EBO 2007/2008 2º S | Versão A | 1º mini-teste | 18/4/2008

MBiol, MQuim

a) Esfera de raio R_1: condutora, logo toda a carga estará na superfície da esfera.

$0 < r < R_1$ $\vec{D} = 0$ pois no interior não há carga.

$R_2 > r > R_1$ $|D| 4\pi r^2 = Q$ [pela lei de Gauss]; $\vec{D} = \frac{1}{4\pi} \frac{Q}{r^2} \vec{e}_r$ [$C m^{-2}$]

Por razões de simetria $\vec{D}$ é radial.

b) $V_1 = V_2 + \int_1^2 (\vec{E} \cdot d\vec{l})$; $V_{R_1} = V_{R_2} + \int_{R_1}^{R_2} \left(\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \vec{e}_r \cdot d\vec{l} \right)$

$$V_{R_1} = V_{bat} + \int_{R_1}^{R_2} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} dr \qquad (d\vec{l} = dr\, \vec{e}_r)$$

$$V_{R_1} = V_{bat} + \frac{1}{4\pi\varepsilon_0} Q \left[-\frac{1}{R_2} + \frac{1}{R_1} \right] \quad (Volt)$$

$$V_{R_1} = 2 + \left\{ (9\times10^9) * 10^{-9} \left[-\frac{1}{0,04} + \frac{1}{0,01} \right] \right\}$$

$$V_{R_1} = +677\ Volt$$

c) $R_2 < r < R_3$ $V_r = \cancel{V_{R_3}}^{=0} + \int_r^{R_3} \frac{1}{4\pi\varepsilon} \frac{Q_{R_2^+}}{r^2} dr$

$$V_r = \frac{1}{4\pi\varepsilon} Q_{R_2^+} \left[-\frac{1}{R_3} + \frac{1}{r} \right]$$

$$V_{R_2} = \frac{1}{4\pi\varepsilon} Q_{R_2^+} \left[-\frac{1}{R_3} + \frac{1}{R_2} \right] = V_{bat}$$

$$Q_{R_2^+} = V_{bat} (4\pi\varepsilon) \frac{R_2 R_3}{R_3 - R_2}$$

$$Q_{R_2^+} = 2 \times \frac{2}{(9\times10^9)} \times \frac{0,04 * 0,06}{0,06 - 0,04} = 53,3\ pC$$

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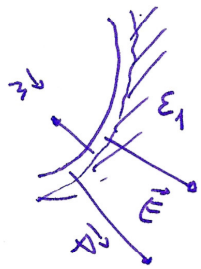
d) (cont.) $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\vec{P} = (\varepsilon_1 - \varepsilon_0)\vec{E} = (2\varepsilon_0 - \varepsilon_0)\vec{E} = \varepsilon_0 \frac{1}{4\pi\varepsilon_0 \times 2} \frac{Q_{R_2}^+}{r^2} \vec{e}_r$

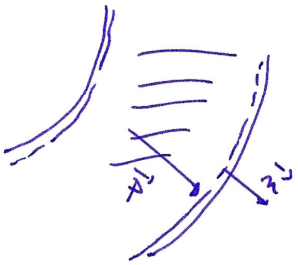


$\sigma'_{R_2} = (\vec{P} \cdot \vec{n})$ normal exterior ao dieléctrico

$\sigma'_1 = -|\vec{P}_{R_2}|$

$\sigma'_{R_2} = -\frac{1}{4\pi \times 2} \frac{Q_{R_2}^+}{R_2^2}$

$\sigma'_{R_2} = -\frac{1}{4\pi \times 2} \frac{3{,}3 \times 10^{-12}}{(0{,}04)^2}$ $\boxed{\sigma'_1 = 1{,}33\ \text{mC m}^{-2}}$



normal exterior ao dieléctrico

$\sigma'_{R_3} = (\vec{P} \cdot \vec{n})$

$\sigma'_{R_3} = +|P_{R_3}| = +\frac{1}{4\pi} \frac{Q_{R_2}^+}{R_3^2}$

$\sigma'_{R_3} = +\frac{1}{4\pi \times 2} \frac{3{,}3 \times 10^{-12}}{(0{,}06)^2}$ $\boxed{\sigma'_{R_3} = +0{,}29\ \text{mC m}^{-2}}$

$Q'_1 = 4\pi R_2^2 \times \sigma'_{R_2} = -4\pi R_2^2 \frac{1}{4\pi} \frac{Q_{R_2}}{R_2^2} = -Q_{R_2}^+$ Total interior

$Q'_2 = 4\pi R_3^2 \times \frac{1}{4\pi} \frac{Q_{R_2}^+}{R_3^2} = +Q_{R_2}^+$ Total exterior