Advanced Quantum Field Theory
1º Semester 2015/2016

I give here a brief description of the proposed topics. You should write a small text on the chosen topic and make an oral presentation. This will be organized in February, with a maximum duration of the oral presentation shall be 30 minutes.

IST, 15 of December, 2015
Jorge C. Romão

1 Renormalization Group and Unified Theories

Objective: Evaluate the evolution of the coupling constants using the renormalization group, for the Standard Model and for the MSSM. The student should do the explicit calculations that are needed.

Bibliography:

- *Advanced Quantum Field Theory* , Jorge C. Romão, Chapter 7.

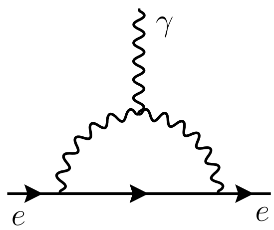
Student:

2 QED in a non-linear gauge

Consider QED with a non-linear gauge condition,

$$F = \partial_\mu A^\mu + \frac{\lambda}{2}\ A_\mu A^\mu\ .$$

1. Write $\mathcal{L}_{eff}$ and show that $s\mathcal{L}_{eff} = 0$, where s is the Slavnov operator.
2. Write the Feynman rules for the new vertices and propagators. Then evaluate at tree level $\gamma + \gamma \to \gamma + \gamma$. Compare with the results in the usual linear gauge.
3. Evaluate the vacuum polarization at one-loop.
4. Show that the diagram of the figure, that would be potentially dangerous for the anomalous magnetic moment of the electron (would be proportional to λ) vanishes.



Student:

3 Vacuum Polarization in QCD

Consider the theory that describes the interactions of quarks with gluons (QCD) given by the Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \sum_{\alpha=1}^{n} \overline{\psi}^\alpha_i (i \not{D} - m_\alpha)_{ij} \psi^\alpha_j$$

where

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \\ (D_\mu)_{ij} &= \delta_{ij} \partial_\mu - ig \left(\frac{\lambda^a}{2} \right)_{ij} A^a_\mu \ . \end{aligned}$$

The index $\alpha = 1, 2, \ldots, n$ denotes the different quark flavours (*up*,*down*,...,*top*). In order to quantize the theory use the linear gauge condition,

$$\mathcal{L}_{GF} = -\frac{1}{2\xi} \ (\partial_\mu A^{\mu a})^2 \ ,$$

that gives the following Lagrangian for the ghosts,

$$\mathcal{L}_G = \partial_\mu \overline{\omega}^a \partial^\mu \omega^a + g f^{abc} \partial^\mu \overline{\omega}^a A^b_\mu \omega^c$$

To renormalize the theory one needs the following counter-term Lagrangian,

$$\begin{aligned} \Delta\mathcal{L} \ = \ & -\frac{1}{4}(Z_3 - 1) \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu \right)^2 - (Z_4 - 1) g f^{abc} \partial_\mu A^a_\nu A^{\mu b} A^{\nu c} \\ & -\frac{1}{4} g^2 (Z_5 - 1) f^{abc} f^{ade} A^b_\mu A^c_\nu A^{\mu d} A^{\nu e} + \sum_\alpha (Z_2 - 1) i \overline{\psi}^\alpha_i \gamma^\mu \partial_\mu \psi^\alpha_i \\ & - \sum_\alpha m_\alpha (Z_{m_\alpha} - 1) \overline{\psi}^\alpha_i \psi^\alpha_i + (Z_1 - 1) g \sum_\alpha \overline{\psi}^\alpha_i \gamma^\mu \left(\frac{\lambda^a}{2} \right)_{ij} \psi^\alpha_j A^a_\mu \\ & + (Z_6 - 1) \partial_\mu \overline{\omega}^a \partial^\mu \omega^a + (Z_7 - 1) g f^{abc} \partial^\mu \overline{\omega}^a A^b_\mu \omega^c \ . \end{aligned}$$

1. Verify the expression for $\mathcal{L}_G$.

2. Verify the Feynman rules given in the text.

3. Evaluate the vacuum polarization.

Student:

4 Renormalization if QCD

Consider the theory that describes the interactions of quarks with gluons (QCD) given by the Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \sum_{\alpha=1}^{n} \overline{\psi}^\alpha_i (i\not{D} - m_\alpha)_{ij} \psi^\alpha_j$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$
$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - ig \left(\frac{\lambda^a}{2}\right)_{ij} A^a_\mu \ .$$

The index $\alpha = 1, 2, \ldots, n$ denotes the different quark flavours (*up,down,...,top*). In order to quantize the theory use the linear gauge condition,

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& -\frac{1}{4} g^2 (Z_5 - 1) f^{abc} f^{ade} A^b_\mu A^c_\nu A^{\mu d} A^{\nu e} + \sum_\alpha (Z_2 - 1) i \overline{\psi}^\alpha_i \gamma^\mu \partial_\mu \psi^\alpha_i \\
& -\sum_\alpha m_\alpha (Z_{m_\alpha} - 1) \overline{\psi}^\alpha_i \psi^\alpha_i + (Z_1 - 1) g \sum_\alpha \overline{\psi}^\alpha_i \gamma^\mu \left(\frac{\lambda^a}{2}\right)_{ij} \psi^\alpha_j A^a_\mu \\
& +(Z_6 - 1) \partial_\mu \overline{\omega}^a \partial^\mu \omega^a + (Z_7 - 1) g f^{abc} \partial^\mu \overline{\omega}^a A^b_\mu \omega^c \ .
\end{aligned}$$

1. Show that the following relations must be true

$$\frac{Z_1}{Z_2} = \frac{Z_4}{Z_3} = \frac{Z_7}{Z_6} = \frac{\sqrt{Z_5}}{\sqrt{Z_3}}$$

2. Evaluate Z_1, Z_2, Z_3, Z_4, Z_6 e Z_7 using Minimal Subtraction (MS). Show explicitly that $Z_1 Z_6 = Z_2 Z_7$.

3. Evaluate the contribution of the fermions to Z_4 and Z_5. Show that they are in agreement with the previous relations.

Student:

5 Renormalization of Scalar Electrodynamics

Consider Scalar Electrodynamics, that is the gauge theory of interactions of photons with charged scalar particles.

1. Write the Lagrangian for this theory.
2. Derive the Feynman rules.
3. Identify the divergent diagrams.
4. Do the on-shell renormalization for the self-energies of the photon and charged scalar particle.

Student:

6 Renormalization of the Wess-Zumino Model

Consider the Wess-Zumino model described by the Lagrangian

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}\partial_\mu A\partial^\mu A + \tfrac{1}{2}\partial_\mu B\partial^\mu B + \frac{i}{2}\overline{\psi}\gamma^\mu\,\partial_\mu\psi - \tfrac{1}{2}m^2A^2 - \tfrac{1}{2}m^2B^2 - \tfrac{1}{2}m\overline{\psi}\psi \\ &\quad -\frac{\lambda^2}{4}\left(A^2+B^2\right)^2 - \frac{m\lambda}{\sqrt{2}}A\left(A^2+B^2\right) - \frac{\lambda}{\sqrt{2}}A\overline{\psi}\psi + \frac{i\lambda}{\sqrt{2}}B\overline{\psi}\gamma_5\psi \end{aligned} \tag{1}$$

where the fermion ψ is a Majorana particle and A and B are real scalar fields.

1. Derive the Feynman rules. Do not forget that the fermion is a Majorana fermion.
2. Identify the divergent diagrams.
3. Evaluate the self-energy of the scalar fields and show that the quadratic divergences cancel.
4. To renormalize this model it is necessary a counter-term Lagrangian of the form,

$$\begin{aligned}\Delta\mathcal{L} &= \frac{1}{2}\delta Z_A\partial_\mu A\partial^\mu A + \frac{1}{2}\delta Z_B\partial_\mu B\partial^\mu B + \frac{i}{2}\delta Z_\psi\overline{\psi}\gamma^\mu\,\partial_\mu\psi \\ &\quad -\frac{1}{2}m^2(2\delta Z_m+\delta Z_A)A^2 - \frac{1}{2}m^2(2\delta Z_m+\delta Z_B)B^2 - \frac{1}{2}m(\delta Z_m+\delta Z_\psi)\overline{\psi}\psi \\ &\quad -\frac{\lambda^2}{4}(2\delta Z_\lambda+2\delta Z_A)A^4 - \frac{\lambda^2}{4}(2\delta Z_\lambda+2\delta Z_B)B^4 \\ &\quad -2\frac{\lambda^2}{4}(2\delta Z_\lambda+\delta Z_A+\delta Z_B)A^2B^2 \\ &\quad -\frac{m\lambda}{\sqrt{2}}(\delta Z_m+\delta Z_\lambda+\frac{3}{2}\delta Z_A)A^3 - \frac{m\lambda}{\sqrt{2}}(\delta Z_m+\delta Z_\lambda+\frac{1}{2}\delta Z_A+\delta Z_B)AB^2 \\ &\quad -\frac{\lambda}{\sqrt{2}}(\delta Z_\lambda+\frac{1}{2}\delta Z_A+\delta Z_\psi)A\overline{\psi}\psi + \frac{i\lambda}{\sqrt{2}}(\delta Z_\lambda+\frac{1}{2}\delta Z_B+\delta Z_\psi)B\overline{\psi}\gamma_5\psi \end{aligned}$$

   Show that the six renormalization constants are related and that there is only one independent, the wave function renormalization. to show this evaluate the renormalization constants in Minimal Subtraction.

Student:

7 Unitarity in Non-Abelian Gauge Theories

Consider the theory that describes the interactions of quarks with gluons (QCD) given by the Lagrangian:

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where

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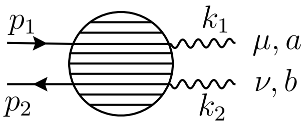
that gives the following Lagrangian for the ghosts,

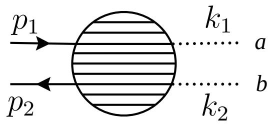
$$\mathcal{L}_G = \partial_\mu \overline{\omega}^a \partial^\mu \omega^a + g f^{abc} \partial^\mu \overline{\omega}^a A^b_\mu \omega^c$$

1. Verify the expression for $\mathcal{L}_G$.

2. Show explicitly that the action is BRS invariant.

   Consider now the amplitudes

$iT^{ab}_{\mu\nu} \equiv$ 

$iT^{ab} \equiv$ 

3. Evaluate $T^{ab}_{\mu\nu}$ at tree level. Verify that, for off-shell gluons, we have $k^\mu_1 T^{ab}_{\mu\nu} \neq 0$. What happens for on-shell gluons?

4. Verify, at tree level, the Ward identities

$$k^\mu_1 \ T^{ab}_{\mu\nu} = k^\nu_2 \ T^{ab}$$

5. Use the above results to explicitly prove unitarity at one-loop level, showing that the optical theorem holds in this case.

Student: