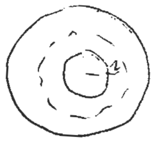
2.º Exame EEO

<u>LEAmb + LEMat + LQuim</u>

I



a) $\oint_{S_f} (\vec{D}\cdot\vec{n})\,dS = q^{int}$ → Gauss; simetria esférica

$$|D|\,4\pi r^2 = Q_1 \longrightarrow \vec{D} = \frac{1}{4\pi}\frac{Q_1}{r^2}\vec{e}_r$$

$$\vec{D}\left(r=\tfrac{3}{4}R_2\right) = \frac{1}{4\pi}\frac{16}{9}\frac{Q_1}{R_2^2}\vec{e}_r$$

b) $V_1 = V_2 + \int_{R_1}^{R_2}(\vec{E}\cdot d\vec{r})$; $V_2(R=R_2)=0$; entre R_1 e R_2 $\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}\vec{e}_r$

$$V_1 = \int_{R_1}^{R_2}\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0}Q_1\int_{R_1}^{R_2}\frac{1}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0}Q_1\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$Q_1 = \frac{1}{1/4\pi\varepsilon_0}\frac{R_1R_2}{R_2-R_1}V_1 \qquad Q_1 = \frac{1}{(9\times10^{9})}\frac{0{,}04\times0{,}06}{0{,}06-0{,}04}\times12 \qquad \boxed{Q_1 = +0{,}16\,nC}$$

c) $\left.\begin{aligned}\vec{D}&=\varepsilon_0\vec{E}+\vec{P}\\ \vec{D}&=\varepsilon\vec{E}\end{aligned}\right\} \longrightarrow \vec{P}=(\varepsilon-\varepsilon_0)\vec{E} = (\varepsilon-\varepsilon_0)\frac{1}{4\pi\varepsilon}\frac{Q_1^*}{R_2^2}\vec{e}_r$

Mantendo V_1, como a capacidade aumenta pela introdução do dieléctrico, a carga passa de $Q_1 \rightarrow Q_1^*$ maior.

$$Q_1^* = \frac{1}{1/4\pi\varepsilon}\frac{R_1R_2}{R_2-R_1}V_1 = \varepsilon_r\frac{1}{(1/4\pi\varepsilon_0)}\frac{R_1R_2}{R_2-R_1}V_1 = \varepsilon_r Q_1$$

$\sigma_2' = (\vec{P}\cdot\vec{n})$ com $\vec{n}$ a normal exterior ao dieléctrico em $r=R_2$.

$$\sigma_2' = |P| = (\varepsilon-\varepsilon_0)\frac{1}{4\pi\varepsilon}\frac{Q_1^*}{R_2^2} = (\varepsilon_r-1)\frac{1}{4\pi\varepsilon_r}\frac{\varepsilon_r Q_1}{R_2^2}$$

$$\sigma_2' = (2-1)\frac{1}{4\pi}\frac{0{,}16\times10^{-9}}{(0{,}06)^2} \qquad \boxed{\sigma_2' = 3{,}5\,nC\,m^{-2}}$$

d) $W_e = \frac{1}{2}QV \qquad \Delta W_e = \dfrac{\frac{1}{2}Q_1^*V_1}{\frac{1}{2}Q_1V_1}\times100 = \varepsilon_r\times100$

$$\Delta W_e = 200\%$$

2° Exame LEAmb + LEMat + LQuim

II

a) Lei de Ampère: $\oint (\vec{B} \cdot \vec{ds}) = \mu I^{int}$

$\oint (\vec{B} \cdot \vec{ds}) = |B| \, 2\pi R_{médio}$ (As linhas de $\vec{B}$ estão no interior do núcleo)

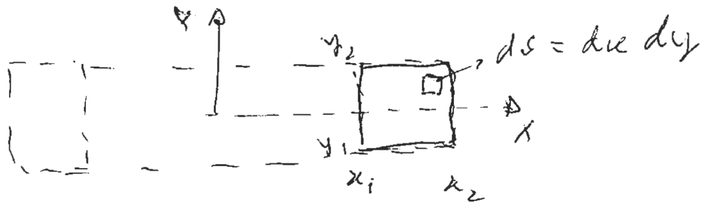
$I^{int} = N I$

$|B| \, 2\pi R_{médio} = \mu N I$ $\qquad |B| = \mu_r \frac{\mu_0}{4\pi} N \frac{2I}{R_{médio}}$

$|B| = (4000) * (10^{-7}) * (2000) * \left(\frac{2 \times 10^{-6}}{0{,}05} \right)$

$|B| = 32 \mu$ Tesla

b)



$\phi_{espira} = \int_S (\vec{B} \cdot \vec{n}) \, ds = \mu_r \frac{\mu_0}{4\pi} N \, 2I \int_{x_1}^{x_2} \frac{1}{x} dx \int_{y_1}^{y_2} dy$

$\phi_{espira} = \mu_r \frac{\mu_0}{4\pi} N \, 2I \ln \frac{x_2}{x_1} (y_2 - y_1)$

$x_1 = R_1 \qquad y_2 - y_1 = (R_3 - R_1)$
$x_2 = R_3$

$\phi_{TOTAL} = N \phi_{espira} \qquad \phi_{TOTAL} = \mu_r \frac{\mu_0}{4\pi} N^2 \, 2 (R_3 - R_1) \ln \frac{R_3}{R_1} I$

$$\phi_{TOTAL} = L I$$

$L = \mu_r \frac{\mu_0}{4\pi} N^2 \, 2 (R_3 - R_1) \ln \frac{R_3}{R_1}$

$L = 4000 * 10^{-7} * 2000^2 * 2 (0{,}07 - 0{,}03) \ln \frac{0{,}07}{0{,}03}$

$L = 108{,}5$ Henry

2° Exame d EAmb + LEFMat + LQuim

III

a) $\phi = \int_S (\vec{B}\cdot\vec{n})\,ds \qquad \phi = \int_S B_0 \cos\omega t\,(\vec{e_z}\cdot\vec{e_z})\,ds \qquad \vec{n} = \vec{e_z}$

$$\phi = B_0 \cos\omega t \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} dy$$

$$\phi = B_0\,(x_2 - x_1)\,(y_2 - y_1)\cos\omega t$$

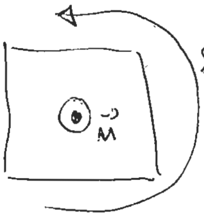
b) $\varepsilon^{ind} = -\dfrac{d\phi}{dt} \qquad I^{ind} = \dfrac{\varepsilon^{ind}}{R}$

$$I^{ind} = \frac{B_0}{R}\,(x_2 - x_1)\,(y_2 - y_1)\,\omega \operatorname{sen}\omega t$$

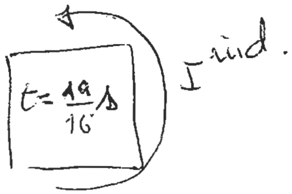
$$I^{ind} = \frac{2\times 10^{-6}}{10\times 10^{3}}\,(0{,}02)\,(0{,}02)\,(4\pi)\operatorname{sen}\Big(\underbrace{4\pi\,\tfrac{19}{16}}_{\text{radianos!...}}\Big)$$

$I^{ind} = 0{,}7$ pico Ampère

c)



$I^{ind} > 0$



d) $W_{mag} = \dfrac{1}{2} L\, I_{ind.}^2$

$$W_{mag} = \frac{1}{2} L \left[\frac{B_0}{R}(x_2 - x_1)(y_2 - y_1)\,\omega \operatorname{sen}\omega t\right]^2$$

$$W_{mag} = \frac{1}{2} L\,\frac{B_0^2}{R^2}(x_2 - x_1)^2 (y_2 - y_1)^2\,\omega^2 \operatorname{sen}^2(\omega t)$$

$$\langle W_{mag}\rangle = \frac{1}{4} L\,\frac{B_0^2}{R^2}(x_2 - x_1)^2 (y_2 - y_1)^2\,\omega^2$$

IV

$$H_x = H_0 \cos\left[\omega t + |k|\left(\alpha x + \frac{1}{\sqrt{5}} z\right)\right]$$
$$H_y = c_1 H_0 \operatorname{sen}\left[\omega t + |k|\left(\alpha x + \frac{1}{\sqrt{5}} z\right)\right]$$
$$H_z = c_2 H_0 \cos\left(\omega t + |k|\left(\alpha x + \frac{1}{\sqrt{5}} z\right)\right]$$

$f = 8{,}44 \times 10^{14}\ Hz \qquad \lambda = 237\ nm$

a) $n = \frac{c}{v} = \frac{c}{\lambda f} = \frac{3\times 10^8}{237\times 10^{-9} * 8{,}44\times 10^{14}} \simeq 1{,}5$

b) $-(\vec{k}\cdot\vec{r}) = -|\vec{k}|(\vec{n}\cdot\vec{r}) = +|k|\left(\alpha x + \frac{1}{\sqrt{5}} z\right)$

$(\vec{n}\cdot\vec{r}) = -\alpha x - \frac{1}{\sqrt{5}} z$

$n_x x + n_y y + n_z z = -\alpha x - \frac{1}{\sqrt{5}} z$ logo: $\vec{n} = -\alpha \vec{e_x} - \frac{1}{\sqrt{5}} \vec{e_z}$

$|\vec{n}| = 1 \qquad (-\alpha)^2 + \left(-\frac{1}{\sqrt{5}}\right)^2 = 1 \qquad \alpha^2 = \frac{4}{5} \qquad \alpha = \pm\frac{2}{\sqrt{5}}$

Enunciado $\alpha \leq 0$; portanto $\boxed{\alpha = -\frac{2}{\sqrt{5}}}$

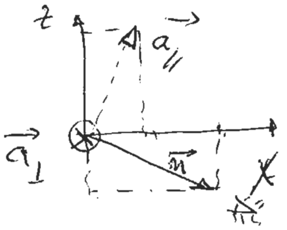
$\vec{n} = \frac{2}{\sqrt{5}} \vec{e_x} - \frac{1}{\sqrt{5}} \vec{e_z}$

c) O.P.EM: $(\vec{n}\cdot\vec{H}) = 0$

$\frac{2}{\sqrt{5}} H_0 \cos(\ ) - \frac{1}{\sqrt{5}} c_2 \cos(\ ) = 0$

$\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}} c_2 = 0 \qquad \boxed{c_2 = 2}$

d)



$\vec{a_{//}} = \frac{1}{\sqrt{5}} \vec{e_x} + \frac{2}{\sqrt{5}} \vec{e_z}$

$\vec{a_\perp} = \vec{e_y}$

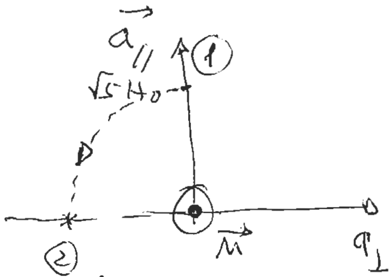
$\vec{H} = H_0 \cos(\ )\, \vec{e_x} + c_1 H_0 \operatorname{sen}(\ )\, \vec{e_y} + 2 H_0 \cos(\ )\, \vec{e_z}$

$\vec{H} = \sqrt{5} H_0 \cos(\ )\left(\frac{1}{\sqrt{5}} \vec{e_x}\right) + \sqrt{5} H_0 \cos(\ )\left(\frac{2}{\sqrt{5}} \vec{e_z}\right) + c_1 H_0 \operatorname{sen}(\ )\, \vec{e_y}$

$\vec{H} = \sqrt{5} H_0 \cos(\ )\left[\frac{1}{\sqrt{5}} \vec{e_x} + \frac{2}{\sqrt{5}} \vec{e_z}\right] + c_1 H_0 \operatorname{sen}(\ )\, \vec{e_y}$

IV (cont.)

$$\vec{H} = \sqrt{5}\, H_0 \cos(\;)\, \vec{a_{\parallel}} + c_1 H_0 \,\text{sen}(\;)\, \vec{a_{\perp}}$$



circular esquerda

$(x = 0 \quad z = 0)$

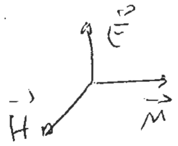
$\omega t = 0$ ① $\left| \begin{array}{l} H_{\parallel} = \sqrt{5} H_0 \\ H_\perp = 0 \end{array}\right.$ $\qquad \omega t = \frac{\pi}{2}$ ② $\left| \begin{array}{l} H_{\parallel} = 0 \\ H_\perp = c_1 H_0 \end{array}\right.$

para a onda ser circular esquerda $\boxed{c_1 = -\sqrt{5}}$

e) $I = \langle |S| \rangle = 0{,}5 \text{ picoW cm}^{-2} = \frac{0{,}5 \times 10^{-12}}{10^{-4}} \text{ W m}^{-2} = 0{,}5 \times 10^{-8} \text{ W m}^{-2}$

$$\vec{S} = [\vec{E} \times \vec{H}]$$

$$\vec{S} = |E|\,|H|\,\vec{n} \qquad \frac{E}{H} = Z$$



$$\vec{S} = \frac{|E|^2}{Z}\vec{n} \quad \text{ou} \quad \vec{S} = Z\,|H|^2\,\vec{n}$$

$$\vec{S} = Z\left[5 H_0^2 \cos^2(\;) + 5 H_0^2 \,\text{sen}^2(\;)\right]\vec{n}$$

$$\vec{S} = Z\, 5 H_0^2\, \vec{n}$$

$$\langle|\vec{S}|\rangle = Z\, 5 H_0^2$$

$$Z = \frac{Z_0}{n}$$

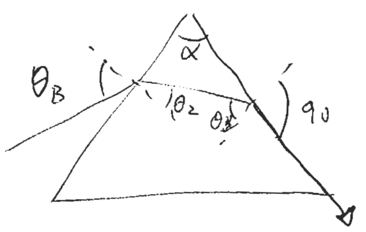
$$\langle|S|\rangle = \frac{Z_0}{n} 5 H_0^2$$

$$H_0 = \sqrt{\frac{n}{5 Z_0}\langle|S|\rangle} \qquad H_0 = \sqrt{\frac{1{,}5}{5 \times 377}\, 0{,}5 \times 10^{-8}}$$

$$\boxed{H_0 \approx 2\,\mu\text{A m}^{-1}}$$

V   ∠EAub, ∠EMaΓ, ∠Cluin

a)



$$\left.\begin{array}{l}\theta_B + \theta_2 = \dfrac{\pi}{2} \\ n_1 \operatorname{sen} \theta_B = n_2 \operatorname{sen} \theta_2\end{array}\right\} \quad \boxed{\operatorname{tg} \theta_B = n_2}$$

$$\alpha + \left(\frac{\pi}{2} - \theta_2\right) + \left(\frac{\pi}{2} - \theta_3\right) = \pi$$

$$\boxed{\alpha - \theta_2 - \theta_3 = 0}$$

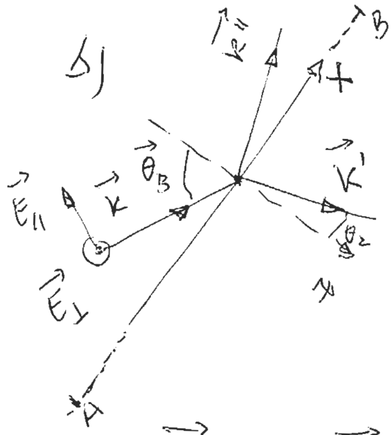
$$n_2 \operatorname{sen} \theta_3 = n_1 \operatorname{sen} 90 \longrightarrow \boxed{\operatorname{sen} \theta_3 = \frac{1}{n_2}}$$

$$n_2 = 1,5 \rightarrow \operatorname{tg} \theta_B = 1,5 \qquad \theta_B = 56,3^\circ$$

$$\left.\begin{array}{l}\theta_2 = 33,7 \\ \theta_3 = 41,8\end{array}\right\} \alpha = \theta_2 + \theta_3 = 75,5^\circ$$

$$\rightarrow \operatorname{sen} \theta_3 = \frac{1}{1,5}$$

b)



$$C = \frac{\omega}{k} \qquad k = \frac{\omega}{c} \qquad k = \frac{3\pi \times 10^5}{3 \times 10^8} \qquad k = \pi \times 10^{-3}\ m^{-1}$$

$$v = \frac{\omega}{k'} \qquad k' = \frac{\omega}{v} = \frac{\omega}{(c/n)} = n k \qquad k' = \frac{3}{2} \pi \times 10^{-3}\ m^{-1}$$

$$\vec{n}' = \operatorname{sen} \theta_2 \vec{e_x} + \cos \theta_2 \vec{e_z}$$

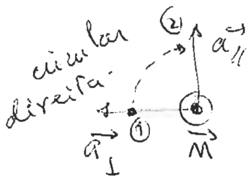
$$\vec{n}' = 0,56\ \vec{e_x} + 0,83\ \vec{e_z}$$

$$\vec{k} = k' \vec{n}' \qquad k' = \frac{3}{2} \pi \times 10^{-3} \left[0,56\ \vec{e_x} + 0,83\ \vec{e_z}\right]\ m^{-1}$$

c)

$$\vec{E}_{inc} = \vec{E}_{\parallel} + \vec{E}_{\perp} \xrightarrow[\text{Brewster}]{\text{âng}} \vec{E}''_{reflect} = \vec{E}''_{\perp}$$

$$\vec{E}_{inc} = E_0 \operatorname{sen}(\omega t - (\vec{k} \cdot \vec{r}))\ \vec{a}_{\parallel} + E_0 \cos(\omega t - (\vec{k}'' \cdot \vec{r}))\ \vec{a}_{\perp}$$



$$E''_{0\perp} = -E_0 \frac{\operatorname{sen}(56,3 - 33,7)}{\operatorname{sen}(56,3 + 33,7)} = -4 \times 10^{-3} \times 0,384 = -1,54 \times 10^{-3}\ V m^{-1}$$

$$\vec{E}''_{refl.} = -1,54 \times 10^{-3} \cos\left(\omega t - (\vec{k}'' \cdot \vec{r})\right) \vec{a}_{\perp}$$

VI

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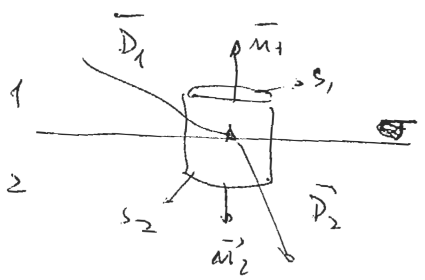
Condições fronteiras meio 1 e meio 2

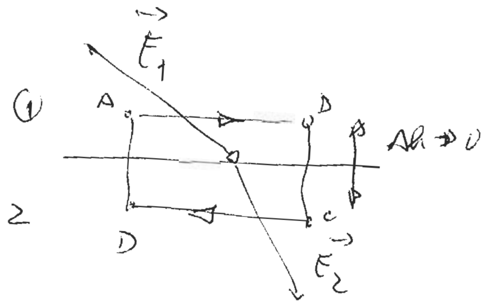


$\sigma \rightarrow$ densidade carga na superfície.

$$\int_{S.f.} (\vec{D}\cdot\vec{n})\, dS = q^{int}$$

$$-D_{n_1} S_1 + D_{n_2} S_2 = \sigma S$$

$$\boxed{D_{n_2} - D_{n_1} = \sigma}$$



$$\oint (\vec{E}\cdot d\vec{s}) = 0$$

$$\boxed{E_{1tg} - E_{2tg} = 0} \qquad \Delta h \rightarrow 0$$

Se meio 1 = condutor $\quad \vec{E} = 0 \quad E_{1tg} = 0 \quad E_{1n} = 0$

Logo $E_{2tg} = 0$ e $E_{2n} = \dfrac{\sigma}{\varepsilon_2}$ logo $\boxed{\vec{E}_2 = \dfrac{\sigma}{\varepsilon_2}\vec{n}_2}$