Teoria de Campo, 1º Exame 2015/2016

(Re) Solução Esquemática

I

a) $\sqrt{s} = 2E_{beam} = 91\ \text{GeV}$

no Lab: $s = (p_1+p_2)^2 = 2m_e^2 + 2m_e E_{e+} \Rightarrow E_{e+} = \dfrac{s-2m_e^2}{2m_e} = 8.1\times10^6\ \text{GeV}$

$= 8100\ \text{TeV}$!!

b) $\bar{u}(p')[p'_\mu + p_\mu]u(p) = \bar{u}(p')[p'^\nu g_{\mu\nu} + p^\nu g_{\mu\nu}] = \bar{u}(p')[p'^\nu g_{\nu\mu} + p^\nu g_{\mu\nu}]u(p)$

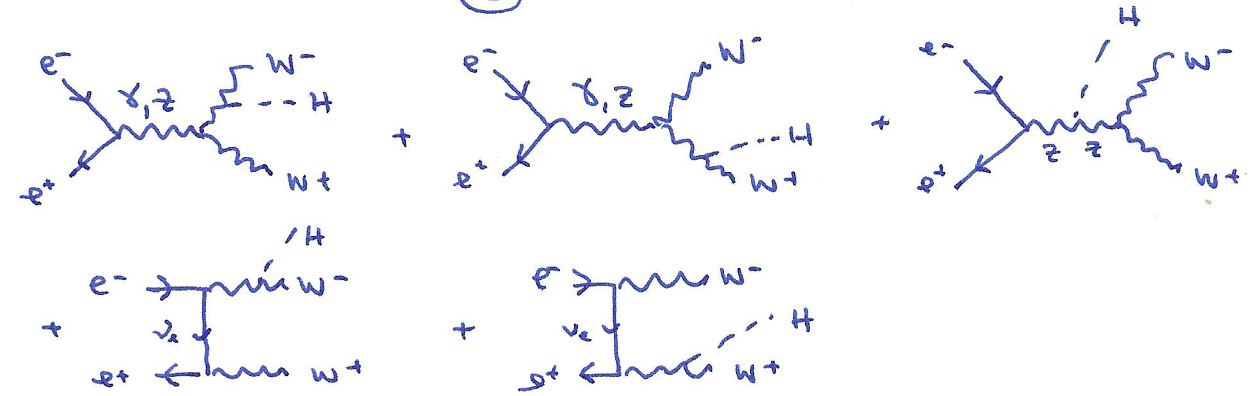
$= \bar{u}(p')[\not{p}'\gamma_\mu + i p'^\nu \sigma_{\nu\mu} + \gamma_\mu \not{p} + i p^\nu \sigma_{\mu\nu}]u(p)$

$= \bar{u}(p')[2m\gamma_\mu + i(p'-p)^\nu \sigma_{\nu\mu}]u(p)$

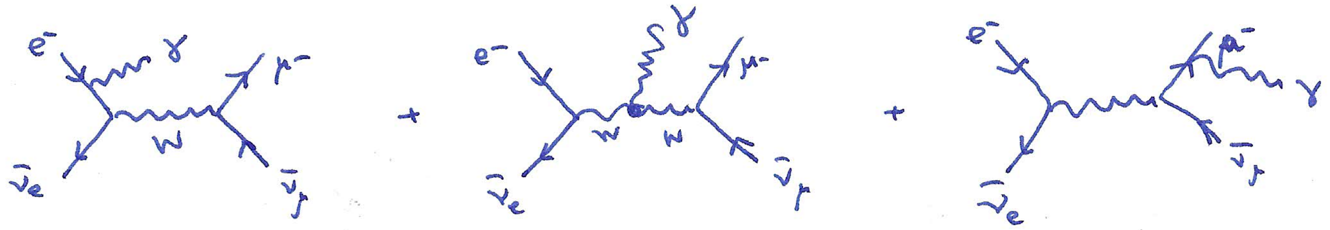
$= \bar{u}(p')[2m\gamma_\mu + i q^\nu \sigma_{\nu\mu}]u(p)$

II

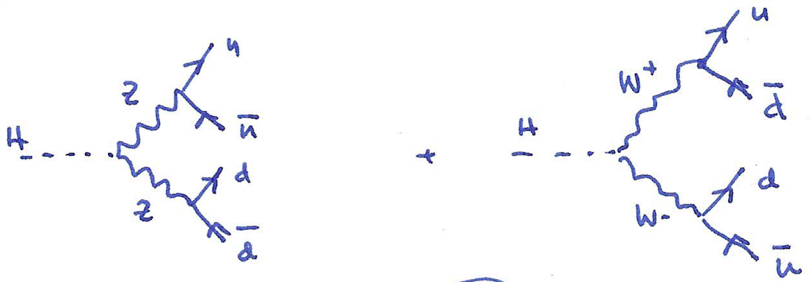
a)



b)

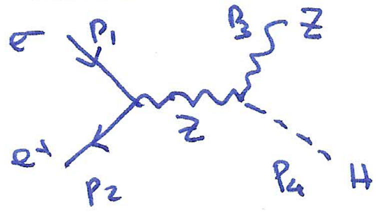


c)



III

a) Desprezando a massa do electrão



b) Desprezando a massa do eletron e usando a equação de Dirac

$$iM = \left(-i\frac{g}{c_W}\right)\left(i\frac{g}{c_W}M_Z\right)\bar{v}(p_2)\gamma^\mu(g_V^e - g_A^e\gamma_5)u(p_1)\frac{(-i)g_{\mu\nu}}{D(s)}\varepsilon^{*\nu}(p_3)$$

ou

$$\boxed{M = -\left(\frac{g}{c_W}\right)^2\frac{M_Z}{D(s)}\bar{v}(p_2)\gamma_\mu(g_V^e - g_A^e\gamma_5)u(p_1)\varepsilon^{*\mu}(p_3)}$$

c) $$\langle|M|^2\rangle = \frac{1}{4}\sum_{\substack{\text{spins,}\\ \text{pol}}}|M|^2$$

$$= 8G_F^2M_Z^4\frac{M_Z^2}{|D(s)|^2}\mathrm{Tr}[\not{p}_2\gamma_\mu(g_V^e - g_A^e\gamma_5)\not{p}_1\gamma_\nu(g_V^e - g_A^e\gamma_5)]\left(-g^{\mu\nu} + \frac{p_3^\mu p_3^\nu}{M_Z^2}\right)$$

$$\mathrm{Tr}[\dots] = (g_V^{e2} + g_A^{e2})\,\mathrm{Tr}[\not{p}_2\gamma_\mu\not{p}_1\gamma_\nu] - 2g_A^eg_V^e\,\mathrm{Tr}[\not{p}_2\gamma_\mu\not{p}_1\gamma_\nu\gamma_5]$$

$$= (g_V^{e2} + g_A^{e2})\,4[p_{2\mu}p_{1\nu} + p_{2\nu}p_{1\mu} - (p_1\cdot p_2)g_{\mu\nu}] + 8i\,\underbrace{g_A^eg_V^e\,\varepsilon_{\alpha\mu\beta\nu}}_{\text{Não contribui contraído com tensor simétrico}}p_2^\alpha p_1^\beta$$

$$\langle|M|^2\rangle = \frac{32G_F^2M_Z^6}{|D(s)|^2}\left[(p_1\cdot p_2) + \frac{2(p_1\cdot p_3)(p_2\cdot p_3)}{M_Z^2}\right](g_V^{e2} + g_A^2)$$

$$p_1 = \frac{\sqrt{s}}{2}(1,0,0,1)\ ;\quad p_2 = \frac{\sqrt{s}}{2}(1,0,0,-1)\ ;\quad p_3 = (E_3, |\vec{p}_3|\sin\theta, 0, |\vec{p}_3|\cos\theta)$$

$$E_3 = \frac{s + M_Z^2 - M_A^2}{2\sqrt{s}}\ ;\quad |\vec{p}_3| = \frac{\sqrt{s}}{2}\lambda$$

$$(p_1\cdot p_2) = \frac{s}{2}\ ;\quad (p_1\cdot p_3) = \frac{\sqrt{s}}{2}(E_3 - |\vec{p}_3|\cos\theta);\quad (p_2\cdot p_3) = \frac{\sqrt{s}}{2}(E_3 + |\vec{p}_3|\cos\theta)$$

$$(p_1\cdot p_3)(p_2\cdot p_3) = \frac{s}{4}(E_3^2 - |\vec{p}_3|^2\cos^2\theta) = \frac{s}{4}(M_Z^2 + |\vec{p}_3|^2(1-\cos^2\theta))$$

$$= \frac{s}{4}\left(M_Z^2 + \frac{s}{4}\lambda^2(1-\cos^2\theta)\right)$$

Logo

$$\langle|M|^2\rangle = \frac{4G_F^2M_Z^4s^2(g_V^{e2} + g_A^{e2})}{|D(s)|^2}\left[\frac{8(p_1\cdot p_2)M_Z^2}{s^2} + \frac{8M_Z^2}{s^2}\frac{2(p_1\cdot p_3)(p_2\cdot p_3)}{M_Z^2}\right]$$

$$\langle |M|^2 \rangle = \frac{4 G_F^2 M_Z^4 s^2 (g_V^{e2} + g_A^{e2})}{|D(s)|^2} \left[\frac{8 M_Z^2}{s} + \lambda^2 (1 - \cos^2\theta) \right]$$

d) $$\frac{d\sigma}{d\Omega} = \frac{G_F^2 M_Z^4}{16\pi^2} \frac{s\lambda}{|D(s)|^2} (g_V^{e2} + g_A^{e2}) \left[\frac{8 M_Z^2}{s} + \lambda^2 (1 - \cos^2\theta) \right]$$

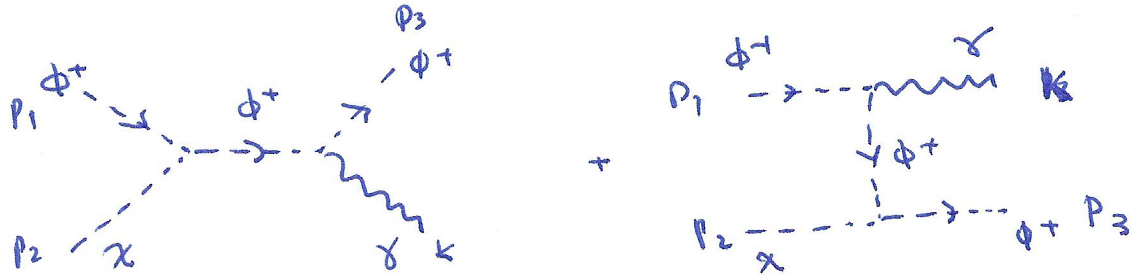
$$\sigma = \frac{G_F^2 M_Z^4 \lambda}{6\pi} \frac{s}{|D(s)|^2} (g_V^{e2} + g_A^{e2}) \left[\frac{12 M_Z^2}{s} + \lambda^2 \right]$$

e) $\sigma(\sqrt{s} = 500\,\text{GeV}) = 56.3\,\text{fb}$

\# Higgs $= \sigma \times \int dt\, \mathcal{L} = 5630.$

IV

a)



1) 2)

$$i M_1 = (-ie)(-i\mu) \frac{i}{(p_3+k)^2 - m_\phi^2} (p_3 + k - (-p_3))^\mu \, \varepsilon^*_\mu(k)$$

$$i M_2 = (-ie)(-i\mu) \frac{i}{(p_1-k)^2 - m_\phi^2} (p_1 - (-p_1 + k))^\mu \, \varepsilon^*_\mu(k)$$

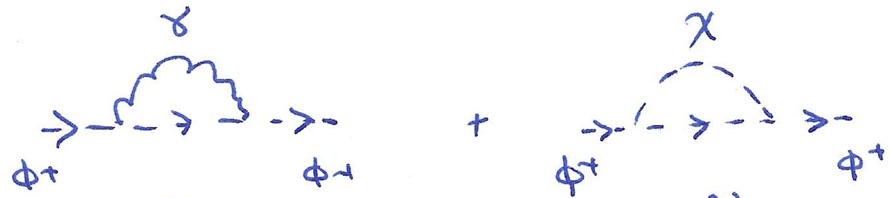
$$M = (-e\mu) \left[\frac{(2p_3 + k)^\mu}{2p_3 \cdot k} + \frac{(2p_1 - k)^\mu}{(-2p_1 \cdot k)} \right] \varepsilon^*_\mu(k)$$

b) $$M_\mu k^\mu = (-e\mu) \left[\frac{(2p_3 + k)\cdot k}{2p_3 \cdot k} - \frac{(2p_1 - k)\cdot k}{2p_1 \cdot k} \right]$$

$$= (-e\mu) [1 - 1] = 0$$

V

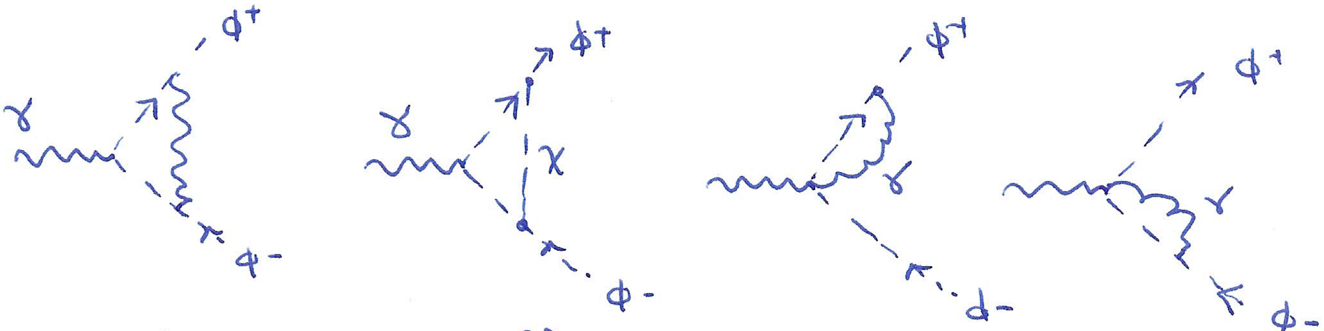
a)



1) 2)

$\omega(G_1) = 2$ (Divergente) $\omega(G_2) = 0$ (Divergente)

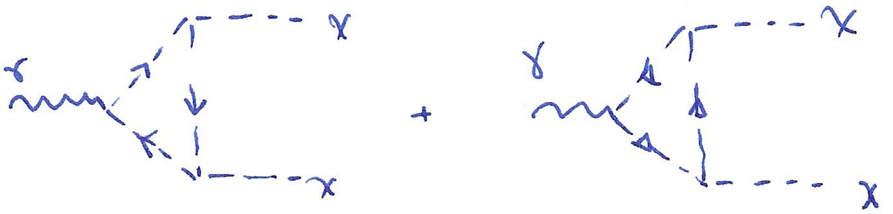
b)



1) 2) 3) 4)

$\omega(G_1) = 1$ (Divergente); $\omega(G_2) = -1$ (Convergente); $\omega(G_3) = \omega(G_4) = 1$ (Divergente)

c)



1) 2)

$\omega(G_1) = \omega(G_2) = -1$, convergente

d) Todos os $\omega_v \leq 4$. A teoria é renormalizável. A única excepção seria se houvesse diagramas divergentes a 1 loop correspondentes a termos que faltassem no Lagrangeano mas que eram compatíveis com a simetria. Nesse caso o Lagrangeano era incompleto e devia ser completado. Não é o caso. Por exemplo o termo χ^3 não existe mas não é divergente.

Nota: os termos divergentes acima correspondem a vértices que existem no Lagrangeano e portanto podem ser absorvidos nos respectivos contratermos.