Soluções dos problemas

Capítulo 1

1.1

a) $|\vec{F}| = \dfrac{Q^2}{4\pi\epsilon_0} \; \dfrac{1}{a(a+L)}$

b) $d = \dfrac{a^2}{2a+L}$

c) $d = \dfrac{1}{2}\left[2a + \dfrac{Q}{\lambda} - \sqrt{\left(\dfrac{Q}{\lambda}\right)^2 + 4a\dfrac{Q}{\lambda}}\right]$

d) $d \to 0$

1.2 $\vec{E} = \dfrac{\sigma z}{2\epsilon_0}\left(\dfrac{1}{z} - \dfrac{1}{\sqrt{a^2+z^2}}\right)\vec{e}_z$

1.3

a) $\phi = \dfrac{Q}{4\pi\epsilon_0}\left[\dfrac{1}{\sqrt{z^2+R^2}} - \dfrac{1}{z}\right]$

b) $\vec{E} = \dfrac{Q}{4\pi\epsilon_0}\left[\dfrac{z}{(z^2+R^2)^{3/2}} - \dfrac{1}{z^2}\right] \vec{e}_z$

c) $\phi \simeq \dfrac{Q}{4\pi\epsilon_0}\left(-\dfrac{R^2}{2z^3}\right)$

1.4

a) $E_z = \dfrac{Q}{4\pi\epsilon_0} \; \dfrac{1}{z\sqrt{z^2+L^2}}$

b) $E_z \simeq \dfrac{Q/L}{2\pi\epsilon_0} \; \dfrac{1}{z} \quad ; \quad E_z \simeq \dfrac{Q}{4\pi\epsilon_0} \; \dfrac{1}{z^2}$

1.6 $|\vec{E}| = \sigma/4\pi\epsilon_0$ e apontando para fora da semiesfera, se a carga for positiva.

1.7

$$E_r = \frac{2a}{r^3} \; \cos\theta + \frac{b}{r^2}$$

$$E_\theta = \frac{a}{r^3} \; \sin\theta$$

1.8

a) $W = \dfrac{Qp}{\pi\epsilon_0}\dfrac{1}{4x^2 - L^2}$

b) $\phi \simeq \dfrac{p}{4\pi\epsilon_0} \; \dfrac{1}{x^2}$

1.10

$$\begin{aligned} \vec{E} &= (\rho a/2\epsilon_0) \; \vec{e}_z, \quad z > a/2 \\ \vec{E} &= -(\rho a/2\epsilon_0) \; \vec{e}_z, \quad z < -a/2 \\ \vec{E} &= \rho z/\epsilon_0 \; \vec{e}_z, \quad -a/2 < z < a/2 \end{aligned}$$

1.11

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\vec{e}_r \; , \; \phi = \frac{Q}{4\pi\epsilon_0 r}, \; r > R$$

$$\vec{E} = \frac{Qr}{4\pi\epsilon_0 R^3}\vec{e}_r, \; r < R$$

$$\phi = \frac{Q}{8\pi\epsilon_0 R^3}(R^2 - r^2) + \frac{Q}{4\pi\epsilon_0 R}, \; r < R$$

1.12

a) $A = \dfrac{3Q}{\pi R^4}$

b) $\begin{cases} \vec{E}_{\text{int}} = \dfrac{Q}{4\pi\epsilon_0 R^4} r(4R - 3r) \; \vec{e}_r \\ \vec{E}_{\text{ext}} = \dfrac{Q}{4\pi\epsilon_0 r^2}\vec{e}_r \end{cases}$

1.14

a) $A = -\dfrac{e}{\pi r_0^3}$

b) $\begin{cases} \vec{E} = \dfrac{e}{4\pi\epsilon_0}\dfrac{2r^2+2rr_0+r_0^2}{r^2 r_0^2}\ e^{-2r/r_0}\vec{e}_r \\ \phi = \dfrac{e}{4\pi\epsilon_0}\dfrac{r+r_0}{rr_0}\ e^{-2r/r_0} \end{cases}$

c) $Q^* = 0,014\ e$

1.15 $\phi = \dfrac{q}{4\pi\epsilon_0}\ \dfrac{1}{R}\ ; \qquad W = \dfrac{Q^2}{8\pi\epsilon_0 R}$

1.16

a) $Q_b = 4\pi\epsilon_0 bV_1 - Q$

b) $r > b,$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q+Q_b}{r^2}\vec{e}_r\ ,\ \phi = \frac{1}{4\pi\epsilon_0}\frac{Q+Q_b}{r}$$

$a < r < b,$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\vec{e}_r\ ,\ \phi = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r}+\frac{Q_b}{b}\right)$$

$r < a,$

$$\vec{E} = 0\ ,\ \phi = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{a}+\frac{Q_b}{b}\right)$$

1.17

a) $\vec{E} = \dfrac{\lambda}{2\pi\epsilon_0}\ \dfrac{1}{r}\ \vec{e}_r\ ,\ \phi = \dfrac{\lambda}{2\pi\epsilon_0}\ \ln\dfrac{R_2}{r}$

b) $\lambda = \dfrac{2\pi\epsilon_0 V}{\ln\frac{R_2}{R_1}}$

c) $W_E = \dfrac{\lambda^2}{4\pi\epsilon_0}\ \ln\dfrac{R_2}{R_1}$

1.20 $C = \dfrac{2\pi\epsilon_0 L}{\ln R_2/R_1}$

1.21 $C' = \dfrac{Cd}{d-a}$

1.25

a) $|\vec{N}| = |\vec{p}||\vec{E}|\sin\theta$ b) $W = 2|\vec{p}||\vec{E}|$

c) $T = \sqrt{\dfrac{4\pi^2\ I}{|\vec{p}||\vec{E}|}}$

1.27 $C_{\text{dieléctrico}} = \dfrac{\epsilon\epsilon_0 A}{(d-a)\epsilon + a\epsilon_0}$

1.30 $V_f = \dfrac{2V_1}{\epsilon_r+1}\ ; \qquad Q = CV_1\ \dfrac{\epsilon_r-1}{\epsilon_r+1}$

1.34

$$dQ = P\cos\theta\ 2\pi r^2\ \sin\theta\ d\theta$$
$$Q^+ = \pi r^2 P\ ; \qquad Q_t = 0$$

1.35

$$W = (1/4\pi\epsilon_0)(q_1q_3/r_{23} - q_1q_3/r_{13} + q_2q_3/r_{13} - q_2q_3/r_{23})$$

1.36 $W_E = 780\text{nJ}\ ; \qquad W_{2q} = 102\ \text{nJ}$

1.37

$$W_f - W_i = -\frac{1}{2}\ Q^2\ \frac{C_2}{C_1(C_1+C_2)}$$

1.40 $v = \sqrt{\dfrac{2qV}{m}}$

1.41 $d = (e/m)\,E_d\ w(L-w/2)/v_0^2$

Capítulo 2

2.1 $v = \dfrac{Im}{e\rho NS} = 7.4\times10^{-5}\ \text{m}\,\text{s}^{-1}$

2.2

$$|J| = \frac{V}{\dfrac{d_1}{\sigma_{c1}}+\dfrac{d_2}{\sigma_{c2}}}$$
$$\sigma = \frac{V\left(\dfrac{\epsilon_2}{\sigma_{c2}}-\dfrac{\epsilon_1}{\sigma_{c1}}\right)}{\dfrac{d_1}{\sigma_{c1}}+\dfrac{d_2}{\sigma_{c2}}}$$

2.3

a) $\rho = 0\,;\rho' = 0\,;\,|\sigma_i| = \dfrac{\epsilon\phi_1 R_1 R_2}{R_i^2(R_2-R_1)}$

b) $I = \dfrac{\sigma_c 4\pi\phi_1 R_1 R_2}{R_2-R_1}$ c) $R = \dfrac{\phi_1}{I}$

2.4 a) $P = R\left(\dfrac{\mathcal{E}}{r+R}\right)^2\ ;\ R = r$

2.5

a) $|F| = 2I_1I_2\frac{\mu_0}{4\pi}l_1(\frac{1}{d} - \frac{1}{d+l_2})$

b) $\vec{M} = 0$

2.6 $\vec{H} = \frac{2\sqrt{2}\ I}{\pi L}\ \vec{e}_z$

2.8 $\vec{B} = \frac{\mu_0}{2}\sigma\omega a\vec{e}_z$

2.9 $\vec{B} = \frac{\mu_0\sigma\omega a^4}{8b^3}\vec{e}_z$

2.10 $|\vec{B}| = \mu_0 I/2R(1/\pi + 1/2)$

2.11

a) $\vec{B} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x-\frac{d}{2}} - \frac{1}{x+\frac{d}{2}}\right)\ \vec{e}_y$

b) $\vec{B} = -\frac{\mu_0 I}{2\pi}\frac{d}{y^2+\frac{d^2}{4}}\ \vec{e}_y$

2.12 $L = \frac{\mu_0 l_1}{2\pi}\ln\frac{d+l_2}{d}$

2.14 $B_{\rm int} = \mu_0\,\sigma\omega\,R,\quad B_{\rm ext} = 0.$

2.15 $M = \mu_0\pi\frac{(r_2r_1)^2}{2(a^2+r_2^2)^{3/2}}$

2.18 $|\vec{B}|_{\rm ext} = \frac{\mu_0}{2\pi}\ \frac{I}{r};\ |\vec{B}|_{\rm int} = \frac{\mu_0}{2\pi}\ I\ \frac{r}{R^2}$

2.19

$$R_2 < r < R_3,\ |\vec{B}| = B_\varphi = \frac{\mu_0\,I}{2\pi}\frac{R_3^2 - r^2}{R_3^2 - R_2^2}\ \frac{1}{r}$$

2.20 $B \simeq \mu_0\,\sigma\,v/2.$

Capítulo 3

3.1 $\mathcal{E}_{\rm AB} = -\frac{\mu_0\,Iv}{2\pi}\ln\frac{b}{a}\ ;\quad V = \epsilon_{\rm AB}$

3.2

$\phi_1 = SB_0\sin(\omega t)\ ;\ \phi_N = NSB_0\sin(\omega t)$

$\mathcal{E} = -NSB_0\,\omega\cos(\omega t)$

3.3 $\epsilon = -Kab\ ;\ E_{\rm m} = -E_{\rm m'} = \frac{Kab}{2(a+b)}$

3.4 a) $\vec{F} = -\frac{B^2a^2}{R}v\vec{e}_x$

3.5

a) $\vec{F} = -qvB\vec{e}_z$ b)$\vec{E} = vB\vec{e}_z$

c) $V = vBL$

3.6 a) $\epsilon = 0.2$ V b) $I = 1$ A

3.7 b) $v_{\rm lim} = \frac{mgR}{B^2l^2}$

3.8 $E = 250$ J ; $\mathcal{E} = 900$ V

3.9

a) $\mathcal{E} = \frac{\mu_0\,Ilvh}{2\pi(D+vt)(D+h+vt)}$

b) $M_{12} = \frac{\mu_0 l}{2\pi}\ln\left(\frac{D+vt+h}{D+vt}\right)$

c) $\mathcal{E} = \frac{\mu_0\,I_0\,\omega}{2\pi}l\ln\left(\frac{D+h}{D}\right)\sin\omega t$

3.14 $f = 1.1$ MHz

Capítulo 4

4.1

a) $A = 6$ cm , b) $\lambda = 1$ m, $f = 2$ Hz

c) $v = 2$ ms^{-1}d) $\vec{n} = -\vec{e}_x$

e) $v_{\rm T}^{\rm max} = 0.75$ ms^{-1}

4.4

a) $f = 3\times10^5$ Hz , b) $f = 3\times10^8$ Hz

c) $f = 10^{10}$ Hz , d) $f = 3\times10^{12}$ Hz

e) $f = 6\times10^{14}$ Hz , f) $f = 3\times10^{19}$ Hz

g) $f = 3\times10^{20}$ Hz

4.6

a) $\vec{n} = \frac{\sqrt{2}}{2}(\vec{e}_y + \vec{e}_z)$, b) $n = 1.2$

c) $\lambda = 3141.6$ m , d) Linear

4.7

a) $\epsilon_r = 1.63$

b) $E_x = 5.9 \times 10^{-1} \sin\left(\omega t - |\vec{k}|z\right)$ V/m

$E_y = 8.8 \times 10^{-1} \cos\left(\omega t - |\vec{k}|z\right)$ V/m

$E_z = 0$

Polarização elíptica

c) $\lambda = 184.8$ m

4.8

a) $n = 1.5$

b) $\vec{n} = \frac{\sqrt{2}}{2}\vec{e}_x - \frac{\sqrt{2}}{2}\vec{e}_y$

c)

$$H_x = -H_0 \cos\left[\omega t - |\vec{k}|\left(\frac{\sqrt{2}}{2}\ x - \frac{\sqrt{2}}{2}\ y\right)\right]$$

$$H_y = -H_0 \cos\left[\omega t - |\vec{k}|\left(\frac{\sqrt{2}}{2}\ x - \frac{\sqrt{2}}{2}\ y\right)\right]$$

$$H_z = 0$$

$$H_0 = 7.3 \times 10^{-9} \text{ A/m}$$

d) Linear

4.9

a) $\vec{n} = \dfrac{\sqrt{3}}{2}\vec{e}_y - \dfrac{1}{2}\vec{e}_z$, b) $n = 1.43$

c) $H_x = 0$

$H_y = -9.5 \times 10^{-4} \cos[\cdots]$ A/m

$H_z = -1.6 \times 10^{-3} \cos[\cdots]$ A/m

d) Linear

e) $\vec{S} = 9.5 \times 10^{-4} \cos^2[\cdots]\ \vec{e}_x$ Wm^{-2}

4.10 a) $\vec{n} = \vec{e}_y$; b) $n = 1.2$; c) Circular direita.

4.11

a) $\lambda = 300$ m

b) $E_0 = 122.8$ V/m ; $H_0 = 0.33$ A/m

c)

$\vec{E} = E_0 \cos(2\pi \times 10^6\ t - 2.09 \times 10^{-2}\ x)\vec{e}_y$ (SI)

$\vec{H} = H_0 \cos(2\pi \times 10^6\ t - 2.09 \times 10^{-2}\ x)\vec{e}_z$ (SI)

4.12

a) Linear; $\vec{n} = \dfrac{\sqrt{3}}{2}\vec{e}_x - \dfrac{1}{2}\vec{e}_y$

b) $d = \dfrac{\lambda_{\text{vazio}}}{4\cos r\ n_{\text{vidro}}}$;

4.14

$\left\langle \vec{S} \right\rangle = 1.1 \times 10^{-14}$ W/m ,

$E_0 = 2.88 \times 10^{-6}$ V/m

4.15 a) $4,24 \times 10^{26}$ W ; b) 1.9×10^{17} W; c) $T = 4.25 \times 10^{11}$ anos. Note-se que $T >$ Idade do Universo $\approx 1.5 \times 10^{10}$ anos.

4.16 a) $r_1 = 40$ m ; b) $P = 5,0 \times 10^5$ W

4.18

$$\Delta = d\frac{|\sin(i-r)|}{\cos r}$$