Exercises Series 1

Note: The resolution of the exercises should be delivered until October 27$^{\text{th}}$ by e-mail to the following address *pimenta@lip.pt*. The file should be in pdf and the name of the file must be serie1_XXXXX.pdf (where XXXXX is the student's IST identifier). If the file is a scan of a manuscript version it should be done with black ink into white paper.

1. Consider an electron in the Hydrogen atom in a state such that the wave function is given by

$$\psi(\vec{r},t) = A\ \psi_{100}(\vec{r},t) + B\ \psi_{211}(\vec{r},t) + C\ \psi_{21,-1}(\vec{r},t) \tag{1}$$

where A, B and C are real and positive. Knowing that

$$< L_z >= \frac{7}{18}\,\hbar \qquad and \qquad < L^2 >= \hbar^2 \tag{2}$$

a) Determine A, B and C.

b) Evaluate $< E >$.

c) Evaluate $< r >$.

2. Consider a simplified model where the Hydrogen atom is described by a one dimensional box of length r with the proton at its centre and where the electron is free to move around. Compute, taking into account the Heisenberg uncertainty principle, the total energy of the electron as a function of r and determine the value of r for which this energy is minimised.

3. The Cosmic Microwave Background fills the Universe with photons with a peak energy of 0.37 meV and a density of $\rho \sim 400\,\mathrm{cm}^{-3}$. Determine:

a) The minimal energy (known as the GZK threshold) that a proton should have in order that the reaction $p\gamma \to \Delta$ may occur.

b) The interaction length of such protons in the Universe considering a mean cross section above the threshold of 0.6 mb.

4. What gain would be required from a photomultiplier in order to resolve the signal produced by 3 photoelectrons from that due to 2 or 4 photoelectrons? Assume the fluctuations in the signal are governed by Poisson statistics, and take two peaks to be resolved when their centres are separated by more than the sum of their respective standard deviations.

5. In the *isothermal* approximation, the depth x of the atmosphere at a height h (i.e., the amount of atmosphere above h) can be approximated as

$$x \simeq X e^{-h/7\,\mathrm{km}} ,$$

with $X \simeq 1030$ g/cm^2. If a shower is generated by a gamma ray of E= 1 TeV penetrating the atmosphere vertically, considering that the radiation length X_0 of air is approximately 36.6 g/cm^2 (440 m) and its critical energy E_c is about 88 MeV:

	Incident electron	Incident photon
Peak of shower t_{max}	$1.0 \times (\ln y - 1)$	$1.0 \times (\ln y - 0.5)$
Centre of gravity t_{med}	$t_{max} + 1.4$	$t_{max} + 1.7$
Number of e^+ and e^- at peak	$0.3y/\sqrt{\ln y - 0.37}$	$0.3y/\sqrt{\ln y - 0.31}$
Total track length	y	y

Table 1: Shower parameters according to Rossi approximation B. $y = E/E_c$; unit of length is the radiation length.

a) Calculate the height h_M of the maximum of the shower in the Rossi approximation B.

b) If 2000 useful Cherenkov photons per radiation length are emitted by charged particles in the visible and near UV, compute the total number N_γ of Cherenkov photons generated by the shower (note: the critical energy is larger than the Cherenkov threshold).

c) Supposing that the Cherenkov photons are all emitted at the centre of gravity of the shower compute how many photons per square meter arrive to a detector at a height h_d of 2000 m, supposing that the average attenuation length of photons in air is 3 km, and that the light pool can be derived by a opening of $\sim$ 1.3° from the shower maximum (1.3° is the Cherenkov angle and 0.5° comes from the intrinsic shower spread). Comment on the size of a Cherenkov telescope, considering an average reflectivity of the mirrors (including absorption in transmission) of 70%, and a photodetection efficiency (including all the chains of acquisition) of 20%.

d) Redo the calculations for $E = 50$ GeV, and comment.