Teoria do Campo – Problem Series 2

Curso de Engenharia Física Tecnológica – 2015/2016

`Due on the 20/5/2016`

`Version of 30/03/2016`

2.1 Show that for the decay, $P \to q_1 + q_2$, the expression for the total width can be written, in the rest frame of the decaying particle, as

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} \frac{|\vec{q}_{1\text{cm}}|}{M^2} \left\langle |\mathcal{M}_{fi}|^2 \right\rangle$$

where $P^2 = M^2$.

2.2 Evaluate the traces necessary for Compton scattering (Eqs. (5.11), (5.12) e (5.13))

$$\sum_{s,s'} |\mathcal{M}_1|^2 = \text{Tr}\left[(\not p' + m)\Gamma_1(\not p + m)\overline{\Gamma}_1\right]$$

$$\sum_{s,s'} |\mathcal{M}_2|^2 = \text{Tr}\left[(\not p' + m)\Gamma_2(\not p + m)\overline{\Gamma}_2\right]$$

$$\sum_{s,s'} (\mathcal{M}_1\mathcal{M}_2^\dagger + \mathcal{M}_1^\dagger\mathcal{M}_2) = \text{Tr}\left[(\not p' + m)\Gamma_1(\not p + m)\overline{\Gamma}_2\right] + \text{Tr}\left[(\not p' + m)\Gamma_2(\not p + m)\overline{\Gamma}_1\right]$$

and show that the final result, Eq. (5.52), can be written as

$$\frac{1}{4}\sum_{s,s'}\sum_{\lambda,\lambda'} \{|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + \mathcal{M}_1\mathcal{M}_2^\dagger + \mathcal{M}_1^\dagger\mathcal{M}_2\} = 2e^4\left[\left(\frac{k}{k'}\right) + \left(\frac{k'}{k}\right) - \sin^2\theta\right]$$

Note: These are complicated traces. You should learn how to use FeynCalc to evaluate these traces.

2.3 Consider the process $e^-(p_1) + e^+(p_2) \to \mu^-(p_3) + \mu^+(p_4)$ in the SM.

a) Evaluate the differential cross section in the CM frame, as a function of the center of mass energy, $\sqrt{s}$, and scattering angle θ defined as the angle between the incoming electron and outgoing muon. Neglect the fermion masses.

b) Make a plot of the total cross section as a function of $\sqrt{s}$, for 10 GeV$< \sqrt{s} <$ 200 GeV.

c) Use CalcHEP to evaluate this same process. Superimpose the points from CalcHEP on your plot. **Note**: You should check that the physical constants are the same in both cases.

2.4 Consider in the SM of electroweak interactions the following processes:

i) $e^-e^+ \to \nu_e\overline{\nu}_e$	*ii)* $e^-e^+ \to \nu_\mu\overline{\nu}_\mu$
iii) $e^-e^+ \to e^-e^+\gamma$	*iv)* $e^-e^+ \to W^-W^+$

a) Use the program QGRAF to find the diagrams that contribute in lowest order.

b) **Draw** the diagrams and indicate the relative signs among the diagrams. Do not do any calculations.

2.5 Consider the process $e^- + e^+ \to \mu^- + \mu^+$ in QED. In class we have discussed the helicity amplitudes in the case of massless fermions. Now do not neglect the masses.

a) Show that

$$\mathcal{M}(\uparrow, \downarrow; \uparrow, \downarrow) = -e^2(1 + \cos\theta)$$

as in the massless case.

b) Show that

$$\mathcal{M}(\uparrow, \downarrow; \uparrow, \uparrow) = -2e^2\, \frac{m_\mu \sin\theta}{\sqrt{s}}, \quad \mathcal{M}(\uparrow, \uparrow; \downarrow, \downarrow) = 4e^2\, \frac{m_e m_\mu \cos\theta}{s}$$

c) Consider the helicity projectors

$$P(h) = \frac{1 + h\gamma_5 \not{s}}{2}$$

where $h = \pm$ for $\uparrow, \downarrow$, respectively, and the spin vector along the direction of motion of the particle is

$$s^\mu = (\gamma\beta, \gamma\, \frac{\vec{\beta}}{\beta})$$

Verify that they are projectors, and that we have, for instance,

$$P(+)u_\uparrow(p) = u_\uparrow(p), \quad P(-)u_\uparrow(p) = 0$$

d) Use the helicity projectors and the trace technique to obtain $|\mathcal{M}(\uparrow, \downarrow; \uparrow, \uparrow)|^2$ and $|\mathcal{M}(\uparrow, \uparrow; \downarrow, \downarrow)|^2$ and check that they are consistent. **Note**: the traces are long, use FeynCalc.