# The 2HDM limit of the C2HDM

J. C. Romão*
*Departamento de Física and CFTP, Instituto Superior Técnico*
*Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal*

We try to look at the real limit of the C2HDM.

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## I. THE C2HDM IN OUR NOTATION

From Ref.[1] we parameterize the scalars $\Phi_1$ and $\Phi_2$ in the original generic basis as

$$\Phi_1 = \begin{bmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{bmatrix} , \qquad \Phi_2 = \begin{bmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{bmatrix} . \tag{1}$$

The massless would-be Goldstone boson is $G^0 = c_\beta \chi_1 + s_\beta \chi_2$. We define the orthogonal state

$$\eta_3 = -s_\beta \chi_1 + c_\beta \chi_2 . \tag{2}$$

where the angle $\beta$ is defined by $\tan\beta = v_2/v_1$ and leads to

$$\begin{bmatrix} G^+ \\ H^+ \end{bmatrix} = \begin{bmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{bmatrix} \begin{bmatrix} \Phi_1^+ \\ \Phi_2^+ \end{bmatrix} \equiv U_\beta^T \begin{bmatrix} \Phi_1^+ \\ \Phi_2^+ \end{bmatrix} \tag{3}$$

where

$$U_\beta = \begin{bmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{bmatrix} \tag{4}$$

The fields $\eta_1$, $\eta_2$, and $\eta_3$ combine into the mass eigenstates $h_1$, $h_2$, and $h_3$ as

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = R \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} , \tag{5}$$

where the orthogonal matrix may be parameterized as

$$R = \begin{bmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ -(c_1 s_2 s_3 + s_1 c_3) & c_1 c_3 - s_1 s_2 s_3 & c_2 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -(c_1 s_3 + s_1 s_2 c_3) & c_2 c_3 \end{bmatrix} . \tag{6}$$

Here, $s_i = \sin\alpha_i$, $c_i = \cos\alpha_i$ $(i = 1, 2, 3)$, and, without loss of generality, the angles may be restricted to

$$-\pi/2 < \alpha_1 \le \pi/2, \qquad -\pi/2 < \alpha_2 \le \pi/2, \qquad 0 \le \alpha_3 \le \pi/2. \tag{7}$$

By definition, we take the masses of the neutral scalars in increasing order: $m_1 < m_2 < m_3$.

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*Electronic address: `jorge.romao@tecnico.ulisboa.pt`

For the following we also need the coupling of the Higgs $h_i$ to the vector bosons and to fermions. For the gauge bosons we take for instance with the $W$. We get from Ref.[2], that

$$\mathcal{L} \supset g M_W W^+ W^- h_i \left(R_{i1} c_\beta + R_{i2} s_\beta\right) \tag{8}$$

therefore the relative coupling to the SM is

$$C_i = R_{i1} c_\beta + R_{i2} s_\beta \tag{9}$$

For the fermions it depends on the type of the model. To be precise we consider down type quarks in Type I model. We have

$$\mathcal{L} \supset -\left(\sqrt{2} G_F\right)^{1/2} m_d\, \overline{d} \left[\frac{R_{i2}}{s_\beta} + i\, c_\beta \frac{R_{i3}}{s_\beta} \gamma_5\right] d\; h_i \tag{10}$$

## II. THE 2HDM IN OUR NOTATION

We do not follow the notation of Ref.[3] but instead that of Silva (notes) and Posch. The doublets are defined as (I changed the names for the real and imaginary parts to be those of the C2HDM)

$$\Phi_1 = \begin{bmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{bmatrix} , \qquad \Phi_2 = \begin{bmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{bmatrix} . \tag{11}$$

The angle $\beta$ has the same meaning, and the angle $\alpha$ is defined through

$$\begin{bmatrix} H \\ h \end{bmatrix} = \begin{bmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \equiv U_\alpha \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \tag{12}$$

where

$$U_\alpha = \begin{bmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{bmatrix} \tag{13}$$

It is also convenient to define the Higgs basis where all the vev is in the first doublet. We have

$$H_1 = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_1^H + iG^0) \end{bmatrix} , \qquad H_2 = \begin{bmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(h_2^H + iA) \end{bmatrix} . \tag{14}$$

where

$$G^+ = c_\beta \varphi_1^+ + s_\beta \varphi_2^+ , \qquad H^+ = -s_\beta \varphi_1^+ + c_\beta \varphi_2^+ \tag{15}$$

$$G^0 = c_\beta \chi_1 + s_\beta \chi_2 , \qquad A = -s_\beta \chi_1 + c_\beta \chi_2 \tag{16}$$

We also have

$$\begin{bmatrix} H \\ h \end{bmatrix} = U_\alpha \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = U_\alpha U_\beta \begin{bmatrix} h_1^H \\ h_2^H \end{bmatrix} = \begin{bmatrix} c_{\alpha-\beta} & s_{\alpha-\beta} \\ -s_{\alpha-\beta} & c_{\alpha-\beta} \end{bmatrix} \begin{bmatrix} h_1^H \\ h_2^H \end{bmatrix} \tag{17}$$

and

$$\begin{bmatrix} h_1^H \\ h_2^H \end{bmatrix} = \begin{bmatrix} c_{\alpha-\beta} & -s_{\alpha-\beta} \\ s_{\alpha-\beta} & c_{\alpha-\beta} \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix} \tag{18}$$

### A. Couplings to the Gauge Bosons in the 2HDM

Using the definition of the covariant derivative in terms of the physical gauge bosons of Ref.[4] we have for the Higgs

$$D_\mu \Phi_i = \left[\partial_\mu + i\frac{g}{\sqrt{2}}\left(\tau^+ W_\mu^+ + \tau^- W_\mu^-\right) + ie\,Q\,A_\mu + i\frac{g}{\cos\theta_W}\left(\frac{\tau_3}{2} - Q\,\sin^2\theta_W\right) Z_\mu\right] \Phi_i \tag{19}$$

where

$$\tau^{+}=\begin{bmatrix}0&1\\0&0\end{bmatrix},\qquad \tau^{-}=\begin{bmatrix}0&0\\1&0\end{bmatrix},\qquad Q=\begin{bmatrix}1&0\\0&0\end{bmatrix} \tag{20}$$

Keeping only the terms without derivatives and only the neutral scalars we have

$$\begin{aligned} D_\mu\Phi_i=&\left[i\frac{g}{\sqrt{2}}\begin{bmatrix}0&W_\mu^{+}\\W_\mu^{-}&0\end{bmatrix}+ie\begin{bmatrix}A_\mu&0\\0&0\end{bmatrix}+i\frac{g}{c_W}\begin{bmatrix}\frac{1}{2}-s_W^2&0\\0&-\frac{1}{2}\end{bmatrix}\right]\begin{bmatrix}0\\\frac{1}{\sqrt{2}}\left(v_i+\eta_i+i\,\chi_i\right)\end{bmatrix}\\ =&\begin{bmatrix}ieA_\mu+i\frac{g}{c_W}(\frac{1}{2}-s_W^2)&i\frac{g}{\sqrt{2}}W_\mu^{+}\\i\frac{g}{\sqrt{2}}W_\mu^{-}&i\frac{g}{c_W}(-\frac{1}{2})Z_\mu\end{bmatrix}\begin{bmatrix}0\\\frac{1}{\sqrt{2}}\left(v_i+\eta_i+i\,\chi_i\right)\end{bmatrix}\\ =&\begin{bmatrix}i\frac{g}{\sqrt{2}}W_\mu^{+}\frac{1}{\sqrt{2}}\left(v_i\eta_i+i\chi_i\right)\\i\frac{g}{c_W}(-\frac{1}{2})Z_\mu\frac{1}{\sqrt{2}}\left(v_i\eta_i+i\chi_i\right)\end{bmatrix} \end{aligned} \tag{21}$$

Therefore

$$\begin{aligned} \mathcal{L}=&\left(\frac{g}{\sqrt{2}}\right)^2W_\mu^{+}W^{-\mu}\frac{1}{2}\sum_i\left[(v_i+\eta_i)^2+\chi_i^2\right]+\left(\frac{g}{c_W}\right)^2\frac{1}{4}Z_\mu Z^\mu\frac{1}{2}\sum_i\left[(v_i+\eta_i)^2+\chi_i^2\right]+\cdots\\ =&\frac{g^2}{4}W_\mu^{+}W^{-}\sum_i\left(v_i^2+2v_i\eta_i+\cdots\right)+\frac{g^2}{8c_W^2}Z_\mu Z^\mu\sum_i\left(v_i^2+2v_i\eta_i+\cdots\right)+\cdots\\ =&\frac{g^2}{4}W_\mu^{+}W^{-}v^2+\frac{g^2}{8c_W^2}Z_\mu Z^\mu v^2+\frac{g^2}{2}W_\mu^{+}W^{-}\left(v_1\eta_1+v_2\eta_2\right)+\frac{g^2}{4c_W^2}Z_\mu Z^\mu\left(v_1\eta_1+v_2\eta_2\right)+\cdots\\ =&M_W^2W_\mu^{+}W^{-}+\frac{1}{2}M_Z^2Z_\mu Z^\mu+gM_WW_\mu^{+}W^{-}\left(c_\beta\eta_1+s_\beta\eta_2\right)+\frac{g}{2}Z_\mu Z^\mu\left(c_\beta\eta_1+s_\beta\eta_2\right) \end{aligned} \tag{22}$$

where we have defined

$$M_W=\frac{1}{2}gv,\quad ,M_Z=\frac{1}{2c_W}gv=\frac{M_W}{c_W},\quad v^2=v_1^2+v_2^2,\quad v_1=vc_\beta,v_2=vs_\beta \tag{23}$$

Now using Eq. (12) we obtain

$$\begin{aligned} c_\beta\eta_1+s_\beta\eta_2=&c_\beta\left(c_\alpha H-s_\alpha h\right)+s_\beta\left(s_\alpha H+c_\alpha h\right)\\ =&s_{\beta-\alpha}h+c_{\beta-\alpha}H \end{aligned} \tag{24}$$

Therefore the coupling of the Higgs to gauge bosons, equivalente to Eq. (9) is now

$$C_h=s_{\beta-\alpha},\quad C_H=c_{\beta-\alpha},\quad C_A=0 \tag{25}$$

### B. Couplings to the Fermions in the 2HDM

Finally the couplings to the fermions. To cover all case we just need to consider up and down fermions coupled to both $\Phi_1$ and $\Phi_2$. We obtain this with the simplified Lagrangian (we assume diagonal couplings from the start)

$$\mathcal{L}_{\rm Yuk}=y_1^d\overline{Q}_L\Phi_1d_R+y_2^d\overline{Q}_L\Phi_2d_R+y_1^u\overline{Q}_L\widetilde{\Phi}_1u_R++y_2^u\overline{Q}_L\widetilde{\Phi}_2u_R+\text{h.c} \tag{26}$$

We now analyze the cases separately (we can consider that all up type couple to $\Phi_2$ and differentiaty only the way the down quarks couple)

### C. All fermions couple only to $\Phi_2$: $y_1^u = y_1^d = 0$

Considering only the neutral components of the Higgs we have

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} =& y_2^d \overline{d}_L d_R \frac{1}{\sqrt{2}} (v_2 + \eta_2) + y_2^u \overline{u}_L u_R \frac{1}{\sqrt{2}} (v_2 + \eta_2) + \text{h.c.} + \cdots \\ =& \frac{y_2^d v s_\beta}{\sqrt{2}} \left(\overline{d}_L d_R + \overline{d}_R d_L\right) \left[1 + \frac{\eta_2}{v s_\beta}\right] + \frac{y_2^u v s_\beta}{\sqrt{2}} \left(\overline{u}_L u_R + \overline{u}_R u_L\right) \left[1 + \frac{\eta_2}{v s_\beta}\right] + \cdots \end{aligned} \tag{27}$$

Now we identify

$$y_2^d = -\frac{\sqrt{2} m_d}{v s_\beta}, \quad y_2^u = -\frac{\sqrt{2} m_u}{v s_\beta} \tag{28}$$

to write

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} =& - m_d \left(\overline{d}_L d_R + \overline{d}_R d_L\right) - m_u \left(\overline{u}_L u_R + \overline{u}_R u_L\right) \\ & - \frac{m_d}{v} \left(\overline{d}_L d_R + \overline{d}_R d_L\right) \left(\frac{s_\alpha}{s_\beta} H + \frac{c_\alpha}{s_\beta} h\right) - \frac{m_u}{v} \left(\overline{u}_L u_R + \overline{u}_R u_L\right) \left(\frac{s_\alpha}{s_\beta} H + \frac{c_\alpha}{s_\beta} h\right) \\ =& - m_d \,\overline{d} d - m_u \,\overline{u} u - \frac{m_d}{v} \overline{d} d \left[\frac{c_\alpha}{s_\beta} h + \frac{s_\alpha}{s_\beta} H\right] - \frac{m_u}{v} \overline{u} u \left[\frac{c_\alpha}{s_\beta} h + \frac{s_\alpha}{s_\beta} H\right] \end{aligned} \tag{29}$$

### D. Up type fermions couple to $\Phi_2$ and Down type couple to $\Phi_1$: $y_1^u = y_2^d = 0$

Considering again only the neutral components of the Higgs we have

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} =& y_1^d \overline{d}_L d_R \frac{1}{\sqrt{2}} (v_1 + \eta_1) + y_2^u \overline{u}_L u_R \frac{1}{\sqrt{2}} (v_2 + \eta_2) + \text{h.c.} + \cdots \\ =& \frac{y_1^d v c_\beta}{\sqrt{2}} \left(\overline{d}_L d_R + \overline{d}_R d_L\right) \left[1 + \frac{\eta_1}{v c_\beta}\right] + \frac{y_2^u v s_\beta}{\sqrt{2}} \left(\overline{u}_L u_R + \overline{u}_R u_L\right) \left[1 + \frac{\eta_2}{v s_\beta}\right] + \cdots \end{aligned} \tag{30}$$

Now we identify

$$y_1^d = -\frac{\sqrt{2} m_d}{v c_\beta}, \quad y_2^u = -\frac{\sqrt{2} m_u}{v s_\beta} \tag{31}$$

to write

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} =& - m_d \left(\overline{d}_L d_R + \overline{d}_R d_L\right) - m_u \left(\overline{u}_L u_R + \overline{u}_R u_L\right) \\ & - \frac{m_d}{v} \left(\overline{d}_L d_R + \overline{d}_R d_L\right) \left(\frac{c_\alpha}{c_\beta} H - \frac{s_\alpha}{c_\beta} h\right) - \frac{m_u}{v} \left(\overline{u}_L u_R + \overline{u}_R u_L\right) \left(\frac{s_\alpha}{s_\beta} H + \frac{c_\alpha}{s_\beta} h\right) \\ =& - m_d \,\overline{d} d - m_u \,\overline{u} u - \frac{m_d}{v} \overline{d} d \left[-\frac{s_\alpha}{c_\beta} h + \frac{c_\alpha}{c_\beta} H\right] - \frac{m_u}{v} \overline{u} u \left[\frac{c_\alpha}{s_\beta} h + \frac{s_\alpha}{s_\beta} H\right] \end{aligned} \tag{32}$$

## III. THE 2HDM LIMIT OF THE C2HDM

We consider the real limit of the C2HDM, by taking

$$\alpha_2 \to 0, \quad \alpha_3 \to 0 \tag{33}$$

Then the $R$ matrix becomes

$$R = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{34}$$

Let us see how this compares with the 2HDM defined in the previous section. We have (as the fields might differ by a sign, we denote in this limit, $h' = h_1$, $H' = h_2$, $A = h_3$),

$$h' = c_1\eta_1 + s_1\eta_2, \quad H' = -s_1\eta_1 + c_1\eta_2, \quad A = -s_\beta\chi_1 + c_\beta\chi_2 \tag{35}$$

The lagrangian for gauge bosons and Type I is then

$$\begin{aligned}\mathcal{L} \supset\, & gM_W W^+W^- \left(c_1 c_\beta + s_1 s_\beta\right) h' + \; gM_W W^+W^- \left(-s_1 c_\beta + c_1 s_\beta\right) H' \\ & - \frac{m_d}{v}\left[\overline{d}d\; h'\; \frac{s_1}{s_\beta} + \overline{d}d\; H'\; \frac{c_1}{s_\beta}\right] - \frac{m_u}{v}\left[\overline{u}u\; h'\; \frac{s_1}{s_\beta} + \overline{u}u\; H'\; \frac{c_1}{s_\beta}\right] \\ =\, & gM_W W^+W^- \left(c_{\beta-\alpha_1} h' + s_{\beta-\alpha_1} H'\right) \\ & - \frac{m_d}{v}\,\overline{d}d\;\left[h'\; \frac{s_1}{s_\beta} + H'\; \frac{c_1}{s_\beta}\right] - \frac{m_u}{v}\,\overline{u}u\;\left[h'\; \frac{s_1}{s_\beta} + H'\; \frac{c_1}{s_\beta}\right]\end{aligned} \tag{36}$$

and for Type II

$$\begin{aligned}\mathcal{L} \supset & gM_W W^+W^- \left(c_{\beta-\alpha_1} h' + s_{\beta-\alpha_1} H'\right) \\ & - \frac{m_d}{v}\,\overline{d}d\;\left[h'\; \frac{c_1}{c_\beta} - H'\; \frac{s_1}{c_\beta}\right] - \frac{m_u}{v}\,\overline{u}u\;\left[h'\; \frac{s_1}{s_\beta} + H'\; \frac{c_1}{s_\beta}\right]\end{aligned} \tag{37}$$

To make contact we have to satisfy the relations

$$s_{\beta-\alpha}\; h = c_{\beta-\alpha_1}\; h' \tag{38}$$

$$c_{\beta-\alpha}\; H = s_{\beta-\alpha_1}\; H' \tag{39}$$

$$\frac{c_\alpha}{s_\beta}\; h = \frac{s_{\alpha_1}}{s_\beta}\; h' \tag{40}$$

$$\frac{s_\alpha}{s_\beta}\; H = \frac{c_{\alpha_1}}{s_\beta}\; H' \tag{41}$$

$$-\frac{s_\alpha}{c_\beta}\; h = \frac{c_{\alpha_1}}{c_\beta}\; h' \tag{42}$$

$$\frac{c_\alpha}{c_\beta}\; H = -\frac{s_{\alpha_1}}{c_\beta}\; H' \tag{43}$$

$$\tag{44}$$

Now these relations can either be solved with $\alpha_1 = \alpha \pm \pi/2$. I get for $\alpha_1 = \alpha + \pi/2$

$$c_{\beta-\alpha_1} = s_{\beta-\alpha}, s_{\beta-\alpha_1} = -c_{\beta-\alpha}, c_1 = -s_\alpha, s_1 = c_\alpha \tag{45}$$

and therefore $h' = h, H' = -H$. On the other hand for $\alpha_1 = \alpha - \pi/2$, I get

$$c_{\beta-\alpha_1} = -s_{\beta-\alpha}, s_{\beta-\alpha_1} = c_{\beta-\alpha}, c_1 = s_\alpha, s_1 = -c_\alpha \tag{46}$$

and therefore $h' = -h, H' = H$.

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