3.º Teste EeO – Versão A (15/12/2006)

a) As ondas são transversais pelo que devemos ter

$$\vec{k}\cdot\vec{E}=0$$

Como (comparando com $\omega t-\vec{k}\cdot\vec{r}$)

$$\begin{cases} k_x = -\alpha \\ k_y = 0 \\ k_z = -\beta \end{cases}$$

obtemos

$$E_0 \cos[\dots]\,(-\alpha-\beta)=0 \quad\Rightarrow\quad \alpha=-\beta=-\sqrt{2}\times10^{-2}\,m^{-1}$$

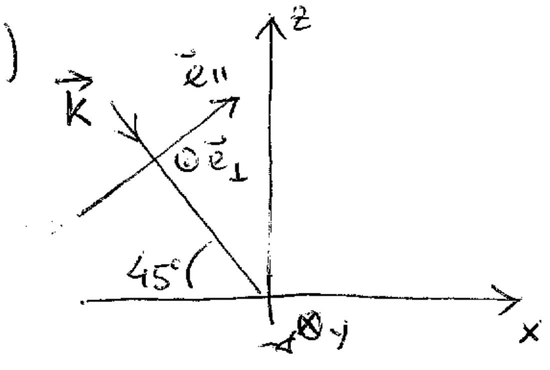
b) $\vec{n}=\dfrac{\vec{k}}{|\vec{k}|}$

Como $|\vec{k}|=\sqrt{k_x^2+k_z^2}=2\times10^{-2}\,m^{-1}$

vem

$$\boxed{\vec{n}=+\frac{1}{\sqrt{2}}\vec{e}_x-\frac{1}{\sqrt{2}}\vec{e}_z}$$

c) $v=\dfrac{\omega}{|\vec{k}|}=2\times10^{8}\,m/s \Rightarrow n=\dfrac{c}{v}=1.5$

d) 

$$\vec{e}_{\parallel}=\frac{1}{\sqrt{2}}\vec{e}_x+\frac{1}{\sqrt{2}}\vec{e}_z \qquad (\vec{e}_{\parallel}\cdot\vec{n}=0)$$

$$\vec{e}_{\perp}=-\vec{e}_y$$

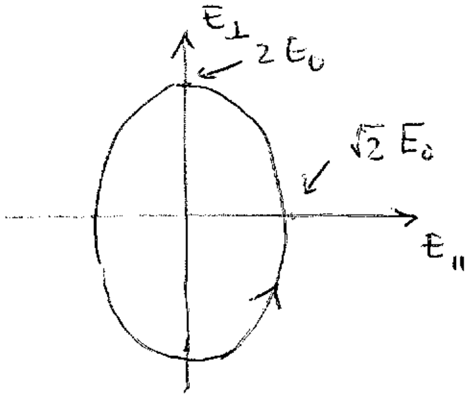
$$\vec{E} = E_0 \cos[\cdot] (\vec{e}_x + \vec{e}_z) - 2E_0 \sin[\cdot\cdot] \vec{e}_y$$

$$= \sqrt{2} E_0 \cos[\cdot] \vec{e}_{\parallel} + 2 E_0 \sin(\cdot\cdot) \vec{e}_{\perp}$$

$$\equiv E_{\parallel} \vec{e}_{\parallel} + E_{\perp} \vec{e}_{\parallel}$$

Com

$$\begin{cases} E_{\parallel} = \sqrt{2} E_0 \cos[\cdot\cdot] \\ E_{\perp} = 2 E_0 \sin[\cdot\cdot] \end{cases}$$



$\boxed{\text{Polarização elíptica esquerda}}$

e) $\vec{S} = \vec{E} \times \vec{H} = \frac{1}{Z} |\vec{E}|^2 \vec{n}$

$$Z = Z_0 \frac{1}{n}$$

$$\langle |\vec{S}| \rangle = \frac{n}{Z_0} \langle |\vec{E}|^2 \rangle$$

$$|\vec{E}|^2 = 2E_0^2 \cos^2[\cdot\cdot] + 4E_0^2 \sin^2[\cdot\cdot]$$

$\boxed{\langle \cos^2 \rangle = \langle \sin^2 \rangle = \frac{1}{2}}$

$$\langle |\vec{E}|^2 \rangle = 2E_0^2 \times \frac{1}{2} + 4E_0^2 \times \frac{1}{2} = 3E_0^2$$

e portanto

$$\boxed{\langle |\vec{S}| \rangle = \frac{n}{Z_0} 3E_0^2 = 1.2 \times 10^{-8}\ W/m^2}$$

3º teste EeO – Versão B (15/12/2016)

(muito igual à versão A). Soluções:

a)

$\beta = \sqrt{2} \times 10^{-2}\ m^{-1}$

b) $\vec{n} = \frac{1}{\sqrt{2}} \vec{e}_x - \frac{1}{\sqrt{2}} \vec{e}_z$

c) $n = 15$

d) Elíptica esquerda

e) $\langle |\vec{S}| \rangle = \frac{n}{Z_0} 3 E_0^2 = 1.2 \times 10^{-8}\ W/m^2$