_{aa} h\left(m_a^2\right) - 8 s_w^2 c_w^2 \sum_a Q_a^2 \, h\left(m_a^2\right) \right\} - \text{SM value}, \tag{4.4a}$$

$$U = -\frac{N_c}{\pi} \left\{ \sum_a \sum_b G\left[(M_L)_{ab}, (M_R)_{ab}, m_W^2, m_a^2, m_b^2 \right] - \frac{1}{2} \sum_{a,a'} G\left[\left(\bar{A}_L\right)_{aa'}, \left(\bar{A}_R\right)_{aa'}, m_Z^2, m_a^2, m_{a'}^2 \right] - 2 s_w^2 \sum_a |Q_a| \left(\bar{A}_L + \bar{A}_R\right)_{aa} h\left(m_a^2\right) - 4 s_w^4 \sum_a Q_a^2 \, h\left(m_a^2\right) \right\} - \text{SM value}, \tag{4.4b}$$

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where

$$G\left(x,y,Q,I,J\right) \equiv -\left(|x|^2+|y|^2\right) g\left(Q,I,J\right)+2\,\mathrm{Re}(xy^*)\ \frac{\sqrt{IJ}}{Q}\ \hat{g}\left(Q,I,J\right), \tag{4.5a}$$

$$g\left(Q,I,J\right) \equiv B_1\left(Q,I,J\right)+B_{11}\left(Q,I,J\right)+2\ \frac{B_{00}\left(Q,I,J\right)-B_{00}\left(0,I,J\right)}{Q}+\frac{1}{6}, \tag{4.5b}$$

$$\hat{g}\left(Q,I,J\right) \equiv B_0\left(Q,I,J\right)-B_0\left(0,I,J\right), \tag{4.5c}$$

$$h\left(I\right) = \frac{B_0\left(0,I,I\right)}{3}. \tag{4.5d}$$

4.4 V and W

We have

$$\begin{aligned} V = {} & \frac{N_c}{8\pi s_w^2 c_w^2} \sum_{a,a'} \left\{ \left[\left| \left(\bar{A}_L\right)_{aa'} \right|^2 + \left| \left(\bar{A}_R\right)_{aa'} \right|^2 \right] k\left(m_Z^2, m_a^2, m_{a'}^2\right) \right. && (4.6a) \\ & \left. -2\left(\bar{A}_L\right)_{aa'}\left(\bar{A}_R\right)_{a'a} \frac{m_a m_{a'}}{m_Z^2}\ j\left(m_Z^2, m_a^2, m_{a'}^2\right) \right\} - \text{SM value}, \\ W = {} & \frac{N_c}{4\pi s_w^2} \sum_a \sum_b \left\{ \left[|(M_L)_{ab}|^2 + |(M_R)_{ab}|^2 \right] k\left(m_W^2, m_a^2, m_b^2\right) \right. && (4.6b) \\ & \left. -2\,\mathrm{Re}\left[(M_L)_{ab}\,(M_R)^*_{ab}\right] \frac{m_a m_b}{m_W^2}\ j\left(m_W^2, m_a^2, m_b^2\right) \right\} - \text{SM value}. \end{aligned}$$

where

$$\begin{aligned} k\left(Q,I,J\right) \equiv {} & Q\,B_1'\left(Q,I,J\right) + Q\,B_{11}'\left(Q,I,J\right) \\ & +2\,B_{00}'\left(Q,I,J\right) - 2\ \frac{B_{00}\left(Q,I,J\right) - B_{00}\left(0,I,J\right)}{Q}, && (4.7a) \\ j\left(Q,I,J\right) \equiv {} & Q\,B_0'\left(Q,I,J\right) - B_0\left(Q,I,J\right) + B_0\left(0,I,J\right). && (4.7b) \end{aligned}$$

4.5 X

We have

$$X = \frac{N_c}{4\pi} \sum_a |Q_a| \left(\bar{A}_L + \bar{A}_R\right)_{aa} l\left(m_Z^2, m_a^2\right) - \text{SM value}, \tag{4.8}$$

where

$$\begin{aligned} l\left(Q,I\right) = {} & I\left[B_0'\left(0,I,I\right) - \frac{B_0\left(Q,I,I\right) - B_0\left(0,I,I\right)}{Q}\right] \\ & +B_1\left(Q,I,I\right) + B_{11}\left(Q,I,I\right) - B_1\left(0,I,I\right) - B_{11}\left(0,I,I\right) \\ & -2\,B_{00}'\left(0,I,I\right) + 2\ \frac{B_{00}\left(Q,I,I\right) - B_{00}\left(0,I,I\right)}{Q}. && (4.9) \end{aligned}$$

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A Technical details

Suppose the fermions f_1 and f_2 with masses m_1 and m_2, respectively, interact with the gauge bosons V_θ and V'_ψ through the Lagrangian

$$\mathcal{L} = V_\theta\, \bar{f}_1\, \gamma^\theta \left(g_V - g_A \gamma_5\right) f_2 + V'_\psi\, \bar{f}_2\, \gamma^\psi \left(g_{V'} - g_{A'} \gamma_5\right) f_1 + \text{H.c.} \tag{A.1}$$

Then, the vacuum polarization between a V_θ and a V'_ψ with four-momenta q caused by a loop of f_1 and f_2 is[4]

$$\begin{aligned} A_{VV'}\left(q^2, m_1^2, m_2^2\right) = \frac{G_V + G_A}{4\pi^2} & \left[q^2\, B_1\left(q^2, m_1^2, m_2^2\right) + q^2\, B_{11}\left(q^2, m_1^2, m_2^2\right) \right. \\ & \left. + 2\, B_{00}\left(q^2, m_1^2, m_2^2\right) + \frac{q^2}{6} - \frac{m_1^2 + m_2^2}{2} \right] \\ & - \frac{G_V - G_A}{4\pi^2}\, m_1 m_2\, B_0\left(q^2, m_1^2, m_2^2\right), \end{aligned} \tag{A.2}$$

where

$$G_V \equiv g_V g_{V'}, \qquad G_A \equiv g_A g_{A'}. \tag{A.3}$$

It follows from the definitions (A.3) that

- In the computation of $A_{\gamma\gamma}\left(q^2\right)$,

$$G_V + G_A = G_V - G_A = g^2 s_w^2\, Q_a^2 \tag{A.4}$$

  for a loop with two identical quarks a with electric charge Q_a. (We use the notation of section 4.2.)

- In the computation of $A_{\gamma Z}\left(q^2\right)$,

$$G_V + G_A = G_V - G_A = -\frac{g^2 s_w}{4 c_w} \left(\bar{A}_L + \bar{A}_R\right)_{aa} Q_a \tag{A.5}$$

  for a loop with two identical a-type quarks. (We use once again the notation of section 4.2.)

- In the computation of $A_{ZZ}\left(q^2\right)$,

$$G_V + G_A = \frac{g^2}{8 c_w^2} \left[\left| \left(\bar{A}_L\right)_{aa'} \right|^2 + \left| \left(\bar{A}_R\right)_{aa'} \right|^2 \right], \tag{A.6a}$$

$$G_V - G_A = \frac{g^2}{4 c_w^2}\, \text{Re} \left[\left(\bar{A}_L\right)_{aa'} \left(\bar{A}_R\right)_{a'a} \right], \tag{A.6b}$$

  in a loop with quarks a and a' carrying identical electric charges.

[4] We assume the gamma matrices to be 4×4 even in a space-time of dimension d; thus, we set $\text{tr}(\gamma^\mu \gamma^\nu) = 4 g^{\mu\nu}$.

- In the computation of $A_{WW}\left(q^2\right)$,

$$G_V + G_A = \frac{g^2}{4}\left[\left|(M_L)_{ab}\right|^2 + \left|(M_R)_{ab}\right|^2\right], \tag{A.7a}$$

$$G_V - G_A = \frac{g^2}{2}\,\mathrm{Re}\left[(M_L)_{ab}\left(M_R^*\right)_{ab}\right], \tag{A.7b}$$

in a loop with quarks a and b carrying electric charges Q_a and $Q_a - 1$, respectively. (We use once more the notation of section 4.2.)

The PV functions defined in eqs. (3.1) are not all independent. Indeed,

$$2\,B_{00}\left(Q,I,J\right) + \frac{Q}{6} - \frac{I+J}{2} = Q\left(B_1 + B_{11}\right)\left(Q,I,J\right) + I\left(B_0 + B_1\right)\left(Q,I,J\right) - J\,B_1\left(Q,I,J\right). \tag{A.8}$$

Setting $Q = 0$ and $I = J$ in eq. (A.8), one obtains

$$2\,B_{00}\left(0,J,J\right) = J\left[1 + B_0\left(0,J,J\right)\right]. \tag{A.9}$$

Taking the derivative relative to Q of eq. (A.8) and then setting $Q = 0$ and $I = J$, one obtains

$$2\,B_{00}^\prime\left(0,J,J\right) + \frac{1}{6} = B_1\left(0,J,J\right) + B_{11}\left(0,J,J\right) + J\,B_0^\prime\left(0,J,J\right). \tag{A.10}$$

Furthermore, explicit computation in eqs. (B.8) yields

$$\left(B_0 + 6\,B_1 + 6\,B_{11}\right)\left(0,J,J\right) = 0. \tag{A.11}$$

From eq. (A.2),

$$\begin{aligned} A_{VV^\prime}\left(0,m_1^2,m_2^2\right) &= \frac{G_V+G_A}{8\pi^2}\left[4\,B_{00}\left(0,m_1^2,m_2^2\right) - m_1^2 - m_2^2\right] \\ &\quad - \frac{G_V-G_A}{4\pi^2}\,m_1 m_2\,B_0\left(0,m_1^2,m_2^2\right). \end{aligned} \tag{A.12}$$

Equation (A.12) leads to the definition of the function F in eq. (4.2).

From eq. (A.2),

$$\frac{A_{VV^\prime}\left(Q,I,J\right) - A_{VV^\prime}\left(0,I,J\right)}{Q} = \frac{G_V+G_A}{4\pi^2}\,g\left(Q,I,J\right) - \frac{G_V-G_A}{4\pi^2}\,\frac{\sqrt{IJ}}{Q}\,\hat{g}\left(Q,I,J\right), \tag{A.13}$$

with the functions g and $\hat{g}$ defined in eqs. (4.5b) and (4.5c), respectively. The function h defined in eq. (4.5d) appears in

$$\left.\frac{\partial A_{\gamma V^\prime}\left(Q,I,I\right)}{\partial Q}\right|_{Q=0} = -\frac{G_V}{4\pi^2}\,h\left(I\right). \tag{A.14}$$

The functions relevant for the computation of the oblique parameters V and W are defined in eqs. (4.7). They appear in

$$\begin{aligned} \frac{\partial A_{VV^\prime}\left(Q,I,J\right)}{\partial Q} - \frac{A_{VV^\prime}\left(Q,I,J\right) - A_{VV^\prime}\left(0,I,J\right)}{Q} &= \frac{G_V+G_A}{4\pi^2}\,k\left(Q,I,J\right) \\ &\quad - \frac{G_V-G_A}{4\pi^2}\,\frac{\sqrt{IJ}}{Q}\,j\left(Q,I,J\right). \end{aligned} \tag{A.15}$$

The function l that appears in the expression for the oblique parameter X is given by eq. (4.9) and originates in

$$\left.\frac{\partial A_{\gamma V'}(Q,I,I)}{\partial Q}\right|_{Q=0} - \frac{A_{\gamma V'}(Q,I,I) - A_{\gamma V'}(0,I,I)}{Q} = -\frac{G_V}{4\pi^2}\, l(Q,I)\,. \tag{A.16}$$

If in eq. (A.12) one sets $G_A = 0$ and $m_1 = m_2$, as happens if $V = \gamma$ is a photon, then one obtains

$$A_{\gamma V'}\left(0, m_1^2, m_1^2\right) = \frac{G_V}{4\pi^2}\left[2\, B_{00}\left(0, m_1^2, m_1^2\right) - m_1^2 - m_1^2\, B_0\left(0, m_1^2, m_1^2\right)\right] = 0, \tag{A.17}$$

because of eq. (A.9). Hence, the contributions to $A_{\gamma\gamma}(0)$ and to $A_{\gamma Z}(0)$ from fermion loops both vanish. Notice, though, that $A_{\gamma\gamma}(0)$ is necessarily zero because of gauge invariance, while $A_{\gamma Z}(0)$ does not need to vanish in general.

B Formulas for the PV functions

In the limit $\epsilon \to 0^+$, we define the divergent quantity

$$\mathrm{div} \equiv \frac{2}{\epsilon} - \gamma + \ln\left(4\pi\mu^2\right), \tag{B.1}$$

where γ is the Euler-Mascheroni constant.

We furthermore define

$$\Delta \equiv Q^2 + I^2 + J^2 - 2\left(QI + QJ + IJ\right). \tag{B.2}$$

The quantity Δ is positive if and only if it is *not* possible to draw a triangle with sides of lengths $\sqrt{Q}$, $\sqrt{I}$, and $\sqrt{J}$, viz. when either $\sqrt{Q} < \left|\sqrt{I} - \sqrt{J}\right|$ or $\sqrt{Q} > \sqrt{I} + \sqrt{J}$. We define the function

$$f(Q,I,J) \equiv \begin{cases} \dfrac{1}{\sqrt{\Delta}} \ln \dfrac{I + J - Q + \sqrt{\Delta}}{I + J - Q - \sqrt{\Delta}} \Leftarrow \Delta > 0, \\ \dfrac{2}{\sqrt{-\Delta}} \left(\arctan \dfrac{I - J + Q}{\sqrt{-\Delta}} + \arctan \dfrac{J - I + Q}{\sqrt{-\Delta}}\right) \Leftarrow \Delta < 0, \\ \dfrac{1}{\sqrt{IJ}} \Leftarrow \sqrt{Q} = \left|\sqrt{I} - \sqrt{J}\right|, \\ \dfrac{-1}{\sqrt{IJ}} \Leftarrow \sqrt{Q} = \sqrt{I} + \sqrt{J}. \end{cases} \tag{B.3}$$

The function $f(Q,I,J)$ is continuous and well-behaved everywhere except at the point $\sqrt{Q} = \sqrt{I} + \sqrt{J}$, namely it diverges when $\sqrt{Q} \to \sqrt{I} + \sqrt{J} - 0^+$.

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The analytic formulas for the relevant PV functions are

$$
\begin{aligned}
A_0\left(I\right) &= I\left(\text{div} - \ln I + 1\right), && \text{(B.4a)}\\
B_0\left(Q,I,J\right) &= \text{div} - \frac{\ln\left(IJ\right)}{2} + 2 + \frac{J-I}{2Q}\ln\frac{I}{J} + \frac{\Delta}{2Q}\,f\left(Q,I,J\right)\\
&\quad + \text{absorptive part}, && \text{(B.4b)}\\
B_1\left(Q,I,J\right) &= -\frac{\text{div}}{2} + \frac{\ln\left(IJ\right)}{4} - 1 + \frac{J-I}{2Q}\\
&\quad + \frac{\left(I-J\right)^2 - 2QJ}{4Q^2}\ln\frac{I}{J} + \frac{\left(J-I-Q\right)\Delta}{4Q^2}\,f\left(Q,I,J\right)\\
&\quad + \text{absorptive part}, && \text{(B.4c)}\\
B_{00}\left(Q,I,J\right) &= \left(\frac{I+J}{4} - \frac{Q}{12}\right)\left[\text{div} - \frac{\ln\left(IJ\right)}{2}\right] - \frac{2}{9}Q + \frac{7}{12}\left(I+J\right) - \frac{\left(I-J\right)^2}{12Q}\\
&\quad + \frac{\left(I-J\right)\left[\Delta - Q\left(I+J\right) - Q^2\right]}{24Q^2}\ln\frac{I}{J} - \frac{\Delta^2}{24Q^2}\,f\left(Q,I,J\right)\\
&\quad + \text{absorptive part}, && \text{(B.4d)}\\
B_{11}\left(Q,I,J\right) &= \frac{\text{div}}{3} - \frac{\ln\left(IJ\right)}{6} + \frac{13}{18} + \frac{I-5J}{6Q} + \frac{\left(I-J\right)^2}{3Q^2}\\
&\quad + \frac{3Q^2J + 3QJ\left(I-J\right) + \left(J-I\right)^3}{6Q^3}\ln\frac{I}{J}\\
&\quad + \frac{Q^2 + Q\left(I-2J\right) + \left(I-J\right)^2}{6Q^3}\,\Delta\,f\left(Q,I,J\right)\\
&\quad + \text{absorptive part}. && \text{(B.4e)}
\end{aligned}
$$

The absorptive parts in eqs. (B.4) exist if and only if $\sqrt{Q} > \sqrt{I} + \sqrt{J}$. The analytic formulas for the relevant derivatives are

$$
\begin{aligned}
B_0'\left(Q,I,J\right) &= -\frac{1}{Q} + \frac{I-J}{2Q^2}\ln\frac{I}{J} + \frac{Q\left(I+J\right) - \left(I-J\right)^2}{2Q^2}\,f\left(Q,I,J\right)\\
&\quad + \text{absorptive part}, && \text{(B.5a)}\\
B_1'\left(Q,I,J\right) &= \frac{1}{2Q} + \frac{I-J}{Q^2} + \frac{QJ - \left(I-J\right)^2}{2Q^3}\ln\frac{I}{J}\\
&\quad + \frac{\left(I-J\right)^3 + Q\left(2J^2 - IJ - I^2\right) - Q^2J}{2Q^3}\,f\left(Q,I,J\right)\\
&\quad + \text{absorptive part}, && \text{(B.5b)}\\
B_{00}'\left(Q,I,J\right) &= -\frac{\text{div}}{12} + \frac{\ln\left(IJ\right)}{24} - \frac{5}{36} - \frac{I+J}{6Q} + \frac{\left(I-J\right)^2}{6Q^2}\\
&\quad + \frac{2\left(J-I\right)^3 + 3Q\left(I^2 - J^2\right)}{24Q^3}\ln\frac{I}{J}\\
&\quad + \frac{\Delta}{24Q^3}\left[2\left(I-J\right)^2 - Q\left(I+J\right) - Q^2\right]f\left(Q,I,J\right)\\
&\quad + \text{absorptive part}. && \text{(B.5c)}
\end{aligned}
$$

When $Q = 0$, the PV functions are

$$B_0(0, I, J) = \text{div} - \frac{\ln(IJ)}{2} + 1 - \frac{I+J}{2(I-J)} \ln\frac{I}{J}, \tag{B.6a}$$

$$B_1(0, I, J) = -\frac{\text{div}}{2} + \frac{\ln I}{2} + \frac{J-3I}{4(I-J)} + \frac{J(2I-J)}{2(I-J)^2} \ln\frac{I}{J}, \tag{B.6b}$$

$$B_{00}(0, I, J) = \frac{I+J}{4}\left[\text{div} - \frac{\ln(IJ)}{2}\right] + \frac{3(I+J)}{8} - \frac{I^2+J^2}{8(I-J)} \ln\frac{I}{J}, \tag{B.6c}$$

$$B_{11}(0, I, J) = \frac{\text{div} - \ln J}{3} + \frac{11I^2 - 7IJ + 2J^2}{18(I-J)^2} - \frac{I^3}{3(I-J)^3} \ln\frac{I}{J}, \tag{B.6d}$$

and their derivatives are

$$B_0'(0, I, J) = \frac{I+J}{2(I-J)^2} - \frac{IJ}{(I-J)^3} \ln\frac{I}{J}, \tag{B.7a}$$

$$B_1'(0, I, J) = -\frac{2I^2 + 5IJ - J^2}{6(I-J)^3} + \frac{I^2 J}{(I-J)^4} \ln\frac{I}{J}, \tag{B.7b}$$

$$B_{00}'(0, I, J) = -\frac{\text{div}}{12} + \frac{\ln(IJ)}{24} - \frac{5I^2 - 22IJ + 5J^2}{72(I-J)^2} + \frac{(I+J)(I^2 - 4IJ + J^2)}{24(I-J)^3} \ln\frac{I}{J}. \tag{B.7c}$$

When both $Q = 0$ and $I = J$ one has

$$B_0(0, J, J) = \text{div} - \ln J, \tag{B.8a}$$

$$B_1(0, J, J) = -\frac{\text{div} - \ln J}{2}, \tag{B.8b}$$

$$B_{00}(0, J, J) = \frac{J(\text{div} - \ln J + 1)}{2}, \tag{B.8c}$$

$$B_{11}(0, J, J) = \frac{\text{div} - \ln J}{3}, \tag{B.8d}$$

$$B_0'(0, J, J) = \frac{1}{6J}, \tag{B.8e}$$

$$B_1'(0, J, J) = -\frac{1}{12J}, \tag{B.8f}$$

$$B_{00}'(0, J, J) = -\frac{\text{div} - \ln J}{12}. \tag{B.8g}$$

All the formulas in this appendix were numerically checked by using `LoopTools`.

C Cancellation of the divergences

In this appendix we demonstrate that the ultraviolet divergences cancel out in the oblique parameters S, T, and U. In the other three parameters such divergences are *a priori* absent.

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C.1 The quark mass terms

The quarks in eqs. (1.1)–(1.7) in general have bare mass terms[5] given by

$$
\begin{aligned}
\mathcal{L}_{\text{bare masses}} = & -\bar{\sigma}_{0,4,L}\, M_1\, \sigma_{0,4,R} \\
& -\bar{\sigma}_{0,-2,L}\, M_2\, \sigma_{0,-2,R} \\
& -\left(\bar{\delta}_{1,7,L}\, M_3\, \delta_{1,7,R} + \bar{\delta}_{-1,7,L}\, M_3\, \delta_{-1,7,R}\right) \\
& -\left(\bar{\delta}_{1,1,L}\, M_4\, \delta_{1,1,R} + \bar{\delta}_{-1,1,L}\, M_4\, \delta_{-1,1,R}\right) \\
& -\left(\bar{\delta}_{1,-5,L}\, M_5\, \delta_{1,-5,R} + \bar{\delta}_{-1,-5,L}\, M_5\, \delta_{-1,-5,R}\right) \\
& -\left(\bar{\tau}_{2,4,L}\, M_6\, \tau_{2,4,R} + \bar{\tau}_{0,4,L}\, M_6\, \tau_{0,4,R} + \bar{\tau}_{-2,4,L}\, M_6\, \tau_{-2,4,R}\right) \\
& -\left(\bar{\tau}_{2,-2,L}\, M_7\, \tau_{2,-2,R} + \bar{\tau}_{0,-2,L}\, M_7\, \tau_{0,-2,R} + \bar{\tau}_{-2,-2,L}\, M_7\, \tau_{-2,-2,R}\right) \\
& + \text{H.c.}
\end{aligned}
\tag{C.1}
$$

The matrices $M_1, \ldots, M_7$ are assumed to have adequate dimensions that we do not, however, specify.

We assume the existence of just one scalar doublet Φ with weak hypercharge $1/2$, and of its conjugate doublet $\tilde{\Phi}$:[6,7]

$$
\Phi = \begin{pmatrix} \varphi_{1,3} \\ \varphi_{-1,3} \end{pmatrix}, \qquad \tilde{\Phi} = \begin{pmatrix} \varphi^*_{-1,3} \\ -\varphi^*_{1,3} \end{pmatrix}. \tag{C.2}
$$

The Yukawa Lagrangian then is

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} = & -\left(\bar{\delta}_{1,7,L},\ \bar{\delta}_{-1,7,L}\right) \Phi\, \Upsilon_1\, \sigma_{0,4,R} - \left(\bar{\delta}_{1,1,L},\ \bar{\delta}_{-1,1,L}\right) \Phi\, \Upsilon_2\, \sigma_{0,-2,R} \\
& -\left(\bar{\delta}_{1,1,L},\ \bar{\delta}_{-1,1,L}\right) \tilde{\Phi}\, \Upsilon_3\, \sigma_{0,4,R} - \left(\bar{\delta}_{1,-5,L},\ \bar{\delta}_{-1,-5,L}\right) \tilde{\Phi}\, \Upsilon_4\, \sigma_{0,-2,R} \\
& -\tilde{\Phi}^\dagger \begin{pmatrix} \bar{\tau}_{0,4,L} & \sqrt{2}\, \bar{\tau}_{-2,4,L} \\ -\sqrt{2}\, \bar{\tau}_{2,4,L} & -\bar{\tau}_{0,4,L} \end{pmatrix} \Upsilon_5 \begin{pmatrix} \delta_{1,1,R} \\ \delta_{-1,1,R} \end{pmatrix} \\
& -\Phi^\dagger \begin{pmatrix} -\bar{\tau}_{0,4,L} & -\sqrt{2}\, \bar{\tau}_{-2,4,L} \\ \sqrt{2}\, \bar{\tau}_{2,4,L} & \bar{\tau}_{0,4,L} \end{pmatrix} \Upsilon_6 \begin{pmatrix} \delta_{1,7,R} \\ \delta_{-1,7,R} \end{pmatrix} \\
& -\tilde{\Phi}^\dagger \begin{pmatrix} \bar{\tau}_{0,-2,L} & \sqrt{2}\, \bar{\tau}_{-2,-2,L} \\ -\sqrt{2}\, \bar{\tau}_{2,-2,L} & -\bar{\tau}_{0,-2,L} \end{pmatrix} \Upsilon_7 \begin{pmatrix} \delta_{1,-5,R} \\ \delta_{-1,-5,R} \end{pmatrix} \\
& -\Phi^\dagger \begin{pmatrix} -\bar{\tau}_{0,-2,L} & -\sqrt{2}\, \bar{\tau}_{-2,-2,L} \\ \sqrt{2}\, \bar{\tau}_{2,-2,L} & \bar{\tau}_{0,-2,L} \end{pmatrix} \Upsilon_8 \begin{pmatrix} \delta_{1,1,R} \\ \delta_{-1,1,R} \end{pmatrix} \\
& -\bar{\sigma}_{0,4,L}\, \Phi^\dagger\, \Upsilon_9 \begin{pmatrix} \delta_{1,7,R} \\ \delta_{-1,7,R} \end{pmatrix} - \bar{\sigma}_{0,-2,L}\, \Phi^\dagger\, \Upsilon_{10} \begin{pmatrix} \delta_{1,1,R} \\ \delta_{-1,1,R} \end{pmatrix}
\end{aligned}
$$

[5]The mass terms (C.1) must be directly written in the Lagrangian, and they may furthermore be generated through the Yukawa couplings of the quarks to scalars that are invariant under the gauge group and acquire a vacuum expectation value (VEV).

[6]If there are several scalar doublets with identical quantum numbers, that does not really affect our work and its final results.

[7]If the gauge symmetry gets broken by VEVs other than those of doublets like the one in eq. (C.2), then the ultraviolet divergences do not cancel out in the parameter T [19].

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$$
\begin{aligned}
&-\bar{\sigma}_{0,4,L}\,\tilde{\Phi}^\dagger\,\Upsilon_{11}\begin{pmatrix}\delta_{1,1,R}\\ \delta_{-1,1,R}\end{pmatrix}-\bar{\sigma}_{0,-2,L}\,\tilde{\Phi}^\dagger\,\Upsilon_{12}\begin{pmatrix}\delta_{1,-5,R}\\ \delta_{-1,-5,R}\end{pmatrix}\\
&-\begin{pmatrix}\bar{\delta}_{1,1,L}, & \bar{\delta}_{-1,1,L}\end{pmatrix}\Upsilon_{13}\begin{pmatrix}\tau_{0,4,R} & -\sqrt{2}\,\tau_{2,4,R}\\ \sqrt{2}\,\tau_{-2,4,R} & -\tau_{0,4,R}\end{pmatrix}\tilde{\Phi}\\
&-\begin{pmatrix}\bar{\delta}_{1,7,L}, & \bar{\delta}_{-1,7,L}\end{pmatrix}\Upsilon_{14}\begin{pmatrix}-\tau_{0,4,R} & \sqrt{2}\,\tau_{2,4,R}\\ -\sqrt{2}\,\tau_{-2,4,R} & \tau_{0,4,R}\end{pmatrix}\Phi\\
&-\begin{pmatrix}\bar{\delta}_{1,-5,L}, & \bar{\delta}_{-1,-5,L}\end{pmatrix}\Upsilon_{15}\begin{pmatrix}\tau_{0,-2,R} & -\sqrt{2}\,\tau_{2,-2,R}\\ \sqrt{2}\,\tau_{-2,-2,R} & -\tau_{0,-2,R}\end{pmatrix}\tilde{\Phi}\\
&-\begin{pmatrix}\bar{\delta}_{1,1,L}, & \bar{\delta}_{-1,1,L}\end{pmatrix}\Upsilon_{16}\begin{pmatrix}-\tau_{0,-2,R} & \sqrt{2}\,\tau_{2,-2,R}\\ -\sqrt{2}\,\tau_{-2,-2,R} & \tau_{0,-2,R}\end{pmatrix}\Phi\\
&+\text{H.c.},
\end{aligned}
\tag{C.3}
$$

with Yukawa-coupling matrices $\Upsilon_1, \dots, \Upsilon_{16}$. When $\varphi_{-1,3}$ acquires a VEV v, one obtains from eq. (C.3) the quark mass terms

$$
\begin{aligned}
\mathcal{L}_{\text{quark masses}} = -v\,\Big(&\bar{\delta}_{-1,7,L}\Upsilon_1\sigma_{0,4,R}+\bar{\delta}_{-1,1,L}\Upsilon_2\sigma_{0,-2,R}\\
&+\bar{\tau}_{0,4,L}\Upsilon_5\delta_{1,1,R}+\sqrt{2}\,\bar{\tau}_{-2,4,L}\Upsilon_5\delta_{-1,1,R}\\
&+\bar{\tau}_{0,-2,L}\Upsilon_7\delta_{1,-5,R}+\sqrt{2}\,\bar{\tau}_{-2,-2,L}\Upsilon_7\delta_{-1,-5,R}\\
&+\bar{\sigma}_{0,4,L}\Upsilon_{11}\delta_{1,1,R}+\bar{\sigma}_{0,-2,L}\Upsilon_{12}\delta_{1,-5,R}\\
&+\bar{\delta}_{-1,7,L}\Upsilon_{14}\tau_{0,4,R}+\sqrt{2}\,\bar{\delta}_{1,7,L}\Upsilon_{14}\tau_{2,4,R}\\
&+\bar{\delta}_{-1,1,L}\Upsilon_{16}\tau_{0,-2,R}+\sqrt{2}\,\bar{\delta}_{1,1,L}\Upsilon_{16}\tau_{2,-2,R}\Big)\\
-v^*\,\Big(&\bar{\delta}_{1,1,L}\Upsilon_3\sigma_{0,4,R}+\bar{\delta}_{1,-5,L}\Upsilon_4\sigma_{0,-2,R}\\
&+\sqrt{2}\,\bar{\tau}_{2,4,L}\Upsilon_6\delta_{1,7,R}+\bar{\tau}_{0,4,L}\Upsilon_6\delta_{-1,7,R}\\
&+\sqrt{2}\,\bar{\tau}_{2,-2,L}\Upsilon_8\delta_{1,1,R}+\bar{\tau}_{0,-2,L}\Upsilon_8\delta_{-1,1,R}\\
&+\bar{\sigma}_{0,4,L}\Upsilon_9\delta_{-1,7,R}+\bar{\sigma}_{0,-2,L}\Upsilon_{10}\delta_{-1,1,R}\\
&+\bar{\delta}_{1,1,L}\Upsilon_{13}\tau_{0,4,R}+\sqrt{2}\,\bar{\delta}_{-1,1,L}\Upsilon_{13}\tau_{-2,4,R}\\
&+\bar{\delta}_{1,-5,L}\Upsilon_{15}\tau_{0,-2,R}+\sqrt{2}\,\bar{\delta}_{-1,-5,L}\Upsilon_{15}\tau_{-2,-2,R}\Big)\\
+\text{H.c.}&
\end{aligned}
\tag{C.4}
$$

Therefore, the complete quark mass terms are given by

$$
\begin{aligned}
\mathcal{L}_{\text{bare masses}}+\mathcal{L}_{\text{quark masses}} = &-\begin{pmatrix}\bar{\delta}_{1,7,L}, & \bar{\tau}_{2,4,L}\end{pmatrix}\bar{M}_h\begin{pmatrix}\delta_{1,7,R}\\ \tau_{2,4,R}\end{pmatrix}\\
&-\begin{pmatrix}\bar{\sigma}_{0,4,L}, & \bar{\delta}_{-1,7,L}, & \bar{\delta}_{1,1,L}, & \bar{\tau}_{0,4,L}, & \bar{\tau}_{2,-2,L}\end{pmatrix}\bar{M}_u\begin{pmatrix}\sigma_{0,4,R}\\ \delta_{-1,7,R}\\ \delta_{1,1,R}\\ \tau_{0,4,R}\\ \tau_{2,-2,R}\end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&-\left(\bar{\sigma}_{0,-2,L},\ \bar{\delta}_{-1,1,L},\ \bar{\delta}_{1,-5,L},\ \bar{\tau}_{-2,4,L},\ \bar{\tau}_{0,-2,L}\right)\bar{M}_d\begin{pmatrix}\sigma_{0,-2,R}\\ \delta_{-1,1,R}\\ \delta_{1,-5,R}\\ \tau_{-2,4,R}\\ \tau_{0,-2,R}\end{pmatrix}\\
&-\left(\bar{\delta}_{-1,-5,L},\ \bar{\tau}_{-2,-2,L}\right)\bar{M}_l\begin{pmatrix}\delta_{-1,-5,R}\\ \tau_{-2,-2,R}\end{pmatrix}\\
&+\text{H.c.},
\end{aligned}
\tag{C.5}
$$

where

$$
\bar{M}_h=\begin{pmatrix}M_3 & \sqrt{2}\,v\Upsilon_{14}\\ \sqrt{2}\,v^*\Upsilon_6 & M_6\end{pmatrix},
\tag{C.6a}
$$

$$
\bar{M}_u=\begin{pmatrix}M_1 & v^*\Upsilon_9 & v\Upsilon_{11} & 0 & 0\\ v\Upsilon_1 & M_3 & 0 & v\Upsilon_{14} & 0\\ v^*\Upsilon_3 & 0 & M_4 & v^*\Upsilon_{13} & \sqrt{2}\,v\Upsilon_{16}\\ 0 & v^*\Upsilon_6 & v\Upsilon_5 & M_6 & 0\\ 0 & 0 & \sqrt{2}\,v^*\Upsilon_8 & 0 & M_7\end{pmatrix},
\tag{C.6b}
$$

$$
\bar{M}_d=\begin{pmatrix}M_2 & v^*\Upsilon_{10} & v\Upsilon_{12} & 0 & 0\\ v\Upsilon_2 & M_4 & 0 & \sqrt{2}\,v^*\Upsilon_{13} & v\Upsilon_{16}\\ v^*\Upsilon_4 & 0 & M_5 & 0 & v^*\Upsilon_{15}\\ 0 & \sqrt{2}\,v\Upsilon_5 & 0 & M_6 & 0\\ 0 & v^*\Upsilon_8 & v\Upsilon_7 & 0 & M_7\end{pmatrix},
\tag{C.6c}
$$

$$
\bar{M}_l=\begin{pmatrix}M_5 & \sqrt{2}\,v^*\Upsilon_{15}\\ \sqrt{2}\,v\Upsilon_7 & M_7\end{pmatrix}
\tag{C.6d}
$$

are the quark mass matrices.

One bi-diagonalizes those mass matrices by writing

$$
\begin{pmatrix}\delta_{1,7,\aleph}\\ \tau_{2,4,\aleph}\end{pmatrix}=\begin{pmatrix}H_{1\aleph}\\ H_{2\aleph}\end{pmatrix}h_\aleph,
\tag{C.7a}
$$

$$
\begin{pmatrix}\sigma_{0,4,\aleph}\\ \delta_{-1,7,\aleph}\\ \delta_{1,1,\aleph}\\ \tau_{0,4,\aleph}\\ \tau_{2,-2,\aleph}\end{pmatrix}=\begin{pmatrix}U_{1\aleph}\\ U_{2\aleph}\\ U_{3\aleph}\\ U_{4\aleph}\\ U_{5\aleph}\end{pmatrix}u_\aleph,
\tag{C.7b}
$$

$$
\begin{pmatrix}\sigma_{0,-2,\aleph}\\ \delta_{-1,1,\aleph}\\ \delta_{1,-5,\aleph}\\ \tau_{-2,4,\aleph}\\ \tau_{0,-2,\aleph}\end{pmatrix}=\begin{pmatrix}D_{1\aleph}\\ D_{2\aleph}\\ D_{3\aleph}\\ D_{4\aleph}\\ D_{5\aleph}\end{pmatrix}d_\aleph,
\tag{C.7c}
$$

$$
\begin{pmatrix}\delta_{-1,-5,\aleph}\\ \tau_{-2,-2,\aleph}\end{pmatrix}=\begin{pmatrix}L_{1\aleph}\\ L_{2\aleph}\end{pmatrix}l_\aleph,
\tag{C.7d}
$$

where $\aleph$ stands for either L or R. The matrices

$$\begin{pmatrix} H_{1\aleph} \\ H_{2\aleph} \end{pmatrix}, \qquad \begin{pmatrix} U_{1\aleph} \\ U_{2\aleph} \\ U_{3\aleph} \\ U_{4\aleph} \\ U_{5\aleph} \end{pmatrix}, \qquad \begin{pmatrix} D_{1\aleph} \\ D_{2\aleph} \\ D_{3\aleph} \\ D_{4\aleph} \\ D_{5\aleph} \end{pmatrix}, \qquad \begin{pmatrix} L_{1\aleph} \\ L_{2\aleph} \end{pmatrix} \tag{C.8}$$

are unitary and satisfy

$$\bar{M}_h = \begin{pmatrix} H_{1L} \\ H_{2L} \end{pmatrix} M_h \left(H_{1R}^\dagger,\ H_{2R}^\dagger \right) \tag{C.9a}$$

$$\bar{M}_u = \begin{pmatrix} U_{1L} \\ U_{2L} \\ U_{3L} \\ U_{4L} \\ U_{5L} \end{pmatrix} M_u \left(U_{1R}^\dagger,\ U_{2R}^\dagger,\ U_{3R}^\dagger,\ U_{4R}^\dagger,\ U_{5R}^\dagger \right), \tag{C.9b}$$

$$\bar{M}_d = \begin{pmatrix} D_{1L} \\ D_{2L} \\ D_{3L} \\ D_{4L} \\ D_{5L} \end{pmatrix} M_d \left(D_{1R}^\dagger,\ D_{2R}^\dagger,\ D_{3R}^\dagger,\ D_{4R}^\dagger,\ D_{5R}^\dagger \right), \tag{C.9c}$$

$$\bar{M}_l = \begin{pmatrix} L_{1L} \\ L_{2L} \end{pmatrix} M_l \left(L_{1R}^\dagger,\ L_{2R}^\dagger \right), \tag{C.9d}$$

where M_h, M_u, M_d, and M_l are the *diagonal* quark mass matrices, that have non-negative real matrix elements.

The unitarity of the matrices (C.8) implies that

$$\begin{matrix} H_{1\aleph}H_{1\aleph}^\dagger, & H_{2\aleph}H_{2\aleph}^\dagger, & U_{1\aleph}U_{1\aleph}^\dagger, & U_{2\aleph}U_{2\aleph}^\dagger, & U_{3\aleph}U_{3\aleph}^\dagger, & U_{4\aleph}U_{4\aleph}^\dagger, & U_{5\aleph}U_{5\aleph}^\dagger, \\ D_{1\aleph}D_{1\aleph}^\dagger, & D_{2\aleph}D_{2\aleph}^\dagger, & D_{3\aleph}D_{3\aleph}^\dagger, & D_{4\aleph}D_{4\aleph}^\dagger, & D_{5\aleph}D_{5\aleph}^\dagger, & L_{1\aleph}L_{1\aleph}^\dagger, & L_{2\aleph}L_{2\aleph}^\dagger \end{matrix} \tag{C.10}$$

are all proportional to unit matrices, of dimensions

$$\begin{matrix} n_{\delta,7,\aleph}, & n_{\tau,4,\aleph}, & n_{\sigma,4,\aleph}, & n_{\delta,7,\aleph}, & n_{\delta,1,\aleph}, & n_{\tau,4,\aleph}, & n_{\delta,-2,\aleph}, \\ n_{\sigma,-2,\aleph}, & n_{\delta,1,\aleph}, & n_{\delta,-5,\aleph}, & n_{\tau,4,\aleph}, & n_{\tau,-2,\aleph}, & n_{\delta,-5,\aleph}, & n_{\tau,-2,\aleph}, \end{matrix} \tag{C.11}$$

respectively.

C.2 The gauge interactions

The interactions of the quarks with the gauge bosons $W^\pm$ are given by

$$\begin{aligned} \mathcal{L}_W = g W_\mu^+ \sum_{\aleph=L,R} \Big(& \frac{1}{\sqrt{2}}\ \bar{\delta}_{1,7,\aleph}\gamma^\mu\gamma_\aleph\delta_{-1,7,\aleph} + \frac{1}{\sqrt{2}}\ \bar{\delta}_{1,1,\aleph}\gamma^\mu\gamma_\aleph\delta_{-1,1,\aleph} + \frac{1}{\sqrt{2}}\ \bar{\delta}_{1,-5,\aleph}\gamma^\mu\gamma_\aleph\delta_{-1,-5,\aleph} \\ & + \bar{\tau}_{2,4,\aleph}\gamma^\mu\gamma_\aleph\tau_{0,4,\aleph} + \bar{\tau}_{0,4,\aleph}\gamma^\mu\gamma_\aleph\tau_{-2,4,\aleph} + \bar{\tau}_{2,-2,\aleph}\gamma^\mu\gamma_\aleph\tau_{0,-2,\aleph} + \bar{\tau}_{0,-2,\aleph}\gamma^\mu\gamma_\aleph\tau_{-2,-2,\aleph} \Big) \\ & + \mathrm{H.c.} \end{aligned} \tag{C.12}$$

We rewrite these interactions using the general notation of eq. (2.2). We obtain

$$N_\aleph = H^\dagger_{1\aleph} U_{2\aleph} + \sqrt{2}\, H^\dagger_{2\aleph} U_{4\aleph}, \tag{C.13a}$$
$$V_\aleph = U^\dagger_{3\aleph} D_{2\aleph} + \sqrt{2}\, U^\dagger_{4\aleph} D_{4\aleph} + \sqrt{2}\, U^\dagger_{5\aleph} D_{5\aleph}, \tag{C.13b}$$
$$Q_\aleph = D^\dagger_{3\aleph} L_{1\aleph} + \sqrt{2}\, D^\dagger_{5\aleph} L_{2\aleph}. \tag{C.13c}$$

The interactions of the quarks with the gauge boson Z are given by

$$\begin{aligned}\mathcal{L}_Z = \frac{g}{c_w} Z_\mu \sum_{\aleph=L,R} \Bigg[& -\frac{2}{3} s_w^2\, \bar\sigma_{0,4,\aleph}\, \gamma^\mu \gamma_\aleph\, \sigma_{0,4,\aleph} + \frac{1}{3} s_w^2\, \bar\sigma_{0,-2,\aleph}\, \gamma^\mu \gamma_\aleph\, \sigma_{0,-2,\aleph} \\ & + \left(\frac{1}{2} - \frac{5}{3} s_w^2\right) \bar\delta_{1,7,\aleph}\, \gamma^\mu \gamma_\aleph\, \delta_{1,7,\aleph} + \left(-\frac{1}{2} - \frac{2}{3} s_w^2\right) \bar\delta_{-1,7,\aleph}\, \gamma^\mu \gamma_\aleph\, \delta_{-1,7,\aleph} \\ & + \left(\frac{1}{2} - \frac{2}{3} s_w^2\right) \bar\delta_{1,1,\aleph}\, \gamma^\mu \gamma_\aleph\, \delta_{1,1,\aleph} + \left(-\frac{1}{2} + \frac{1}{3} s_w^2\right) \bar\delta_{-1,1,\aleph}\, \gamma^\mu \gamma_\aleph\, \delta_{-1,1,\aleph} \\ & + \left(\frac{1}{2} + \frac{1}{3} s_w^2\right) \bar\delta_{1,-5,\aleph}\, \gamma^\mu \gamma_\aleph\, \delta_{1,-5,\aleph} + \left(-\frac{1}{2} + \frac{4}{3} s_w^2\right) \bar\delta_{-1,-5,\aleph}\, \gamma^\mu \gamma_\aleph\, \delta_{-1,-5,\aleph} \\ & + \left(1 - \frac{5}{3} s_w^2\right) \bar\tau_{2,4,\aleph}\, \gamma^\mu \gamma_\aleph\, \tau_{2,4,\aleph} - \frac{2}{3} s_w^2\, \bar\tau_{0,4,\aleph}\, \gamma^\mu \gamma_\aleph\, \tau_{0,4,\aleph} \\ & + \left(-1 + \frac{1}{3} s_w^2\right) \bar\tau_{-2,4,\aleph}\, \gamma^\mu \gamma_\aleph\, \tau_{-2,4,\aleph} + \left(1 - \frac{2}{3} s_w^2\right) \bar\tau_{2,-2,\aleph}\, \gamma^\mu \gamma_\aleph\, \tau_{2,-2,\aleph} \\ & + \frac{1}{3} s_w^2\, \bar\tau_{0,-2,\aleph}\, \gamma^\mu \gamma_\aleph\, \tau_{0,-2,\aleph} + \left(-1 + \frac{4}{3} s_w^2\right) \bar\tau_{-2,-2,\aleph}\, \gamma^\mu \gamma_\aleph\, \tau_{-2,-2,\aleph} \Bigg]. \end{aligned} \tag{C.14}$$

Rewriting these interactions by using the general notation of eq. (2.3), we obtain

$$H_\aleph = H^\dagger_{1\aleph} H_{1\aleph} + 2\, H^\dagger_{2\aleph} H_{2\aleph}, \tag{C.15a}$$
$$U_\aleph = -U^\dagger_{2\aleph} U_{2\aleph} + U^\dagger_{3\aleph} U_{3\aleph} + 2\, U^\dagger_{5\aleph} U_{5\aleph}, \tag{C.15b}$$
$$D_\aleph = D^\dagger_{2\aleph} D_{2\aleph} - D^\dagger_{3\aleph} D_{3\aleph} + 2\, D^\dagger_{4\aleph} D_{4\aleph}, \tag{C.15c}$$
$$L_\aleph = L^\dagger_{1\aleph} L_{1\aleph} + 2\, L^\dagger_{2\aleph} L_{2\aleph}. \tag{C.15d}$$

C.3 The finiteness of S

From appendix B, one gathers that the ultraviolet-divergent parts of the functions G and h in eqs. (4.5) are

$$G\left(x, y, Q, I, J\right) = \frac{\mathrm{div}}{3} \left(|x|^2 + |y|^2\right) + \text{finite parts}, \qquad h\left(I\right) = \frac{\mathrm{div}}{3} + \text{finite parts}. \tag{C.16}$$

Therefore, the contribution of the a-type quarks to the ultraviolet divergence in S in eq. (4.4a) is proportional to

$$S = -\frac{N_c}{2\pi} \frac{\mathrm{div}}{3} \left[\mathrm{tr}\left(\bar A_L^2 + \bar A_R^2\right) + 2\left(s_w^2 - c_w^2\right) |Q_a|\, \mathrm{tr}\left(\bar A_L + \bar A_R\right) - 8 s_w^2 c_w^2 Q_a^2 n_a \right] + \cdots, \tag{C.17}$$

where we have used the notation of section 4.2. According to eqs. (2.4), $\bar A_\aleph = A_\aleph - 2\,|Q_a|\, s_w^2\, \mathbb{1}$ for $\aleph = L, R$. Therefore, the quantity inside square brackets in eq. (C.17) is

$$\begin{aligned} & \mathrm{tr}\left[A_L^2 + A_R^2 - 4\,|Q_a|\, s_w^2 \left(A_L + A_R\right) + 8 Q_a^2 s_w^4\, \mathbb{1}\right] \\ & +2\left(s_w^2 - c_w^2\right) |Q_a|\, \mathrm{tr}\left(A_L + A_R - 4\,|Q_a|\, s_w^2\, \mathbb{1}\right) - 8 c_w^2 s_w^2 Q_a^2\, \mathrm{tr}\, \mathbb{1} \\ & \qquad = \mathrm{tr}\left[A_L^2 + A_R^2 - 2\,|Q_a| \left(A_L + A_R\right)\right]. \end{aligned} \tag{C.18}$$

Thus, the a-type quarks produce in S a divergence proportional to

$$\mathrm{tr}\left[A_L^2 + A_R^2 - 2\,|Q_a|\,(A_L + A_R)\right]. \tag{C.19}$$

Therefore, the oblique parameter S is finite if the equation

$$0 = 3\,\mathrm{tr}\Big[(H_\aleph)^2 + (U_\aleph)^2 + (D_\aleph)^2 + (L_\aleph)^2\Big] - 10\,\mathrm{tr}\,H_\aleph - 4\,\mathrm{tr}\,U_\aleph - 2\,\mathrm{tr}\,D_\aleph - 8\,\mathrm{tr}\,L_\aleph \tag{C.20}$$

holds for both $\aleph = L$ and $\aleph = R$. Now, according to eqs. (C.15),

$$(H_\aleph)^2 = H_{1\aleph}^\dagger H_{1\aleph} + 4\,H_{2\aleph}^\dagger H_{2\aleph}, \tag{C.21a}$$
$$(U_\aleph)^2 = U_{2\aleph}^\dagger U_{2\aleph} + U_{3\aleph}^\dagger U_{3\aleph} + 4\,U_{5\aleph}^\dagger U_{5\aleph}, \tag{C.21b}$$
$$(D_\aleph)^2 = D_{2\aleph}^\dagger D_{2\aleph} + D_{3\aleph}^\dagger D_{3\aleph} + 4\,D_{4\aleph}^\dagger D_{4\aleph}, \tag{C.21c}$$
$$(L_\aleph)^2 = L_{1\aleph}^\dagger L_{1\aleph} + 4\,L_{2\aleph}^\dagger L_{2\aleph}. \tag{C.21d}$$

Therefore, the right-hand side of eq. (C.20) is equal to

$$\begin{aligned}
&\mathrm{tr}\Big(3\,H_{1\aleph}^\dagger H_{1\aleph} + 12\,H_{2\aleph}^\dagger H_{2\aleph} + 3\,U_{2\aleph}^\dagger U_{2\aleph} + 3\,U_{3\aleph}^\dagger U_{3\aleph} + 12\,U_{5\aleph}^\dagger U_{5\aleph} \\
&\quad + 3\,D_{2\aleph}^\dagger D_{2\aleph} + 3\,D_{3\aleph}^\dagger D_{3\aleph} + 12\,D_{4\aleph}^\dagger D_{4\aleph} + 3\,L_{1\aleph}^\dagger L_{1\aleph} + 12\,L_{2\aleph}^\dagger L_{2\aleph} \\
&\quad - 10\,H_{1\aleph}^\dagger H_{1\aleph} - 20\,H_{2\aleph}^\dagger H_{2\aleph} + 4\,U_{2\aleph}^\dagger U_{2\aleph} - 4\,U_{3\aleph}^\dagger U_{3\aleph} - 8\,U_{5\aleph}^\dagger U_{5\aleph} \\
&\quad -2\,D_{2\aleph}^\dagger D_{2\aleph} + 2\,D_{3\aleph}^\dagger D_{3\aleph} - 4\,D_{4\aleph}^\dagger D_{4\aleph} - 8\,L_{1\aleph}^\dagger L_{1\aleph} - 16\,L_{2\aleph}^\dagger L_{2\aleph}\Big) \\
&\qquad = \mathrm{tr}\,\Big(-7\,H_{1\aleph} H_{1\aleph}^\dagger - 8\,H_{2\aleph} H_{2\aleph}^\dagger + 7\,U_{2\aleph} U_{2\aleph}^\dagger - U_{3\aleph} U_{3\aleph}^\dagger + 4\,U_{5\aleph} U_{5\aleph}^\dagger \\
&\qquad\qquad + D_{2\aleph} D_{2\aleph}^\dagger + 5\,D_{3\aleph} D_{3\aleph}^\dagger + 8\,D_{4\aleph} D_{4\aleph}^\dagger - 5\,L_{1\aleph} L_{1\aleph}^\dagger - 4\,L_{2\aleph} L_{2\aleph}^\dagger\Big).
\end{aligned} \tag{C.22}$$

Using the fact that all the matrices in the right-hand side of eq. (C.22) are proportional to unit matrices of the appropriate dimensions, cf. eq. (C.11), eq. (C.22) is equal to

$$-7\,n_{\delta,7,\aleph} - 8\,n_{\tau,4,\aleph} + 7\,n_{\delta,7,\aleph} - n_{\delta,1,\aleph} + 4\,n_{\tau,-2,\aleph} + n_{\delta,1,\aleph} + 5\,n_{\delta,-5,\aleph} + 8\,n_{\tau,4,\aleph} - 5\,n_{\delta,-5,\aleph} - 4\,n_{\tau,-2,\aleph}, \tag{C.23}$$

which is zero, Q.E.D.

The Standard Model. In the SM there are three u-type and three d-type quarks, there are neither h-type nor l-type quarks, the matrix V_R is zero, and the matrix V_L is 3×3 unitary. Hence,

$$\bar U_L = \left(1 - \frac{4}{3}\,s_w^2\right) \times \mathbb{1}_3,\ \bar U_R = \left(-\frac{4}{3}\,s_w^2\right) \times \mathbb{1}_3,\ \bar D_L = \left(1 - \frac{2}{3}\,s_w^2\right) \times \mathbb{1}_3,\ \bar D_R = \left(-\frac{2}{3}\,s_w^2\right) \times \mathbb{1}_3 \tag{C.24}$$

are all proportional to the unit matrix. Using eqs. (4.4a) and (C.16), the divergence in S is therefore

$$\begin{aligned}
S = -\frac{N_c}{2\pi}\,\frac{\mathrm{div}}{3}\,\Bigg\{&3\left[\left(1 - \frac{4}{3}\,s_w^2\right)^2 + \left(-\frac{4}{3}\,s_w^2\right)^2 + \left(1 - \frac{2}{3}\,s_w^2\right)^2 + \left(-\frac{2}{3}\,s_w^2\right)^2\right] \\
&+2\left(s_w^2 - c_w^2\right) \times 3\left[\frac{2}{3}\left(1 - \frac{8}{3}\,s_w^2\right) + \frac{1}{3}\left(1 - \frac{4}{3}\,s_w^2\right)\right] \\
&-8 s_w^2 c_w^2 \times 3\left(\frac{4}{9} + \frac{1}{9}\right)\Bigg\} + \text{finite terms}.
\end{aligned} \tag{C.25}$$

Thus,

$$S=-\frac{N_c}{2\pi}\,\mathrm{div}\left[2-4s_w^2+\frac{40}{9}\,s_w^4+2\left(2s_w^2-1\right)\left(1-\frac{20}{9}\,s_w^2\right)+8\left(s_w^4-s_w^2\right)\frac{5}{9}\right]+\text{finite terms.} \tag{C.26}$$

The terms inside the square brackets in eq. (C.26) clearly cancel out.

C.4 The finiteness of U

In the oblique parameter U, by using eq. (C.16) we find that the contribution of the a-type quarks to the ultraviolet divergence in eq. (4.4b) is

$$U=\frac{N_c}{2\pi}\frac{\mathrm{div}}{3}\left\{\mathrm{tr}\left[\left(\bar{A}_L\right)^2+\left(\bar{A}_R\right)^2\right]+4s_w^2\left|Q_a\right|\mathrm{tr}\left(\bar{A}_L+\bar{A}_R\right)+8s_w^4Q_a^2\,\mathrm{tr}\,\mathbb{1}\right\}+\cdots. \tag{C.27}$$

Using eqs. (2.4), the quantity inside square brackets in eq. (C.27) is

$$\begin{aligned}&\mathrm{tr}\left(A_L^2+A_R^2\right)-4\left|Q_a\right|s_w^2\,\mathrm{tr}\left(A_L+A_R\right)+8Q_a^2s_w^4\,\mathrm{tr}\,\mathbb{1}\\&\quad+4s_w^2\left|Q_a\right|\mathrm{tr}\left(A_L+A_R-4\left|Q_a\right|s_w^2\right)+8s_w^4Q_a^2\,\mathrm{tr}\,\mathbb{1}=\mathrm{tr}\left(A_L^2+A_R^2\right).\end{aligned} \tag{C.28}$$

Therefore, the oblique parameter U is finite if

$$0=\mathrm{tr}\left[\left(H_\aleph\right)^2+\left(U_\aleph\right)^2+\left(D_\aleph\right)^2+\left(L_\aleph\right)^2\right]-2\,\mathrm{tr}\left(N_\aleph N_\aleph^\dagger+V_\aleph V_\aleph^\dagger+Q_\aleph Q_\aleph^\dagger\right), \tag{C.29}$$

for both $\aleph=L$ and $\aleph=R$. According to eqs. (C.13) and (C.21), the right-hand side of eq. (C.29) is equal to

$$\begin{aligned}&\mathrm{tr}\left(H_{1\aleph}^\dagger H_{1\aleph}+4\,H_{2\aleph}^\dagger H_{2\aleph}+U_{2\aleph}^\dagger U_{2\aleph}+U_{3\aleph}^\dagger U_{3\aleph}+4\,U_{5\aleph}^\dagger U_{5\aleph}\right.\\&\qquad\left.+D_{2\aleph}^\dagger D_{2\aleph}+D_{3\aleph}^\dagger D_{3\aleph}+4\,D_{4\aleph}^\dagger D_{4\aleph}+L_{1\aleph}^\dagger L_{1\aleph}+4\,L_{2\aleph}^\dagger L_{2\aleph}\right)\\&-2\,\mathrm{tr}\left(H_{1\aleph}^\dagger H_{1\aleph}+2\,H_{2\aleph}^\dagger H_{2\aleph}+U_{3\aleph}^\dagger U_{3\aleph}+2\,U_{4\aleph}^\dagger U_{4\aleph}+2\,U_{5\aleph}^\dagger U_{5\aleph}+D_{3\aleph}^\dagger D_{3\aleph}+2\,D_{5\aleph}^\dagger D_{5\aleph}\right)\\&\quad=n_{\delta,7,\aleph}+4\,n_{\tau,4,\aleph}+n_{\delta,7,\aleph}+n_{\delta,1,\aleph}+4\,n_{\tau,-2,\aleph}\\&\qquad+n_{\delta,1,\aleph}+n_{\delta,-5,\aleph}+4\,n_{\tau,4,\aleph}+n_{\delta,-5,\aleph}+4\,n_{\tau,-2,\aleph}\\&\qquad-2\left(n_{\delta,7,\aleph}+2\,n_{\tau,4,\aleph}+n_{\delta,1,\aleph}+2\,n_{\tau,4,\aleph}+2\,n_{\tau,-2,\aleph}+n_{\delta,-5,\aleph}+2\,n_{\tau,-2,\aleph}\right)\\&\quad=0,\end{aligned} \tag{C.30}$$

Q.E.D.

The Standard Model. Using eqs. (4.4b), (C.16), and (C.24), in the SM the divergence in U is

$$\begin{aligned}U=-\frac{N_c}{\pi}\frac{\mathrm{div}}{3}&\left\{\sum_u\sum_d\left|\left(V_L\right)_{ud}\right|^2\right.\\&\quad-\frac{1}{2}\times 3\left[\left(1-\frac{4}{3}\,s_w^2\right)^2+\left(-\frac{4}{3}\,s_w^2\right)^2+\left(1-\frac{2}{3}\,s_w^2\right)^2+\left(-\frac{2}{3}\,s_w^2\right)^2\right]\\&\quad-2s_w^2\times 3\left[\frac{2}{3}\left(1-\frac{8}{3}\,s_w^2\right)+\frac{1}{3}\left(1-\frac{4}{3}\,s_w^2\right)\right]\\&\quad\left.-4s_w^4\times 3\left(\frac{4}{9}+\frac{1}{9}\right)\right\}+\text{finite terms,}\end{aligned} \tag{C.31}$$

cf. eq. (C.25). Thus,

$$U=-\frac{N_c}{\pi}\,\mathrm{div}\left[1-\frac{1}{2}\left(2-4s_w^2+\frac{40}{9}\,s_w^4\right)-2s_w^2\left(1-\frac{20}{9}\,s_w^2\right)-\frac{20}{9}\,s_w^4\right]+\text{finite terms.} \tag{C.32}$$

The terms inside the square brackets in eq. (C.32) clearly cancel out.

C.5 The finiteness of T

According to appendix B, the ultraviolet-divergent part of the function F in eq. (4.2) is

$$F\left(x,y,I,J\right)=\frac{\mathrm{div}}{4m_Z^2}\left[\left(|x|^2+|y|^2\right)\left(I+J\right)-4\,\mathrm{Re}(xy^*)\,\sqrt{IJ}\right]. \tag{C.33}$$

Then, the oblique parameter T is finite because

$$\begin{aligned}0 &= \mathrm{tr}\left[N_LN_L^\dagger M_h^2+N_L^\dagger N_LM_u^2+V_LV_L^\dagger M_u^2+V_L^\dagger V_LM_d^2+Q_LQ_L^\dagger M_d^2+Q_L^\dagger Q_LM_l^2+(L\to R)\right]\\ &\quad-\mathrm{tr}\left[H_L^2M_h^2+U_L^2M_u^2+D_L^2M_d^2+L_L^2M_l^2+(L\to R)\right], \qquad \text{(C.34a)}\\ 0 &= \mathrm{tr}\left[N_LM_uN_R^\dagger M_h+V_LM_dV_R^\dagger M_u+Q_LM_lQ_R^\dagger M_d+(L\leftrightarrow R)\right]\\ &\quad-\mathrm{tr}\left(H_LM_hH_RM_h+U_LM_uU_RM_u+D_LM_dD_RM_d+L_LM_lL_RM_l\right). \qquad \text{(C.34b)}\end{aligned}$$

Let us demonstrate each of the two identities (C.34) in turn.

1. We start with eq. (C.34a). We note that

$$\begin{aligned}N_\aleph N_\aleph^\dagger-H_\aleph^2 &= -2\,H_{2\aleph}^\dagger H_{2\aleph}, & \text{(C.35a)}\\ N_\aleph^\dagger N_\aleph+V_\aleph V_\aleph^\dagger-U_\aleph^2 &= 4\,U_{4\aleph}^\dagger U_{4\aleph}-2\,U_{5\aleph}^\dagger U_{5\aleph}, & \text{(C.35b)}\\ V_\aleph^\dagger V_\aleph+Q_\aleph Q_\aleph^\dagger-D_\aleph^2 &= 4\,D_{5\aleph}^\dagger D_{5\aleph}-2\,D_{4\aleph}^\dagger D_{4\aleph}, & \text{(C.35c)}\\ Q_\aleph^\dagger Q_\aleph-L_\aleph^2 &= -2\,L_{2\aleph}^\dagger L_{2\aleph}. & \text{(C.35d)}\end{aligned}$$

Therefore, eq. (C.34a) reads

$$\begin{aligned}0 &= \sum_{\aleph=L,R}\mathrm{tr}\left[-H_{2\aleph}^\dagger H_{2\aleph}M_h^2+\left(2\,U_{4\aleph}^\dagger U_{4\aleph}-U_{5\aleph}^\dagger U_{5\aleph}\right)M_u^2\right.\\ &\qquad\left.+\left(2D_{5\aleph}^\dagger D_{5\aleph}-D_{4\aleph}^\dagger D_{4\aleph}\right)M_d^2-L_{2\aleph}^\dagger L_{2\aleph}M_l^2\right]\\ &= \sum_{\aleph=L,R}\mathrm{tr}\left(-H_{2\aleph}M_h^2H_{2\aleph}^\dagger+2\,U_{4\aleph}M_u^2\,U_{4\aleph}^\dagger-U_{5\aleph}M_u^2\,U_{5\aleph}^\dagger\right.\\ &\qquad\left.+2\,D_{5\aleph}M_d^2\,D_{5\aleph}^\dagger-D_{4\aleph}M_d^2D_{4\aleph}^\dagger-L_{2\aleph}M_l^2L_{2\aleph}^\dagger\right)\\ &= \mathrm{tr}\left[-\left(\bar M_h\bar M_h^\dagger\right)_{22}+2\left(\bar M_u\bar M_u^\dagger\right)_{44}-\left(\bar M_u\bar M_u^\dagger\right)_{55}\right.\\ &\qquad+2\left(\bar M_d\bar M_d^\dagger\right)_{55}-\left(\bar M_d\bar M_d^\dagger\right)_{44}-\left(\bar M_l\bar M_l^\dagger\right)_{22}\\ &\qquad-\left(\bar M_h^\dagger\bar M_h\right)_{22}+2\left(\bar M_u^\dagger\bar M_u\right)_{44}-\left(\bar M_u^\dagger\bar M_u\right)_{55}\\ &\qquad\left.+2\left(\bar M_d^\dagger\bar M_d\right)_{44}-\left(\bar M_d^\dagger\bar M_d\right)_{55}-\left(\bar M_l^\dagger\bar M_l\right)_{22}\right].\end{aligned} \tag{C.36}$$

We now utilize eqs. (C.6) to ascertain that

$$-\left(\bar{M}_h\bar{M}_h^\dagger\right)_{22} = -2\,|v|^2\,\Upsilon_6\Upsilon_6^\dagger - M_6M_6^\dagger, \tag{C.37a}$$
$$2\left(\bar{M}_u\bar{M}_u^\dagger\right)_{44} = 2\,|v|^2\,\Upsilon_6\Upsilon_6^\dagger + 2\,|v|^2\,\Upsilon_5\Upsilon_5^\dagger + 2M_6M_6^\dagger, \tag{C.37b}$$
$$-\left(\bar{M}_u\bar{M}_u^\dagger\right)_{55} = -2\,|v|^2\,\Upsilon_8\Upsilon_8^\dagger - M_7M_7^\dagger, \tag{C.37c}$$
$$2\left(\bar{M}_d\bar{M}_d^\dagger\right)_{55} = 2\,|v|^2\,\Upsilon_7\Upsilon_7^\dagger + 2\,|v|^2\,\Upsilon_8\Upsilon_8^\dagger + 2M_7M_7^\dagger, \tag{C.37d}$$
$$-\left(\bar{M}_d\bar{M}_d^\dagger\right)_{44} = -2\,|v|^2\,\Upsilon_5\Upsilon_5^\dagger - M_6M_6^\dagger, \tag{C.37e}$$
$$-\left(\bar{M}_l\bar{M}_l^\dagger\right)_{22} = -2\,|v|^2\,\Upsilon_7\Upsilon_7^\dagger - M_7M_7^\dagger, \tag{C.37f}$$

and that

$$-\left(\bar{M}_h^\dagger\bar{M}_h\right)_{22} = -2\,|v|^2\,\Upsilon_{14}^\dagger\Upsilon_{14} - M_6^\dagger M_6, \tag{C.38a}$$
$$2\left(\bar{M}_u^\dagger\bar{M}_u\right)_{44} = 2\,|v|^2\,\Upsilon_{14}^\dagger\Upsilon_{14} + 2\,|v|^2\,\Upsilon_{13}^\dagger\Upsilon_{13} + 2M_6^\dagger M_6, \tag{C.38b}$$
$$-\left(\bar{M}_u^\dagger\bar{M}_u\right)_{55} = -2\,|v|^2\,\Upsilon_{16}^\dagger\Upsilon_{16} - M_7^\dagger M_7, \tag{C.38c}$$
$$2\left(\bar{M}_d^\dagger\bar{M}_d\right)_{55} = 2\,|v|^2\,\Upsilon_{16}^\dagger\Upsilon_{16} + 2\,|v|^2\,\Upsilon_{15}^\dagger\Upsilon_{15} + 2M_7^\dagger M_7, \tag{C.38d}$$
$$-\left(\bar{M}_d^\dagger\bar{M}_d\right)_{44} = -2\,|v|^2\,\Upsilon_{13}^\dagger\Upsilon_{13} - M_6^\dagger M_6, \tag{C.38e}$$
$$-\left(\bar{M}_l^\dagger\bar{M}_l\right)_{22} = -2\,|v|^2\,\Upsilon_{15}^\dagger\Upsilon_{15} - M_7^\dagger M_7, \tag{C.38f}$$

Q.E.D.

2. We next turn to eq. (C.34b). We notice that

$$\begin{aligned}
\mathrm{tr}\left(L_LM_lL_RM_l\right) &= \mathrm{tr}\left(L_{1L}M_lL_{1R}^\dagger L_{1R}M_lL_{1L}^\dagger + 4\,L_{2L}M_lL_{2R}^\dagger L_{2R}M_lL_{2L}^\dagger\right.\\
&\qquad\left.+2\,L_{1L}M_lL_{2R}^\dagger L_{2R}M_lL_{1L}^\dagger + 2\,L_{2L}M_lL_{1R}^\dagger L_{1R}M_lL_{2L}^\dagger\right)\\
&= \mathrm{tr}\left\{\left(\bar{M}_l\right)_{11}\left[\left(\bar{M}_l\right)_{11}\right]^\dagger + 4\left(\bar{M}_l\right)_{22}\left[\left(\bar{M}_l\right)_{22}\right]^\dagger\right.\\
&\qquad\left.+2\left(\bar{M}_l\right)_{12}\left[\left(\bar{M}_l\right)_{12}\right]^\dagger + 2\left(\bar{M}_l\right)_{21}\left[\left(\bar{M}_l\right)_{21}\right]^\dagger\right\}\\
&= \mathrm{tr}\left(M_5M_5^\dagger + 4M_7M_7^\dagger + 4\,|v|^2\,\Upsilon_{15}\Upsilon_{15}^\dagger + 4\,|v|^2\,\Upsilon_7\Upsilon_7^\dagger\right).
\end{aligned} \tag{C.39}$$

Similarly,

$$\mathrm{tr}(H_LM_hH_RM_h)=\mathrm{tr}\left(M_3M_3^\dagger+4M_6M_6^\dagger+4|v|^2\Upsilon_{14}\Upsilon_{14}^\dagger+4|v|^2\Upsilon_6\Upsilon_6^\dagger\right), \tag{C.40a}$$
$$\mathrm{tr}(U_LM_uU_RM_u)=\mathrm{tr}\left(M_3M_3^\dagger+M_4M_4^\dagger+4M_7M_7^\dagger+4|v|^2\Upsilon_{16}\Upsilon_{16}^\dagger+4|v|^2\Upsilon_8\Upsilon_8^\dagger\right), \tag{C.40b}$$
$$\mathrm{tr}(D_LM_dD_RM_d)=\mathrm{tr}\left(M_4M_4^\dagger+M_5M_5^\dagger+4M_6M_6^\dagger+4|v|^2\Upsilon_{13}\Upsilon_{13}^\dagger+4|v|^2\Upsilon_5\Upsilon_5^\dagger\right). \tag{C.40c}$$

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Therefore, eq. (C.34b) reads

$$\begin{aligned}\operatorname{tr}\Big[&N_L M_u N_R^\dagger M_h + V_L M_d V_R^\dagger M_u \\ &+Q_L M_l Q_R^\dagger M_d + (L \leftrightarrow R)\Big] = \operatorname{tr}\Big[2M_3M_3^\dagger + 2M_4M_4^\dagger + 2M_5M_5^\dagger \\ &\qquad +8M_6M_6^\dagger + 8M_7M_7^\dagger \\ &\qquad +4\,|v|^2\Big(\Upsilon_5\Upsilon_5^\dagger + \Upsilon_6\Upsilon_6^\dagger + \Upsilon_7\Upsilon_7^\dagger \\ &\qquad\qquad + \Upsilon_8\Upsilon_8^\dagger + \Upsilon_{13}\Upsilon_{13}^\dagger + \Upsilon_{14}\Upsilon_{14}^\dagger \\ &\qquad\qquad +\Upsilon_{15}\Upsilon_{15}^\dagger + \Upsilon_{16}\Upsilon_{16}^\dagger\Big)\Big]. \end{aligned}\tag{C.41}$$

Equation (C.41) holds because

$$\begin{aligned}\operatorname{tr}\left(N_L M_u N_R^\dagger M_h\right) &= \operatorname{tr}\left(N_R M_u N_L^\dagger M_h\right) \\ &= M_3M_3^\dagger + 2M_6M_6^\dagger + 2\,|v|^2\left(\Upsilon_6\Upsilon_6^\dagger + \Upsilon_{14}\Upsilon_{14}^\dagger\right),\end{aligned}\tag{C.42a}$$

$$\begin{aligned}\operatorname{tr}\left(V_L M_d V_R^\dagger M_u\right) &= \operatorname{tr}\left(V_R M_d V_L^\dagger M_u\right) \\ &= M_4M_4^\dagger + 2M_6M_6^\dagger + 2M_7M_7^\dagger \\ &\quad +2\,|v|^2\left(\Upsilon_{13}\Upsilon_{13}^\dagger + \Upsilon_5\Upsilon_5^\dagger + \Upsilon_{16}\Upsilon_{16}^\dagger + \Upsilon_8\Upsilon_8^\dagger\right),\end{aligned}\tag{C.42b}$$

$$\begin{aligned}\operatorname{tr}\left(Q_L M_l Q_R^\dagger M_d\right) &= \operatorname{tr}\left(Q_R M_l Q_L^\dagger M_d\right) \\ &= M_5M_5^\dagger + 2M_7M_7^\dagger + 2\,|v|^2\left(\Upsilon_7\Upsilon_7^\dagger + \Upsilon_{15}\Upsilon_{15}^\dagger\right),\end{aligned}\tag{C.42c}$$

Q.E.D.

The Standard Model. In the SM

$$T = \frac{N_c}{4\pi c_w^2 s_w^2}\left[2\sum_u\sum_d F\left(V_{ud}, 0, m_u^2 . m_d^2\right) - \sum_{u,u'} F\left(\delta_{uu'}, 0, m_u^2, m_{u'}^2\right) - \sum_{d,d'} F\left(\delta_{dd'}, 0, m_d^2, m_{d'}^2\right)\right],\tag{C.43}$$

where V is the 3×3 unitary CKM matrix. Using eq. (C.33), one then has

$$\begin{aligned}T = \frac{N_c}{4\pi c_w^2 s_w^2}\,\frac{\operatorname{div}}{4m_Z^2}&\left[2\sum_u\sum_d |V_{ud}|^2\left(m_u^2 + m_d^2\right) - \sum_{u,u'}\delta_{uu'}\left(m_u^2 + m_{u'}^2\right) - \sum_{d,d'}\delta_{dd'}\left(m_d^2 + m_{d'}^2\right)\right] \\ &+\text{finite terms.}\end{aligned}\tag{C.44}$$

Since V is an unitary matrix, the terms inside the square brackets in eq. (C.44) cancel out.

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