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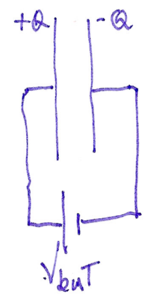
I

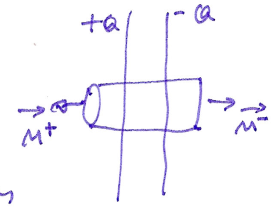
a)

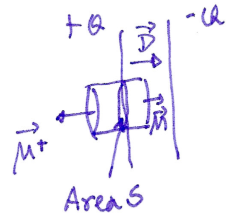


$$\int_{S_f} (\vec{D}.\vec{n})\, ds = Q^{int}$$

$$Q^{int} = +Q - Q = 0$$

$\vec{D} = 0$ para pontos exteriores.



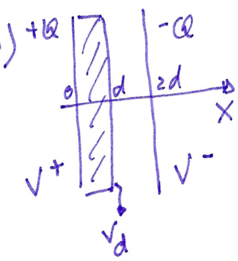


$$Q^{int} = \sigma \text{ Area } S$$

$$\underbrace{|D|}_{(\vec{D}.\vec{n}) \equiv |D|} \cancel{*}S = \sigma S$$

$$\boxed{\vec{D} = \sigma\, \vec{e}_x}$$

b)



$$V_d = V_- + \int_d^{(-)} (\vec{E}.\vec{ds})$$

$$V_d - V^- = \int_d^{2d} \frac{\sigma_3}{\varepsilon_0} (\vec{e_x} . dx\, \vec{e_x})$$

$$(V_d - V^-) = \frac{\sigma_3}{\varepsilon_0} d$$

$$\frac{1}{C_{eq_3}} = \frac{1}{C_a} + \frac{1}{C_b}\ ; \quad C_a = \frac{3\varepsilon_0 A}{d} \text{ e } C_b = \frac{\varepsilon_0 A}{d}\ ; \quad \frac{1}{C_{eq_3}} = \frac{1}{3\frac{\varepsilon_0 A}{d}} + \frac{1}{\frac{\varepsilon_0 A}{d}}$$

$$C_{eq_3} = \frac{3}{4} \frac{\varepsilon_0 A}{d} \qquad \begin{cases} d = 1\,mm \\ A = 4\,cm^2 \\ \varepsilon_0 = 8{,}854 \times 10^{-12}\, F m^{-1} \end{cases} \qquad C_{eq_3} = \frac{3}{4} * 3{,}54\, pF$$

$$C_{eq_3} = 2{,}66\, pF$$

$$C_{eq_3} = \frac{Q_3}{V_{bat}} \qquad Q_3 = C_{eq_3} V_{bat}$$

$$V_{bat} = 2V \qquad Q_3 = 2{,}66 \times 10^{-12} \times 2 \qquad Q_3 = 5{,}3 \times 10^{-12}\, C$$

$$\sigma_3 = \frac{Q_3}{Area} = \frac{5{,}3 \times 10^{-12}}{4 \times 10^{-4}} = 1{,}3 \times 10^{-8}\, C m^{-2}$$

$$(V_d - V^-) = \frac{\sigma}{\varepsilon_0} d = 1{,}5 V$$

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b) outra hipótese:

$V_b = (V_d - V^-) = \frac{\sigma}{\varepsilon_0} d$

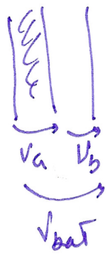
$V_{bat} = V_a + V_b \qquad V_a = \frac{Q_3}{C_a} \qquad V_b = \frac{Q_3}{C_b}$

$C_a = 3\frac{\varepsilon_0 A}{d} \qquad C_b = \frac{\varepsilon_0 A}{d}$

$V_a = \frac{Q_3}{3C_b} = \frac{1}{3}V_b$

$V_b = \frac{Q_3}{C_b} = 3V_a$

$C_{eq_3} = \frac{3}{4}\frac{\varepsilon_0 A}{d}$



$V_{bat} = \frac{1}{3}V_b + V_b$

$V_{bat} = \frac{4}{3}V_b \qquad V_b = \frac{3}{4}V_{bat} \qquad V_b = 0{,}75\,V_{bat}$

$\boxed{V_b = 1{,}5\ V}$

d) A carga Q_5 é a que está em Q_4 porque só foi retirado o dieléctrico. A carga em Q_4 é a que está em Q_3 pois só foi desligada a bateria.

Como vimos na b) $Q_3 = 5{,}3\times10^{-12}\,C \equiv Q_5$

Quanto a $Q_1 = \frac{\varepsilon_0 A}{d} * V_{bat} = 3{,}54\,pF \times 2\,V = 7{,}1\times10^{-12}\,C$

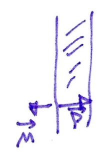
$\frac{Q_5}{Q_1} = \frac{5{,}3\times10^{-12}}{7{,}1\times10^{-12}} = 0{,}75 \qquad \boxed{\frac{Q_5}{Q_1} = 75\%}$

c) $\vec{D} = \varepsilon_0\vec{E} + \vec{P} \qquad \sigma' = (\vec{P}\cdot\vec{n})$ $\vec{n}$ normal exterior do dieléctrico

$\varepsilon\vec{E} = \varepsilon_0\vec{E} + \vec{P}$

$\vec{P} = (3\varepsilon_0 - \varepsilon_0)\frac{\sigma_3}{3\varepsilon_0}\vec{e}_x$

$\vec{P} = \frac{2}{3}\sigma_3\,\vec{e}_x$



$\sigma' = -|P|$

$\sigma' = -\frac{2}{3}\sigma_3 \qquad \sigma_3 = 1{,}3\times10^{-8}\,C\,m^{-2}$ (alínea b)

$\boxed{\sigma' = -8{,}7\times10^{-9}\,C\,m^{-2}}$

e) $W_{e_5} = \frac{1}{2}Q_5V_5 = \frac{1}{2}C_5V_5^2 = \frac{1}{2}\frac{\varepsilon_0 A}{2d}V_{bat}^2\left(\frac{3}{2}\right)^2$

$W_{e_1} = \frac{1}{2}Q_1V_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V_{bat}^2$

$\Delta W_e = W_{e_5} - W_{e_1} = \frac{1}{2}\frac{\varepsilon_0 A}{d}V_{bat}^2\left(\frac{9}{8} - 1\right) = +\frac{1}{2}\,3{,}54\times10^{-12}\times4\times\frac{1}{8}$

$\Delta W_e = +8{,}85\times10^{-13}$ Joules ; no final há aumento da energia armazenada electricamente

I

e) (cont.) 1º $\underline{1 \to 2}$ $Q_1 < Q_2$ porque $C_2 = 3C_1$; a bateria põe mais carga

$C_1 = \frac{\varepsilon_0 A}{d}$ $W_{e_1} = \frac{1}{2} C_1 V_{bat}^2$

$W_{e_2} = \frac{1}{2} C_2 V_{bat}^2 = 3 * W_{e_1}$ $\quad W_{e_2} = 3 * W_{e_1}$

2º $\underline{2 \to 3}$ $Q_2 > Q_3$ porque $C_{eq_3} = \frac{3}{4} C_1 = \frac{1}{4} C_2$; a bateria retira carga.

$W_{e_3} = \frac{1}{2} C_{eq_3} V_{bat}^2 = \frac{1}{4} C_2 \frac{1}{2} V_{bat}^2 = \frac{1}{4} W_{e_2}$ $\quad W_{e_3} = \frac{1}{4} W_{e_2}$

Ao afastar as placas a capacidade diminui e a bateria "foi buscar" carga às placas.

3º $\underline{3 \to 4}$ não há alteração porque só a bateria é desligada e é montado o voltímetro. $Q_4 = Q_3$; $V_4 = V_{bat}$; $W_{e_4} = W_{e_3}$

4º $\underline{4 \to 5}$ Como a bateria não está ligada a carga mantém-se:

$Q_5 = Q_4 = Q_3$

Mas ao retirar o dieléctrico a capacidade diminui.

$C_5 = \frac{\varepsilon_0 A}{2d} = \frac{1}{2} C_1 = \frac{1}{2} \frac{4}{3} C_{eq_3} = \frac{2}{3} C_{eq_3}$

$V_5 = \frac{Q_5}{C_5} = \frac{Q_3}{\frac{2}{3} C_{eq_3}} = \frac{3}{2} V_{bat}$ a diferença de potencial aumenta

$W_{e_5} = \frac{1}{2} C_5 V_5^2 = \frac{1}{2} \frac{2}{3} C_{eq_3} \left(\frac{3}{2} V_3\right)^2 = \frac{3}{2}\left(\frac{1}{2} C_{eq_3} V_3^2\right) = \frac{3}{2} W_{e_3}$

O trabalho realizado ao ser retirado o dieléctrico ficará "armazenado" electricamente no condensador.

Resumindo

$\Delta W_{e_{51}} = \Delta W_{e_{12}} + \Delta W_{23} + \Delta W_{e_{34}} + \Delta W_{45}$

$\Delta W_{e_{12}} = 3W_{e_1} - W_{e_1} = 2W_{e_1}$

$\Delta W_{e_{23}} = \frac{1}{4} W_{e_2} - W_{e_2} = -\frac{3}{4} W_{e_2}$

$\Delta W_{e_{34}} = 0$

$\Delta W_{e_{45}} = \frac{3}{2} W_{e_3} - W_{e_3} = \frac{1}{2} W_{e_3}$

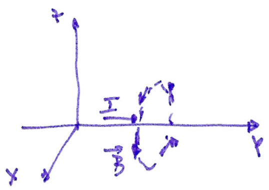
$\Delta W_{e_{51}} = 2W_{e_1} - \frac{3}{4} W_{e_2} + \frac{1}{2} W_{e_3}$

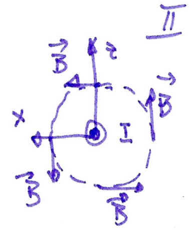
$= 2W_{e_1} - \frac{3}{4} W_{e_2} + \frac{1}{2} \frac{1}{4} W_{e_2}$

$= 2W_{e_1} - \frac{5}{8} W_{e_2}$

$= 2W_{e_1} - \frac{5}{8} 3W_{e_1} \Rightarrow \boxed{\Delta W_{e_{51}} = \frac{1}{8} W_{e_1}}$

a)





Plano YOZ

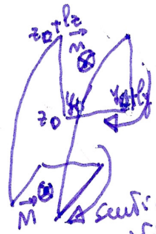
$$\vec{B} = |B|\,\vec{e_x}$$

$$\oint (\vec{B}\cdot\vec{ds}) = \mu_0 I$$

$$|B|\,2\pi z = \mu_0 I$$

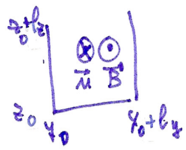
$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{z}\,\vec{e_x}}$$

b)



sentido positivo de circulação

sentido positivo de circulação.

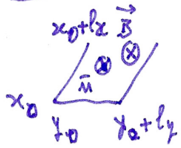


1) No Plano YOZ (por espira)

$$\int_S (\vec{B}\cdot\vec{n})\,ds = -\frac{\mu_0}{4\pi} 2I \int_{z_0}^{z_0+l_z} \frac{1}{z}\,dz \int_{y_0}^{y_0+l_y} dy$$

$$= -\frac{\mu_0}{4\pi} 2I \left(\ln\frac{z_0+l_z}{z_0}\right) l_y$$

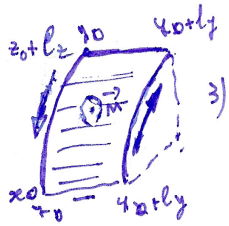
2) No Plano XOY (por espira)



$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{x}(-\vec{e_z})$$

$$\int_S (\vec{B}\cdot\vec{n})\,ds = \frac{\mu_0}{4\pi} 2I \int_{x_0}^{x_0+l_x} \frac{dx}{x} \int_{y_0}^{y_0+l_y} dy$$

$$= \frac{\mu_0}{4\pi} 2I \left(\ln\frac{x_0+l_x}{x_0}\right) l_y$$



3) Superfície definida pelos dois arcos de círculo: $\vec{B}$ é tangente à superfície logo $\int_S (\vec{B}\cdot\vec{n})\,ds = 0$ pois $\vec{B} \perp \vec{n}$

4) $\phi_{TOTAL} = (\phi_{XOY} + \phi_{YOZ}) \times N_{espiras}$

$$\phi_{TOTAL} = \frac{\mu_0}{4\pi} 2IN\,l_y * \left[+\ln\frac{x_0+l_x}{x_0} - \ln\frac{z_0+l_z}{z_0}\right] \text{ (Weber)}$$

5) $|\phi_{TOTAL}| = LI$

$$L = \frac{\mu_0}{4\pi} 2N(y_2-y_1)\left|\ln\frac{x_2}{x_1} - \ln\frac{z_2}{z_1}\right| \text{ Henry}$$

II (cont)

c) cont.

$$L = 10^{-7} \times 2 \times 100 \times 0{,}02 \left| \ln \frac{5}{2} - \ln \frac{5}{0{,}5} \right|$$

$$L = 0{,}55 \times 10^{-6} \text{ Henry} \qquad \boxed{L = 0{,}55\ \mu H}$$

d) $\phi_{TOTAL} = -L\,I$

$$\phi_{TOTAL} = -0{,}55 \times 10^{-6} \times 2 \times 10^{-3}$$

$$\phi_{TOTAL} = -1{,}1 \times 10^{-9} \text{ Weber} \qquad \boxed{\phi_{TOTAL} = -1{,}1\ n\,Wb}$$

e) $\phi_{TOTAL} = -L\,I \qquad I = I_0 + \alpha t^2$

$$\frac{d\phi_{TOTAL}}{dt} = -L\frac{dI}{dt} \qquad \frac{dI}{dt} = 2\alpha t$$

$$\frac{d\phi_{TOTAL}}{dt} = -L\,2\alpha t \qquad \varepsilon^{ind} = -\frac{d\phi}{dt}$$

$$I^{ind} = \frac{\varepsilon^{ind}}{R}$$

$$\boxed{I^{ind} = +\frac{2L\alpha t}{R}}$$

f)

$$I^{ind}(t=3s) = +\frac{2 \times 0{,}55 \times 10^{-6} \times 0{,}2 \times 10^{-3} \times 3}{75}$$

$$I^{ind} = +8{,}8 \times 10^{-12}\,A \qquad I^{ind} = +8{,}8\ pA$$

sentido positivo de circulação → que é o sentido da corrente induzida.

IV

a) $-(\vec{k}\cdot\vec{r}) = \alpha x + \beta z$ $\quad (\vec{k}\cdot\vec{E}) = 0$

$$\begin{cases} k_x = -\alpha \\ k_y = 0 \\ k_z = -\beta \end{cases} \qquad \begin{array}{l} -\alpha\, E_0 \cos(\dots) - \beta\, E_0 \cos(\ ) = 0 \\ -\alpha - \beta = 0 \quad \alpha = -\beta \Rightarrow \boxed{\alpha = -\sqrt{2}\times 10^{-2}\, m^{-1}} \end{array}$$

b) $\vec{k} = +\sqrt{2}\times 10^{-2}\,\vec{e_x} - \sqrt{2}\times 10^{-2}\,\vec{e_z}$ (m^{-1})

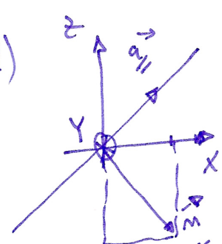
$\vec{n} = \dfrac{\vec{k}}{|\vec{k}|}$ $\qquad |k|^2 = 2\times 10^{-4} \times 2 = 4\times 10^{-4}$

$|k| = 2\times 10^{-2}\, m^{-1}$

$$\vec{n} = \frac{\sqrt{2}\times 10^{-2}}{2\times 10^{-2}}\vec{e_x} - \frac{\sqrt{2}\times 10^{-2}}{2\times 10^{-2}}\vec{e_z} \qquad \boxed{\vec{n} = \frac{1}{\sqrt{2}}\vec{e_x} - \frac{1}{\sqrt{2}}\vec{e_z}}$$

c) $$n = \frac{c}{v} = \frac{ck}{\omega} = \frac{3\times 10^8 \times 2\times 10^{-2}}{4\times 10^6} \qquad \boxed{n = 1{,}5}$$

d)

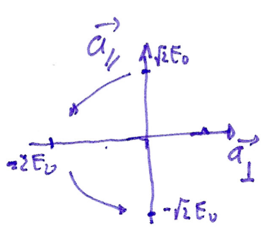


$\vec{a_{//}} = \dfrac{1}{\sqrt{2}}\vec{e_x} + \dfrac{1}{\sqrt{2}}\vec{e_z}$ $\qquad \vec{a_\perp} = \vec{e_y}$

$\vec{E} = E_0 \cos(\ )\,\vec{e_x} - 2E_0 \operatorname{sen}(\ )\,\vec{e_y} + E_0\cos(\ )\,\vec{e_z}$

$\vec{E} = \sqrt{2}E_0\cos(\ )\left(\frac{1}{\sqrt{2}}\vec{e_x}\right) + \sqrt{2}E_0\cos(\ )\left(\frac{1}{\sqrt{2}}\vec{e_z}\right) - 2E_0\operatorname{sen}(\ )\,\vec{e_y}$

$\vec{E} = \sqrt{2}E_0\cos(\ )\,\vec{a_{//}} - 2E_0\operatorname{sen}(\ )\,\vec{a_\perp}$



$x = 0$; $z = 0$

$\omega t = 0 \ \left| \begin{array}{l} E_{//} = \sqrt{2}E_0 \\ E_\perp = 0 \end{array}\right.$ $\quad \omega t = \frac{\pi}{2} \ \left| \begin{array}{l} E_{//} = 0 \\ E_\perp = -2E_0 \end{array}\right.$ $\quad \omega t = \pi \ \left| \begin{array}{l} E_{//} = -\sqrt{2}E_0 \\ E_\perp = 0 \end{array}\right.$

polarização elíptica esquerda.

e) $$|\vec{S}| = \frac{E^2}{Z} = \frac{n}{Z_0}E^2 = \frac{n}{Z_0}\left[2E_0^2\cos^2(\ ) + 4E_0^2\operatorname{sen}^2(\ )\right]$$

$$\langle|\vec{S}|\rangle = \frac{n}{Z_0}\left(2E_0^2\,\frac{1}{2} + 4E_0^2\,\frac{1}{2}\right) = \frac{n}{Z_0}3E_0^2 = \frac{1{,}5}{377}\times 3\times(10^{-3})^2 = 12\times 10^{-9}\, W m^{-2}$$

$$\boxed{\langle|\vec{S}|\rangle = 12\, nW\, m^{-2}}$$

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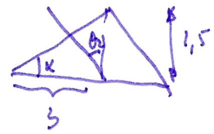
IV

a)



$tg\,\alpha = \frac{2,5}{3}$ $\alpha = 39,8$

$\theta_2 = \alpha$ $\boxed{\theta_2 = 39,8^\circ}$

Reflexão total; ângulo limite; $n_2 \,sen\, \theta_{2\,lim} = n_1 \,sen\, \frac{\pi}{2}$ $\boxed{n_2 = 1,56}$

b) Por razões de simetria $\theta_3 = \theta_2 = 39,8^\circ$

Ângulo de Brewster: $\theta_3 + \theta_4 = 90^\circ$ $\boxed{\theta_4 = 50,2^\circ}$

$n_3 \,sen\, \theta_3 = n_1 \,sen\, \theta_4$ $n_3 = \frac{n_1}{tg\,\theta_3}$ $\boxed{n_3 = 1,2}$

c) À entrada a onda é polarizada circular. A energia total da onda será igualmente repartida pela componente perpendicular e pela paralela. Só pensamos na componente perpendic.

Transmissão em A: $T_A = \frac{4 n_1 n_2}{(n_1+n_2)^2} = \frac{4 \times 1,56}{(1+1,56)^2} = 0,95_2$

Reflexão em B (TOTAL): $R_B = \frac{sen^2(\theta_2 - \frac{\pi}{2})}{sen^2(\theta_2 + \frac{\pi}{2})} = 1$ como seria de esperar.

Transmissão em C: $T_C = \frac{4 n_2 n_3}{(n_2+n_3)^2} = \frac{4 \times 1,56 \times 1,2}{(1,56+1,2)^2} = 0,98_3$

Reflexão em D: $R_D = \frac{sen^2(\theta_3 - \theta_4)}{sen^2(\theta_3 + \theta_4)} = \frac{sen^2(39,8 - 50,2)}{1} = 0,03_2$

Transmissão em E: $T_E = \frac{4 n_3 n_1}{(n_1+n_3)^2} = \frac{4 \times 1,2 \times 1}{(1+1,2)^2} = 0,99_2$

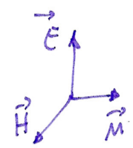
TOTAL $= T_A \times R_B \times T_C * R_D * T_E$

TOTAL $= 0,95 \times 1 * 0,98 \times 0,03 \times 0,99 = 0,027_7$

TOTAL $= 2,8\,\%$

V

$$\vec{S} = [\vec{E} \times \vec{H}]$$



$$\vec{H} = \frac{1}{z}[\vec{n} \times \vec{E}]$$

$$\vec{S} = \frac{1}{z}\left[\vec{E} \times [\vec{n} \times \vec{E}]\right]$$

$$\vec{S} = \frac{1}{z}(\vec{E}\cdot\vec{E})\,\vec{n}$$

$$z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu\varepsilon}{\varepsilon^2}} = \frac{\sqrt{\mu\varepsilon}}{\varepsilon} = \frac{1}{v}\,\frac{1}{\varepsilon}$$

$$\vec{S} = v\,\varepsilon\,(\vec{E}\cdot\vec{E})\,\vec{n} \qquad (A)$$

Densidade de energia eléctrica: $u_{elec} = \frac{1}{2}(\vec{D}\cdot\vec{E})$

Densidade de energia magnética: $u_{mag} = \frac{1}{2}(\vec{B}\cdot\vec{H})$

$$u_e = \frac{1}{2}\varepsilon(\vec{E}\cdot\vec{E})$$

$$u_m = \frac{1}{2}\mu(\vec{H}\cdot\vec{H}) = \frac{1}{2}\mu\,\frac{1}{z^2}(\vec{E}\cdot\vec{E}) = \frac{1}{2}\varepsilon(\vec{E}\cdot\vec{E}) = u_e$$

$$u_{em} = u_e + u_m = \varepsilon(\vec{E}\cdot\vec{E}) = \mu(\vec{H}\cdot\vec{H})$$

De (A) $\vec{S} = v\,u_{em}\,\vec{n}$