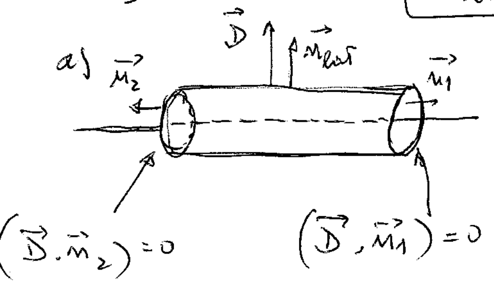
I | 1º Exame A | LEAmb + LEMat + LQuim. | 19/1/2008

a) 

Por Gauss: $\int_{s.f} (\vec{D}\cdot\vec{n})\, ds = q^{livre}$

Por razões de simetria: $\vec{D}$ tem simetria radial e é $\perp$ ao fio condutor.

$|D|\, 2\pi r\, \cancel{l} = \lambda \cancel{l}$

$|D| = \frac{1}{4\pi}\frac{2\lambda}{r}$

$$\vec{D} = \frac{1}{4\pi}\frac{2\lambda}{\frac{1}{2}R_2}\vec{e}_r$$

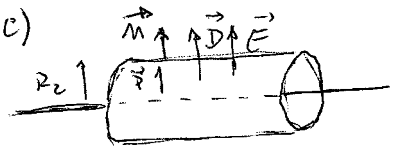
b) $V_A = V_B + \int_A^B (\vec{E}\cdot d\vec{r})$ $V_{R_3} = 0$: está ligado à Terra.; $\vec{D} = \varepsilon\vec{E}$

$$V_{R_1} = V_{R_3} + \int_{R_1}^{R_3}(\vec{E}\cdot d\vec{r}) = \int_{R_1}^{R_2}\frac{1}{4\pi\varepsilon_1}\frac{2\lambda}{r}\,dr + \int_{R_2}^{R_3}\frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}\,dr$$

$$V_{R_1} = \frac{1}{4\pi\varepsilon_0}\,2\lambda\left[\frac{1}{\varepsilon_{r_1}}\ln\frac{R_2}{R_1} + \ln\frac{R_3}{R_2}\right]$$

$$V_{R_1} = (9\times10^9)\times 2 * (2\times10^{-9})\left[\frac{1}{2}\ln\frac{0{,}01}{0{,}002} + \ln\frac{0{,}06}{0{,}01}\right]$$

$$\boxed{V_{R_1} = 93{,}47\ V}$$

c) 

$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

$\vec{P} = (\varepsilon_1 - \varepsilon_0)\vec{E}$

$\vec{P} = \left(1 - \frac{1}{\varepsilon_{r_1}}\right)\vec{D} \rightarrow$

$\vec{D}_{R_2} = \frac{1}{4\pi}\frac{2\lambda}{R_2}\vec{e}_r$

$\vec{P}_{R_2} = \left(1 - \frac{1}{\varepsilon_{r_1}}\right)\frac{1}{4\pi}\frac{2\lambda}{R_2}\vec{e}_r$

$\sigma_2' = (\vec{P}\cdot\vec{n})$ $\vec{n}$ é a normal <u>exterior</u> ao dieléctrico

$$\sigma_2' = \left(1 - \frac{1}{\varepsilon_{r_1}}\right)\frac{1}{4\pi}\frac{2\lambda}{R_2} = \left(1 - \frac{1}{2}\right)\frac{1}{4\pi}\frac{2\times2\times10^{-9}}{0{,}01}$$ $\boxed{\sigma' \doteq +16\,nC\,m^{-2}}$

d) $\vec{E}_{R_2^+} = \frac{\vec{D}}{\varepsilon_0}$ $\vec{E}_{R_2^-} = \frac{\vec{D}}{\varepsilon_1}$ (do lado do dieléctrico)

$$\vec{E}_{R_2^+} - \vec{E}_{R_2^-} = \left(\frac{1}{\varepsilon_0} - \frac{1}{\varepsilon_1}\right)\frac{1}{4\pi}\frac{2\lambda}{R_2}\vec{e}_r$$

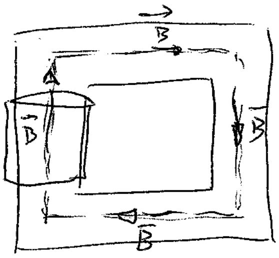
$$\vec{E}_{R_2^+} - \vec{E}_{R_2^-} = \frac{1}{\varepsilon_0}\left[1 - \frac{1}{\varepsilon_{r_1}}\right]\frac{1}{4\pi}\frac{2\lambda}{R_2}\vec{e}_r$$

$$(\vec{E}_{R_2^+} - \vec{E}_{R_2^-})\cdot\vec{n} = \frac{1}{\varepsilon_0}\left[1 - \frac{1}{\varepsilon_{r_1}}\right]\frac{1}{4\pi}\frac{2\lambda}{R_2}$$

$$\boxed{E_{R_2^+} - E_{R_2^-} = \frac{\sigma_2'}{\varepsilon_0}}$$

19/1/2008

a)



$$i = \frac{dq}{dt} \qquad dq = \sigma\, dS \qquad dS = R_1\, d\theta\, dz$$

$$i = \frac{\sigma R_1\, d\theta\, dz}{dt} \qquad i = \sigma R_1 \omega\, dz$$

$$I^{int} = \sigma R_1 \omega b$$

$$\oint (\vec{B} \cdot \vec{ds}) = \mu I^{int}$$

$$|B| * 4a = \mu \sigma R_1 \omega b \qquad B = \frac{\sigma R_1 \omega b}{4a} \mu_r \mu_0$$

$$|B| = \frac{(50 \times 10^{-9})(3 \times 10^{-2}) \left(\frac{5000 \times 2\pi}{60}\right) * 0,1 * 4000 \times 4\pi \times 10^{-7}}{0,6}$$

$$|B| = 0,66\ nT$$

b) Tomando 1) $\vec{B}$ na linha média.

2) $|B| \approx$ cte na secção média do solenoide.

$$\phi_{espira} = \int (\vec{B} \cdot \vec{n})\, dS \qquad |B|\, 4a = \mu N I$$

$$\phi_{espira} = |B| \pi R_1^2 \qquad |B| = \mu \frac{NI}{4a}$$

$$\phi_{TOTAL} = N |B| \pi R_1^2 \longrightarrow \phi_{TOTAL} = \mu N \frac{NI}{4a} \pi R_1^2$$

$$\phi_{TOTAL} = \mu \frac{N^2}{4a} \pi R_1^2\, I$$

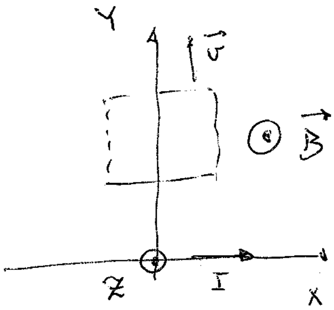
$$\phi_{TOTAL} = L I$$

$$L = \mu_r \mu_0 \frac{N^2}{4a} \times \pi R_1^2$$

$$L = 4000 * 4\pi \times 10^{-7} * \frac{10^4}{0,6} \times \pi * (3 \times 10^{-2})^2$$

$L = 0,24$ Henry.

$$\underline{L = 237\ mH}$$



a) $\oint (\vec{B} \cdot \vec{ds}) = \mu_0 I$ fio infinito

$$|B|\, 2\pi y = \mu_0 I$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{2I}{y} \vec{e_z}$$

b) $\phi = \int_S (\vec{B} \cdot \vec{n})\, ds = \int_S |B|\, ds = \int_{L+vt}^{2L+vt} |B|\, dy \int_{-L/2}^{L/2} dx = \frac{\mu_0}{4\pi} 2I \int_{L+vt}^{2L+vt} \frac{dy}{y} \int_{-L/2}^{L/2} dx$

$$\phi = \frac{\mu_0}{4\pi} 2IL \ln \frac{2L+vt}{L+vt} \qquad \vec{n} \parallel \vec{B}$$

c) $\frac{d\phi}{dt} = \frac{\mu_0}{4\pi} 2IL \frac{1}{\frac{2L+vt}{L+vt}} \times \frac{v(L+vt) - (2L+vt)v}{(L+vt)^2}$

$$\frac{d\phi}{dt} = \frac{\mu_0}{4\pi} 2IL \frac{vL + \cancel{v^2 t} - 2Lv - \cancel{v^2 t}}{(2L+vt)(L+vt)}$$

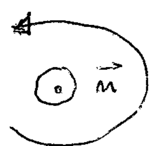
$$\frac{d\phi}{dt} = -\frac{\mu_0}{4\pi} 2IL \frac{Lv}{(2L+vt)(L+vt)} \qquad \varepsilon^{ind} = -\frac{d\phi}{dt}$$

$$\varepsilon^{ind} = +\frac{\mu_0}{4\pi} 2IL \frac{Lv}{(2L+vt)(L+vt)} \qquad \varepsilon^{ind} = R I^{ind}$$

$$I^{ind} = \frac{\mu_0}{4\pi} \frac{2IL^2 v}{R} \frac{1}{(2L+vt)(L+vt)}$$

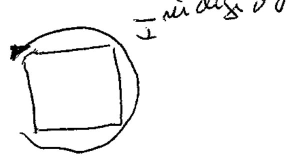
$$I^{ind} = 10^{-7} \frac{2 \times 4 \times (4 \times 10^{-2})^2 \times 2}{(10 \times 10^3)(2 \times 0{,}04 + 2 \times 2)(0{,}04 + 2 \times 2)}$$

$$I^{ind} = 15{,}5 \times 10^{-15}\, A$$



sentido positivo

Como $I^{ind} > 0$



$\vec{n} = \vec{e_z}$

IV

$E_x = E_0 \cos\left[\omega t + |k|\left(\alpha x + \frac{1}{\sqrt{3}} y\right)\right]$ $\qquad \alpha \geq 0$

$E_y = c_1 E_0 \cos\left[\omega t + |k|\left(\alpha x + \frac{1}{\sqrt{3}} y\right)\right]$

$E_z = c_2 E_0 \,\text{sen}\left[\omega t + |k|\left(\alpha x + \frac{1}{\sqrt{3}} y\right)\right]$

$f_0 = 5 \times 10^{14}\ Hz$ $\qquad \lambda = 400\ nm$ (no meio)

a) $(\vec{k} \cdot \vec{r}) = -|k|\left(\alpha x + \frac{1}{\sqrt{3}} y\right)$ $\qquad \vec{n} = \frac{\vec{k}}{|k|}$

$\begin{cases} n_x = -\alpha \\ n_y = -\frac{1}{\sqrt{3}} \\ n_z = 0 \end{cases}$ $\qquad \alpha^2 + \frac{1}{3} = 1$ $\qquad \alpha^2 = \frac{2}{3}$ $\qquad \alpha = \pm\sqrt{\frac{2}{3}}$

para $|\vec{n}| = 1$ $\qquad$ mas no enunciado $\alpha > 0$ $\qquad \boxed{\alpha = \sqrt{\frac{2}{3}}}$

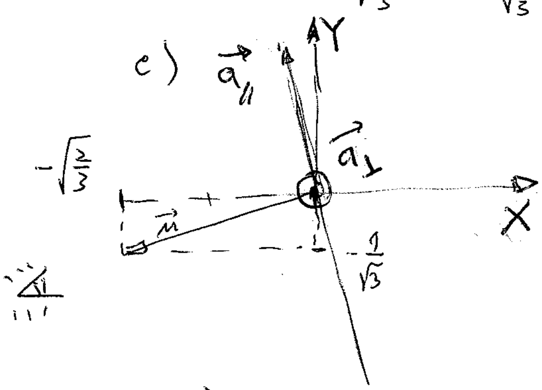
$\vec{n} = -\sqrt{\frac{2}{3}}\,\vec{e_x} - \frac{1}{\sqrt{3}}\,\vec{e_y}$

b) O. plana E.M. $(\vec{E} \cdot \vec{n}) = 0$

$E_0 \cos(\ )\left(-\sqrt{\frac{2}{3}}\right) + c_1 E_0 \cos[\ ]\left(-\frac{1}{\sqrt{3}}\right) = 0$

$\sqrt{\frac{2}{3}} + c_1 \frac{1}{\sqrt{3}} = 0$ $\qquad \boxed{c_1 = -\sqrt{2}}$

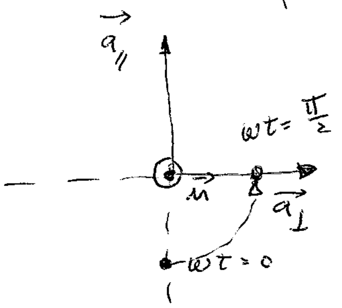
c)



$\vec{a}_{\parallel} = -\sqrt{\frac{1}{3}}\,\vec{e_x} + \sqrt{\frac{2}{3}}\,\vec{e_y}$ $\qquad \vec{a}_{\perp} = \vec{e_z}$

$\vec{E} = E_0 \cos(\dots)\,\vec{e_x} - \sqrt{2}\,E_0 \cos[\dots]\,\vec{e_y} + c_2 E_0 \,\text{sen}[\dots]\,\vec{e_z}$

$\vec{E} = -\sqrt{3}\,E_0 \cos(\dots)\left(-\frac{1}{\sqrt{3}}\vec{e_x}\right) - \sqrt{3}\,E_0 \cos[\dots]\left(\sqrt{\frac{2}{3}}\,\vec{e_y}\right) + c_2 E_0 \,\text{sen}[\dots]\,\vec{e_z}$



$\vec{E} = -\sqrt{3}\,E_0 \cos[\dots]\,\vec{a}_{\parallel} + c_2 E_0 \,\text{sen}[\ \ ]\,\vec{a}_{\perp}$

$(x = 0,\ y = 0,\ z = 0)$

$\omega t = 0 \ \Big|\ E_{\parallel} = -\sqrt{3}\,E_0$, $\ E_{\perp} = 0$ $\qquad \omega t = \frac{\pi}{2} \ \Big|\ E_{\parallel} = 0$, $\ E_{\perp} = c_2 E_0$ $\qquad \boxed{c_2 = \sqrt{3}}$

d) $\vec{S} = \vec{E} \times \vec{H}$ $\qquad \frac{E}{H} = Z$ $\qquad Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\mu_0}}\sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\varepsilon_0}{\varepsilon}} = \frac{Z_0}{\sqrt{\varepsilon_r}} = \frac{Z_0}{n}$ $\qquad (\mu \approx \mu_0)$

$|S| = 0{,}6 \times 10^{-12} / 10^{-4} = 0{,}6 \times 10^{-8}\ W m^{-2}$

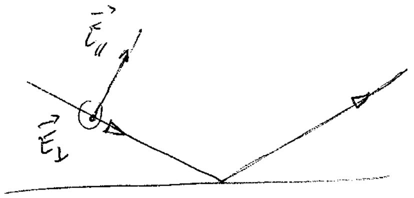
$|S| = \frac{n}{Z_0}|E|^2$ $\qquad |S| = \frac{n}{Z_0}\left[3E_0^2 \cos^2(\ ) + 3E_0^2 \,\text{sen}^2(\ )\right]$ $\qquad \langle |S| \rangle = \frac{n}{Z_0} 3E_0^2$

$n = \frac{c}{v} = \frac{\lambda_0 f_0}{\lambda f} = \frac{\lambda_0}{\lambda} = \frac{c}{\lambda f_0}$ $\qquad \langle |S| \rangle = \frac{c}{\lambda f_0 Z_0} 3E_0^2$ $\qquad \boxed{E_0 = 0{,}7\ mV\,m^{-1}}$

V

$i_{Brewster}$: $\quad tg\,(i_B + r_B) = \infty \qquad i_B + r_B = \frac{\pi}{2}$

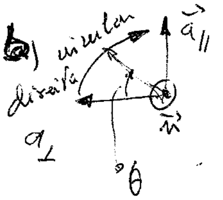
$1 \times \operatorname{sen} i_B = n \operatorname{sen} r_B \qquad \operatorname{sen} i_B = n \operatorname{sen}\left(\frac{\pi}{2} - i_B\right)$



$tg\, i_B = n = \sqrt{\varepsilon_r}$

$tg\, i_B = \sqrt{2} \qquad i_B = 54{,}7^\circ \qquad r_B = 35{,}3^\circ$

b)



$$\vec{E} = E_0 \cos\underbrace{(\omega t - (\bar{k}.\bar{r}))}_{\theta}\vec{a}_{\perp} + E_0 \operatorname{sen}(\omega t - (\bar{k}.\bar{r}))\,\vec{a}_{\parallel}$$

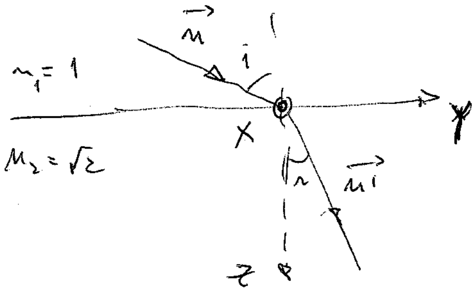
Fresnel onda reflectida $\rightarrow$
$$\begin{cases} E''_{0\perp} = -E_{0\perp}\dfrac{\operatorname{sen}(i-r)}{\operatorname{sen}(i+r)} = -E_{0\perp}\dfrac{\operatorname{sen}(54{,}7-35{,}3)}{\operatorname{sen}(54{,}7+35{,}3)} \\ E''_{0\parallel} = 0 \end{cases}$$

$$\vec{E}'' = \underbrace{-E_{0\perp}\frac{\operatorname{sen}(54{,}7-35{,}3)}{\operatorname{sen}(54{,}7+35{,}3)}}\cos(\omega t - (\bar{k}''.\bar{r}))\,\vec{a}_{\perp}$$

$$\boxed{E''_0 = 1{,}33 \times 10^{-3}\ \mathrm{V\,m^{-1}}}$$

$$\vec{E}'' = -1{,}33 \times 10^{-3} \cos(\omega t - (\vec{k}''.\vec{r}))\,\bar{a}_{\perp}$$

d)



$n'_z = \cos i_B \qquad n'_y = \operatorname{sen} i_B$

$\vec{n}' = 0{,}82\,\vec{e}_z + 0{,}58\,\vec{e}_y$

$\vec{k}' = |k'|\,\vec{n}'$

$|k'| = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0} n' = \frac{2\pi}{\frac{c}{f_0}} n' = \frac{2\pi f_0}{c} n' = \frac{\omega}{c} n'$

$|\bar{k}'| = \frac{3\pi \times 10^5}{3 \times 10^8} \times \sqrt{2} = 4{,}44 \times 10^{-3}\ \mathrm{m^{-1}}$

$$\boxed{\vec{k}' = 4{,}44 \times 10^{-3}\,[0{,}82\,\bar{e}_z + 0{,}58\,\bar{e}_y]\ (\mathrm{m^{-1}})}$$

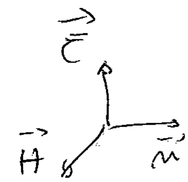
$$\boxed{k' = 3{,}64 \times 10^{-3}\,\vec{e}_z + 2{,}58 \times 10^{-3}\,\vec{e}_y \quad \mathrm{m^{-1}}}$$

VI

$$\vec{S} = (\vec{E} \times \vec{H})$$

$$\vec{E} = Z(\vec{H} \times \vec{n})$$

$$\vec{H} = \frac{1}{Z}(\vec{n} \times \vec{E})$$



$$\vec{S} = Z\left[(\vec{H} \times \vec{n}) \times \vec{H}\right]$$

$$\vec{S} = Z(\vec{H} \cdot \vec{H})\,\vec{n}$$

$$\boxed{S = v\mu(\vec{H} \cdot \vec{H})\,\vec{n}}$$

$$\vec{S} = \frac{1}{Z}\left[\vec{E} \times (\vec{n} \times \vec{E})\right]$$

$$\vec{S} = \frac{1}{Z}(\vec{E} \cdot \vec{E})\,\vec{n}$$

$$\boxed{\vec{S} = v\varepsilon(\vec{E} \cdot \vec{E})\,\vec{n}}$$

$$\left.\begin{array}{l} Z = \sqrt{\dfrac{\mu}{\varepsilon}} \\ v = \dfrac{1}{\sqrt{\varepsilon\mu}} \end{array}\right\} \quad \frac{1}{Z} = v\varepsilon \quad ; \quad Z = v\mu$$

$$D = \varepsilon \vec{E} \qquad \vec{B} = \mu \vec{H}$$

<u>Densidades de energía:</u>

$$\begin{cases} u_{elect} = \frac{1}{2}(\vec{E} \cdot \vec{D}) = \frac{1}{2}\varepsilon(\vec{E} \cdot \vec{E}) \\ u_{magn} = \frac{1}{2}(\vec{H} \cdot \vec{B}) = \frac{1}{2}\mu(\vec{H} \cdot \vec{H}) \end{cases}$$

$$u_{e.m.} = u_{elect} + u_{magn}$$

$$u_{em} = \frac{1}{2}\varepsilon(\vec{E} \cdot \vec{E}) + \frac{1}{2}\mu(\vec{H} \cdot \vec{H})$$

$$u_{elect} = \frac{1}{2}\varepsilon(\vec{E} \cdot \vec{E}) = \frac{1}{2}\varepsilon(Z\vec{H} \cdot Z\vec{H})$$

$$= \frac{1}{2}\varepsilon Z^2(\vec{H} \cdot \vec{H})$$

$$= \frac{1}{2}\varepsilon \frac{\mu}{\varepsilon}(\vec{H} \cdot \vec{H})$$

$$= \frac{1}{2}\mu(\vec{H} \cdot \vec{H}) = u_{magn}.$$

$$u_{em} = \varepsilon(\vec{E} \cdot \vec{E}) = \mu(\vec{H} \cdot \vec{H})$$

$$\boxed{\vec{S} = v\, u_{em}\, \vec{n}}$$

NOTA: irradiance $\Rightarrow$ $\langle |S| \rangle = v \langle u_{em} \rangle$

$$\langle |S| \rangle \equiv v \frac{1}{2}\varepsilon E_0^2 = v\frac{1}{2}\mu H_0^2$$