The Need for the Higgs in the SM

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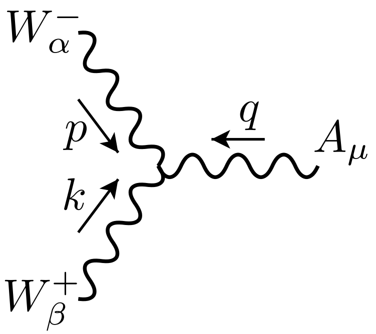
Summary

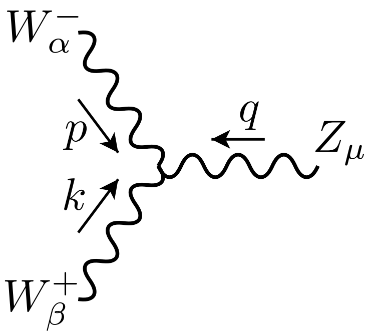
- In this talk we review the role of the Higgs boson in preserving unitarity of the scattering amplitudes in the Standard Model.
- We will look at the processes
  - $\nu_e + \overline{\nu}_e \to W_L^- + W_L^+$
  - $W_L^- + W_L^+ \to W_L^- + W_L^+$
  - $e^- + e^+ \to W_L^- + W_L^+$

  for longitudinally polarized gauge bosons.
- Special emphasis will be put in using algebraic methods to evaluate the amplitudes and cross sections.

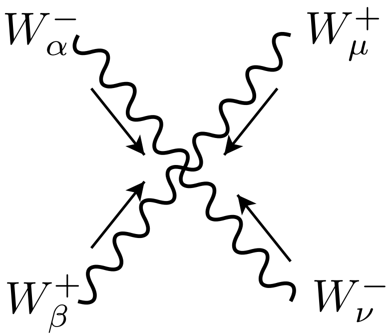
Gauge Boson Self-Couplings



$$-ie\left[g_{\alpha\beta}(p-k)_\mu + g_{\beta\mu}(k-q)_\alpha + g_{\mu\alpha}(q-p)_\beta\right] \equiv -ie\Gamma_{\alpha\beta\mu}(p,k,q)$$



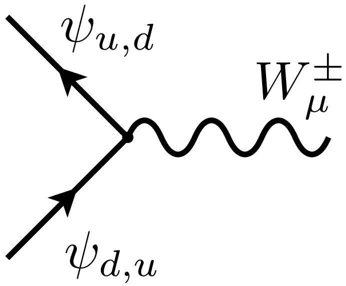
$$ig\cos\theta_W\left[g_{\alpha\beta}(p-k)_\mu + g_{\beta\mu}(k-q)_\alpha + g_{\mu\alpha}(q-p)_\beta\right] \equiv ig\Gamma_{\alpha\beta\mu}(p,k,q)$$



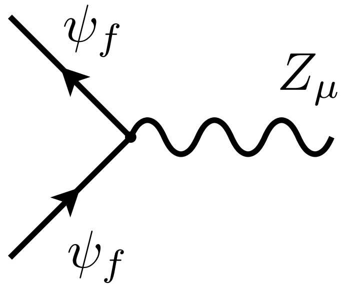
$$ig^2\left[2g_{\alpha\nu}g_{\beta\mu} - g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}\right]$$

$$\boxed{\Gamma_{\alpha\beta\mu}(p,k,q) \equiv \left[g_{\alpha\beta}(p-k)_\mu + g_{\beta\mu}(k-q)_\alpha + g_{\mu\alpha}(q-p)_\beta\right]}$$

Gauge Couplings to Fermions



$$i\frac{g}{\sqrt{2}}\gamma_\mu \frac{1-\gamma_5}{2}$$



$$i\frac{g}{\cos\theta_W}\gamma_\mu \left(g_V^f - g_A^f\gamma_5\right)$$

where

$$g_V^f = \frac{1}{2}T_3^f - Q_f \sin^2\theta_W, \qquad g_A^f = \frac{1}{2}T_3^f$$

Using

$$P_L = \frac{1-\gamma_5}{2}, \qquad P_R = \frac{1+\gamma_5}{2},$$

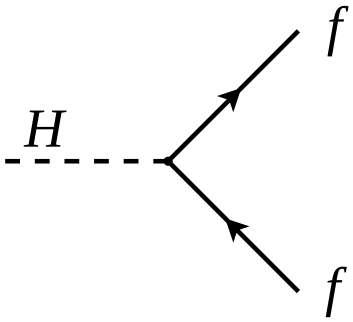
We get

$$g_V^f - g_A^f\gamma_5 = g_L^f P_L + g_R^f P_R$$

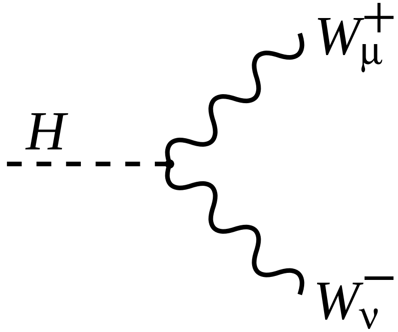
where

$$g_L^f = g_V^f + g_A^f = T_3^f - Q_f\sin^2\theta_W, \qquad g_R^f = g_V^f - g_A^f = -Q_f \sin^2\theta_W$$

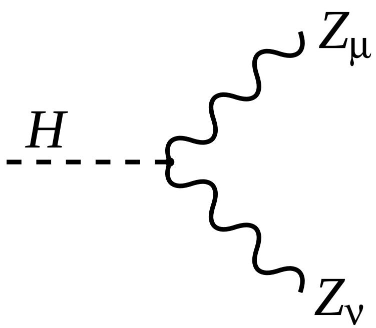
Higgs Boson Couplings



$$-i\frac{g}{2}\,\frac{m_f}{m_W}$$



$$i g\, m_W\, g_{\mu\nu}$$



$$i\frac{g}{\cos\theta_W}\, m_Z\, g_{\mu\nu}$$

Polarization Vectors

- In many problems with massive gauge bosons we do not measure their polarization, and therefore we sum over all polarizations using the well known result,

$$\sum_\lambda \varepsilon^\mu(k,\lambda)\varepsilon^{*\nu}(k,\lambda) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{M_W^2}$$

where we used the W boson as an example.

- As we will be considering the case of longitudinal polarized gauge bosons, we have to review the expressions for the polarization vectors.
- Let us start with the case of longitudinal polarization. In the gauge boson rest frame where $p^\mu = (M_W, 0, 0, 0)$, the longitudinal polarization vector is

$$\varepsilon_L^\mu(p) = (0,0,0,1), \quad \text{satisfying} \quad \varepsilon_L(p)\cdot\varepsilon_L(p) = -1, \varepsilon_L(p)\cdot p = 0\ .$$

Polarization Vectors $\cdots$

- In the frame where the gauge boson is moving with velocity $\vec{\beta}$ we have

$$\varepsilon_L^\mu(p) = (\gamma\beta, \gamma\hat{\beta})$$

where, $\vec{\beta} = \vec{p}/E$, $\gamma^{-1} = \sqrt{1-\beta^2}$ e $\hat{\beta} = \vec{\beta}/\beta$, verifying the invariant relations $\varepsilon_L(p) \cdot \varepsilon_L(p) = -1$ e $\varepsilon_L(p) \cdot p = 0$. For $E \gg M_W$ we have approximately

$$\varepsilon_L^\mu(p) \simeq \frac{p^\mu}{M_W}$$

- We should notice that this relation is only approximate (the invariant relations are violated by terms of the order of M_W^2/E^2). A better approximation is

$$\varepsilon_L^\mu(p) = \frac{p^\mu}{M_W} + \frac{1}{2\gamma^2}\frac{Q^\mu}{M_W} + \cdots$$

where

$$Q^\mu = (-E, \vec{p})$$

Polarization Vectors · · ·

- With this relation one can show that

$$\varepsilon_L(p) \cdot \varepsilon_L(p) = -1 + \mathcal{O}\left(\frac{1}{\gamma^4}\right), \quad \varepsilon_L(p) \cdot p = 0 + \mathcal{O}\left(\frac{1}{\gamma^4}\right)$$

Depending on the case, one can use the simpler, the more correct or the exact expression.

- Now we consider the case of transverse polarized gauge bosons. Here we have in the frame where the gauge boson moves along the z axis, that is, $p^\mu = E_W(1,0,0,\beta)$ that the two degrees of transverse polarization can be written as

$$\varepsilon_T^\mu = (0, \vec{\varepsilon}_{T_{1,2}})$$

where $\vec{\varepsilon}_{T_{1,2}}$ lie in the plane perpendicular to the z axis. Any combination of linearly independent vectors will do, but there are obviously two preferred choices.

Polarization Vectors $\cdots$

- The first one, corresponds to two linearly polarized perpendicular vectors,

$$\varepsilon_{T_1}^{\mu} = (0,1,0,0), \quad \varepsilon_{T_1} = (0,0,1,0)$$

and the other choice corresponds to helicity $\pm$,

$$\varepsilon_{\pm}^{\mu} = (0, \frac{1}{\sqrt{2}}, \pm\frac{i}{\sqrt{2}}, 0)$$

satisfying the correct normalization conditions $\varepsilon(p) \cdot \varepsilon^*(p) = -1, \varepsilon(p) \cdot p = 0$.

- Note that for the case of helicity vectors, as they are complex, we have to take the complex conjugate in the normalization condition. In real situations the gauge bosons will be moving along some direction $\vec{\beta} = \vec{k}/E_W$ with respect to some reference frame, usually defined by the incident particles.

Polarization Vectors $\cdots$

If we have the situation indicated in Fig. 1

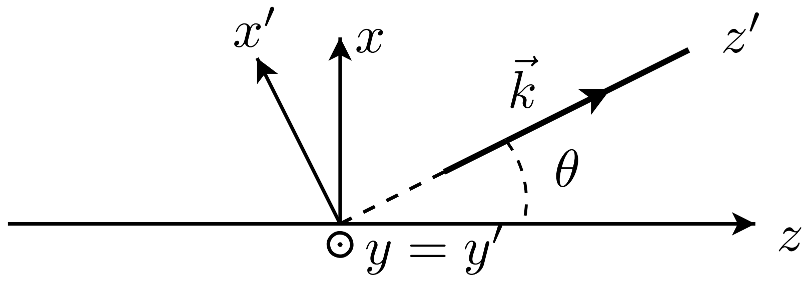


Figure 1: Kinematics of the gauge boson

then we will have for the two choices of transverse polarization vectors,

$$\varepsilon^\mu(\lambda = \pm) = (0, \frac{1}{\sqrt{2}}\cos\theta, \pm\frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\sin\theta)$$

and

$$\varepsilon^\mu(\lambda = \vec{e}_x) = (0, \cos\theta, 0, -\sin\theta), \quad \varepsilon^\mu(\lambda = \vec{e}_y) = (0, 0, 1, 0)$$

Polarization Vectors ···

❐ We can verify that using these definitions of we can reproduce the general result of the sum over all polarizations

$$\sum_{\lambda} \varepsilon^{\mu}(k,\lambda)\varepsilon^{*\nu}(k,\lambda) = -g^{\mu\nu} + \frac{k^{\mu}k^{\nu}}{M_W^2}$$

Take for instance

- $\varepsilon_T^{\mu}(1) = (0, \cos\theta, 0, \sin\theta)$
- $\varepsilon_T^{\mu}(2) = (0, 0, 1, 0)$
- $\varepsilon_L^{\mu} = (\gamma\beta, \gamma\sin\theta, 0, \gamma\cos\theta)$

❐ Also a more compact expression for the polarization vector, valid for all polarizations, can be shown to be given by,

$$\varepsilon^{\mu} = \left[\gamma(\vec{\beta}\cdot\vec{\varepsilon}), \vec{\varepsilon} + (\gamma-1)(\hat{\beta}\cdot\vec{\varepsilon})\hat{\beta}\right]$$

where $\vec{\varepsilon}$ is the polarization vector in the rest frame of the gauge boson.

Unitarity and Growth of the Amplitudes with $\sqrt{s}$

- The last topic we want to discuss in this introduction is the behavior of the amplitudes with the growth of the center of mass energy.
- For these $1 + 2 \to 3 + 4$ processes, it can be shown, that the amplitudes for large values of $\sqrt{s}$ should, at most, be constant with the energy,

$$\lim_{\sqrt{s}\to\infty} \mathcal{M} = \text{constant}.$$

- This in turn will imply that the cross sections given by,

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2\, s}\, \frac{|\vec{p}_{3CM}|}{|\vec{p}_{1CM}|}\, \overline{|\mathcal{M}|}^2$$

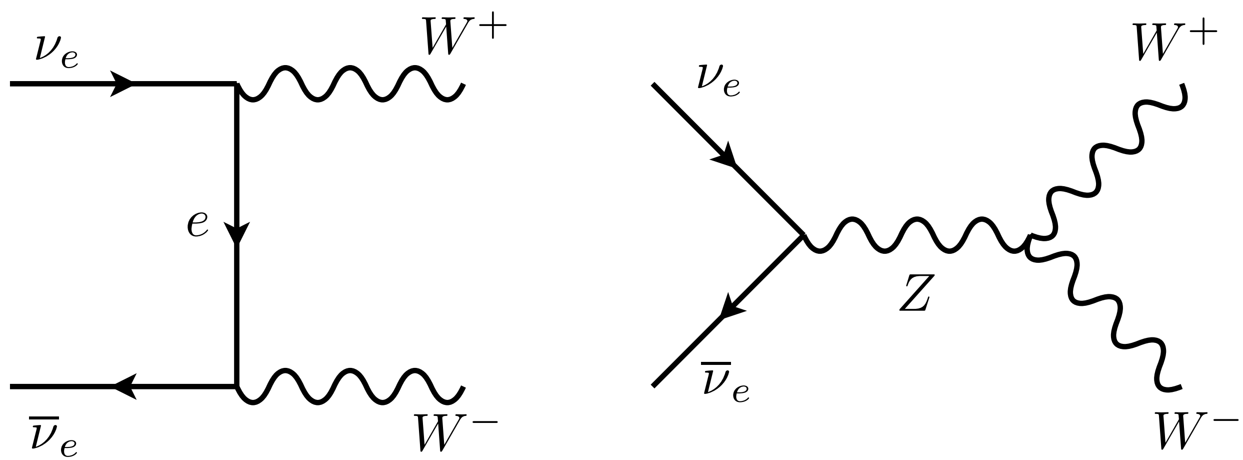
will decrease for values of $\sqrt{s} \gg M$, where M is any mass in the problem.

The scattering $\nu_e + \overline{\nu}_e \to W_L^+ + W_L^-$

- As a first exercise on the structure of the SM we will look at the process

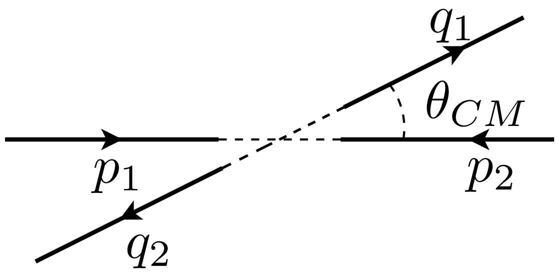
$$\nu_e(p_1) + \overline{\nu}_e(p_2) \to W_L^+(q_1) + W_L^-(q_2)$$

- In the SM this process has the tree-level diagrams



Kinematics for $\nu_e + \overline{\nu}_e \to W_L^+ + W_L^-$

The kinematics in the CM frame is given in the figure, where θ_{CM} is the scattering angle in the CM frame.



which can be written as

$$\begin{cases} p_1 = \dfrac{\sqrt{s}}{2}(1,0,0,1) \\ p_2 = \dfrac{\sqrt{s}}{2}(1,0,0,-1) \\ q_1 = \dfrac{\sqrt{s}}{2}(1,\beta\sin\theta_{\rm CM},0,\beta\cos\theta_{CM}) \\ q_2 = \dfrac{\sqrt{s}}{2}(1,-\beta\sin\theta_{\rm CM},0,-\beta\cos\theta_{CM}) \end{cases}$$

where β is the velocity of the W in the CM frame,

$$\beta = \sqrt{1 - \frac{4M_W^2}{s}}$$

It is also important to write down the explicit expressions for the longitudinal polarization vectors. We have

$$\varepsilon_L^\mu(q_1) = \frac{\sqrt{s}}{2M_W}(\beta, \sin\theta_{\text{CM}}, 0, \cos\theta_{CM})$$

$$\varepsilon_L^\mu(q_2) = \frac{\sqrt{s}}{2M_W}(\beta, -\sin\theta_{\text{CM}}, 0, -\cos\theta_{CM})$$

The Amplitudes

- Let us begin by writing the amplitudes for the two diagrams. We have (in this case it is safe to neglect the electron mass in the propagator)

$$\mathcal{M}_t = -\frac{g^2}{2}\overline{v}(p_2)\gamma_\nu(\not{p}_1 - \not{q}_1)\gamma_\mu u(p_1)\frac{1}{t}\ \varepsilon_L^\mu(q_1)\varepsilon_L^\nu(q_2)$$

$$\begin{aligned}\mathcal{M}_s &= -\frac{g^2}{2}\overline{v}(p_2)\gamma_\alpha P_L u(p_1)\left[-g^{\alpha\beta} + \frac{(p_1+p_2)^\alpha(p_1+p_2)^\beta}{M_W^2/c_W^2}\right] \\ &\quad \times \frac{\Gamma_{\mu\nu\beta}(-q_1,-q_2,q_1+q_2)}{s - M_W^2/c_W^2}\ \varepsilon_L^\mu(q_1)\varepsilon_L^\nu(q_2) \\ &= \frac{g^2}{2}\overline{v}(p_2)\gamma^\alpha P_L u(p_1)\frac{\Gamma_{\mu\nu\alpha}(-q_1,-q_2,q_1+q_2)}{s - M_W^2/c_W^2}\ \varepsilon_L^\mu(q_1)\varepsilon_L^\nu(q_2)\end{aligned}$$

- We have used the SM relation $M_W = M_Z c_W$ with $c_W = \cos\theta_W$
- In the last step, we have used the Dirac equation and the fact that the neutrinos (in the SM) have no mass to simplify the numerator of the Z propagator.

Unitarity and the cancellation of the bad behavior

- Each of the amplitudes do not obey the constraint from unitarity.
- To see this one has to understand that at high energy we have

$$\overline{v}(p_2)u(p_1) \propto \sqrt{s}, \quad p_i, q_i \propto \sqrt{s}, \quad \varepsilon_L^\mu(q_i) \propto \frac{\sqrt{s}}{M_W}$$

and therefore

$$\mathcal{M}_t, \mathcal{M}_s \propto s$$

- Then, unless there is a cancellation between the fastest growing terms with $\sqrt{s}$, we will have a problem. This cancellation comes about because gauge invariance of the theory forces the couplings to be related.
- To see this let us start by the simplest formula for the longitudinal polarization, $\varepsilon_L^\mu \simeq p^\mu / M_W$.

- Using this relation we obtain

$$\mathcal{M}_t = \frac{g^2}{2M_W^2}\overline{v}(p_2)\not{q}_2 P_L u(p_1) + \mathcal{O}(1) \ .$$

which indeed shows that the amplitude grows with s.

- If we do the same for the s-channel diagram of the Z, we obtain,

$$\begin{aligned}\mathcal{M}_s &= \frac{g^2}{4M_W^2}\overline{v}(p_2)\not{q}_1 P_L u(p_1) - \frac{g^2}{4M_W^2}\overline{v}(p_2)\not{q}_2 P_L u(p_1) + \mathcal{O}(1)\\ &= -\frac{g^2}{2M_W^2}\overline{v}(p_2)\not{q}_2 P_L u(p_1) + \mathcal{O}(1)\end{aligned}$$

where we have used $q_1 = -q_2 + p_1 + p_2$ and the Dirac equation for massless neutrinos. We therefore obtain,

$$\mathcal{M} = \mathcal{M}_\nu + \mathcal{M}_Z = \mathcal{O}(1)$$

in agreement with the unitarity argument. It should be stressed that the cancellation depends on the relation between the couplings of different particles and these were dictated by gauge invariance.

Let us calculate the cross section to display the effect of the cancellation. Writing, in an obvious notation,

$$|\mathcal{M}|^2 = |\mathcal{M}_t|^2 + |\mathcal{M}_s|^2 + (\mathcal{M}_t\mathcal{M}_s^* + \mathcal{M}_t^*\mathcal{M}_s)$$

we get (we used the `Mathematica` package `FeynCalc`),

- t-channel

$$\begin{aligned}|\mathcal{M}_t|^2 &= -\frac{g^4\left(4M_W^4 + st\right)^2\left[M_W^4 - 2M_W^2 t + t(s+t)\right]}{4M_W^4 t^2\left(s - 4M_W^2\right)^2}\\ &= g^4\left[\sin^2\theta\ x^2 + \sin^2\theta\ x - \cos^2\theta - \cos\theta + \mathcal{O}\left(x^{-1}\right)\right]\end{aligned}$$

where we have defined $x = \dfrac{s}{4M_W^2}$ and the expansion is for $x \gg 1$.

Cross section ⋯

- For the s-channel Z exchange we get

$$|\mathcal{M}_s|^2 = -\frac{g^4\left(2M_W^2+s\right)^2\left(M_W^4-2M_W^2t+t(s+t)\right)}{4M_W^4\left(M_Z^2-s\right)^2}$$

$$= g^4\left[\sin^2\theta\ x^2 + \frac{M_Z^2\sin^2\theta}{2M_W^2}\ x + \frac{3M_Z^4\sin^2\theta}{16M_W^4} - \frac{3\sin^2\theta}{4} + \mathcal{O}\left(x^{-1}\right)\right]$$

- For the interference term

$$(\mathcal{M}_t\mathcal{M}_s^* + \mathcal{M}_t^*\mathcal{M}_s) =$$

$$= \frac{g^4\left(8M_W^{10}+4M_W^8(s-4t)+2M_W^6t(s+4t)+5M_W^4s^2t+2M_W^2st^3+s^2t^2(s+t)\right)}{2M_W^4t\left(4M_W^2-s\right)\left(M_Z^2-s\right)}$$

$$= g^4\left[-2\sin^2\theta\ x^2 - \sin^2\theta\ x - \frac{M_Z^2\sin^2\theta}{2M_W^2}\ x + \cos\theta + 1 - \frac{M_Z^4\sin^2\theta}{8M_W^4}\right.$$

$$\left. - \frac{M_Z^2\sin^2\theta}{4M_W^2} + \mathcal{O}\left(x^{-1}\right)\right]$$

- One can see that the terms proportional to x^2 and x exactly cancel.

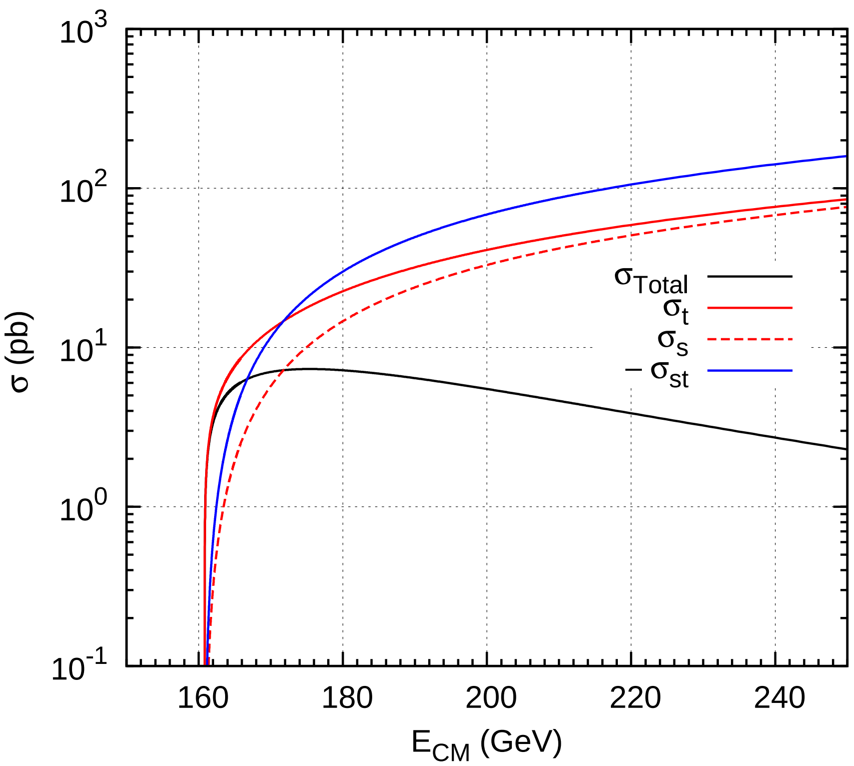
Cross section · · ·

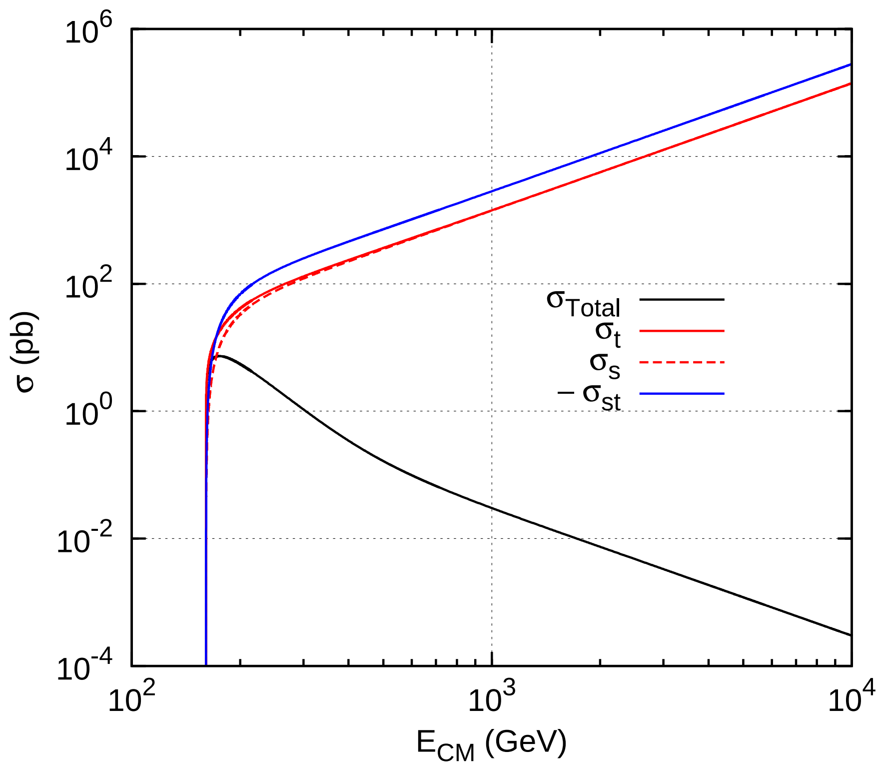
- The high energy the behavior is

$$|\mathcal{M}|^2 = \frac{g^4 \sin^2\theta \left(M_Z^2 - 2M_W^2\right)^2}{16 M_W^4} + \mathcal{O}\left(x^{-1}\right)$$

consistent with unitarity.

- The behavior of the various terms is shown in the figure.





The scattering $W_L^- + W_L^+ \to W_L^- + W_L^+$

- The next process that we will consider is the scattering of longitudinal $W_L^\pm$.

$$W_L^-(p_1) + W_L^+(p_2) \to W_L^-(q_1) + W_L^+(q_2)$$

where the momenta are as indicated and the subscript L means that the gauge bosons $W^\pm$ are longitudinally polarized.

- In the SM this process has seven tree-level diagrams

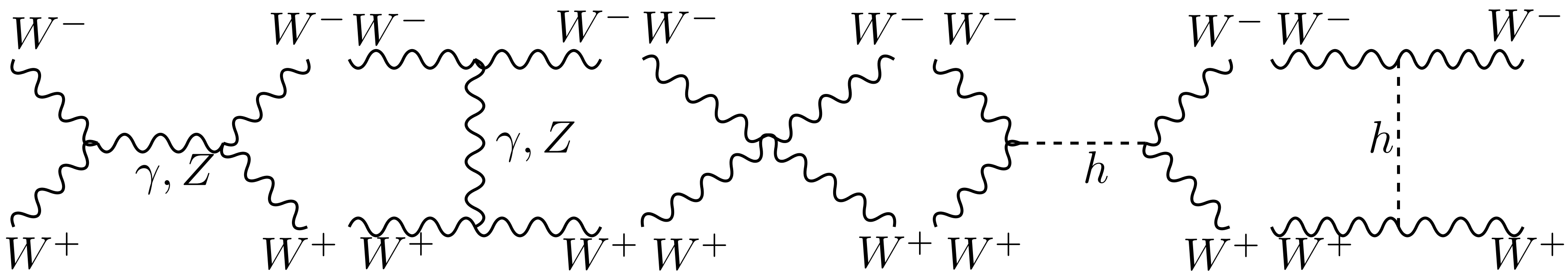


Figure 2: Diagrams contributing to $W_L^- + W_L^+ \to W_L^- + W_L^+$.

Kinematics for $W_L^- + W_L^+ \to W_L^- + W_L^+$

$$
\begin{cases}
p_1 = \dfrac{\sqrt{s}}{2}(1,0,0,\beta) \\
p_2 = \dfrac{\sqrt{s}}{2}(1,0,0,-\beta) \\
q_1 = \dfrac{\sqrt{s}}{2}(1,\beta\sin\theta_{\rm CM},0,\beta\cos\theta_{CM}) \\
q_2 = \dfrac{\sqrt{s}}{2}(1,-\beta\sin\theta_{\rm CM},0,-\beta\cos\theta_{CM})
\end{cases}
\begin{cases}
\varepsilon_L(p_1) = \dfrac{\sqrt{s}}{2M_W}(\beta,0,0,1) \\
\varepsilon_L(p_2) = \dfrac{\sqrt{s}}{2M_W}(\beta,0,0,-1) \\
\varepsilon_L(q_1) = \dfrac{\sqrt{s}}{2M_W}(\beta,\sin\theta_{\rm CM},0,\cos\theta_{CM}) \\
\varepsilon_L(q_2) = \dfrac{\sqrt{s}}{2M_W}(\beta,-\sin\theta_{\rm CM},0,-\cos\theta_{CM})
\end{cases}
$$

- $\beta = \sqrt{1 - 4M_W^2/s}$
- Notice that the invariant relations of the type $\varepsilon_L(p_1)\cdot\varepsilon_L(p_1) = -1$ and $\varepsilon_L(p_1)\cdot p_1 = 0$ are verified for all cases.
- This case is very interesting, because not only the gauge structure is necessary for the amplitudes to have the correct behavior, as in the previous case, but also the Higgs boson is fundamental.

The Amplitudes

Let us denote, in an obvious notation, the amplitudes as

$$\mathcal{M} = \mathcal{M}^s_{\gamma+Z} + \mathcal{M}^t_{\gamma+Z} + \mathcal{M}_{4W} + \mathcal{M}^{s+t}_H$$

We have then,

$$\mathcal{M}^s_\gamma = \frac{g^2 s_W^2}{s}\ \epsilon_L^\alpha(p_1)\epsilon_L^\beta(p_2)\epsilon_L^\gamma(q_1)\epsilon_L^\delta(q_2)\ \Gamma_{\alpha,\beta,\mu}(p_1,p_2,-p_1-p_2) \\ \times \Gamma_{\delta,\gamma,\nu}(-q_2,-q_1,p_1+p_2)\ g^{\mu\nu}$$

$$\mathcal{M}^s_Z = \frac{g^2 c_W^2}{s - M_W^2/c_W^2}\ \epsilon_L^\alpha(p_1)\epsilon_L^\beta(p_2)\epsilon_L^\gamma(q_1)\epsilon_L^\delta(q_2)\ \Gamma_{\alpha,\beta,\mu}(p_1,p_2,-p_1-p_2) \\ \times \Gamma_{\delta,\gamma,\nu}(-q_2,-q_1,p_1+p_2)\left[g^{\mu\nu} - \frac{(p_1+p_2)^\mu(p_1+p_2)^\nu}{M_W^2/c_W^2}\right]$$

$$\mathcal{M}^t_\gamma = \frac{g^2 s_W^2}{t}\ \epsilon_L^\alpha(p_1)\epsilon_L^\beta(p_2)\epsilon_L^\gamma(q_1)\epsilon_L^\delta(q_2)\ \Gamma_{\alpha,\gamma,\mu}(p_1,-q_1,q_1-p_1) \\ \times \Gamma_{\delta,\beta,\nu}(-q_2,p_2,q_2-p_2)\ g^{\mu\nu}$$

The Amplitudes ···

$$\mathcal{M}_Z^t = \frac{g^2 c_W^2}{t - M_W^2/c_W^2}\, \epsilon_L^\alpha(p_1)\epsilon_L^\beta(p_2)\epsilon_L^\gamma(q_1)\epsilon_L^\delta(q_2)\ \Gamma_{\alpha,\gamma,\mu}(p_1,-q_1,q_1-p_1)$$

$$\times\, \Gamma_{\delta,\beta,\nu}(-q_2,p_2,q_2-p_2)\left[g^{\mu\nu} - \frac{(p_1-q_1)^\mu (p_1-q_1)^\nu}{M_W^2/c_W^2}\right]$$

$$\mathcal{M}_{4W} = g^2\ \epsilon_L^\alpha(p_1)\epsilon_L^\beta(p_2)\epsilon_L^\gamma(q_1)\epsilon_L^\delta(q_2)\ \left[2g_{\alpha\delta}\, g_{\beta\gamma} - g_{\alpha\beta}\, g_{\delta\gamma} - g_{\alpha\gamma}\, g_{\delta\beta}\right]$$

$$\mathcal{M}_H^s = -\frac{g^2 M_W^2}{s - M_H^2}\, \epsilon_L^\alpha(p_1)\epsilon_L^\beta(p_2)\epsilon_L^\gamma(q_1)\epsilon_L^\delta(q_2)\ g_{\alpha\beta}\, g_{\gamma\delta}$$

$$\mathcal{M}_H^t = -\frac{g^2 M_W^2}{t - M_H^2}\, \epsilon_L^\alpha(p_1)\epsilon_L^\beta(p_2)\epsilon_L^\gamma(q_1)\epsilon_L^\delta(q_2)\ g_{\alpha\gamma}\, g_{\beta\delta}$$

- Where $s_W^2 = \sin^2\theta_W$, $c_W^2 = \cos^2\theta_W$, and use the SM relations $M_W = c_W M_Z$ and $e = g s_W$.
- If we insert the $\epsilon_L^\mu \simeq p^\mu/M_W$ we see that the amplitudes can grow potentially as s^2 or even s^3.
- We calculate the amplitudes using the exact expressions for ϵ_L^μ

The Amplitudes $\cdots$

$$\mathcal{M}_\gamma^s = \frac{g^2 s_W^2}{4M_W^4 s}\left(2M_W^2 + s\right)^2\left(4M_W^2 - s - 2t\right)$$

$$\mathcal{M}_Z^s = \frac{g^2\, c_W^2}{4M_W^4\left(s - M_W^2/c_W^2\right)}\left(2M_W^2 + s\right)^2\left(4M_W^2 - s - 2t\right)$$

$$\begin{aligned}\mathcal{M}_\gamma^t = \frac{g^2 s_W^2}{4M_W^4 t\left(s - 4M_W^2\right)^2}\Big[&256M_W^{10} - 64M_W^8(4s+t) + 16M_W^6 s(5s+14t)\\ &- 4M_W^4 s\left(2s^2 + 21st + 20t^2\right) + 8M_W^2 s^2 t(s+3t) - s^2t^2(2s+t)\Big]\end{aligned}$$

$$\begin{aligned}\mathcal{M}_Z^t = \frac{g^2 c_W^2}{4M_W^4\left(s - 4M_W^2\right)^2\left(t - M_W^2/c_W^2\right)}\Big[&256M_W^{10} - 64M_W^8(4s+t) + 16M_W^6 s(5s+14t)\\ &- 4M_W^4 s\left(2s^2 + 21st + 20t^2\right) + 8M_W^2 s^2 t(s+3t) - s^2t^2(2s+t)\Big]\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{4W} = \frac{g^2 s}{4M_W^4\left(s - 4M_W^2\right)^2}\Big[&-64M_W^6 + 48M_W^4(s+t) - 4M_W^2 s(3s+7t)\\ &+s\left(s^2 + 4st + t^2\right)\Big]\end{aligned}$$

The Amplitudes ···

And finally for the Higgs mediated diagrams,

$$\mathcal{M}_H^s = -\,g^2 \frac{\left(s - 2M_W^2\right)^2}{4M_W^2\,(s - M_H^2)}$$

$$\mathcal{M}_H^t = -\,g^2 \frac{\left(-8M_W^4 + 2M_W^2 s + st\right)^2}{4\,(t - M_H^2)\,(M_W s - 4M_W^3)^2}$$

- For $\sqrt{s} \gg M_W$ the first five amplitudes grow as s^2 and the last two (from the Higgs exchange) as s.
- We define, as before, the dimensionless variable $x = s/(4M_W^2)$, for $x \gg 1$ we should be able to write all amplitudes in the form

$$\mathcal{M}_i = A_i x^2 + B_i x + C_i + \mathcal{O}(1/x)$$

Unitarity and the cancellation of the bad behavior

$$\boxed{\mathcal{M}_i = A_i x^2 + B_i x + C_i + \mathcal{O}(1/x)}$$

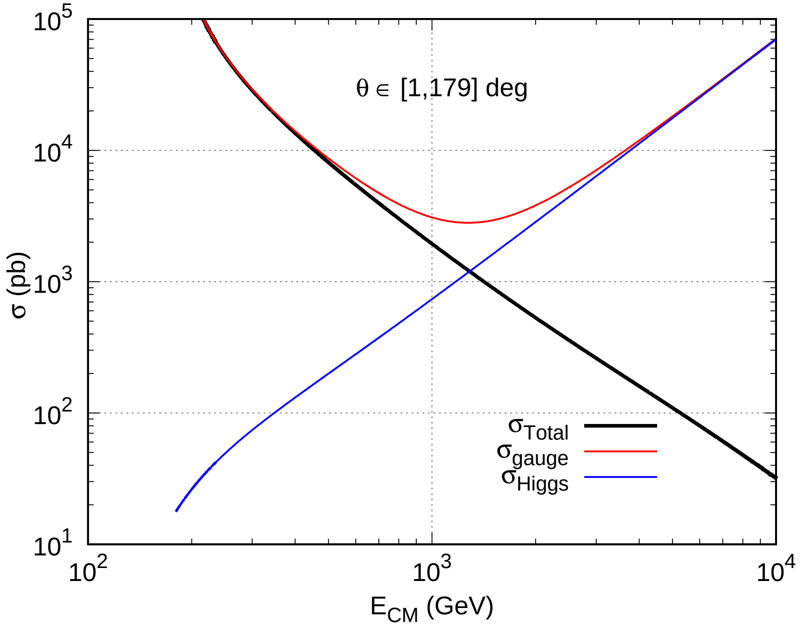
	A_i	B_i	C_i
$\mathcal{M}_\gamma^s$	$-g^2 4 s_W^2 \cos\theta$	0	$g^2 3 s_W^2 \cos\theta$
$\mathcal{M}_Z^s$	$-g^2 4 c_W^2 \cos\theta$	$-g^2 \cos\theta$	$g^2 \left[3\cos\theta\ c_W^2 - \dfrac{\cos\theta}{4 c_W^2}\right]$
$\mathcal{M}_\gamma^t$	$g^2\, s_W^2 \Big(-\cos^2\theta - 2\cos\theta + 3\Big)$	$g^2\, 8 s_W^2 \cos\theta$	$\dfrac{g^2 s_W^2}{\cos\theta - 1}\Big(-2\cos^2\theta - \cos\theta - 1\Big)$
$\mathcal{M}_Z^t$	$g^2\, c_W^2 \Big(-\cos^2\theta - 2\cos\theta + 3\Big)$	$g^2 \left(8\cos\theta c_W^2 - \dfrac{\cos\theta}{2} - \dfrac{3}{2}\right)$	$\dfrac{g^2}{\cos\theta - 1}\Big[-2\cos^2\theta c_W^2 - \dfrac{\cos^2\theta}{2} - \cos\theta c_W^2 - \dfrac{\cos\theta}{4c_W^2} + 3\cos\theta - c_W^2 - \frac{3}{4c_W^2} + \dfrac{3}{2}\Big]$
$\mathcal{M}_{4W}$	$g^2 \Big(\cos^2\theta + 6\cos\theta - 3\Big)$	$g^2\,(2 - 6\cos\theta)$	0
$\sum_{\gamma Z}$	0	$g^2 \dfrac{1+\cos\theta}{2}$	$g^2 3\cos\theta + \cdots$
$\mathcal{M}_H^s$	0	$-g^2$	$g^2 \left(1 - \dfrac{M_H^2}{4M_W^2}\right)$
$\mathcal{M}_H^t$	0	$g^2 \left(\dfrac{1}{2} - \dfrac{\cos\theta}{2}\right)$	$-g^2 \left[\dfrac{1+\cos\theta}{2} + \dfrac{M_H^2}{4M_W^2}\right]$
$\sum_{\gamma Z H}$	0	0	$\neq 0$

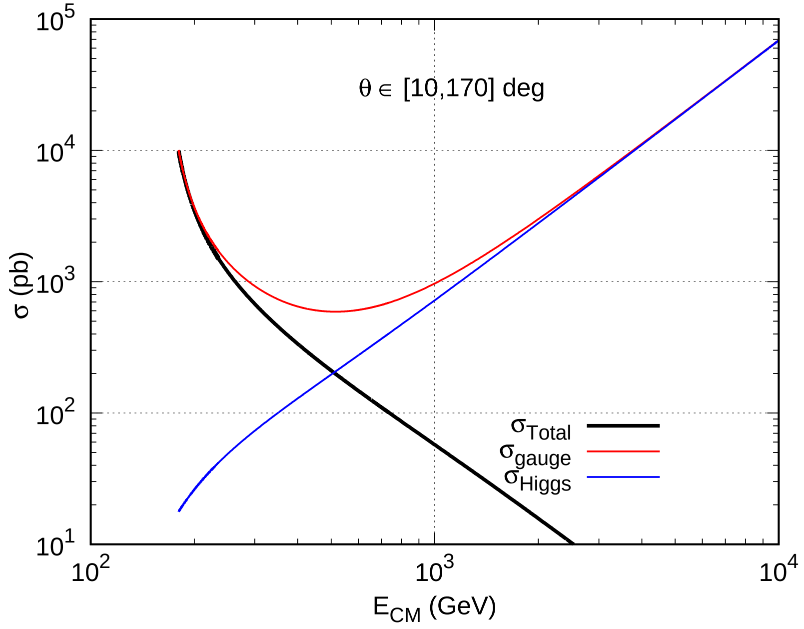
Unitarity $\cdots$

- We see that the terms proportional to x^2 cancel among the first five diagrams involving only the gauge bosons.
- The term proportional do x remains after we sum over the gauge part. So, if we consider only a gauge theory of intermediate gauge bosons, we are in trouble.
- This trouble can be traced back to the fact that with mass the gauge invariance is lost, and the theory is inconsistent if the diagrams involving the Higgs boson field are not taken in account.
- In conclusion, the Higgs boson is crucial to make the SM consistent.

Cross section

- As, in this case, the amplitudes are pure c-numbers with no spinor part, the cross section is simply obtained by summing all the amplitudes and taking the absolute value of the result to obtain $\mathcal{M}|^2$.
- As it is well known the t-channel contribution of the photon has a colinear divergence. We avoid that by making cuts in the scattering angle θ. We consider two cases, $\theta_{\min} = 1^\circ, 10^\circ$
- We get



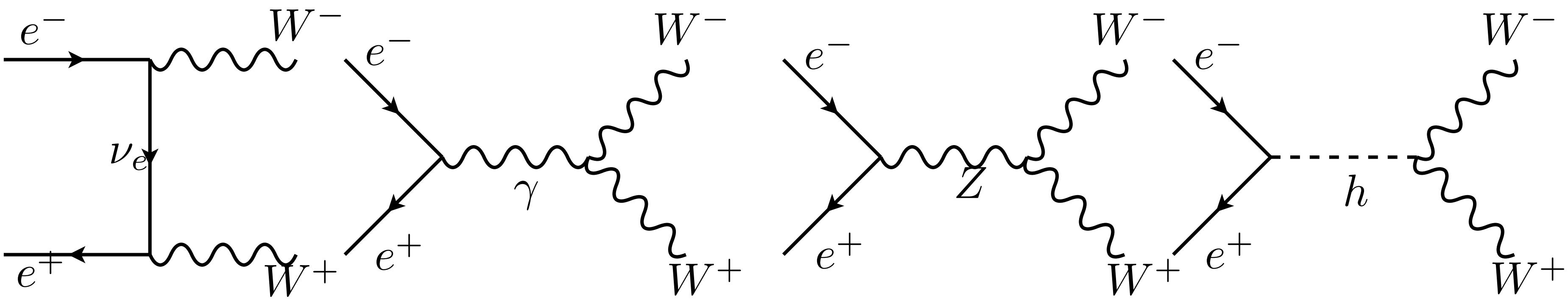


The scattering $e^- + e^+ \to W_L^- + W_L^+$

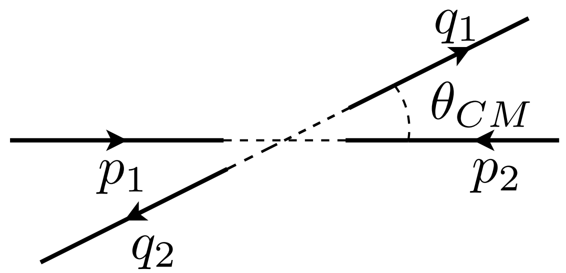
- As a last example of the importance of the Higgs boson for the consistency of the SM we consider the process,

$$e^-(p_1) + e^+(p_2) \to W_L^-(q_1) + W_L^+(q_2)$$

- Compared to other two, this process has the advantage of not being an academic problem, it has in fact already been tested at the LEP experiments.
- In the SM this process has four tree-level diagrams



Kinematics for $e^- + e^+ \to W_L^- + W_L^+$



$$\begin{cases} p_1 = \dfrac{\sqrt{s}}{2}(1,0,0,\beta_e) \\ p_2 = \dfrac{\sqrt{s}}{2}(1,0,0,-\beta_e) \\ q_1 = \dfrac{\sqrt{s}}{2}(1,\beta\sin\theta_{\rm CM},0,\beta\cos\theta_{CM}) \\ q_2 = \dfrac{\sqrt{s}}{2}(1,-\beta\sin\theta_{\rm CM},0,-\beta\cos\theta_{CM}) \end{cases}$$

$$\begin{cases} \varepsilon_L(q_1) = \dfrac{\sqrt{s}}{2M_W}(\beta,\sin\theta_{\rm CM},0,\cos\theta_{CM}) \\ \varepsilon_L(q_2) = \dfrac{\sqrt{s}}{2M_W}(\beta,-\sin\theta_{\rm CM},0,-\cos\theta_{CM}) \end{cases}$$

- Where, as before, $\beta = \sqrt{1-4M_W^2/s}$
- And, (we keep $m_e \neq 0$), $\beta_e = \sqrt{1-4m_e^2/s}$

The Amplitudes

- As the Higgs coupling to the electrons is proportional to the electron mass, we expect, that for sufficiently high energy there will be a piece of the first three diagrams proportional to m_e that will increase with energy.
- Therefore, we are not making any approximation with regard to m_e.

$$\mathcal{M}_\nu^t = -\frac{g^2}{2t}\overline{v}(p_2)\gamma_\nu P_L\left(\not{p}_1 - \not{q}_1\right)\gamma_\mu P_L u(p_1)\ \epsilon_L^\mu(q_1)\epsilon_L^\nu(q_2)$$

$$\mathcal{M}_\gamma^s = -\frac{g^2 s_W^2}{s}\overline{v}(p_2)\gamma^\alpha u(p_1)\ \Gamma_{\nu\mu\alpha}(-q_2, -q_1, q_1 + q_2)\ \epsilon_L^\mu(q_1)\epsilon_L^\nu(q_2)$$

$$\mathcal{M}_Z^s = -\frac{g^2}{s - M_W^2/c_W^2}\left[-g_{\alpha\beta} + \frac{Q^\alpha Q^\beta}{M_W^2/c_W^2}\right]\overline{v}(p_2)\gamma^\beta\left(g_L P_L + g_R P_R\right)u(p_1)$$
$$\times\,\Gamma_{\nu\mu\alpha}(-q_2, -q_1, q_1 + q_2)\ \epsilon_L^\mu(q_1)\epsilon_L^\nu(q_2)$$

$$\mathcal{M}_H^s = \frac{g^2 m_e}{2}\frac{1}{s - M_H^2}\ \overline{v}(p_2)u(p_1)\ g_{\mu\nu}\ \epsilon_L^\mu(q_1)\epsilon_L^\nu(q_2)$$

The Amplitudes $\cdots$

- Where we have defined $Q = p_1 + p_2$, and made use again of the SM relations like $e = g s_W$.
- If we neglect for the moment m_e this process is very much like the neutrino scattering.
- One can convince ourselves that the first three amplitudes are proportional to s for $\sqrt{s} \gg m_e, M_W$.
- However we should be careful, because the Higgs exchange diagram is proportional to $m_e\sqrt{s}$ and that these terms, although subleading, will be important for sufficiently high energy.

Cancellation of bad high energy behavior

With the help of `FeynCalc` we obtain

$$\mathcal{M}_\nu = \frac{g^2}{2M_W^2} \left[-\overline{v}(p_2)\not{\epsilon}_1 P_L u(p_1) - m_e \overline{v}(p_2) P_L u(p_1) \right] + \mathcal{O}(1/x)$$

$$\mathcal{M}_\gamma = \frac{g^2 s_W^2}{M_W^2}\, \overline{v}(p_2)\not{\epsilon}_1 \left(P_L + P_R\right) u(p_1) + \mathcal{O}(1/x)$$

$$\mathcal{M}_Z = -\frac{g^2 s_W^2}{M_W^2}\, \overline{v}(p_2)\not{\epsilon}_1 \left(P_L + P_R\right) u(p_1) + \frac{g^2}{2M_W^2}\, \overline{v}(p_2)\not{\epsilon}_1 P_L u(p_1)$$

$$+ \frac{g^2}{4M_W^2}\, m_e \left[\overline{v}(p_2) P_L u(p_1) - \overline{v}(p_2) P_R u(p_1)\right] + \mathcal{O}(1/x)$$

$$\mathcal{M}_H = \frac{g^2}{4M_W^2}\, m_e \left[\overline{v}(p_2) P_L u(p_1) + \overline{v}(p_2) P_R u(p_1)\right] + \mathcal{O}(1/x)$$

Cancellation of bad high energy behavior ···

$$\mathcal{M}_\nu + \mathcal{M}_\gamma + \mathcal{M}_Z = -\frac{g^2}{4M_W^2}\, m_e \left[\overline{v}(p_2)P_L u(p_1) + \overline{v}(p_2)P_R u(p_1)\right] + \mathcal{O}(1/x)$$

$$\mathcal{M}_H = \frac{g^2}{4M_W^2}\, m_e \left[\overline{v}(p_2)P_L u(p_1) + \overline{v}(p_2)P_R u(p_1)\right] + \mathcal{O}(1/x)$$

$$\mathcal{M}_\nu + \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_H = \mathcal{O}(1/x)$$

Hence the Higgs boson exchange is needed to cancel the contribution of the amplitude that grows like $\dfrac{m_e\sqrt{s}}{M_W^2}$.

Cross section

To evaluate the cross section it is simpler to write the total amplitude as

$$\mathcal{M} = g^2 \overline{v}(p_2)\Gamma_{\mu\nu} u(p_1)\ \epsilon_L^{\mu}(q_1)\epsilon_L^{\nu}(q_2), \qquad \Gamma_{\mu\nu} = \sum_i \Gamma^i_{\mu\nu}$$

where the $\Gamma^i_{\mu\nu}$ are

$$\Gamma^{\nu t}_{\mu\nu} = -\frac{1}{2t}\gamma_\nu P_L \left(\not{p}_1 - \not{q}_1\right)\gamma_\mu P_L u(p_1)$$

$$\Gamma^{\gamma s}_{\mu\nu} = -\frac{s_W^2}{s}\gamma^\alpha \Gamma_{\nu\mu\alpha}(-q_2, -q_1, q_1 + q_2)$$

$$\Gamma^{Zs}_{\mu\nu} = -\frac{1}{s - M_W^2/c_W^2}\left[-g_{\alpha\beta} + \frac{Q^\alpha Q^\beta}{M_W^2/c_W^2}\right]\gamma^\beta \left(g_L P_L + g_R P_R\right) \times \Gamma_{\nu\mu\alpha}(-q_2, -q_1, q_1 + q_2)$$

$$\Gamma^{Hs}_{\mu\nu} = \frac{m_e}{2}\frac{1}{s - M_H^2} g_{\mu\nu}$$

- With the help of `FeynCalc` we evaluate

$$|\mathcal{M}|^2 = \frac{1}{4}\text{Tr}\left[(\not p_2 - m_e)\Gamma(\not p_1 + m_e)\overline{\Gamma}\right] \text{ where } \overline{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$$

- We get:

$$\begin{aligned}|\mathcal{M}_\nu|^2 = &-\frac{g^4}{16M_W^4 t^2\left(s-4M_W^2\right)^2}\Big[m_e^8 s^2 - 2m_e^6 s\left(4M_W^4 + M_W^2 s + 2st\right)\\ &+ m_e^4\left(16M_W^8 + 16M_W^6 s + M_W^4 s(s+24t) + 2M_W^2 s^2 t + s^2 t(s+6t)\right)\\ &- 2m_e^2\left(16M_W^{10} + 4M_W^8 s + 8M_W^6 st + 4M_W^4 st(s+3t) - M_W^2 s^2 t^2 + s^2t^2(s+2t)\right)\\ &+ \left(4M_W^4 + st\right)^2\left(M_W^4 - 2M_W^2 t + t(s+t)\right)\Big]\end{aligned}$$

$$|\mathcal{M}_\gamma|^2 = -\frac{g^4 s_W^4}{2M_W^4 s^2}\left(2M_W^2 + s\right)^2\left[m_e^4 + m_e^2\left(2M_W^2 - s - 2t\right) + M_W^4 - 2M_W^2 t + t(s+t)\right]$$

Cross section ···

$$
\begin{aligned}
|\mathcal{M}_Z|^2 =& \frac{g^4 c_W^4}{32 M_W^4 \left(M_W^2 - c_W^2 s\right)^2} \left(2M_W^2 + s\right)^2 \left[-2m_e^4 \left(8s_W^4 - 4s_W^2 + 1\right)\right.\\
&+ m_e^2 \left(4\left(-8M_W^2 s_W^4 + 4M_W^2 s_W^2 + 8s_W^4 t - 4s_W^2 t + t\right) + s\left(1 - 4s_W^2\right)^2\right)\\
&\left.-2\left(8s_W^4 - 4s_W^2 + 1\right)\left(M_W^4 - 2M_W^2 t + t(s+t)\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_\nu \mathcal{M}_\gamma^* =& \mathcal{M}_\nu^* \mathcal{M}_\gamma\\
=& \frac{g^4 s_W^2}{8 M_W^4 s t \left(4M_W^2 - s\right)} \left[m_e^6 s \left(2M_W^2 + s\right) + m_e^4 \left(2M_W^2 + s\right)\left(4M_W^4 - 4M_W^2 s - 3st\right)\right.\\
&+ m_e^2 s \left(6M_W^6 + M_W^4(3s + 4t) + 2M_W^2 t(2s + 3t) + st(s + 3t)\right)\\
&\left.- 8M_W^{10} - 4M_W^8(s - 4t) - 2M_W^6 t(s + 4t) - 5M_W^4 s^2 t - 2M_W^2 s t^3 - s^2 t^2 (s + t)\right]
\end{aligned}
$$

Cross section · · ·

$$
\begin{aligned}
\mathcal{M}_\nu \mathcal{M}_Z^* =& \mathcal{M}_\nu^* \mathcal{M}_Z = -\frac{g^4 c_W^2 \left(2M_W^2 + s\right)}{16 M_W^4 t \left(4M_W^2 - s\right)\left(M_W^2 - c_W^2 s\right)} \\
& \times \Big[m_e^6 \left(s - 2 s s_W^2\right) + m_e^4 \left(-8 M_W^4 s_W^2 + M_W^2 s \left(8 s_W^2 - 3\right) + 3s \left(2 s_W^2 - 1\right) t\right) \\
& + m_e^2 \left(4 M_W^6 + M_W^4 \left(-6 s s_W^2 + 2s + 4t\right) + M_W^2 s \left(1 - 4 s_W^2\right) t - s \left(2 s_W^2 - 1\right) t (s + 3t)\right) \\
& + \left(2 s_W^2 - 1\right)\left(4 M_W^8 - 8 M_W^6 t + M_W^4 t (5s + 4t) - 2 M_W^2 s t^2 + s t^2 (s + t)\right)\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_\gamma \mathcal{M}_Z^* =& \mathcal{M}_\gamma^* \mathcal{M}_Z = -\frac{g^4 c_W^2 s_W^2 \left(4 s_W^2 - 1\right)\left(2M_W^2 + s\right)^2}{8 M_W^4 s \left(M_W^2 - c_W^2 s\right)} \\
& \times \Big[m_e^4 + m_e^2 \left(2M_W^2 - s - 2t\right) + M_W^4 - 2 M_W^2 t + t(s+t)\Big]
\end{aligned}
$$

$$
|\mathcal{M}_H|^2 = -\frac{g^4 m_e^2 \left(4 m_e^2 - s\right)\left(s - 2M_W^2\right)^2}{32 M_W^4 \left(M_H^2 - s\right)^2}
$$

Cross section $\cdots$

$$
\begin{aligned}
\mathcal{M}_\nu \mathcal{M}_H^* =& \mathcal{M}_\nu^* \mathcal{M}_H = -\frac{g^4 m_e^2 \left(2M_W^2 - s\right)}{16 M_W^4 t \left(M_H^2 - s\right)\left(s - 4M_W^2\right)} \\
& \times \Big[m_e^4 s + m_e^2 \left(4M_W^4 - 3M_W^2 s - 2st\right) - 4M_W^6 + 2M_W^4 (s + 2t) - 3M_W^2 st \\
& + st(s+t) \Big] \\
\mathcal{M}_\gamma \mathcal{M}_H^* =& \mathcal{M}_\gamma^* \mathcal{M}_H = \frac{g^4 m_e^2 s_W^2 \left(2M_W^2 - s\right)\left(2M_W^2 + s\right)\left(2m_e^2 + 2M_W^2 - s - 2t\right)}{8 M_W^4 s \left(s - M_H^2\right)} \\
\mathcal{M}_Z \mathcal{M}_H^* =& \mathcal{M}_Z^* \mathcal{M}_H \\
=& \frac{g^4 c_W^2 m_e^2 \left(4s_W^2 - 1\right)\left(2M_W^2 - s\right)\left(2M_W^2 + s\right)\left(2m_e^2 + 2M_W^2 - s - 2t\right)}{32 M_W^4 \left(s - M_H^2\right)\left(M_W^2 - c_W^2 s\right)}
\end{aligned}
$$

The total $|\mathcal{M}|^2$ is

$$
\begin{aligned}
|\mathcal{M}|^2 =& |\mathcal{M}_\nu|^2 + |\mathcal{M}_\gamma|^2 + |\mathcal{M}_Z|^2 + |\mathcal{M}_H|^2 \\
& + \left(\mathcal{M}_\nu \mathcal{M}_\gamma^* + \mathcal{M}_\nu^* \mathcal{M}_\gamma\right) + \left(\mathcal{M}_\nu \mathcal{M}_Z^* + \mathcal{M}_\nu^* \mathcal{M}_Z\right) + \left(\mathcal{M}_\gamma \mathcal{M}_Z^* + \mathcal{M}_\gamma^* \mathcal{M}_Z\right) \\
& + \left(\mathcal{M}_\nu \mathcal{M}_H^* + \mathcal{M}_\nu^* \mathcal{M}_H\right) + \left(\mathcal{M}_\gamma \mathcal{M}_H^* + \mathcal{M}_\gamma^* \mathcal{M}_H\right) + \left(\mathcal{M}_Z \mathcal{M}_H^* + \mathcal{M}_Z^* \mathcal{M}_H\right)
\end{aligned}
$$

Cross section · · ·

- All these complicated expressions were obtained with the package `FeynCalc` for `Mathematica`.
- We manipulate the expressions using the functions `TeXForm` for `LaTeX` output and `FortranForm` for the `Fortran` output.
- In this way we minimize the errors of handling complicated expressions.

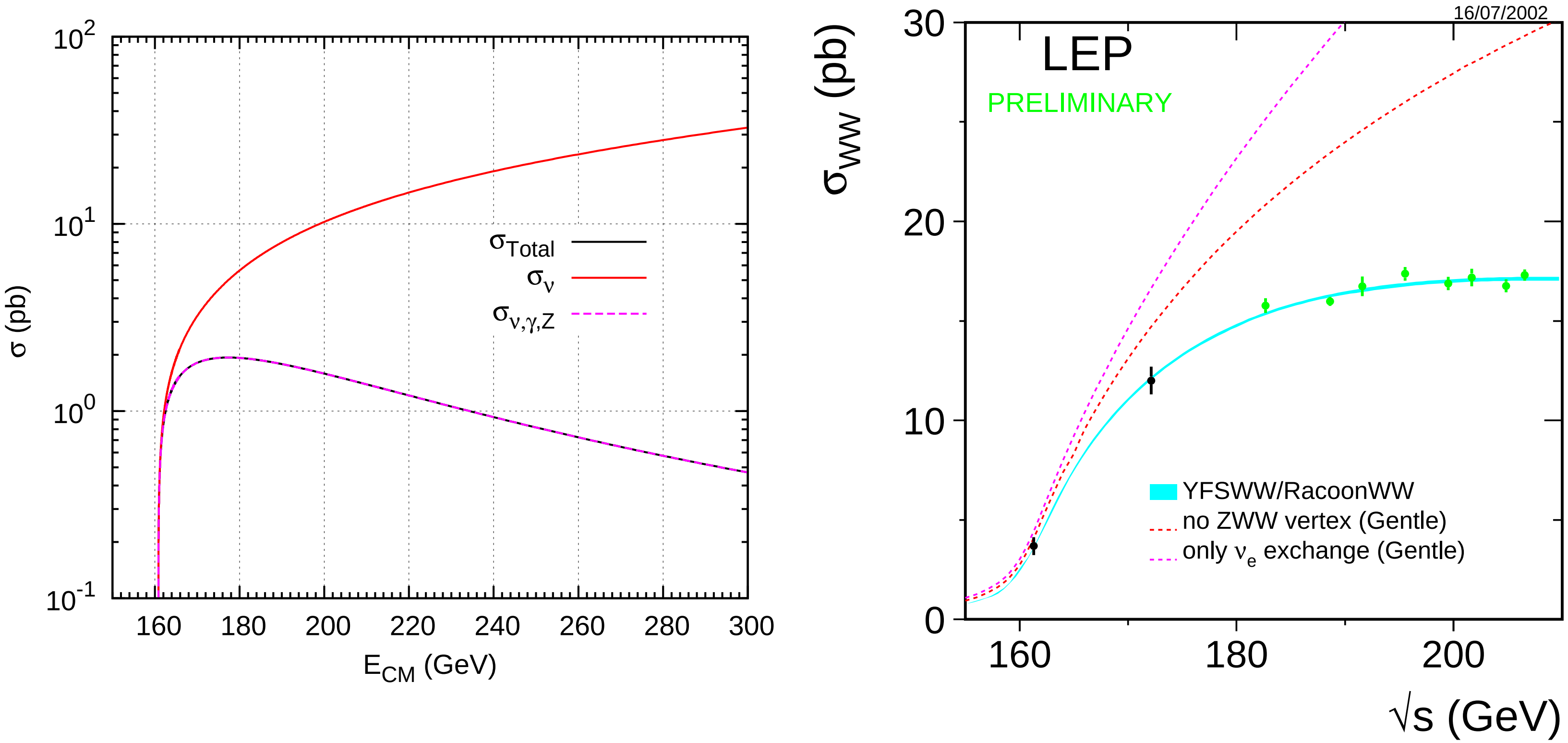


Figure 3: On the left panel it is shown the total cross section for $e^- + e^+ \to W_L^- + W_L^+$ (black). Also shown are the ν exchange cross section (red) and the sum of the ν exchange with the s-channel exchange of gauge bosons (magenta). On the right panel it is shown the LEP result for $e^- + e^+ \to W^- + W^+$.

Cross section $\cdots$

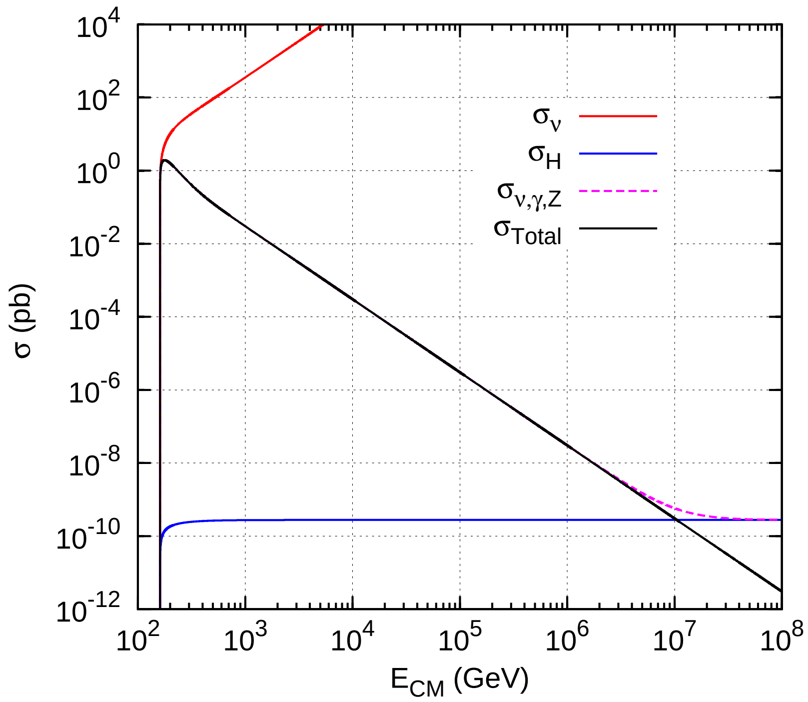
- At these energies the contribution of the diagrams involving the Higgs boson is very small, due to the smallness of the cross section the electron mass and can be completed neglected at LEP energies.
- One can estimate that the importance of the Higgs boson diagrams will show up when

$$\frac{m_e\sqrt{s}}{M_W^2} \simeq 1, \to \sqrt{s} \simeq 10^7 \text{GeV}$$

an energy completely outside the reach of man-made accelerators.

- However, from the consistency point of view, the cancellation of the bad behavior at those energies should be there.

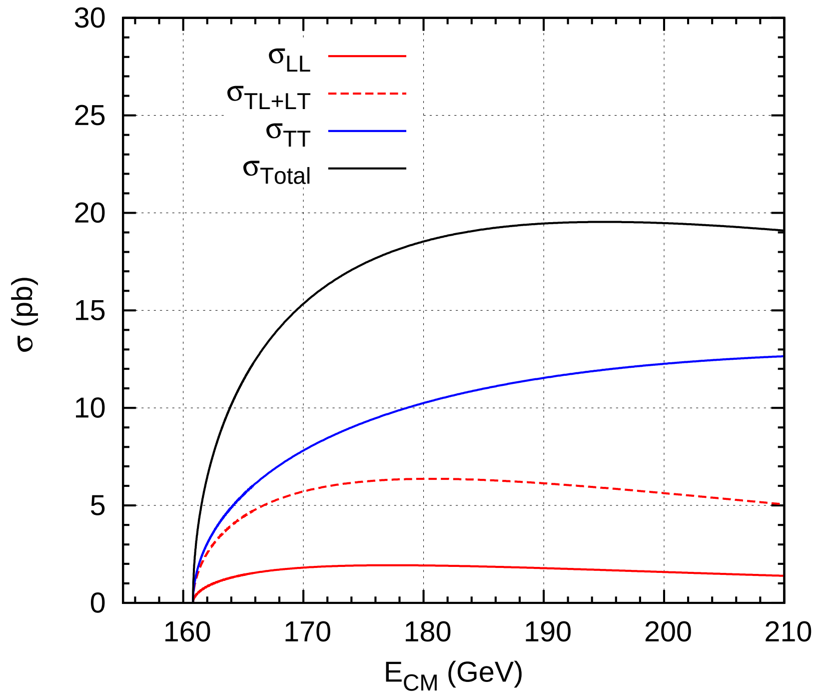
Cross section · · ·

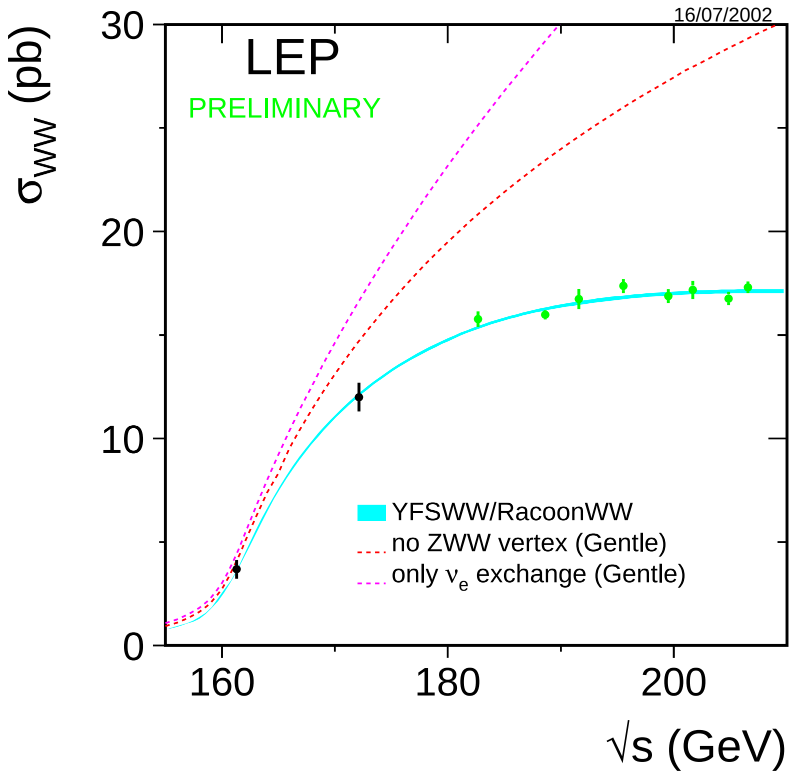


- In the Figure we show the contribution of the various pieces to the cross section of the process $e^- + e^+ \to W_L^- + W_L^+$. We see that for high enough energy, $\sqrt{s} \simeq M_W^2/m_e \simeq 10^7$ GeV,
- The Higgs boson contribution is crucial to avoid the cross section to be constant. If the electron mass was not so small, the effect would be seen much earlier as in the case of $W_L^- + W_L^+ \to W_L^- + W_L^+$.

Other Polarizations of the W's

As a last exercise let us calculate the cross sections for various final polarizations of the W bosons. First, let us look at the total cross sections for the various possibilities. This is shown in the Figure, confirming what we said before concerning the relative importance of the various polarizations. On the right panel of that figure we show LEP results for comparison.





Other Polarizations of the W's $\cdots$

One can also learn something from the angular distributions of the differential cross sections. These are shown in Fig. 4 for two values of the center of mass energy, the first one close to threshold, the second one close to the maximum of the cross section.

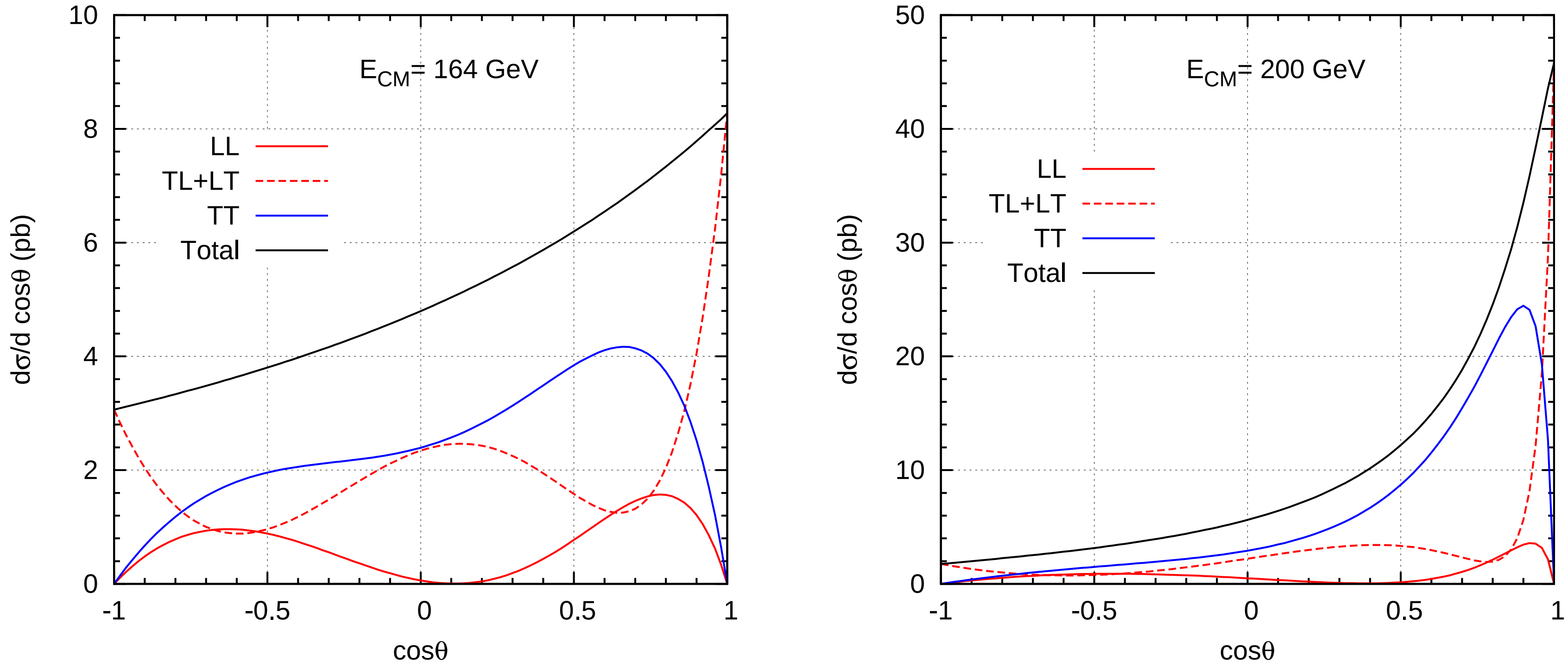


Figure 4: Angular dependence of the differential cross section for two values of the center of mass energy.

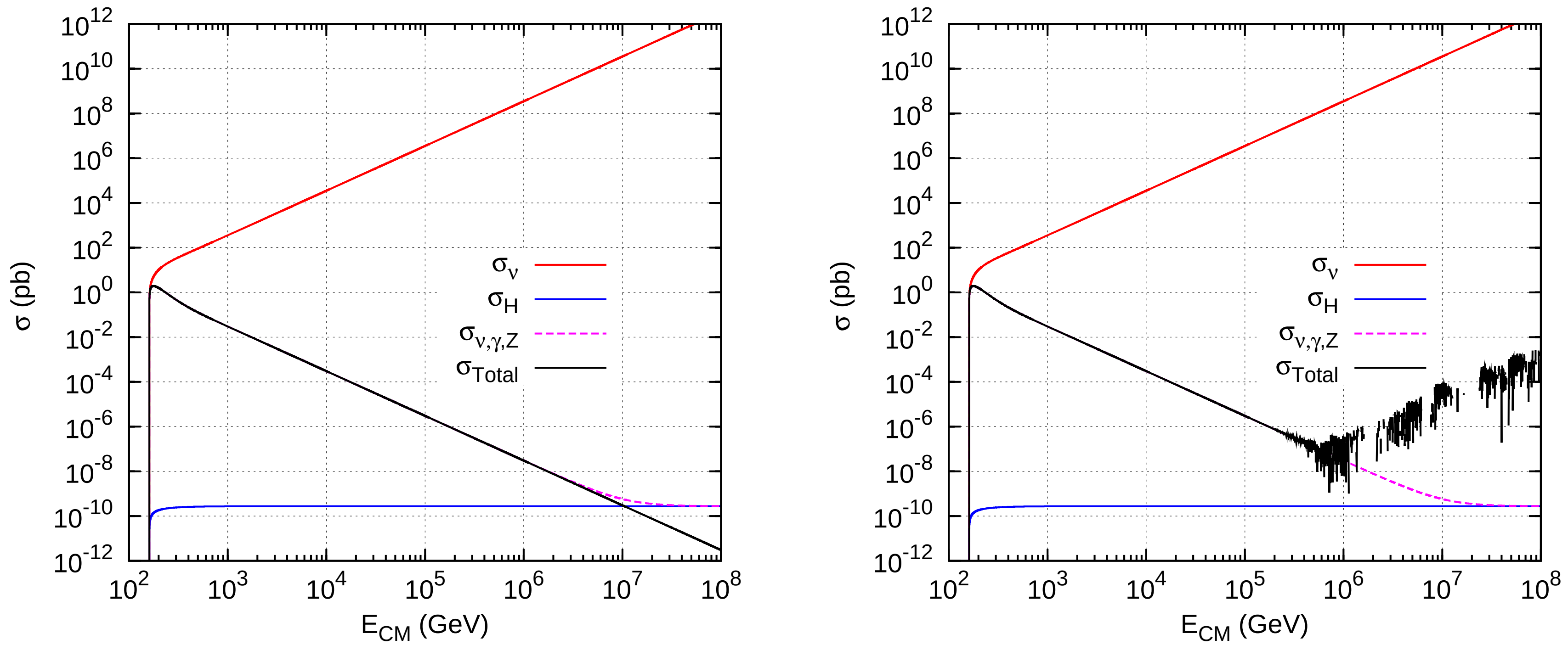
Other Polarizations of the W's $\cdots$

- The more relevant fact is that the cross section is peaked in the forward direction and this feature increases with increasing beam energy.
- The other relevant piece of information has to do with the fact that for $\theta = 0$, meaning the W^- in the same direction as the incident electron, both the TT and LL differential cross section vanish, while the TL+LT has a maximum.
- This can be understood as follows. Due to the (V-A) nature of the charged current interaction, the electron will have helicity $-1/2$ while the positron helicity $+1/2$, resulting in a total helicity -1 in the direction of the electron. So if the W bosons are produced in the same direction their total helicity must be -1 and that can only be achieved if one is transverse and the other longitudinal polarization, more precisely,

$$W^-(\lambda = -1), W^+(\lambda = 0), \quad \text{or} \quad W^+(\lambda = +1), W^-(\lambda = 0)$$

Precision and speed: `Mathematica`, `FORM` and `Fortran`

In the previous discussion the more experienced should have been a bit puzzled. To make the point, let us enlarge the plot.



Precision and speed $\cdots$

- We see that for $\sqrt{s} = 10^7$ GeV the cancellation has to be achieved better than one part in 10^{20}. For numerical calculations in `double precision` in `C` or `Fortran` this is a problem. How come that we could achieve this precision without resorting to `quadrupole precision`?.
- The key to the answer lies in the fact that our formula for the total cross section already has the bad behaviour cancelled *before* we insert it into the `Fortran` program. This is achieved with `Mathematica`. One could think that this makes `Mathematica` a better choice to make the calculation of $|\mathcal{M}|^2$.
- However `Mathematica` is quite slow when compared with, for instance, `FORM`. For this problem in an `Intel Core-2` at 2.56 MHz, it takes close to 350 s. The same problem with `FORM` takes less than 7 s, a factor of 50!. For larger problems, one has to use `FORM`.

Precision and speed $\cdots$

- The problem with `FORM` is that has poor capabilities for simplifying expressions. So with `FORM` we can get (in 7 s) a `Fortran` output that is the *sum* of the all the contributions to $|\mathcal{M}|^2$ without great simplification. If we use this output we get the situtation in the right panel.
- The precision problems appear exactly where they should, at 1 part in 10^{13}. But we can get the best of both programs, we can use `FORM` to evaluate the traces and input it into `Mathematica` to simplify the expressions and make a `Fortran` output.
- In this way we can get the same result as with `Mathematica` with a total time of around 20 s instead of 350 s! The whole process can be automatized [**?**].

Conclusions

We have discussed the importance of the Higgs boson in making the `SM` consistent.
We have used symbolic calculations with the package `FeynCalc` for `Mathematica`.
The programs can be found at the web page
`http://porthos.tecnico.ulisboa.pt/CTQFT`