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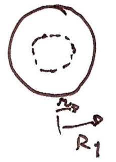
a) $0 < r < R_1$



Gauss: $\displaystyle \oint_{s.f} (\vec{D}.\vec{n})\, ds = Q^{int}$ $\qquad Q^{int} = \displaystyle\int_{Vol.} \rho\, dV$

Razões de simetria: $\vec{D}$ radial ; s.f.: s. esférica

$$\rho = \frac{Q_{TOTAL}}{\frac{4}{3}\pi R_1^3}$$

$$|D|\, 4\pi r^2 = \rho\, \frac{4}{3}\pi r^3$$

$$|D|\, 4\pi r^2 = \frac{Q}{\frac{4}{3}\pi R_1^3}\, \frac{4}{3}\pi r^3$$

$$\boxed{\vec{D} = \frac{1}{4\pi}\, \frac{Q}{R_1^3}\, r\, \vec{e_r} \qquad [C\, m^{-2}]}$$

b) $\displaystyle V_A = V_B + \int_A^B (\vec{E}.\vec{dl})$

$R_1 < r < R_2$ $\qquad \displaystyle\oint_{s.f} (\vec{E}.\vec{dl})\, ds = \frac{1}{\varepsilon_0} Q^{int}$ $\qquad \vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}\vec{e_r}$

$$V_r = V_{R_2} + \int_r^{R_2} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}(\vec{e_r}.\vec{dl}) \qquad V_{R_2} = V_{bat}$$

$$V_r = V_{bat} + \frac{1}{4\pi\varepsilon_0} Q \int_r^{R_2} \frac{1}{r^2}\, dr$$

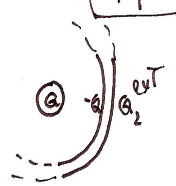
$$V_r = V_{bat} + \frac{1}{4\pi\varepsilon_0} Q \left(-\frac{1}{R_2} + \frac{1}{r}\right)$$

Para $r = R_1$ $\qquad \displaystyle V_{R_1} = V_{bat} + \frac{1}{4\pi\varepsilon_0}\, Q\, \frac{R_2 - R_1}{R_1 R_2} \qquad [V]$

$$V_{R_1} = 2 + (9\times10^9)(10^{-12})\,\frac{0{,}04 - 0{,}01}{0{,}01 * 0{,}04}$$

$$\boxed{V_{R_1} = 2{,}675\ V}$$

c)



$$\oint_{S.f.} (\vec{E}.\vec{n})\, ds = \frac{1}{\varepsilon_2} Q^{int}$$

NOTA: face interna R_2 tem $(-Q)$ pois é atraida pela carg $(+Q)$ da esfera de raio R_1

$$|E|\, 4\pi r^2 = \frac{1}{\varepsilon_2}\left(Q - Q + Q_2^{ext}\right)$$

I| c) (cont.) $\vec{E} = \dfrac{1}{4\pi\varepsilon_2}\dfrac{Q_2^{ext}}{r^2}\vec{e_r}$

$R_2 < r < R_3$ $V_r = V_{R_3} + \displaystyle\int_r^{R_3} \frac{1}{4\pi\varepsilon_2} Q_2^{ext} \frac{1}{r^2}(\vec{e_r}\cdot d\vec{r})$

$$V_r = V_{R_3} + \frac{1}{4\pi\varepsilon_2} Q_2^{ext}\left[-\frac{1}{R_3} + \frac{1}{r}\right]$$

$V_{R_3} = 0$ (ligado à Terra)

Se $r = R_2$
$V_{R_2} = V_{bat}$

$$V_{bat} = \frac{1}{4\pi\varepsilon_2} Q_2^{ext} \frac{R_3 - R_2}{R_2 R_3}$$

$\varepsilon_2 = 3\varepsilon_0$

$$Q_2^{ext} = V_{bat}\frac{1}{\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{1}{3}}\frac{R_2 R_3}{R_3 - R_2}$$

$$Q_2^{ext} = 2\,\frac{3}{9\times10^9}\,\frac{0{,}04\;\,0{,}06}{0{,}06 - 0{,}04}$$

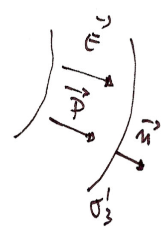
$$\boxed{Q_2^{ext} = 8\times10^{-11}\,C}$$

d) $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

$\varepsilon_2\vec{E} = \varepsilon_0\vec{E} + \vec{P}$

$\vec{P} = (\varepsilon_2 - \varepsilon_0)\vec{E}$

$\sigma' = (\vec{P}\cdot\vec{n})$ sendo $\vec{n}$ a normal exterior ao dieléctrico.



$\sigma_3' = |\vec{P}|$

$\sigma_3' = (\varepsilon_2 - \varepsilon_0)\,|\vec{E}(r = R_3)|$

$$\sigma_3' = (3\varepsilon_0 - \varepsilon_0)\frac{1}{4\pi(3\varepsilon_0)}\frac{Q_2^{ext}}{R_3^2}$$

$$\sigma_3' = \frac{2}{3}\,\frac{1}{4\pi}\,\frac{8\times10^{-11}}{(0{,}06)^2}$$

$$\boxed{\sigma_3' = 1{,}18\times10^{-9}\;C\,m^{-2}}$$

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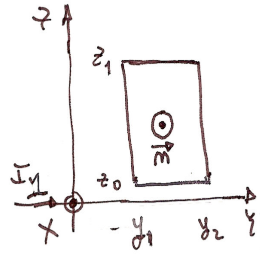
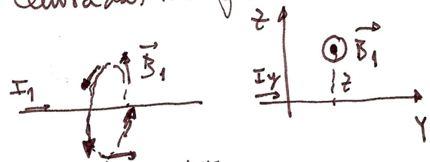
II



a) Linhas de força de $\vec{B}$: circunferências centradas no fio condutor.

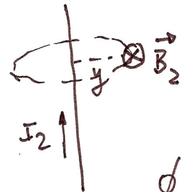


$$\oint(\vec{B}\cdot d\vec{s}) = \mu_0 I^{int}$$

$$|\vec{B}_1| 2\pi z = \mu_0 I_1$$

$$\boxed{\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{z}\vec{e}_x}$$

b)



$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I_2}{y}(-\vec{e}_x)$$

$$\phi_2 = \int_S(\vec{B}_2\cdot\vec{m})dS = -\frac{\mu_0}{4\pi}2I_2\int_{y_1}^{y_2}\frac{1}{y}dy\int_{z_0}^{z_1}dz \quad \text{por espira}$$

$$\phi_2 = -\frac{\mu_0}{4\pi}2I_2\left(\ln\frac{y_2}{y_1}\right)(z_1-z_0) \qquad L_{2\,circuito} = \frac{|\phi_2|}{I_2}$$

$$L_{2\,circuito} = \frac{\mu_0}{4\pi}2(z_1-z_0)\left(\ln\frac{y_2}{y_1}\right)$$

$$L_{2\,circuito} = 10^{-7}*2*0{,}03*\left(\ln\frac{0{,}03}{0{,}01}\right)$$

$$L_{2\,circuito} = 6{,}6\times10^{-9}\ \text{Henry. por espira}$$

$$\boxed{L_{2\,circuito_T} = 6{,}6\times10^{-7}\ \text{Henry}}$$

c)

$$\phi_1 = \int_S(\vec{B}_1\cdot\vec{m})dS = \frac{\mu_0}{4\pi}2I_1\int_{z_0}^{z_1}\frac{1}{z}dz\int_{y_1}^{y_2}dy \quad \text{por espira}$$

$$\phi_1 = \frac{\mu_0}{4\pi}2I_1\ln\frac{z_1}{z_0}(y_2-y_1)$$

$z_0 = 0{,}5$ cm
$z_1 = z_0 + l_z = 3{,}5$ cm

$$\phi_1 = \frac{\mu_0}{4\pi}2\times2\times10^{-3}\left(\ln\frac{3{,}5\times10^{-2}}{0{,}5\times10^{-2}}\right)(3-1)\times10^{-2}$$

$y_2 = 3$ cm
$y_1 = 1$ cm
$I_y = 2$ mA
$N = 100$ espiras

$$\phi_1 = 1{,}56\times10^{-11}\ \text{Wb por espira.}$$

$$\boxed{\phi_{1T} = +1{,}56\times10^{-9}\ \text{Wb.}}$$

II

c) cont. $\phi_{2T} = - L_{2\,circ} * I_2$

$\phi_{2T} = -6,6 \times 10^{-7} \times 3 \times 10^{-3}$ $\phi_{2T} = -1,98 \times 10^{-9}$ Wb

$$\phi_{TOTAL} = (+1,56 \times 10^{-9} - 1,98 \times 10^{-9})\ Wb$$

$$\boxed{\phi_{TOTAL} = -0,42 \times 10^{-9}\ Wb}$$

d) $\varepsilon^{ind} = -\dfrac{d\phi_T}{dt}$ $\phi_T = \phi_{1\,Total} + \phi_{2\,Total}$; mas $\phi_{2\,Total}$ = cte no tempo

$$\varepsilon^{ind} = -\frac{d\phi_{1T}}{dt} = -\frac{d}{dt}\left[\frac{\mu_0}{4\pi} 2NI_1 \left(\ln\frac{z_f}{z_i}\right)(y_2 - y_1)\right]$$

$$\begin{cases} z_i = z_0 + vt \\ z_f = z_i + l_z \end{cases} \qquad \frac{d}{dt}\ln\left(\frac{z_0 + vt + l_z}{z_0 + vt}\right) =$$

$$= \frac{1}{\frac{z_0 + vt + l_z}{z_0 + vt}}\left[\frac{v(z_0 + vt) - (z_0 + vt + l_z)v}{(z_0 + vt)^2}\right]$$

$$= \frac{v}{z_0 + vt + l_z}\left[1 - \frac{z_0 + vt + l_z}{z_0 + vt}\right]$$

$$= \frac{v}{z_0 + vt + l_z} - \frac{v}{z_0 + vt}$$

$$= \frac{[\cancel{z_0} + \cancel{vt} - \cancel{z_0} - \cancel{vt} - l_z]\,v}{(z_0 + vt + l_z)(z_0 + vt)}$$

$$= \frac{-vl_z}{(z_0 + vt + l_z)(z_0 + vt)}$$

$$\varepsilon^{ind} = \frac{\mu_0}{4\pi} 2NI_1 (y_2 - y_1) \frac{v\,l_z}{(z_0 + vt + l_z)(z_0 + vt)}$$

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II d) cont

$$I^{ind} = \frac{\varepsilon^{ind}}{R}$$

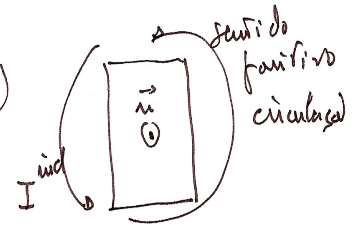
$$I^{ind} = \frac{\mu_0}{4\pi}\,\frac{2 N I_1 (y_2 - y_1)}{R}\,\frac{v\, l_z}{(z_0 + v t + l_z)(z_0 + v t)}$$

e)

$$I^{ind}(t=2s) = 10^{-7}\,\frac{2\times10^{2}\times 2\times10^{-3}\times(3-1)\times10^{-2}}{26}\,\frac{10^{-2} * 0{,}03}{[0{,}005 + (10^{-2}\times 2) + 0{,}03] * (0{,}005 + 10^{-2}\times 2)}$$

$$I^{ind}(t=2s) = \frac{10^{-7}\times 3\times10^{-4}\times 3\times10^{-4}}{1{,}375\times10^{-3}} = 6{,}7\times10^{-12}\ A$$

$$I^{ind} = +6{,}7\ pA \quad (p = \text{pico} \Rightarrow 10^{-12})$$



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IV

$\gamma \geq 0$

$$H_x = H_0 \operatorname{sen}\left[\omega t + |k|\left(\gamma x + \frac{1}{\sqrt{3}} z\right)\right]$$

$$H_y = H_0 \cos\left[\omega t + |k|\left(\gamma x + \frac{1}{\sqrt{3}} z\right)\right]$$

$$H_z = \beta H_0 \operatorname{sen}\left[\omega t + |k|\left(\gamma x + \frac{1}{\sqrt{3}} z\right)\right]$$

$f = 3 \times 10^{14}\ Hz$ $\qquad \lambda = 600\ nm$

a) $n = \frac{c}{v} = \frac{c}{\lambda f} = \frac{3 \times 10^8}{3 \times 10^{14} \times 600 \times 10^{-9}}$ $\qquad n = 1{,}67$

b) $-|k|(\vec{n} \cdot \vec{r}) = +|k|\left(\gamma x + \frac{1}{\sqrt{3}} z\right)$

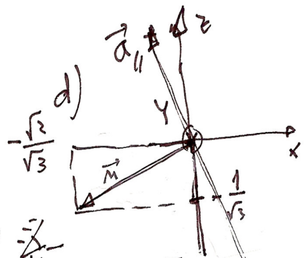
$$\begin{cases} n_x = -\gamma \\ n_y = 0 \\ n_z = -\frac{1}{\sqrt{3}} \end{cases} \qquad \begin{aligned} &|\vec{n}| = 1 \\ &n_x^2 + n_y^2 + n_z^2 = 1 \\ &\gamma^2 + 0 + \frac{1}{3} = 1 \end{aligned} \qquad \gamma^2 = \frac{2}{3} \qquad \boxed{\gamma = +\sqrt{\frac{2}{3}}}$$

$$\vec{n} = -\sqrt{\frac{2}{3}}\,\vec{e_x} - \frac{1}{\sqrt{3}}\,\vec{e_z}$$

c) $(\vec{H} \cdot \vec{n}) = 0 \qquad -H_0 \operatorname{sen}(\dots)\sqrt{\frac{2}{3}} - \beta H_0 \operatorname{sen}(\dots)\frac{1}{\sqrt{3}} = 0$

$$-\sqrt{\frac{2}{3}} - \beta\frac{1}{\sqrt{3}} = 0 \qquad \boxed{\beta = -\sqrt{2}}$$

d)

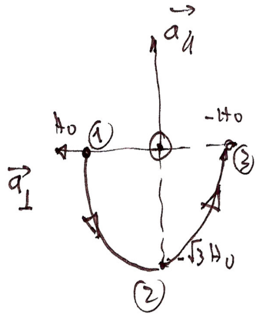


$$\vec{a}_{//} = +\sqrt{\frac{2}{3}}\,\vec{e_z} - \frac{1}{\sqrt{3}}\,\vec{e_x} \qquad \vec{a}_\perp = \vec{e_y}$$

$$\vec{H} = \sqrt{3} H_0 \operatorname{sen}(\,)\frac{1}{\sqrt{3}}\vec{e_x} + H_0\cos(\,)\,\vec{e_y} + (-\sqrt{2})(\sqrt{3}) H_0 \operatorname{sen}(\,)\frac{1}{\sqrt{3}}\vec{e_z}$$

$$\vec{H} = -\sqrt{3} H_0 \operatorname{sen}(\dots)\left[-\frac{1}{\sqrt{3}}\vec{e_x} + \sqrt{\frac{2}{3}}\vec{e_z}\right] + H_0\cos(\dots)\,\vec{e_y}$$

$$\vec{H} = -\sqrt{3} H_0 \operatorname{sen}(\dots)\,\vec{a}_{//} + H_0 \cos(\dots)\,\vec{a}_\perp$$



em $x = 0$ e $z = 0$

$\omega t = 0$ (1): $H_{//} = 0$, $H_\perp = H_0$

$\omega t = \frac{\pi}{2}$ (2): $H_{//} = -\sqrt{3} H_0$, $H_\perp = 0$

$\omega t = \pi$ (3): $H_{//} = 0$, $H_\perp = -H_0$

polarização elíptica esquerda.

III

e) $\vec{S} = (\vec{E} \times \vec{H})$ $\quad \frac{E}{H} = z \quad\quad z = \frac{z_0}{n}$

$|\vec{S}| = z H^2$

$\langle|\vec{S}|\rangle = \frac{z_0}{n} \langle| 3H_0^2 \operatorname{sen}^2(\dots) + H_0^2 \cos^2(\dots) |$

$\langle|S|\rangle = \frac{z_0}{n}\left(3H_0^2 \frac{1}{2} + H_0^2 \frac{1}{2}\right)$

$\langle|S|\rangle = \frac{z_0}{n} 2H_0^2$

$\langle|S|\rangle = 0{,}5 \text{ pW cm}^{-2} = \frac{0{,}5 \times 10^{-12} \text{ W}}{10^{-4} \text{ m}^2}$

$\langle|S|\rangle = 0{,}5 \times 10^{-8} \text{ W m}^{-2}$

$z_0 = 377 \,\Omega$

$n = 1{,}67$ (da a))

$H_0^2 = \frac{n}{z_0} \frac{\langle|S|\rangle}{2}$

$H_0^2 = \frac{1{,}67}{377} \times \frac{0{,}5 \times 10^{-8}}{2}$

$\boxed{H_0 = 3{,}3 \times 10^{-6} \text{ A m}^{-1}}$

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IV

θ_2 é o ângulo de incidência em B. É o ângulo de Brewster: $\theta_2 + \theta_3 = \frac{\pi}{2}$; $\theta_2 = \frac{\pi}{2} - \theta_3$; $\theta_2 = 30^\circ$

$$n_2 \operatorname{sen} \theta_2 = n_3 \operatorname{sen} \theta_3$$

$$n_2 \cos \theta_3 = n_3 \operatorname{sen} \theta_3 \; ; \; n_3 \operatorname{tg} \theta_3 = n_2 \quad \text{logo:}$$

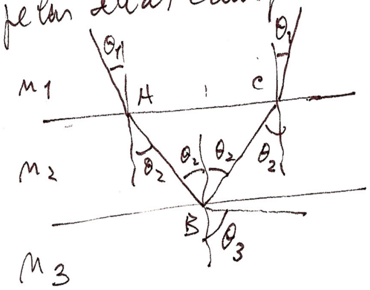
$$n_2 = 1 \operatorname{tg} \pi/3 \qquad \boxed{n_2 = 1{,}73}$$

Em A: $n_1 \operatorname{sen} \theta_1 = n_2 \operatorname{sen} \theta_2$

$$\operatorname{sen} \theta_1 = \frac{n_2}{n_1} \cos \theta_3 \qquad \theta_1 = \operatorname{arc\,sen}\left(\frac{1{,}73}{2} \cos \frac{\pi}{3}\right)$$

$$\boxed{\theta_1 = 25{,}6^\circ}$$

b) Estamos só a pensar na componente perpendicular de uma onda polarizada circularmente. A energia Total incidente em A irá ser distribuída igualmente pelas duas componentes perpendic. e paralela.



em A: $$T_A = \frac{4 \operatorname{sen} \theta_1 \cos \theta_1 \operatorname{sen} \theta_2 \cos \theta_2}{\operatorname{sen}^2(\theta_1 + \theta_2)}$$

em B: $$R_B = \frac{\operatorname{sen}^2(\theta_2 - \theta_3)}{\operatorname{sen}^2(\theta_2 + \theta_3)} = \operatorname{sen}^2(\theta_2 - \theta_3)$$

em C: $$T_C = \frac{4 \operatorname{sen} \theta_2 \cos \theta_2 \operatorname{sen} \theta_1 \cos \theta_1}{\operatorname{sen}^2(\theta_2 + \theta_1)} = T_A$$

No Total $= T_A * R_B * T_C = T_A^2 * R_B$

$T_A = 0{,}965 \qquad R_B = 0{,}25$

TOTAL $= 0{,}24$ TOTAL $= 24\%$

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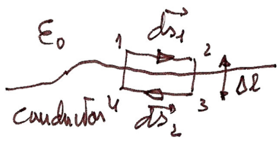
V

$\text{div}\, D = \rho$ ou $\int_{S.f} (\vec{D}.\vec{n})\, ds = Q_{livre}^{interior.}$

$\text{rot}\, E = -\dfrac{\partial B}{\partial t}$ ou $\oint (\vec{E}.\vec{dl}) = -\dfrac{d}{dt}\int_S (\vec{B}.\vec{n})\, ds.$

Electrostática: $\oint (\vec{E}.\vec{dl}) = 0$

Condutor : $\vec{D}_{condutor} = 0$



$\int_{12} (\vec{E_1}.\vec{ds_1}) + \int_{34} (\vec{E_2}.\vec{ds_2}) = 0$

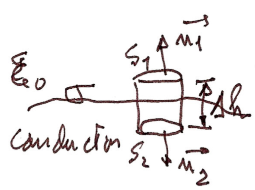
condição fronteira $\Delta l \to 0$

$l_{12} = l_{34}$

$(\vec{E}.\vec{ds}) = E_{tg}$

$E_{tg_1} - E_{tg_2} = 0$ mas $\vec{E}_{condutor} = 0$

$E_{tg_2} = 0 \Rightarrow \boxed{E_{tg_1} = 0}$



$\int_{S_1} (\vec{D_1}.\vec{n_1})\, ds_1 + \int_{S_2} (\vec{D_2}.\vec{n_2})\, ds_2 = \int_S \sigma\, ds$

$\sigma \to$ densidade de carga na superfície do condutor.

condição fronteira : $\Delta h \to 0$

$S_1 = S_2 = S$

$(\vec{D}.\vec{n}) = D_n$

$D_{n_1} - D_{n_2} = \sigma$ mas $D_{n_2} = 0$ pois $\vec{D}_{condutor} = 0$

$D_{n_1} = \sigma$

$\boxed{E_{n_1} = \dfrac{\sigma}{\varepsilon_0}}$ concluindo $\boxed{\vec{E_1} = \dfrac{\sigma}{\varepsilon_0}\vec{n_1}}$

Logo o campo $\vec{E}$ será $\perp$ à superfície do condutor.

NOTA: A superfície do condutor tenha ela a forma que tiver é uma equipotencial. Logo $\vec{E}$ tem de ser $\perp$ à sup.