Teoria de Campo – Série 1

Curso de Engenharia Física Tecnológica – 2013/2014

`Until 4/4/2014`

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1.1 An electron beam with energy $E_e = 50$ GeV, collides head-on with a laser beam with energy $E_\gamma = 2$ eV. What is the energy of the photons that are scattered backwards, that is, in the direction of the electron beam? In these conditions what is the energy of the scattered electrons.

1.2 For a collision $1 + 2 \to 3 + 4$ we define, in the center of mass frame (`CM`),

$$P_{\text{CM}} = (\sqrt{s}, \vec{0}) = p_1 + p_2 = p_3 + p_4$$

where $\sqrt{s}$ is the total energy in the `CM` frame. Show that

$$p^0_{1\text{CM}} = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \qquad p^0_{2\text{CM}} = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}}$$

$$p^0_{3\text{CM}} = \frac{s + m_3^2 - m_4^2}{2\sqrt{s}}, \qquad p^0_{4\text{CM}} = \frac{s + m_4^2 - m_3^2}{2\sqrt{s}}$$

$$|\vec{p}_1|_{\text{CM}} = \frac{\lambda(\sqrt{s}, m_1, m_2)}{2\sqrt{s}}, \quad |\vec{p}_3|_{\text{CM}} = \frac{\lambda(\sqrt{s}, m_3, m_4)}{2\sqrt{s}}$$

where

$$\lambda(x, y, z) = \sqrt{(x^2 - y^2 - z^2)^2 - 4y^2 z^2}$$

Note: We are going to use these results quite often.

1.3 Consider an electron that obeys the free Dirac equation. Show that in this case we have

$$\frac{d(\vec{\Sigma} \cdot \vec{p})}{dt} = 0$$

where

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

What is the meaning of this conservation law?

Note: For an operator $\mathcal{O}$ that does not depend on time we have

$$\frac{d\mathcal{O}}{dt} = i\Big[H, \mathcal{O}\Big]$$

where H is the Hamiltonian of the system.

1.4 Fill in the entries of the *multiplication table* for the γ matrices as shown in Table 1. This table is very useful in actual calculations. To do this, note that any product can be expanded as a linear combination of the 16 matrices in the basis. Also use the fact that the Lorentz structure has to maintained. Useful relations are,

$$\varepsilon^{0123} = +1\,, \qquad \varepsilon_{\alpha\beta_1\gamma_1\delta_1}\varepsilon^{\alpha\beta_2\gamma_2\delta_2} = -\sum_P (-1)^P g^{P[\beta_2}_{\beta_1} g^{\gamma_2}_{\gamma_1} g^{\delta_2]}_{\delta_1}$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma_0\gamma_1\gamma_2\gamma_3\,, \qquad \varepsilon_{\alpha\beta\gamma_1\delta_1}\varepsilon^{\alpha\beta\gamma_2\delta_2} = -2\left(g^{\gamma_2}_{\gamma_1} g^{\delta_2}_{\delta_1} - g^{\delta_2}_{\gamma_1} g^{\gamma_2}_{\delta_1}\right)$$

$$\varepsilon_{\alpha\beta\gamma\delta_1}\varepsilon^{\alpha\beta\gamma\delta_2} = -6 g^{\delta_2}_{\delta_1}\,.$$

	1	γ_5	γ^μ	$\gamma_5\gamma^\mu$	$\sigma^{\mu\nu}$
1	1				
γ_5					
γ^α					
$\gamma_5\gamma^\alpha$					
$\sigma^{\alpha\beta}$					

Table 1: Multiplication table for γ matrices.

1.5 Choose to demonstrate two of the following relations at your choice.

$$\overline{u}(p,s)u(p,s') = 2m\ \delta_{ss'} \qquad \overline{v}(p,s)v(p,s') = -2m\ \delta_{ss'}$$

$$u^\dagger(p,s)u(p,s') = 2E_p\ \delta_{ss'} \qquad v^\dagger(p,s)v(p,s') = 2E_p\ \delta_{ss'}$$

$$\overline{v}(p,s)u(p,s') = 0 \qquad v^\dagger(p,s)u(-p,s') = 0$$

$$\sum_s [u_\alpha(p,s)\overline{u}_\beta(p,s)] = (\not{p}+m)_{\alpha\beta} \qquad \sum_s [v_\alpha(p,s)\overline{v}_\beta(p,s)] = -(-\not{p}+m)_{\alpha\beta}$$

$$\sum_s [u_\alpha(p,s)\overline{u}_\beta(p,s) - v_\alpha(p,s)\overline{v}_\beta(p,s)] = 2m\ \delta_{\alpha\beta}$$