1º Mini-teste EO (31/10/2007) ①

VERSÃO A

a) Por razões de simetria (enquanto $L >> R_3$) o campo é radial. Usando a lei de Gauss para uma superfície cilíndrica de raio r tal que $R_1 < r < R_3$ e altura $h << L$ obtemos

$$\int_S (\vec{D} \cdot \vec{n})\, dS = Q_{int} = \lambda h$$

$$|\vec{D}|\, 2\pi r h = \lambda h \Rightarrow |\vec{D}| = \frac{\lambda}{2\pi}\frac{1}{r}$$

$$\vec{D}_\phi = \frac{\lambda}{2\pi}\frac{1}{r}\vec{e}_r$$

Para $r = 3/4 R_2$ vem

$$\boxed{\vec{D} = \frac{\lambda}{2\pi}\frac{4}{3R_2}\vec{e}_r}$$

b) $$\phi(R_1) - \phi(R_3) = \int_{R_1}^{R_3} dr\, \vec{E} \cdot d\vec{r}$$

$$= \int_{R_1}^{R_2} dr\, \vec{E} \cdot d\vec{r} + \int_{R_2}^{R_3} dr\, \vec{E} \cdot d\vec{r}$$

Usando $\phi(R_3) = 0$ e

$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_1}\frac{1}{r} \qquad R_1 < r < R_2 \qquad \varepsilon_1 = \varepsilon_{r_1}\varepsilon_0$$

$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r} \qquad R_2 < r < R_3$$

$$\phi(R_1) = \frac{\lambda}{2\pi\varepsilon_1}\int_{R_1}^{R_2}\frac{1}{r}\,dr + \frac{\lambda}{2\pi\varepsilon_0}\int_{R_2}^{R_3}\frac{1}{r}\,dr$$

$$\phi(R_1) = \frac{\lambda}{2\pi\varepsilon_1}\ln\left(\frac{R_2}{R_1}\right) + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_3}{R_2}\right)$$

$$= 35.3\ \mathrm{V}$$

c) Em $r = R_1$; $\vec{n} = -\vec{e}_r$

$$\sigma'_1 = -|\vec{P}| = -\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_1}|\vec{D}| = -\frac{3}{4}\frac{\lambda}{2\pi}\frac{1}{R_1} = -1.2\times10^{-7}\ \mathrm{C/m^2}$$

d) $$E_r(r=R_2^+) = \frac{|\vec{D}|}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{\lambda}{2\pi}\frac{1}{R_2}$$

$$E_r(r=R_2^-) = \frac{|\vec{D}|}{\varepsilon_1} = \frac{1}{\varepsilon_1}\frac{\lambda}{2\pi}\frac{1}{R_2}$$

$$E_r(r=R_2^+) - E_r(r=R_2^-) = \frac{\lambda}{2\pi}\frac{1}{R_2}\left(\frac{1}{\varepsilon_0}-\frac{1}{\varepsilon_1}\right)$$

$$= \frac{\lambda}{2\pi}\frac{1}{R_2}\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0\varepsilon_1}$$

$$= \frac{3}{4}\frac{\lambda}{2\pi}\frac{1}{R_2}\frac{1}{\varepsilon_0}$$

$$\sigma'_2 = \vec{P}\cdot\vec{n} = |\vec{P}| = \frac{3}{4}\frac{\lambda}{2\pi}\frac{1}{R_2}$$

logo

$$\boxed{E_r(r=R_2^+) - E_r(r=R_2^-) = \frac{1}{\varepsilon_0}\sigma'_2}$$