Teoria do Campo – Problem Set 2

Curso de Engenharia Física Tecnológica – 2013/2014

`Hand in until 23/5/2014`

`Version of 20/04/2014`

2.1 Starting from the definition

$$S_{fi} = \lim_{t\to\varepsilon_f\infty} \int d^3x\ \psi_f^\dagger(x)\Psi_i(x)$$

get the fundamental expression for Chapter 2, Eq. (2.50),

$$S_{fi} = \delta_{fi} - ie\varepsilon_f \int d^4y\ \overline{\psi}_f(y)\not{A}(y)\Psi_i(y)\,. \tag{1}$$

To do this follow the steps:

a) Show that (Eq. (2.40))

$$S_F(x'-x) = \theta(t'-t)\int d^3p\ \sum_{r=1}^{2}\psi_p^r(x')\overline{\psi}_p^r(x) - \theta(t-t')\int d^3p\ \sum_{r=3}^{4}\psi_p^r(x')\overline{\psi}_p^r(x)$$

where

$$\psi_p^r(x) = \frac{1}{\sqrt{2E}}\ (2\pi)^{-3/2}\ w^r(\vec{p})\ e^{-i\varepsilon_r p\cdot x}$$

b) Obtain Eqs. (2.48) e (2.49),

$$\lim_{t\to+\infty}\Psi(x) - \psi(x) = \int d^3p\ \sum_{r=1}^{2}\psi_p^r(x)\left[-ie\int d^4y\ \overline{\psi}_p^r(y)\not{A}(y)\Psi(y)\right]$$

$$\lim_{t\to-\infty}\Psi(x) - \psi(x) = \int d^3p\ \sum_{r=3}^{4}\psi_p^r(x)\left[+ie\int d^4y\ \overline{\psi}_p^r(y)\not{A}(y)\Psi(y)\right]$$

c) Use the previous results to obtain Eq. (1).

2.2 Show that for the decay $P \to p_1 + p_2$ the total width is given in the rest frame of the decaying particle by

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2}\,\frac{|\vec{p}_{1\mathrm{cm}}|}{M^2}\,\overline{|M_{fi}|^2}$$

where $P^2 = M^2$.

2.3 Evaluate one of the traces needed for the Compton scattering (Eq. 4.12)

$$\mathrm{T}_2 = \mathrm{Tr}\left[(\not{p}' + m)\Gamma_2(\not{p} + m)\overline{\Gamma}_2\right]$$

where (see Eq. 4.4)

$$\Gamma_2 = \frac{-e^2}{2p\cdot k'}\gamma_\mu(\not{p} - \not{k}' + m)\gamma_\nu\ \varepsilon^\mu(k,\lambda)\varepsilon'^{\nu *}(k',\lambda')$$

2.4 Consider in QED the process $\gamma(k_1) + \gamma(k_2) \to e^-(p_1) + e^+(p_2)$.

a) Write the amplitude for the process,

$$M \equiv M_{\mu\nu}\ \epsilon^\mu(k_1)\epsilon^\nu(k_2)$$

b) Show the gauge invariance of the process, that is

$$k_1^\mu M_{\mu\nu} = k_2^\nu M_{\mu\nu} = 0$$

Just show for one case.

2.5 Consider the electroweak part of the standard model. For the following processes **draw** the diagrams that contribute in lowest order in perturbation theory.

a) $e^+ + e^- \to \nu_e + \overline{\nu}_e$

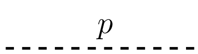
b) $e^+ + \nu_\mu \to e^+ + \nu_\mu$

c) $e^+ + e^- \to \nu_e + \overline{\nu}_e + \gamma$

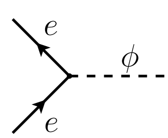
2.6 Consider the process $\phi \to e^+ + e^-$ in a theory described by the following Lagrangian,

$$\mathcal{L} = \mathcal{L}_{\mathrm{QED}} + \frac{1}{2}\partial_\mu\phi\, \partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - \beta\, \overline{\psi}\gamma_5\psi\, \phi$$

where ϕ is a neutral (pseudo)-scalar field (spin 0) and ψ is the electron. Besides QED we have the following Feynman rules:



$$\frac{i}{p^2 - m_\phi^2}$$



$$-i\beta\,\gamma_5$$

a) Write the amplitude for the process.

b) Find the decay width $\Gamma(\phi \to e^+ + e^-)$ as a function of the model parameters.

c) Suppose that one measures $m_\phi = 1.8$ GeV with a lifetime $\tau_\phi = 8.5 \times 10^{-23}$ s. Find the value of β? $m_e = 0.511$ MeV